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A model-independent measure of aggregate idiosyncratic risk ☆

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ABSTRACT

This paper introduces a model-independent measure of aggregate idiosyncratic risk, which does not require estimation of market betas or correlations and is based on the concept of gain from portfolio diversification. The statistical results and graphical analyses provide strong evidence that there are significant level and trend differences between the average idiosyncratic volatility measures of Campbell et al. [Campbell, J.Y., Lettau, M., Malkiel, B.G., and Xu, Y., 2001, Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk, Journal of Finance 56, 1–43.] and the new methodology. Although both approaches indicate a noticeable increase in the firm-level idiosyncratic risk, the volatility measure of CLMX is greater and has a stronger upward trend than the new idiosyncratic volatility measure. For both measures of idiosyncratic risk, the upward trend is found to be stronger for smaller, lower-priced, and younger firms. The analytical and empirical results show that the significant upward trend in the differences of the two idiosyncratic volatility measures is related to the increase in the cross-sectional dispersion of the volatility of individual stocks.

1. Introduction

Campbell, Lettau, Malkiel and Xu (2001) (hereafter CLMX) decompose the return of an individual stock into three components to study the volatility of stock returns at the market, industry, and firm levels. CLMX use the firm-level return data to examine the volatility of the value-weighted NYSE/AMEX/Nasdaq composite index and the value-weighted average idiosyncratic volatility. Their results provide strong evidence for a positive deterministic trend in the firm-level idiosyncratic volatility for the period of July 1962 to December 1997.

Schwert (1989) finds that the market volatility has no significant trend over the sample period of 1859–1987. Although there has been a strong belief that stock market volatility has increased over time CLMX confirm and update Schwert's finding that aggregate stock market volatility does not exhibit any visible upward slope from July 1962 to December 1997. CLMX conclude that

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the stock market has become more volatile over the sample period of 1962–1997 but on a firm level instead of a market or industry level. The trend increase in idiosyncratic volatility relative to market volatility implies that the correlations among individual stock returns and the explanatory power of the market model for a typical stock have declined. It is also more difficult to diversify away idiosyncratic risk with a limited number of stocks in a portfolio.

Xu and Malkiel (2003) decompose the total volatility of individual stocks into systematic volatility and idiosyncratic volatility and provide strong evidence that the average idiosyncratic volatility has trended upwards. They find that the increase in idiosyncratic volatility is associated with the increasing share of trading in the Nasdaq market, the degree of institutional ownership in the stock market, and the expected earnings growth of individual stocks. Irvine and Pontiff (2005) show that the upward trend in aggregate idiosyncratic risk is related to an increase in the idiosyncratic volatility of fundamental cash flows. They also argue that the rise in cash flow volatility can be explained by increasing competition in various industries.

Brandt, Brav, and Graham (2005) and Bekaert, Hodrick, and Zhang (2005) cast doubts on the existence of significant upward trend in aggregate idiosyncratic volatility. They find that the positive trend is specific to the 1962–1997 sample period, and the episode of very high idiosyncratic volatility in the 1990s is limited to stocks with low institutional ownership and low stock price. Cao, Simin, and Zhao (in press) indicate that controlling for growth options eliminates or even reverses the upward trend in firm-level volatility. Brown and Kapadia (2007) show that the trend in idiosyncratic volatility is driven by new listings. Wei and Zhang (2006) provide evidence that the upward trend in idiosyncratic volatility is explained by a drop in return-on-equity and an increase in the volatility of the return-on-equity, especially in new listings. These articles indicate that firm size, price, and age are important characteristics to explain the upward trend in idiosyncratic volatility.

The aforementioned studies compute the average idiosyncratic volatility based on the variance decomposition or using the residuals from the one-factor capital asset pricing model (CAPM) or three-factor model of Fama and French (1993). The excess return of an individual stock is defined in previous studies as the sum of its systematic excess return component and its idiosyncratic return component. Then, the idiosyncratic risk is measured by the variance decomposition or by factor models assuming a certain parametric specification for the return generating process. In other words, the estimation of aggregate idiosyncratic risk has so far been model-dependent.

Earlier studies including Campbell et al. (2001) and Xu and Malkiel (2003) define total risk of an individual stock i as $\sigma_i^2 = f(M) + g(\varepsilon_i)$, where $f(M)$ is the portion of total variance explained some model M. If the model is correct and we are fully diversified then $g(\varepsilon_i)$ is irrelevant as in the CAPM. However, measuring the idiosyncratic risk by $g(\varepsilon_i)$ is meaningful only if the model M is correct. If we have various models M_1, M_2, M_3,… we end up with different measures of idiosyncratic risk for the same individual stock i. Unless we know the correct model we cannot obtain an accurate and unique measure of idiosyncratic volatility. For example, if we need to calculate the number of stocks that should be held to achieve a certain level of diversification, model-dependent measures of idiosyncratic volatility will generate different numbers.

