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Co-movements of index options and futures quotes[☆]

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ABSTRACT

We report evidence that the co-movements of index options and index futures quotes differ sharply from perfect correlation in periods with option trades. In half-hour intervals with (without) option trades 25% (12%) of call option quote changes have either the opposite sign or are larger in magnitude than the corresponding index futures quote changes. We calibrate a stochastic volatility model that allows for trade and no-trade periods using real data and simulate the joint co-movements of index quotes and option quotes in this model. We show that for trade intervals the observed co-movements differ from the benchmark case established by our simulations approximately three times too often. We provide empirical evidence that market microstructure effects – specifically, stale quotes and aggressive quotes – explain the majority of the deviations from the benchmark. Our findings are relevant for techniques that use estimates of local co-movements as inputs to price or hedge options.

1. Introduction

High-frequency data hold the promise of new insights about the co-movements of option prices and the prices of the underlying asset. But high-frequency data are also frequently affected by market microstructure effects. This is a challenge for empirical research since any departure from perfect correlation between quote changes for options and quote changes for the underlying asset may be driven by both the stochastic process of the asset and market microstructure effects. Bakshi et al. (2000) report that co-movements of S&P 500 index options and futures often deviate from perfect positive correlation for call options and perfect negative correlation for put options. Bossaerts and Hillion (2003) apply a new methodology that uses only local information on co-movements of index options and futures to price and hedge options on the DAX index. Their methodology displays superior performance on simulated data (see Bossaerts and Hillion (1997)) but performs poorly on actual data, a finding that the authors attribute to the anomalous co-movements reported by Bakshi et al. (2000). Both studies suggest that the observed co-movements may reflect a stochastic volatility process for the asset. They also suggest that there may be market microstructure effects that cause these co-movements. The existing evidence on the relative importance and the nature of such market microstructure effects is incomplete because variables that play a key role in most microstructure models have not been considered. We use the methodology of Bakshi et al. (2000) and show that two important microstructure effects, namely trades and signed orders, have the potential to explain a significant fraction of instances in which the co-movement of the quotes of options and the underlying asset deviate from the perfect correlation benchmark.

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We use a one-year sample of quotes and trades for European-style options and futures on the Financial Times Stock Exchange 100 stock index (FTSE 100) from the London International Financial Futures and Options Exchange (Liffe). We frequently observe anomalous quote co-movements. For call options such co-movements would include call option quote changes and index futures quote changes with opposite signs and call option quote changes that are larger in magnitude than the corresponding index futures quote changes. We compute the frequency of such co-movements in periods with and without option trades and find large differences. Are there market microstructure effects that can explain such co-movements around option trades? Alternatively, do option trades coincide with innovations in the price or volatility of the underlying asset indirectly causing these co-movements?

We use simulations to determine how often we should observe anomalous co-movements in a stochastic volatility framework with a trade and no-trade process calibrated to match the data. We show that the observed patterns of co-movements would require trades to be strongly negatively correlated with the magnitude of the innovations in the index and positively correlated with innovations in the volatility. But, in the data, trades are positively correlated with the magnitude of the index innovations. As a consequence, contrary to what we observe empirically the simulated data generate departures from perfect correlation less frequently in trade than in no-trade periods. Using an empirically plausible correlation between trades and the magnitude of the index innovations we can rationalize approximately one-third of the observed frequency of quote co-movements that depart from perfect correlation.

Another potential explanation for the observed co-movements is market microstructure effects, which are more likely to be important around trades. But what types of market microstructure effects would lead to the quotes for calls to be lowered and quotes for puts to be raised when the underlying stock index goes up? We identify two types of trader behavior that are associated with different patterns of co-movements.

In the first case, a buyer-initiated trade in a call option occurs shortly after a trader submits an aggressive sell limit order that undercuts the current best ask quote by, on average, six ticks. The aggressive quotes cause the call option mid-quote to temporarily decrease while the futures mid-quote is virtually unchanged. The quotes therefore move in opposite directions in 30% of the observations both before and after the trade as the option's mid-quote is temporarily pushed down before the trade and reverts back after the trade. In the first case traders seek liquidity by placing aggressive limit orders that are likely to be executed quickly offering them a better price than a market order. In the second case, the change in the futures mid-quote over the 10 min before the buyer-initiated trade implies, on average, a two to three tick increase in the value of the call option but the call option's mid-quote is virtually unchanged and adjusts upwards only after the trade. The quotes therefore move in opposite directions for approximately 12% of the observations before and 30% of the observations after the trade because of the stale quote before and the delayed quote adjustment after the trade. We find analogous patterns for seller-initiated trades in call options and buyer- and seller-initiated trades in put options.

We interpret our findings as evidence that the above market microstructure effects explain a significant fraction of the observed anomalous co-movements of index options and futures quotes around trades. Moreover, it is plausible that aggressive limit orders that fail to execute and stale quotes that are not picked off also explain some of the anomalous co-movements in intervals without trades.

Our findings are relevant for data-driven techniques for pricing or hedging derivatives that rely on measures of co-movements of option prices and the underlying price such as Hutchinson et al. (1994), Bossaerts and Hillion (1997, 2003), Hayashi and Mykland (2005), Brandt (2003), Gondzio et al. (2003), and Crépey (2004). The optimal frequency for such techniques is problem-specific and partly unresolved. One of the impediments to the use of higher frequency data is the concern about microstructure effects. If one can develop methods to filter out microstructure effects such as the ones identified in this paper, the use of higher frequency data is more attractive. Our results also complement the simulation results of Dennis and Mayhew (2003). They quantify the importance of microstructure noise for option pricing tests.

2. Description of the market and the sample

Our sample was obtained from Liffe's market data services. It consists of time-series of all quotes and trades for all European-style FTSE 100 index option contracts and the FTSE 100 index futures contracts. We also obtained the daily time-series of closing prices, open interest, and implied volatilities for all contracts. Our sample period covers 236 trading days between August 1, 2001 and July 30, 2002. We construct a proxy for the risk free interest rate using the overnight, one week, one month, three months and one year Sterling London Interbank Offered Rates (Libor), provided by Datastream. We obtain daily dividend yields on the FTSE 100 index from Datastream.

Trading of the FTSE 100 index options and index futures is fully electronic and takes place in a common trading system, Liffe Connect, which brings two advantages for empirical work. First, trades can be classified as buyer- or seller-initiated without error [1]. Second, option and futures quotes are synchronous, allowing us to determine the exact order and timing of quote changes around trades.

We drop all observations from our sample that violate one or more of the following criteria: (i) the bid or ask quote differs from the mid-quote by more than £30, (ii) the current bid or ask quote has not changed for more than 30 min, and (iii) the bid or the ask quote does not satisfy either the upper or lower bounds for rational call and put option prices. Applying the above criteria leaves us with 8,239,375 observations or approximately 95% of the original sample.

[1] Savickas and Wilson (2003) show, for a sample from CBOE's floor-based market, that trade classification algorithms that are often used for equity data correctly classify between 59 and 83% of option trades.

Table 1
Descriptive statistics

	Bid-ask spread	Mid-quote	Quoted depth	Trade size	Average daily	
					Quotes	Trades
Index futures	1.5	5003.1	3.1	3.4	25.2	10,309.8
Call options						
All	8.7	126.9	17.3	17.0	17.5	124.8
By moneyness						
Out-of-the-money	8.9	81.4	19.3	20.9	10.1	73.1
At-the-money	8.4	140.1	16.2	10.8	14.9	46.6
In-the-money	12.0	373.7	14.9	19.5	1.4	5.7
By time to expiration						
Less than 2 months	7.4	103.0	16.4	15.1	19.0	99.3
2 to 6 months	10.2	171.6	18.8	20.6	5.5	19.6
More than 6 months	18.6	238.4	22.6	40.5	1.8	6.1
Put options						
All	8.6	128.8	18.8	19.7	17.4	153.5
By moneyness						
Out-of-the-money	8.4	75.2	20.8	24.7	10.4	87.2
At-the-money	8.3	139.1	17.6	13.6	14.0	53.6
In-the-money	12.0	377.3	16.8	9.6	2.0	13.0
By time to expiration						
Less than 2 months	7.4	106.1	18.0	19.7	19.3	117.6
2 to 6 months	9.7	177.1	20.1	19.6	5.3	28.1
More than 6 months	18.0	228.6	23.6	20.1	1.6	8.2

The table reports the means of the following six variables: the quoted bid-ask spread, the mid-quote, the quoted depth, the trade size, the daily number of new quotes in thousands, and the daily number of trades. The units are pounds for the bid-ask spread and the mid-quote, number of contracts for the quoted depth and the trade size, and thousands of updates for the quote updates. Means are also reported for sub-samples based on the options' moneyness and time to expiration. A call option's moneyness depends on the ratio of its strike price to the underlying; if the ratio is less than 0.97 the call is out-of-the-money; if the ratio is between 0.97 and 1.03 it is at-the-money; and if the ratio is above 1.03 it is in-the-money. Put options are classified similarly using the underlying divided by the strike price. A Wilcoxon test rejects the null hypothesis of equal median values for each variable across the moneyness and time to expiration categories for all cases with p-values less than 0.001.

Table 1 reports descriptive statistics for our sample. The average spread is £1.5 for index futures compared to £8.7 for call and £8.6 for the put options. The minimum tick size of £0.5 is therefore not binding for a typical observation in our sample [2]. The mean quoted depth and trade size are fairly close for call and put options. The mean depth and trade size are approximately equal for the index futures and both are approximately one fifth of the corresponding means for the index options. The index futures are actively traded; there are almost 40 index futures trades for every index option trade. The quoted bid-ask spread varies systematically with moneyness and time to expiration; in general, at-the-money options and short time to expiration options have narrower spreads. Quotes are updated more frequently for at-the-money options compared to out-of-the-money and in-the-money options.

3. Basic properties of option prices

We take some of the properties for option prices derived by Bergman et al. (1996) as a starting point for our analysis. They show that when the underlying asset price follows a diffusion whose volatility depends only on time and the concurrent asset price, then a call option price is always increasing and convex in the underlying asset price. One implication of their results is that the delta of a European-style option is bounded at any time $t<T$ before the option's expiration date T:

$$0 \leq \frac{\partial c(S,t)}{\partial S} \leq 1 \tag{1}$$

$$-1 \leq \frac{\partial p(S,t)}{\partial S} \leq 0, \tag{2}$$

in which $c(\cdot)$ denotes the value of the call and $p(\cdot)$ denotes the value of the put option and S denotes the value of the underlying asset. Bergman et al. (1996) also show that as long as the option has some positive time value the inequalities in Eqs. (1) and (2) are strict. When volatility is stochastic, however, or the underlying asset price does not follow a diffusion, then the call price can be a decreasing or concave function of the stock price.

Following Bakshi et al. (2000) we use the mid-quotes of the options and the index futures as proxies for the value of the option and the underlying index. There can be four types of violations of the inequalities in Eqs. (1) and (2). Type I violations occur when the call option mid-quote moves in the opposite direction of the index futures mid-quote or when the put option mid-quote moves in the same direction as the index mid-quote. Type II violations occur when the option mid-quote is unchanged and the index futures mid-quote changes. Type III violations occur when the index futures mid-quote is unchanged and the option mid-quote

[2] The FTSE 100 index options are quoted in index points. Each contract is valued at £10 per index point. The tick size is 0.5 index points or £5. We omit the index-point multiplier of ten in our empirical analysis.

Table 2
Mean violation rates

	N	Type [%]	
		I	IV
Call options			
30-minute non-overlapping intervals			
All	176,308	8.9	4.5
Only no-trade	161,433	8.2	4.2
Only trade	14,875	16.6**	8.4**
Thirty 1-minute intervals centered around each trade			
All	725,064	8.4	4.7
Minute before trade	25,449	27.5**	14.4**
Minute after trade	26,383	32.7**	17.7**
All other 28 intervals	673,232	6.7	3.8
Put options			
30-minute non-overlapping intervals			
All	236,545	9.2	4.3
Only no-trade	219,863	8.5	3.9
Only trade	16,682	17.9**	10.3**
Thirty 1-minute intervals centered around each trade			
All	837,449	8.5	4.9
Minute before trade	29,870	26.3**	15.6**
Minute after trade	31,329	32.2**	18.2**
All other 28 intervals	776,250	6.9	3.9

The table reports the mean rates of type I and type IV violations. The top panel reports mean rates for call options and the bottom panel reports mean rates for put options. In each panel, the first three rows report results using non-overlapping 30-minute intervals and the next four rows report results using thirty 1-minute intervals, possibly overlapping, centered around trades. In the latter case the intervals are split into fifteen 1-minute sub-intervals before and fifteen 1-minute sub-intervals after a trade. The first row reports the mean rate for all intervals (per 30 min), the second row reports the mean rate for all intervals in which no trade occurred, and the third row reports the mean rate for all intervals in which a trade occurred. The fourth row reports the mean rate for all intervals (per minute), the fifth and sixth row reports the mean rate for the middle two intervals (the minute before and after a trade), and the seventh row reports the mean rate for all other 28 intervals around a trade. The different types of violations are presented in Section 3. A superscript ** indicates that the mean is statistically significantly different for the trade vs. no-trade, or before/after trade versus all other intervals at the 1% level.

changes. Type IV violations occur when the magnitude of change in the option mid-quote is greater than the magnitude of the change in the index futures mid-quote. We focus on violations of type I and IV, which are more puzzling than violations of type II and III because they involve changes in both the option and the index futures mid-quotes. We adopt a stochastic volatility model as our benchmark option pricing model which implies that even in a frictionless world a non-zero frequency of violations of type I and IV is expected [3].