Two basic related properties implied by the CAPM are that all investors hold in their portfolio all the risky securities available in the market, and that investors hold the risky assets in the same proportions, as these assets are available in the market, independent of the investors' preference. However, these properties contradict with the market experience. First, investors differ in their investment strategy and do not necessarily adhere to the same risky portfolio. Second, the typical investor usually does not hold many risky assets in his portfolio for various reasons, such as transaction costs, incomplete information, indivisibility of investment, institutional restrictions, liquidity constraints, or any other exogenous reasons. Therefore, when calculating the number of stocks that should be held to achieve a given level of diversification it is inappropriate to use the CAPM common decomposition of the variance as it is assumed that one holds small number of stocks. In other words, using the factor or CAPM based models in estimating idiosyncratic volatility may lead to inaccurate or inconsistent measures of diversifiable risk.

If investors held all the risky securities available in the market, the expected return of individual stocks would be determined solely by their contribution to the risk of the market portfolio (i.e., only systematic risk would affect expected returns). However, Blume and Friend (1975) find that the average number of securities held in a portfolio of typical investor is about 3.41. Barber and Odean (2000) report that the mean household's portfolio contains only 4.3 stocks and the median household invests in 2.61 stocks. Both studies indicate that most individuals hold very small number of stocks in their portfolios. With a constraint on the number of securities in the portfolio idiosyncratic risk becomes an important component of the investor's asset allocation decision as it affects the expected return and risk of the portfolio.

Furthermore, there are several asset pricing models in the literature that take idiosyncratic risk into account. For example, Levy (1978) shows in a framework with limited diversification that the equilibrium asset pricing equation relates the returns of individual stocks to their beta with the market and their beta with respect to a market-wide measure of idiosyncratic risk. Hence, measuring and investigating the aggregate idiosyncratic risk have important implications for asset pricing models where the investors hold undiversified portfolios.

This paper introduces a model-independent measure of aggregate idiosyncratic risk based on the mean-variance portfolio theory and the concept of gain from portfolio diversification. With the new approach, there is no need to estimate the covariance terms or the industry-level or firm-level beta coefficients when constructing the average idiosyncratic risk at the industry- or firm-level.

Since there is no gain from diversification when the correlations among individual stocks equal one, the variance of the portfolio with perfectly correlated securities contains systematic risk and idiosyncratic risk of the securities in the portfolio.[3] We also think that the stock market index can be viewed as a fully diversified portfolio, which does not contain any idiosyncratic risk.

[3] In this paper, the portfolio with perfectly correlated securities is called the “non-diversified” portfolio.

Since the market portfolio contains a large number of stocks, enough diversification gains are achieved and the idiosyncratic risk contributes nothing to the total risk of the market portfolio. That is, the risk of this well-diversified portfolio is due solely to the systematic risk of the securities in the portfolio.

The new measure of average idiosyncratic volatility is defined as the difference between the variance of the non-diversified portfolio and the variance of the fully diversified portfolio. We present two versions of the new methodology; one decomposing total risk into firm and market variance, and the other decomposing total risk into firm, industry, and market variance. Although the average idiosyncratic volatility measures of CLMX and the new methodology have very similar fluctuations through time, there are some significant differences.

Similar to the findings of CLMX and Xu and Malkiel, we detect a significant increase in the average firm-level idiosyncratic volatility for the NYSE/AMEX/Nasdaq stocks. The statistical results and graphical analyses from the new approach decomposing total risk into firm and market variance, and decomposing total risk into firm, industry, and market variance indicate that the average idiosyncratic volatility has trended upwards for the whole sample, but the trend is more pronounced for the Nasdaq stocks and relatively weaker for the NYSE/AMEX and NYSE sample.

The results from testing the differences in means, medians, and variances and the graphical analyses provide strong evidence that there are significant level and trend differences between the average idiosyncratic volatility measures of CLMX and the new methodology. It will be shown both theoretically and empirically that the value-weighted average idiosyncratic variances estimated with the CLMX methodology are greater than those of the new methodology. The upward trend in the new idiosyncratic volatility measure is not as strong as in the idiosyncratic volatility measure of CLMX. The level and trend differences are more noticeable for the Nasdaq stocks, but the differences are rather weak for the NYSE/AMEX and NYSE sample. These results hold for the sample period of CLMX (1962:07-1997:12) and for the extended sample period of 1962:07-2005:12.

We show both analytically and empirically that the differences between the idiosyncratic volatility measures of CLMX and new methodology are more significant when the cross-sectional dispersion in the standard deviation of individual stocks is larger. Since there is an increase in the cross-sectional dispersion of the volatility of individual stocks through time we find an upward trend in the differences of the two aggregate idiosyncratic volatility measures. A notable point is that there is a significant time trend in the cross-sectional dispersion of the standard deviation (and variance) of the NYSE/AMEX/Nasdaq and Nasdaq stocks for both sample periods. However, the plots for the NYSE/AMEX and NYSE sample do not exhibit such a strong upward slope.

We also investigate the significance of trend in idiosyncratic volatility for a group of stocks formed based on firm size, price, and age. Specifically, we test the presence and significance of trend in aggregate idiosyncratic volatility of large, small, high-priced, low-priced, young, and old firms. The statistical results indicate that the upward trend in idiosyncratic volatility is stronger for smaller, lower-priced, and younger firms.

This paper is organized as follows. Section 2 introduces alternative measures of aggregate idiosyncratic risk. Section 3 describes the data and estimation methodology. Section 4 presents the empirical findings for groups of stocks based on the exchange. Section 5 provides results for groups of stocks based on market capitalization, price, and firm age. Section 6 concludes the paper.

2. Aggregate idiosyncratic volatility measures

2.1. CLMX methodology decomposing total risk into firm and market variance

The idiosyncratic risk of a stock is unobservable, and is generally estimated using a return generating process. The previous research calculates idiosyncratic volatility of a stock relative to the systematic (or market-wide) returns of the stock.[4] The excess return for each stock relative to the risk-free rate is defined as the sum of its systematic excess return component and its idiosyncratic return component. Assuming that the capital asset pricing model (CAPM) holds, the systematic excess return is a function of the excess market return:

$$R_{i,t} = \beta_i R_{m,t} + \varepsilon_{i,t}, \tag{1}$$

where $R_{i,t}$ is the excess return on a stock i in period t, $\beta_i R_{m,t}$ is the systematic excess return with β_i denoting the beta for stock i and $R_{m,t}$ denoting the excess market return in period t, and $\varepsilon_{i,t}$ is the idiosyncratic return component. Based on this return decomposition, total risk of the stock can be decomposed into systematic risk and idiosyncratic risk. We can define the market portfolio as a value-weighted portfolio of n stocks, and its excess return will be written as $R_{m,t}=\sum_{i=1}^{n} w_{i,t} R_{i,t}$, where $w_{i,t}$ represents the weight of each stock i in the market portfolio, and satisfies the constraints $0 \le w_{i,t} \le 1$ and $\sum_{i=1}^{n} w_{i,t}=1$.

In the case where the return generating process is assumed to follow a one-factor model in Eq. (1), we have the following relationship:

$$Var(R_{i,t}) = \beta_i^2 Var(R_{m,t}) + Var(\varepsilon_{i,t}) + 2\beta_i Cov(R_{m,t}, \varepsilon_{i,t}). \tag{2}$$

[4] Xu and Malkiel (2003) construct the firm-level average idiosyncratic volatility by decomposing total risk into firm and market variance. Therefore, the CLMX methodology decomposing total risk into firm and market variance is originally proposed and used by Xu and Malkiel (2003). However, this methodology can be viewed as a subset of CLMX's methodology decomposing total risk into firm, industry, and market variance.

Taking the weighted sums across individual stocks gives

$$\sum_{i=1}^{n} w_{i,t} Var(R_{i,t}) = \beta^2 Var(R_{m,t}) + \sum_{i=1}^{n} w_{i,t} Var(\varepsilon_{i,t}), \tag{3}$$

where $\beta^2=\sum_{i=1}^{n} w_i \beta_i^2$ and $\mathrm{Cov}(R_{m,t}, \varepsilon_{i,t})=0$. Eq. (3) indicates that the aggregate idiosyncratic volatility can be defined as the difference between the value-weighted average volatility of individual stocks and the volatility that is attributable to fluctuations in the broad market index.

Xu and Malkiel (2003) investigate the time-series behavior of aggregate idiosyncratic volatility using a restricted version of Eq. (3). They assume the market betas for firms to be equal to one, and define the aggregate idiosyncratic variance as the difference between the value-weighted average stock variance and the market variance:

$$\sum_{i=1}^{n} w_{i,t} \mathrm{Var}(\varepsilon_{i,t}) = \sum_{i=1}^{n} w_{i,t} \mathrm{Var}(R_{i,t}) - \mathrm{Var}(R_{m,t}). \tag{4}$$

When using Eq. (4) instead of Eq. (3) to estimate aggregate idiosyncratic volatility, the volatility estimate will be biased by (β^2-1) $\mathrm{Var}(R_{m,t})$. If, as an approximation, we treat the weights w_i as some probability measure, then $\beta^2-1=\sum_{i=1}^{n} w_i \beta_i^2-1$ can be viewed as the variance of β_i. In this case, the volatility measure will be biased upward. However, based on the construction of 100 portfolios of Fama and French (1992) for the sample period of July 1962 to December 1998, Xu and Malkiel (2003) indicate that the estimated bias is likely to be small, in the neighborhood of 4%–5% of the market volatility.[5]

2.2. New methodology decomposing total risk into firm and market variance

Idiosyncratic risk represents the stock's variance that is not attributable to overall market volatility, but is related to the firm's specific volatility. Idiosyncratic risk is unique to a stock because it is related to the part of a stock's return that does not vary with returns on other stocks or the market. With many stocks in a portfolio, idiosyncratic risk becomes less important because when the idiosyncratic part of one stock's return increases, it is likely that the idiosyncratic part of another stock has decreased, movements that cancel each other out. Hence with enough diversification as a result of a portfolio containing a large number of different stocks, the idiosyncratic risk contributes nothing to the total risk of the portfolio. In other words, the risk of a well-diversified portfolio is due solely to the systematic risk of stocks in the portfolio.