4. Empirical results

4.1. Violation rates: trade versus no-trade intervals

Table 2 reports the mean rates of type I and IV violations across all non-overlapping 30-minute intervals in our sample for calls and puts. The top row of each panel reports the overall mean violation rates. The overall mean rates of type I and IV violations are of the same order of magnitude as the ones reported in Table 3 of Bakshi et al. (2000) suggesting that the violations may be a regularity of options data that holds across different markets, trading systems, and time periods. For example, we find that type I violations occur in 8.9% of all call option intervals and in 9.2% of all put option intervals compared with 9.3% and 9.9% in their study.

The second and third row of each panel report mean violation rates across all intervals in which no trades occur and all intervals in which at least one trade occurs. The rates of type I and IV violations approximately double and a test of the null hypothesis that the violation rates are equal across trade and no-trade intervals strongly rejects the null in all cases with p-values less than 0.1%.

We next examine finer time intervals to establish a closer link between the violations of type I and IV and trades. The bottom halves of each panel of Table 2 report the mean violation rates for 30 one-minute intervals centered around each option trade.

Overall, Type I and IV violations are approximately four times as likely during the two one-minute intervals before and after a trade than during the other one-minute intervals. The violation rates in the 2 min surrounding the trade are statistically significantly higher than the violation rates in all other intervals. For example, in the one-minute interval before a put option trade the option mid-quote

[3] In our empirical analysis, we use the FTSE 100 index futures as a proxy for the underlying asset price. One advantage of the index futures price is that it is more likely to reflect all available information about the component stocks than the underlying index itself which may be based on individual stock quotes that are stale. Our proxy should not affect the frequency of type I violations at all, but it may bias our result against finding evidence of violation IV. During our sample period, the interpolated interest rate r was always larger than the dividend yield δ on the FTSE 100 index. Therefore, the theoretical futures price $F_{T,t}=S_t\exp(r-\delta)(T-t)$ is larger than the underlying index inflating the denominators in the inequalities for Type IV violations.

Table 3
Violation rates and moneyness and time to expiration—call options

Moneyness		Time to expiration					
		Less than 2 months		2–6 months		More than 6 months	
Out-of-the-money							
	Violations						
	I	11.1	(18.7)	8.6	(22.6)	6.8	(22.6)
	IV	2.4	(4.1)	2.8	(8.8)	3.1	(8.1)
	Option	1.1	(2.6)	1.3	(3.4)	1.4	(3.6)
	Index futures	9.5	(12.3)	9.3	(10.9)	8.0	(10.9)
At-the-money							
	Violations						
	I	7.9	(11.9)	7.5	(14.4)	6.4	(20.7)
	IV	6.7	(9.8)	6.8	(15.3)	6.8	(17.2)
	Mean of absolute mid-quote change						
	Option	3.0	(4.9)	2.8	(5.8)	2.3	(7.7)
	Index futures	8.6	(10.2)	8.7	(10.1)	7.8	(8.7)
In-the-money							
	Violations						
	I	4.3	(20.3)	3.5	(23.0)	3.1	(*)
	IV	4.3	(31.8)	3.6	(27.9)	3.9	(*)
	Mean of absolute mid-quote change						
	Option	1.7	(14.5)	1.3	(10.0)	1.0	(*)
	Index futures	9.1	(13.6)	9.3	(10.3)	8.0	(*)

* Less than 25 observations and thus omitted.

The table reports mean rates of type I and type IV violations for call options by moneyness and time to expiration categories, each as a percentage of total number of 30-minutes intervals. A call option's moneyness depends on the ratio of its strike price to the underlying; if the ratio is less than 0.97 the call is out-of-the-money; if the ratio is between 0.97 and 1.03 it is at-the-money; and if the ratio is above 1.03 it is in-the-money. The mean rate for intervals with a trade are reported in parentheses next to the mean rates for intervals with no trade. The mean absolute mid-quote changes for the call options and the index futures are reported below the total violation rates. The mean absolute mid-quote change for intervals with a trade is reported in parentheses next to the mean for no-trade intervals. The mid-quote changes are measured in pounds and for the index futures the nearest-to-maturity index futures are used. A test of equal means for all pairs of trade vs. no-trade violation rates rejects at the 1% level with the exception of the ones for the at-the-money and more than 6 months to expiration category for which the tests reject at the 2% level.

moves in the same direction as the index futures 26.3% of the time and moves in the opposite direction by more than the change in the index futures 15.6% of the time. The corresponding violation rates for the one-minute intervals away from the trade are 6.9% and 3.9%.

One possibility is that the correlation between trades and violations is indirect; perhaps the violation rates depend on the option characteristics such as moneyness or time to expiration which in turn are correlated with the trading activity. Table 3 reports, for call options, the mean rate of violation by type for different sub-groups formed based on the options' moneyness and time to expiration categories [4]. Table 3 shows the non-overlapping 30-minute interval violation rates for nine different categories that consist of three moneyness and three time to expiration categories. The first number in each column is the violation rate for no-trade intervals, and the number in parentheses is the violation rate for trade intervals.

We find that for all moneyness and all time to expiration categories the violations of type I and IV occur more frequently in trade than in no-trade intervals for both call and put options. On average, across all categories type I and IV violations occur about three times as often in trade as in no-trade intervals. These differences are statistically significant with all but two *p*-values less than 1%. At-the-money call options with the shortest time to expiration have the lowest frequency of type I violations in trade intervals. One possible explanation for this regularity is that quotes for the at-the-money and short time to expiration options are updated more frequently (see Table 1) and therefore may be less likely to become stale. The frequency of type IV violations increases sharply in moneyness. One explanation for this regularity is that as the option's moneyness (and hence delta) increases then even in the frictionless benchmark case its expected quote revision approaches that of the underlying asset. Therefore a relatively small friction is needed to cause in-the-money options to have a quote revision that exceeds the magnitude of the quote revision in the underlying asset causing a violation of type IV.