Our new methodology in estimating aggregate idiosyncratic risk depends on the concept of *gain from portfolio diversification* introduced by Markowitz (1952, 1959). According to the mean-variance portfolio theory of Markowitz, the risk of the portfolio of n securities

$$R_{p,t} = \sum_{i=1}^{n} w_{i,t} R_{i,t} \tag{5}$$

is measured by the variance

$$\sigma_{p,t}^2 = \sum_{i=1}^{n} w_{i,t}^2 \sigma_{i,t}^2 + 2 \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j>i}}^{n} w_{i,t} w_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}, \tag{6}$$

where $\sigma_{i,t}^2$ is the variance of excess return on stock i, $\rho_{ij,t}$ is the correlation of excess returns on stock i and j, $\sigma_{i,t}$ and $\sigma_{j,t}$ are the standard deviations of excess returns on stock i and j, and $w_{i,t}$ and $w_{j,t}$ represent the weights of stock i and j in the portfolio.

It is well known that for a given set of weights the lower the correlations ρ_{ij}, the smaller the portfolio variance and, hence, the larger the gain from diversification. If the securities are perfectly correlated ($\rho_{ij}=1$), no diversification gains are achieved. Diversification is beneficial to the risk-averse investor because it reduces portfolio risk except in the extremely rare case where returns on securities move perfectly together. Since there is no gain from diversification when the securities are perfectly correlated, we can find the variance of the non-diversified portfolio, which contains systematic risk and idiosyncratic risk of the securities in the portfolio. When $\rho_{ij}=1$ in Eq. (6), the variance of the non-diversified portfolio is written as

$$\sigma_{p,t}^2 = \left(\sum_{i=1}^{n} w_{i,t} \sigma_{i,t} \right)^2. \tag{7}$$

Eq. (7) defines the variance of the non-diversified portfolio as the square of the value-weighted average standard deviations of individual stocks.

The stock market index can be viewed as a fully diversified portfolio which does not contain any idiosyncratic risk. Since the market portfolio contains a large number of stocks, enough diversification gains are achieved and the idiosyncratic risk contributes

[5] Schwert and Seguin (1990) find that the cross-sectional dispersion in betas is correlated with the level of market volatility. This conclusion is based on a comparison of an equal-weighted group of small-stock portfolios with an equal-weighted group of large-stock portfolios. In order to account for this effect, Xu and Malkiel (2003) use value weighting since large dispersion in betas is much more significant for small stocks than for large stocks.

nothing to the total risk of the market portfolio. In other words, the risk of this well-diversified portfolio is due solely to the systematic risk of the securities in the portfolio.

The concept of gain from portfolio diversification yields a model-independent measure of aggregate idiosyncratic risk which does not require estimation of market betas or correlations:

$$\sigma_{\varepsilon,t}^2 = \left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2 - \text{Var}\ (R_{m,t}) \tag{8}$$

where $\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}$ is the value-weighted average standard deviation of individual stocks. The difference between the variance of the non-diversified portfolio, $\left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2$, and the variance of the fully diversified portfolio (i.e., market variance), Var $(R_{m,t})$, yields the value-weighted average idiosyncratic variance $\sigma_{\varepsilon,t}^2$ in Eq. (8).

The difference between the aggregate idiosyncratic volatility measures of CLMX and new methodology can be seen in Eqs. (4) and (8). The CLMX methodology uses the value-weighted average variance of individual stocks $\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}^2$ as shown in Eq. (4), whereas the new methodology uses the square of the value-weighted average standard deviation of individual stocks $\left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2$ as presented in Eq. (8). Since the same market variance, Var$(R_{m,t})$, is subtracted from both quantities it is easy to see which measure is greater than the other.[6]

Since the portfolio weights satisfy the following constraints $0 \le w_{i,t} \le 1$ and $\sum_{i=1}^{n} w_{i,t} = 1$, we show at an earlier stage of the study that $\left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2$ is smaller than $\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}^2$ as long as the standard deviations (or variances) of individual stocks are different from each other. Although it cannot be observed in practice, if the standard deviations (or variances) of all stocks in the portfolio are the same, then both measures will be equal to each other. This implies that the aggregate idiosyncratic volatility measure of CLMX is greater by definition than the volatility measure obtained from the new methodology. Therefore, empirically we should be able to observe a difference in the levels through time. However, whether the difference between the two measures is statistically or economically significant is an empirical question, and can be sensitive to the choice of data set or sample period. This paper investigates the differences between these two idiosyncratic volatility measures.

At an earlier stage of the study, we analytically show that the differences in the idiosyncratic volatility measures will be more significant when the cross-sectional dispersion in the standard deviation of individual stocks is larger. If there is an increase in the cross-sectional dispersion in the volatility of individual stocks through time we expect an upward trend in the differences of the two aggregate idiosyncratic volatility measures.[7]

The new measure of aggregate idiosyncratic risk in Eq. (8) can further be analyzed with an index of the benefits of diversification:

$$I = \frac{\left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2 - \text{Var}\ (R_{m,t})}{\text{Var}\ (R_{m,t})},$$

where the index I equals zero if there is no benefit of diversification (i.e., $\rho_{ij} = 1$). We can easily demonstrate this assuming two stocks in the portfolio:

$$I = \frac{2w_1w_2\sigma_1\sigma_2(1-\rho_{12})}{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}},$$

where $I = 0$ if $\rho_{12} = 1$. It is also obvious that as ρ_{ij} decreases the index I increases. For example, if $\rho_{ij} = -1$ the variance of the market, Var $(R_{m,t})$, becomes very small especially as compared to the variance of the non-diversified portfolio, $\left(\sum_{i=1}^{n} w_{i,t}\sigma_{i,t}\right)^2$, and hence the index I becomes very large. Larger values of I indicate greater benefits of diversification. The index I tells us how far we are away from the worst case where $\rho_{ij} = 1$. Suppose that for a given period of time with given σ_i and σ_j we have a portfolio of securities with $\rho_{ij} < 1$. Using the index I we can find how far we are away from the worst case where we have a portfolio of perfectly correlated securities. Therefore, the index I measures the total benefits of diversification from holding the portfolio of securities with hypothetical portfolio of perfectly correlated securities. $\rho_{ij} < 1$ as opposed to holding the hypothetical portfolio of perfectly correlated securities.

2.3. CLMX methodology decomposing total risk into firm, industry, and market variance

CLMX (2001) decompose the return of an individual stock into three components: the market-wide return, an industry-specific residual, and a firm-specific residual. Their objective is to define volatility measures that sum to the total return volatility of an individual stock, without having to estimate covariances and betas for firms and industries.

[6] To save space, we delete one of the appendices that provides an analytical proof based on the Cauchy–Schwarz inequality that as long as the volatility of all individual stocks in the portfolio are not equal to each other, the average idiosyncratic volatility measure of CLMX in Eq. (4) is greater than the new idiosyncratic volatility measure in Eq. (8). These results are available from the authors upon request.

[7] To preserve space, we drop two of our appendices that show the differences in the idiosyncratic volatility measures will be more significant when the cross-sectional dispersion in the standard deviation of individual stocks is larger and that also provide an alternative comparison of CLMX and new measures under the one-factor CAPM framework.

They denote industries by an i subscript and individual firms by a j subscript. The excess return of industry i in period t is written as $R_{i,t}=\sum_{i=1}^{n} w_{ji,t}R_{ji,t}$ where $R_{ji,t}$ is the excess return of firm j that belongs to industry i and $w_{ji,t}$ is the weight of firm j in industry i. Similarly, the excess market return is $R_{m,t}=\sum_{i=1}^{n} w_{i,t}R_{i,t}$ where total market. $R_{i,t}$ is the excess return of industry i and $w_{i,t}$ is the weight of industry i in the total market.

They decompose industry and firm returns based on the CAPM that sets intercepts to zero. The excess return of industry i is defined as a function of the excess market return,

$$R_{i,t} = \beta_{im}R_{m,t} + \varepsilon_{i,t}, \tag{9}$$

where β_{im} denotes the beta for industry i with respect to the market return, and $\varepsilon_{i,t}$ is the industry-specific residual. The excess return of firm j in industry i is defined as a function of the excess return on its industry,

$$R_{ji,t} = \beta_{ji}R_{i,t} + \eta_{ji,t} = \beta_{ji}\beta_{im}R_{m,t} + \beta_{ji}\varepsilon_{i,t} + \eta_{ji,t}, \tag{10}$$

where β_{ji} is the beta of firm j in industry i with respect to its industry, and $\eta_{ji,t}$ is the firm-specific residual.

$\eta_{ji,t}$ is orthogonal by construction to the industry return $R_{i,t}$, and CLMX also assume that $\eta_{ji,t}$ is orthogonal to the components $R_{m,t}$ and $\varepsilon_{i,t}$. In other words, they assume that $\beta_{jm}=\beta_{ji}\beta_{im}$, and the weighted sums of the different betas equal one: $\sum_i w_{i,t}\beta_{im}=1$ and $\sum_{j\in i} w_{ji,t}\beta_{ji}=1$. The CAPM decomposition in Eqs. (9) and (10) yields a simple variance decomposition in which all covariance terms are zero:

$$\mathrm{Var}\left(R_{i,t}\right) = \beta_{im}^2\ \mathrm{Var}\left(R_{m,t}\right) + \ \mathrm{Var}\left(\varepsilon_{i,t}\right) \tag{11}$$

$$\mathrm{Var}\left(R_{ji,t}\right) = \beta_{jm}^2\ \mathrm{Var}\left(R_{m,t}\right) + \beta_{ji}^2\ \mathrm{Var}\left(\varepsilon_{i,t}\right) + \ \mathrm{Var}\left(\eta_{ji,t}\right) \tag{12}$$

Since this variance decomposition requires estimation of firm-specific and industry-specific betas, CLMX use a simplified model that assumes betas to be equal to unity, i.e., $\beta_{im}=\beta_{ji}=\beta_{jm}=1$.