The bottom two rows in each panel report the mean of the absolute change in the mid-quote of the option and the index futures for each category and for no-trade and trade intervals. For both call and put options and index futures the mean absolute mid-quote change is greater in trade intervals than in no-trade intervals across all categories and the difference is statistically and economically significant. The difference is on average approximately 2.5 index points for the index futures and 5.5 index points for the index options. One possible explanation for this finding is that the underlying stock index follows a process that is characterized by stochastic volatility and a negative correlation between innovations in the index and the volatility. It is possible that trades are correlated with innovations in such a way as to explain the different patterns in quote co-movements for trade and no-trade periods.

[4] For brevity, we concentrate on call options in most tests. The results are qualitatively and quantitatively similar for put options.

4.2. Simulation results for our benchmark case

We design a joint simulation exercise to explore to what extent a two-dimensional Markov diffusion model can help explain the different violation frequencies we observe for trade and no-trade intervals. We use the following specification of Heston's (1993) stochastic volatility option pricing model:

$$dS(t)/S(t) = rdt + \sqrt{\nu(t)}dW_s(t) \quad (3)$$

$$d\nu(t) = [\theta - \kappa\nu(t)]dt + \sigma\sqrt{\nu(t)}dW_\nu(t), \quad (4)$$

in which dW_S and dW_v are two Brownian motions with an instantaneous correlation coefficient ρ. A stochastic volatility model can generate both violations of type I and type IV. A small stock price innovation combined with a large volatility innovation of the opposite sign can cause a violation of type I. A small stock price innovation combined with a large volatility innovation of the same sign can cause a violation of type IV.

We construct daily estimates of the parameters of the stochastic volatility model using end-of-day closing prices of call options. Each day, we use the closing prices of all options with a positive trading volume across the same moneyness and time-to-expiration categories we report in Table 3 and the closing index value to estimate daily parameters. Let Θ denote the parameter vector $\Theta=\{\kappa, \theta, \sigma, \rho, v\}$. We minimize the mean squared pricing error:

$$\min_{\Theta} \frac{1}{N} \sum_{n=1}^{N} \left(\hat{C}_n(t,\tau,K,S) - C_n(t,\tau,K,S)\right)^2, \quad (5)$$

where $\hat{C}_n$ is the observed option price for the nth call, and C_n is its model price. On average, we have 30 call options per day. In our simulations we use the time-series averages of these parameters [5]. The parameters values used are: $\kappa=4.19$, $\theta=0.029$, $\sigma=0.463$, $\rho=-0.527$, and $\sqrt{\nu}=0.176$.

In all simulations, we set the initial price for the underlying asset equal to 5000, the risk-free interest rate equal to 0.0411, and the time-to-expiration equal to 0.22. These values roughly correspond to sample averages. We calculate prices for at-the-money, out-of-the-money, and in-the-money call options.

For the above parameterizations, we generate changes in S and v, using the discretized versions of the continuous time stochastic processes for dS and dv:

$$S(t+dt) - S(t) = rS(t)dt + \sqrt{\nu(t)}S(t)\,\varepsilon_1(t)\sqrt{dt} \quad (6)$$

$$\nu(t+dt) - \nu(t) = [\theta - \kappa\nu(t)]dt + \sigma\sqrt{\nu(t)}\,\varepsilon_2(t)\sqrt{dt}, \quad (7)$$

with dt set to 30 min ($dt=1/(365\times24\times2)$). We generate 1 million changes and compute the corresponding changes in the underlying asset price, $dS=S(t+dt)-S(t)$, and the changes in the call option price, $dC=C(t+dt)-C(t)$.

We simulate trades to compare trade and no-trade periods. The results on mid-quote changes reported in Table 3 suggest that trades are positively correlated with the absolute value of mid-quote changes for the underlying index. Our baseline simulation generates a correlation of 0.16 between the squared index innovations (dS^2) and the trade indicator and we have verified that the results are robust to changes in the correlation. We verify that across dS deciles we approximately match the observed trade frequencies [6].

Table 4 reports the simulated and observed violation rates for different options of different moneyness and for trade and no trade intervals. The top panel of the table reports the results for violations of type I and the bottom panel reports the results for violations of type IV. In each panel, the first row reports the rate of violations in simulations. For out-of the-money options the type I violation rate is 7.6% in intervals with a trade and 11.3% in intervals with no trade. The corresponding figures are 5.7% and 8.4% for at-the-money options and 4.0% and 6.1% for in-the- money options. The second row reports the simulated violation rates after accounting for rounding using a tick size of 0.5 units. The net effect of rounding is to reduce the violation rates for both type I and type IV. The last row of the table reports the violation rates observed in our sample for call options with 2–6 months to expiration (see Table 3).

The simulation results in Table 4 suggest that, for no-trade intervals and after accounting for price discreteness, the simulated violation rates of type I and type IV are both within a few percent of the observed rates. For trade intervals, however, the gap between the simulated rates, with or without accounting for price discreteness, are large. For example, we simulate for at-the-

[5] Bakshi et al. (2000) use one day of their sample to extract the parameters of a stochastic volatility model from half-hour intervals. We have arbitrarily picked one day in the middle of our sample period and used their estimation technique. The conclusions we obtain from these parameter estimates are qualitatively similar to the ones reported here. Also, we have produced simulation results that are based on the parameter estimates of Bakshi et al. (2000), and all our conclusions hold for these parameters estimates.

[6] We generate a normally distributed random variable, λ, defined as $\lambda=\text{trade-corr}\times\varepsilon_1(t)+\sqrt{1-trade-corr^2}\,\varepsilon_3(T)$ where $\varepsilon_1(t)$ is the innovation for S, $\varepsilon_3(t)$ is a standard normal random variable, and trade-corr is a parameter. The trade indicator is one ($I_{trade}(t)=1$) if $\Phi(\lambda)\leq\alpha$ or $\Phi(\lambda)\geq 1-\alpha$ where we choose α to generate the desired probability of trade (2α) and $\Phi(\cdot)$ is the standard normal cdf. By changing the trade-corr parameter we can change the correlation between I_{trade} and dS^2 , which is 0.16 for the baseline simulation.

Table 4
Simulated vs. observed violation rates

Moneyness	OTM		ATM		ITM	
	Trade	No-Trade	Trade	No-Trade	Trade	No-Trade
Type I						
Simulated						
No rounding (tick size=0)	7.6%	11.3%	5.7%	8.4%	4.0%	6.1%
Rounding (tick size=0.5)	4.6%	6.8%	3.1%	4.7%	2.0%	3.0%
Observed	22.6%	8.6%	14.4%	7.5%	23.0%	3.5%
Type IV						
Simulated						
No rounding (tick size=0)	5.1%	7.5%	6.4%	9.4%	7.3%	11.0%
Rounding (tick size=0.5)	2.8%	4.1%	3.7%	5.4%	4.1%	6.2%
Observed	8.8%	2.8%	15.3%	6.8%	27.9%	3.6%

The table reports the simulated rates of type I (top panel) and type IV (bottom panel) violations for out-of-the money, at-the-money, and in-the-money options, with and without accounting for price discreteness. For each moneyness group we generate one million quote changes for the options and the index. The last row reports the violation rates observed in the data for trade and no-trade intervals. We use the observed violations from the 2–6 months time-to-expiration category from Table 3 as a comparison.

money call options during trade periods 3.1% of type I violations but we observe 14.4% of type I violations in the data. For type IV violations, the corresponding rates are 3.7% (simulated) versus 15.3% (observed) [7].

In order to better understand why the stochastic volatility model is unable to match the higher rates of violations in trade intervals we examine the distribution of violations across the innovations in the volatility process (dv) and the underlying asset price (dS). The top panel of Table 5 reports the distribution of type I violation rates across deciles of dS and dv. The simulated data are sorted into deciles according to dv and then according to deciles of dS so that each category has 10,000 observations. The last column reports the column averages and the last row of the panel reports the row averages.

There are virtually no violations of type I for the three extreme deciles for dS but the violation rates increase to 31.3% and 32.5% for the 5th and 6th deciles. For the volatility process the highest rates of violation are observed for extreme values of dv; the 1st and 2nd and 9th and 10th deciles have average violation rates of 11% and 17%. In other words, the violation rates are negatively correlated with the magnitude of the innovation in the underlying asset price and positively correlated with the magnitude of the innovation in the volatility process.

The bottom panel of Table 5 reports the distribution of type IV violation rates across deciles of dS and dv. Similar to the case of type I violations we observe very few violations for the top and bottom three deciles of dS. The violations are concentrated in the middle two deciles for dS and the extreme three deciles for dv.

Our simulation results show that the stochastic volatility model generates violations of type I when small changes in the underlying asset price are combined with large changes in the volatility prices of the opposite sign. Similarly, the stochastic volatility model generates violations of type IV when small changes in the underlying asset price are combined with large changes in the volatility of the same sign. Empirically, trades tend to occur more often when there are large changes in the underlying asset price. A consequence of this is that our simulations of the stochastic volatility model can only generate one-third or less of the violations of either type I or type IV observed in intervals with trades. In contrast, for at-the-money options the simulated and the observed violations rates are fairly close in no-trade periods.

4.3. Empirical evidence on the market microstructure effects

Stale quotes may explain why violations of type I and IV are more frequent around trades than at other times. If the index futures but not the option quotes reflect recent changes in the index we may observe a delayed quote revision for the options. A delayed quote revision may cause violations because the quote changes for the options and the index futures are asynchronous. For example, suppose the index futures quotes change by several ticks before the trade, while the option quotes remain constant. After the trade, the option quotes are revised to reflect the cumulative change in the index, which may cause the options quotes to change more than the index futures quotes or in the opposite direction of the last index futures quote revision after a trade.

Fig. 1 plots, in event time, the average changes in the mid-quotes of the index futures (solid line) and the index options (dash-dot line) around buyer-and seller-initiated trades in call options. For each trade the series of mid-quotes are normalized so that changes are measured relative to the index futures mid-quote and the option mid-quote at the time of each trade. Changes are plotted for every 10 second interval from 15 min before a trade to 15 min after a trade. The normalized bid and ask quotes for the index option are plotted with a dashed line [8].

[7] Unreported simulation results show that our findings above are robust to varying the ρ parameter and the correlation between trades and the squared innovation in the underlying asset price.

[8] Unconditionally, the index futures mid-quotes are slightly positively autocorrelated for very short horizons. The first-order autocorrelation coefficient for minute-by-minute mid-quote returns is 0.08 and we reject the null of zero autocorrelation at the 1% level or better. Higher-order autocorrelations are close to zero.

Table 5
Distribution of simulated violation rates of type I and type IV

dS	*dv*—deciles										
Deciles	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	Avg.
Type I											
D1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
D2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%
D3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.7%	0.6%
D4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.6%	63.7%	7.5%
D5	0.0%	0.0%	0.0%	0.0%	0.5%	16.6%	38.9%	63.4%	93.8%	100.0%	31.3%
D6	99.5%	94.6%	67.0%	41.3%	19.1%	1.6%	0.5%	0.5%	0.6%	0.5%	32.5%
D7	66.0%	13.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.0%
D8	7.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%
D9	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
D10	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Avg.	17.3%	10.8%	6.7%	4.1%	2.0%	1.8%	3.9%	6.4%	10.6%	17.0%	8.1%
Type IV											
D1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
D2	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
D3	14.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.4%
D4	84.4%	16.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.1%
D5	100.0%	100.0%	85.9%	47.0%	9.6%	0.0%	0.0%	0.0%	0.0%	0.0%	34.3%
D6	0.5%	0.6%	0.4%	0.6%	0.4%	8.6%	43.3%	83.0%	99.4%	99.5%	33.6%
D7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	13.9%	82.3%	9.6%
D8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	12.3%	1.2%
D9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%
D10	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Avg.	19.9%	11.7%	8.6%	4.8%	1.0%	0.9%	4.3%	8.3%	11.3%	19.5%	9.0%

The table reports the distribution of simulated violation rates of type I (top panel) and type IV (bottom panel). The percentage of observations with a violation of a given type is reported for the simulated data for at-the-money options. The observations are double-sorted into deciles for the asset innovations (dS) and the volatility innovations (dv). Each cell has 10,000 observations or 1% of the total of one million observations. The last column and row reports the average across dv and dS categories, respectively.