Computing the value-weighted average variance of industry returns gives

$$\sum_{i=1}^{N} w_{i,t}\ \mathrm{Var}\left(R_{i,t}\right) = \ \mathrm{Var}\left(R_{m,t}\right) + \sum_{i=1}^{N} w_{i,t}\ \mathrm{Var}\left(\varepsilon_{i,t}\right) = \sigma_{m,t}^2 + \sigma_{\varepsilon,t}^2, \tag{13}$$

where N is the number of industries in the total market and the terms involving betas aggregate out because $\sum_i w_{i,t}\beta_{im}=1$. $\sigma_{m,t}^2=\mathrm{Var}(R_{m,t})$ is the market variance and $\sigma_{\varepsilon,t}^2=\sum_{i=1}^{N} w_{i,t}\ \mathrm{Var}\left(\varepsilon_{i,t}\right)$ is the value-weighted average idiosyncratic variance of industry returns.

Similarly, the value-weighted average of firm variances in industry i is

$$\sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(R_{ji,t}\right) = \ \mathrm{Var}\left(R_{i,t}\right) + \sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(\eta_{ji,t}\right) = \ \mathrm{Var}\left(R_{i,t}\right) + \sigma_{\eta i,t}^2, \tag{14}$$

where n is the number of firms in industry i, and the terms involving betas aggregate out because $\sum_i w_{i,t}\beta_{im} = 1$ and $\sum_{j\in i} w_{ji,t}\beta_{ji} = 1$. $\sigma_{\eta i,t}^2 = \sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(\eta_{ji,t}\right)$ is the value-weighted average of firm-level volatility in industry i.

Computing the value-weighted average across industries using Eq. (9), yields again a beta-free variance decomposition:

$$\sum_{i=1}^{N} w_{i,t}\sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(R_{ji,t}\right) = \sum_{i=1}^{N} w_{i,t}\ \mathrm{Var}\left(R_{i,t}\right) + \sum_{i=1}^{N} w_{i,t}\sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(\eta_{ji,t}\right) = \ \mathrm{Var}\left(R_{m,t}\right) + \sum_{i=1}^{N} w_{i,t}\ \mathrm{Var}\left(\varepsilon_{i,t}\right) + \sum_{i=1}^{N} w_{i,t}\sigma_{\eta i,t}^2 = \sigma_{m,t}^2 + \sigma_{\varepsilon,t}^2 + \sigma_{\eta,t}^2, \tag{15}$$

$\sigma_{\eta,t}^2 = \sum_{i=1}^{N} w_{i,t}\sigma_{\eta i,t}^2 = \sum_{i=1}^{N} w_{i,t}\sum_{j=1}^{n} w_{ji,t}\mathrm{Var}(\eta_{ji,t})$ is the value-weighted average of firm-level idiosyncratic volatility across all firms. Eq. (15) shows that the value-weighted average firm idiosyncratic variance, $\sigma_{\eta,t}^2$, is the value-weighted average total stock variance, $\sum_{i=1}^{N} w_{i,t}\sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(R_{ji,t}\right)$, less the value-weighted average industry-specific idiosyncratic variance, $\sigma_{\varepsilon,t}^2$, and the market variance $\sigma_{m,t}^2$:

$$\sigma_{\eta,t}^2 = \sum_{i=1}^{N} w_{i,t}\sum_{j=1}^{n} w_{ji,t}\ \mathrm{Var}\left(R_{ji,t}\right) - \sigma_{\varepsilon,t}^2 - \sigma_{m,t}^2. \tag{16}$$

2.4. New methodology decomposing total risk into firm, industry, and market variance

Assuming that we have N industries in the total market, the excess return on the market is written as

$$R_{m,t} = \sum_{i=1}^{N} w_{i,t}R_{i,t}, \tag{17}$$

where $R_{i,t}$ is the excess return on industry i and $w_{i,t}$ is the weight of industry i in the market. When the excess returns of all industries are perfectly correlated, there is no gain from diversification. That is, the variance of the non-diversified portfolio in Eq. (17) becomes

$$\left(\sum_{i=1}^{N} w_{i,t}\sigma_{i,t}\right)^2 \tag{18}$$

where $\sigma_{i,t}$ is the standard deviation of excess returns of industry i.

The value-weighted average idiosyncratic variance at the industry-level is the difference between the variance of the non-diversified portfolio in Eq. (18) and the variance of the fully diversified market portfolio which is the variance of the market index returns:

$$\sigma^2_{\varepsilon,t} = \left(\sum_{i=1}^{N} w_{i,t}\sigma_{i,t}\right)^2 - \mathrm{Var}\left(R^{\mathrm{index}}_{m,t}\right). \tag{19}$$

Assuming that we have n firms in industry i, the excess return on industry i is written as

$$R_{i,t} = \sum_{j=1}^{n} w_{ji,t}R_{ji,t}, \tag{20}$$

where $R_{ji,t}$ is the excess return of firm j that belongs to industry i and $w_{ji,t}$ is the weight of firm j in industry i. When the excess returns of all firms in industry i are perfectly correlated, there is no gain from diversification, and the variance of the non-diversified portfolio in Eq. (20) becomes

$$\left(\sum_{j=1}^{n} w_{ji,t}\sigma_{ji,t}\right)^2, \tag{21}$$

where $\sigma_{ji,t}$ is the standard deviation of excess return of firm j that belongs to industry i.