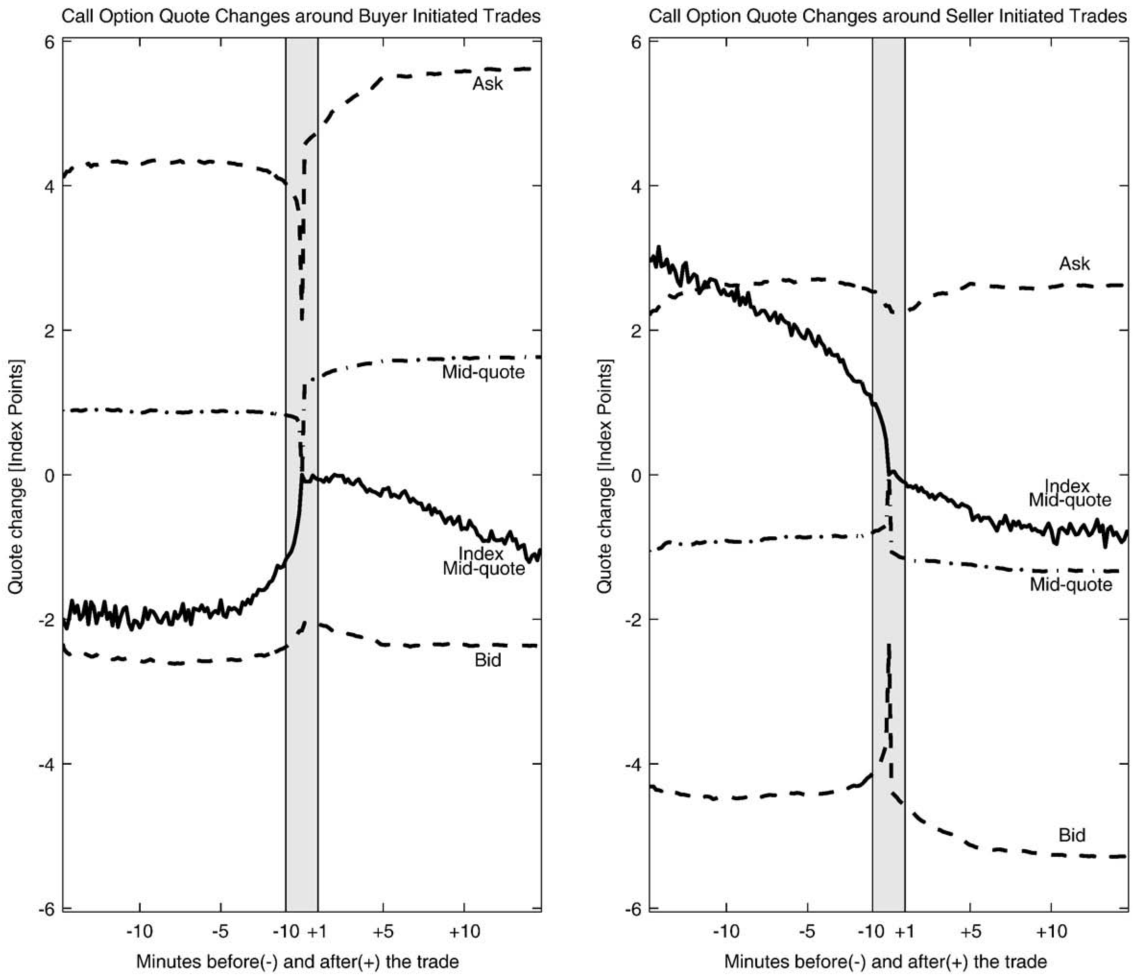

Fig. 1. The subplot on the left plots the normalized series of the mid-quotes for the index futures (solid line) and the option (dash–dot line) for all buyer-initiated trades in the call options. The series of mid-quotes are normalized by subtracting the respective mid-quote values at the time of the trade. The normalized bid and ask quotes series are plotted with dashed lines. The subplot on the right plots the corresponding quote series for the seller-initiated trades.

Before a buyer-initiated call option trade the index mid-quote increases and before a seller-initiated trade it decreases. The increase starts at about 5 min before the purchase and up to 15 min before the sale although the most rapid change occurs in the last few minutes before the trade. If the value of the call option is increasing in the underlying index, the observed movements are consistent with the trade resulting from a stale limit order being picked off by a market order.

The first two rows of Table 6 show that the average index futures quote changes multiplied with the options' delta from 5 min prior to the trade to the time of the trade are economically and statistically large and positive for buyer-initiated trades and large and negative for seller-initiated trades. From Fig. 1 we infer that after the trade, the change in index mid-quotes is small. Table 6, rows three and four, confirm that the delta-weighted index futures movements over the 5 min after the trade are economically small and with one exception statistically insignificant.

The increase in the option mid-quote after a buyer-initiated trade is at least partly due to the ask quote shifting up; on average the bid quote has already adjusted before the buyer-initiated trade. Similarly for seller-initiated trades the decrease in the mid-quote after a trade is partly driven by a decrease in the bid quote. The above evidence is consistent with the quotes being stale at the time of the trade. Table 6, rows seven and eight, confirm that both call and put option mid-quotes are revised upwards in statistically and economically significant magnitudes after buyer-initiated trades. Similarly, option mid-quotes incur large downward revisions after seller-initiated trades. These large quote-revisions in the direction of the index movement are consistent with stale-quotes being picked off and being adjusted to the true price after the trade.

But the discrete negative jump in the ask quote within the last minute before a buyer-initiated trade and the discrete positive jump in the bid quote before a seller-initiated trade evident from Fig. 1 are not consistent with stale quotes. In the case of stale quotes we would expect that the passive side of the trade—the ask quote for buyer-initiated trades and the bid quote for seller-initiated trades—would be a quote that has been in effect for some time and therefore does not reflect all available information including the recent changes in the index futures quote. But the plot shows that instead the ask quote for buyer-initiated trades and

Table 6
Option and futures mid-quote changes around option trades

	Buyer initiated			Seller initiated		
	N	Spread	Change	N	Spread	Change
	Index mid-quote changes 5 min before option trades					
Call options	13,910	1.98	0.51**	14,740	2.23	−0.28**
Put options	17,444	2.13	0.77**	18,284	2.22	−0.45**
	Index mid-quote changes 5 min after option trades					
Call options	14,190	2.01	0.05**	15,110	2.25	−0.04
Put options	17,700	2.14	0.08	18,553	2.23	0.01
	Option mid-quote changes 5 min before option trades					
Call options	10,944	1.97	−0.94**	11,540	2.18	0.90**
Put options	12,409	2.23	−0.55**	13,016	2.23	0.93**
	Option mid-quote changes 5 min after option trades					
Call options	13,795	2.15	1.60**	14,705	2.31	−1.23**
Put options	17,040	2.40	1.57**	17,888	2.45	−1.44**

The table reports for all option trades the mean change in the index futures' mid-quote and the option's mid-quote around the option trade. The first two rows contain the index futures mid-quote changes from 5 min before a trade to the time of a trade. Rows 3 and 4 contain the index futures mid-quote changes from the time of a trade to 5 min after the trade. Rows 5 through 8 contain the corresponding changes for the options' mid-quotes. The index futures changes are multiplied by the average option's delta to make the magnitudes comparable. The means for buyer initiated trades are reported on the left and the means for seller initiated trades on the right. The number of observations and the mean quoted half-spread at the time of a trade are reported next to the mean changes for each category. A ** indicates that the change is statistically significant at the 1% level.

the bid quote for seller-initiated trades are submitted shortly before the trade so that quotes are not stale. Table 6 confirms this result and shows that the economic effects are sizeable. The option mid-quote decreases before buyer-initiated trades by 36% of the quoted spread and increases before seller-initiated trades by approximately 41%.

The economically large discrete negative jump in the ask quote before a buyer-initiated trade and the discrete positive jump in the bid quote before a seller-initiated trade are puzzling. They contradict the stale quote explanation because the quotes involved in the trades are evidently not stale. Yet, the jumps are hard to reconcile with standard quote updating since they are in a direction opposite to that implied by the change in the underlying index.

4.3.1. Aggressive quotes versus stale quotes

To potentially distinguish between different explanations we separate our sample of trades into three categories: The first category contains all buyer-initiated trades for which the option's ask quote was not updated during the 30 s prior to the trade. The second category contains all seller-initiated trades for which the option's bid quote was not updated during the 30 s prior to the trade, and the third category contains all remaining options [9]. We refer to the first two categories as the stale quote categories. The third category, which we refer to as the aggressive quote category, contains approximately 35% of all call option trades, and 37% of all put option trades.

Fig. 2 plots the mean of the normalized mid-quotes series for the index futures and the mean of the normalized ask, bid, and mid-quotes for call options. Each option quote series is normalized by subtracting the option mid-quote at the time of the trade, and each index futures mid-quote series is normalized by subtracting the index future mid-quote value at the time of the trade.

The index movements prior to the trade and the behavior of the ask and bid quote series around the trade are different for the stale quote and aggressive quote categories. The top sub-plots of Fig. 2 show the series for buyer-initiated and seller-initiated trades for which the option's mid- quote changed in the 30 s prior to the trade. The index movements before and after the trade exhibit a slight downward trend but there are no drastic changes over the 30-minute window. The ask quote series (for buyer-initiated trades) and the bid quote series (for seller-initiated trades) contain on average a jump of almost 6 ticks at the time of the trade. Compared to the jump of Fig. 1, it is more than 50% larger. But, the ask and bid revert to their pre-trade level shortly after the trade; consistent with that, there does not appear to be a systematic revision in the option's mid-quote after the trade, ignoring the jump.

The bottom sub-plots of Fig. 2 show that for this sub-sample, on average, the index decreases by approximately 12 ticks before seller-initiated trades and increases by approximately 12 ticks before buyer-initiated trades in the calls. Consistent with a stale quotes explanation, the quotes are revised after the trade.

By splitting the sample into the two groups, we demonstrate that there are two scenarios for trades. In the first scenario, stale quotes are picked off. In the second scenario, traders post aggressive bid and ask quotes right before a trade.

4.3.2. Frequency of violations revisited

Table 7 reports the violation frequencies of one-minute intervals centered around trades, similar to the lower half of the call and put panel of Table 2. We use the three categories obtained by the separation of trades into trades most likely caused by stale quotes and trades caused by liquidity seeking behavior, either through an aggressive ask or bid quote. For each option trade, we again construct 30 one-minute intervals centered around the trade (which are potentially overlapping for different trades). We aggregate the intervals into three groups, the one-minute intervals immediately before the trade, immediately after the trade, and all other one-minute intervals, and document average violation frequencies for all such intervals.

[9] The choice of thirty seconds is arbitrary. We have also used 60 seconds and 45 s, with virtually the same results.