We think that the industry index portfolio, $R^{\mathrm{index}}_{i,t}$, can be viewed as a fully diversified portfolio which does not contain any idiosyncratic risk. Since the industry portfolio contains a large number of different stocks, enough diversification gains are achieved and the idiosyncratic risk contributes nothing to the total risk of the industry portfolio. In other words, the risk of this well-diversified portfolio is due solely to the systematic risk of the securities in the portfolio.

The concept of gain from portfolio diversification yields a model-independent measure of idiosyncratic volatility for industry i which does not require estimation of betas or correlations:

$$\sigma^2_{\varepsilon i,t} = \left(\sum_{j=1}^{n} w_{ji,t}\sigma_{ji,t}\right)^2 - \mathrm{Var}\left(R^{\mathrm{index}}_{i,t}\right), \tag{22}$$

where $\sum_{j=1}^{n} w_{ji,t}\sigma_{ji,t}$ is the value-weighted average standard deviation of excess returns of firm j that belongs to industry i, and Var($R^{\mathrm{index}}_{i,t}$) is the variance of excess returns on the fully diversified industry index portfolio.

Eqs.(19) and (22) indicate that the value-weighted average idiosyncratic variance at the firm-level is the value-weighted average idiosyncratic variance of all industries in the market, $\left(\sum_{i=1}^{n} w_{i,t}\sigma_{\varepsilon i,t}\right)^2$, less the variance of the excess market return, Var($R^{\mathrm{index}}_{m,t}$):

$$\sigma^2_{\eta,t} = \left(\sum_{i=1}^{N} w_{i,t}\sigma_{\varepsilon i,t}\right)^2 - \mathrm{Var}\left(R^{\mathrm{index}}_{m,t}\right) = \left(\sum_{i=1}^{N} w_{i,t}\sqrt{\left(\sum_{j=1}^{n} w_{ji,t}\sigma_{ji,t}\right)^2 - \mathrm{Var}\left(R^{\mathrm{index}}_{i,t}\right)}\right)^2 - \mathrm{Var}\left(R^{\mathrm{index}}_{m,t}\right). \tag{23}$$

3. Data and estimation

Aggregate idiosyncratic volatility measures are constructed from the Center for Research in Security Prices (CRSP) tape that includes firm-level return data on the NYSE, AMEX, and Nasdaq stocks. The monthly idiosyncratic volatility measures are calculated using the daily data that cover the period from July 2, 1962 to December 31, 2005. To generate daily excess return, we subtract the 30-day Treasury bill return divided by the number of trading days in a month. Since firms traded on the NYSE, AMEX, and Nasdaq have diverse market capitalizations and firm characteristics, we also estimate the aggregate idiosyncratic volatility measures for the NYSE/AMEX, NYSE, and Nasdaq stocks separately.

Following CLMX, we calculate the monthly variance of excess market returns using the within-month daily data as:[8]

$$\mathrm{Var}\left(R_{m,t}\right) = \sigma^2_{mt} = \sum_{d=1}^{D_t}\left(R_{m,d} - \mu_m\right)^2 \tag{24}$$

where D_t is the number of trading days in month t, $R_{m,d}$ is the excess market return on day d, and μ_m is the mean of $R_{m,d}$.

For the CLMX and new methodology decomposing total risk into firm and market variance, we need to calculate the value-weighted average stock volatility. As shown in Eqs. (4) and (8), the CLMX methodology uses the value-weighted average variance of individual stocks, whereas the new methodology uses the square of the value-weighted average standard deviation of individual stocks.

We use within month daily returns to compute the value-weighted average stock variance for each month from July 1962 to December 2005:

$$\mathrm{CLMX}: \sum_{i=1}^{n_t} w_{i,t}\sigma^2_{i,t}, \quad \mathrm{New}: \left(\sum_{i=1}^{n_t} w_{i,t}\sigma_{i,t}\right)^2$$

where $\mathrm{Var}\left(R_{i,t}\right) = \sigma^2_{i,t} = \sum_{d=1}^{D_t}\left(R_{i,d} - \mu_d\right)^2$ is the monthly variance of stock i, $\sigma_{i,t} = \sqrt{\sum_{d=1}^{D_t}\left(R_{i,d} - \mu_d\right)^2}$ is the monthly standard deviation of stock i, $R_{i,d}$ is the return on stock i in day d, μ_d is the mean of $R_{i,d}$, and n_t is the number of stocks that exist in month t. To be consistent with CLMX, for weights in period t, $w_{i,t}$, we use the market capitalization of a firm in period $t-1$ and take the weights constant within period t.