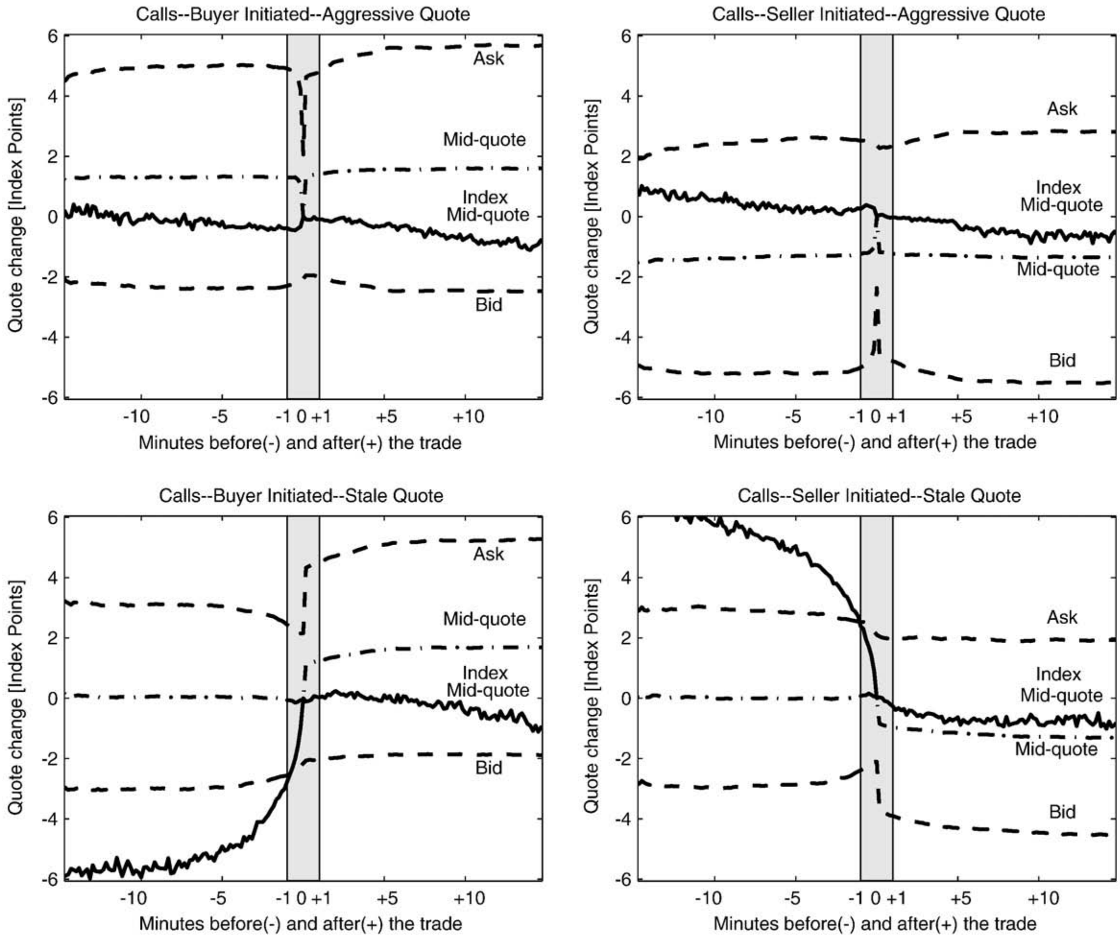

Fig. 2. The top left subplot plots the normalized series of the mid-quotes for the index futures (solid line) and the options (dotted line) for all buyer-initiated trades in the call options for which the ask quote changed in the last 30 s before the trade. The series of mid-quotes are normalized by subtracting the respective mid-quote values at the time of the trade. The normalized bid and ask quotes series are plotted with dashed lines. The bottom left subplot plots the corresponding series for call options for which the best ask quote did not change in the last 30 s before the trade. The subplots on the right plot the corresponding quote series for the seller-initiated trades.

Table 2 demonstrates that there are different violation patterns for quotes in the 2 min immediately surrounding the trades and the other 28 one-minute intervals, but that there is no substantial difference in the one-minute pre- and post-trade. Table 7 shows that there is a considerable difference in violations within the two one-minute intervals surrounding the trade once we separate them into trades against stale quotes and trades against aggressive quotes. Violations I and IV are much less likely in the minute prior to the trade for both call and put options when quotes are stale, but in the minute after the trades, the quotes often generate such violations consistent with the quotes being adjusted to the index changes plotted in Fig. 2. A test of equal means rejects, at the 1% level, the null hypothesis that the violation rates (I or IV) are equal for the minute before and after trades in the stale quotes case and for the null that the rates are equal for the minute before trades for the stale versus the aggressive quote cases. We focus on stale quotes in the 30 s prior to the trade, but we tabulate in Table 7 violations that occur during the last minute prior to the trade. Option quotes can therefore change from 60 s to 31 s prior to the trade, leading to type I violations in stale quote periods.

The last six rows of the call and put panel of Table 7 show that around trades preceded by aggressive quotes violations of type I and IV occur both before and after a trade with approximately the same frequencies. The average rates are higher compared to Table 2 and we reject the null of equal means for the minute before or after periods versus all other 28-minute intervals at 0.1%. Together with the top subplots of Fig. 2 this makes sense because the aggressive quotes appear to be submitted independently of index movements and they undercut the current ask or bid by multiple tick sizes. After the trade, the ask or bid revert to their pre-trade level, and if the index movement surrounding the trade is small, or is in the opposite direction of the option's movement, we observe a type IV or type I violation.

5. Conclusions

We study to what extent market microstructure effects can help explain high-frequency co-movements of index options and index futures quotes. We apply the methodology of Bakshi et al. (2000) to intervals with and without trades. We show that the

Table 7
Violation rates for trades against stale and aggressive quotes

	Call options			Put options		
		Type [%]			Type [%]	
	N	I	IV	*N*	I	IV
Trades against stale quotes						
Minute before trade	8769	12.4	3.9	11,170	12.9	4.5
Minute after trade	8914	30.2	16.5	11,450	29.7	16.2
All other 28 intervals	230,048	6.8	3.4	287,264	7.0	3.7
Buyer-initiated trades against aggressive quotes						
Minute before trade	7796	36.8	20.3	8971	33.9	22.8
Minute after trade	8156	35.2	19.0	9550	34.1	20.7
All other 28 intervals	207,188	6.7	3.8	234,774	6.7	4.0
Seller-initiated trades against aggressive quotes						
Minute before trade	8884	34.3	19.7	9729	34.7	21.7
Minute after trade	9313	33.0	17.5	10,329	33.3	18.2
All other 28 intervals	235,996	6.7	4.1	254,212	6.8	3.9

The table reports the mean rates of type I and type IV violations for buyer- and seller-initiated trades in call and put options split into three groups depending on whether there was a change in the best ask quote (buyer initiated) or bid quote (seller initiated) or no change in either within the last 30 s before the trade. The first three columns report number of observations, the mean rates for type I and IV violations for call options and the next three columns report the corresponding figures for put options. The table reports results using thirty 1-minute intervals, possibly overlapping, centered around trades. The intervals are split into fifteen 1-minute sub-intervals before and fifteen 1-minute sub-intervals after a trade. The first three rows report the mean rate for all trades that were classified as potentially stale, the following three rows report results for buyer-initiated trades in which there was a change in the best ask and the last three rows show results for trades in which there was a change in the best bid in the 30 s prior to a trade. Within each block, the first row shows average values for the minute prior to the trade, the middle row shows averages for the minute after the trade, and the last row shows averages for all other 1-minute intervals. A test of equal means for the violation rates of each type for intervals before or after trades versus all other intervals rejects at the 1% level or better. The null hypothesis of equal mean violation rates before and after trades against stale quotes is rejected at the 1% level.

frequency of quote co-movements that deviate from the benchmark of perfect correlation doubles for trade versus no-trade periods on a half-hour scale and quadruples for trade versus no-trade periods on a minute-by-minute scale. We extract the parameters of a stochastic volatility model from our data and show that for a stochastic volatility model to explain the observed differences between trade and no-trade intervals, the probability of a trade would have to be strongly negatively correlated with the absolute index price change. Yet, in our data, trades occur more frequently after larger index movements. Overall, we show that approximately one-third of the co-movement benchmark deviations can be rationalized by stochastic volatility and up to two-thirds may be due to market microstructure effects. We demonstrate empirically that two patterns of trader behavior, aggressive quotes and stale quotes, can explain the observed deviations.

In unreported simulations we extend the framework to incorporate stylized versions of stale quotes and aggressive quotes. These simulation results confirm that a combination of microstructure and asset pricing effects can generate the approximate magnitude of observed anomalous quote co- movements. An explicit microstructure model that yields an expression that relates trades and quotes and the underlying asset price could form the basis for more sophisticated simulations.

One promising direction for future research is to combine option pricing models and a microstructure model of trades and quotes. There has been a considerable interest in developing better and improved methods to hedge options under transaction costs and stochastic volatility and jump-diffusion models. One critical ingredient for these estimation techniques is high-frequency data that is free of microstructure effects. Ideally, one would like to filter out the microstructure effects in order to obtain estimates of the true hedge ratios from high-frequency data. Our evidence suggests that techniques for filtering out these effects need to incorporate a quote process that is correlated with the innovations in the true value of the option.

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