For the CLMX and new methodology decomposing total risk into firm, industry and market variance, we group individual firms into 49 industries according to the SIC classification scheme of Fama and French (1997).[9]

For idiosyncratic volatility in industry i, CLMX sum the squares of the industry-specific residual in Eq. (9) within a month t:

$$\sigma^2_{\varepsilon i,t} = \sum_{d=1}^{D_t} \varepsilon^2_{i,d}. \tag{25}$$

As indicated by CLMX, they have to average over industries so that the covariances of individual industries cancel out. This gives a measure of average industry-level idiosyncratic volatility IND_t:

$$\mathrm{IND}_t = \sum_{i=1}^{N_t} w_{i,t}\sigma^2_{\varepsilon i,t}, \tag{26}$$

where N_t is the number of industries in the total market that exist in month t.

According to the new methodology, the aggregate idiosyncratic risk at the industry-level is defined as the difference between the value-weighted average variance of 49 industries and the market variance. As shown in Eq. (19), we first calculate the square of the value-weighted average standard deviation of the 49 industries that belong to the market and then take the difference between this squared term and the market variance.

To estimate firm-specific idiosyncratic volatility, CLMX first sum the squares of the firm-specific residual in Eq. (10) for each firm in the sample:

$$\sigma^2_{\eta ji,t} = \sum_{d=1}^{D_t} \eta^2_{ji,d}. \tag{27}$$

Next, they calculate the value-weighted average of the firm-specific volatilities within an industry:

$$\sigma^2_{\eta i,t} = \sum_{j=1}^{n_t} w_{ji,t}\sigma^2_{\eta ji,t}, \tag{28}$$

where n_t is the number of firms in industry i that exist in month t. Finally, they average over industries to obtain a measure of average firm-level idiosyncratic volatility FIRM_t

$$\mathrm{FIRM}_t = \sum_{i=1}^{N_t} w_{i,t}\sigma^2_{\eta i,t}, \tag{29}$$

where the firm-specific covariances cancel out.

To find the firm-specific idiosyncratic volatility with the new methodology, we first compute idiosyncratic volatility for each industry in the market, which is defined in Eq. (22). Then, the value-weighted average idiosyncratic variance at the firm-level is

[8] Goyal and Santa-Clara (2003) and Bali, Cakici, Yan, and Zhang (2005) also use within month daily data to compute monthly idiosyncratic and total volatility measures.

[9] The same SIC classification scheme is used by CLMX.

Table 1
Descriptive statistics of idiosyncratic risk measures

	NYSE/AMEX/NASDAQ		NYSE/AMEX		NYSE		NASDAQ	
	CLMX	New	CLMX	New	CLMX	New	CLMX	New
Panel A. Sample period of CLMX (1962:07–1997:12)								
Mean	6.44	4.85	5.52	4.22	4.97	3.89	14.54	9.95
Median	6.28	4.75	5.15	3.99	4.66	3.69	14.14	9.20
Standard deviation	2.91	2.16	2.30	1.79	2.14	1.66	8.24	6.22
Maximum	40.95	31.16	33.69	25.25	31.59	23.49	64.54	46.32
Minimum	2.08	1.75	2.41	1.75	1.98	1.53	2.79	1.39
Skewness	4.28	4.70	5.08	4.59	5.15	4.55	1.01	0.95
Kurtosis	48.63	54.10	56.56	48.59	59.16	48.57	6.42	5.75
AC(1)	0.574	0.570	0.532	0.570	0.534	0.573	0.826	0.834
AC(2)	0.535	0.531	0.484	0.525	0.486	0.527	0.809	0.821
AC(3)	0.486	0.482	0.433	0.474	0.445	0.482	0.777	0.798
AC(6)	0.383	0.375	0.308	0.347	0.317	0.351	0.719	0.746
AC(12)	0.303	0.271	0.233	0.241	0.246	0.245	0.665	0.681
ADF unit root	−5.10	−4.80	−4.69	−4.49	−4.73	−4.54	−3.83	−3.62
Panel B. Extended sample period (1962:07–2005:12)								
Mean	7.63	5.76	6.05	4.80	5.65	4.59	20.63	15.13
Median	6.59	4.96	5.47	4.33	5.05	4.12	16.28	11.15
Standard deviation	5.42	4.07	3.07	2.52	3.02	2.48	14.94	12.67
Maximum	47.06	37.59	39.05	31.51	38.13	30.84	126.24	103.89
Minimum	2.08	1.82	1.54	1.17	1.47	1.12	5.92	3.42
Skewness	3.57	3.99	3.98	3.92	4.09	4.01	3.22	3.43
Kurtosis	20.29	24.83	33.52	32.68	34.27	33.21	17.30	18.81
AC(1)	0.807	0.778	0.525	0.542	0.526	0.541	0.829	0.850
AC(2)	0.754	0.715	0.459	0.475	0.460	0.474	0.761	0.784
AC(3)	0.743	0.690	0.463	0.490	0.473	0.495	0.747	0.767
AC(6)	0.645	0.583	0.307	0.336	0.317	0.341	0.654	0.679
AC(12)	0.538	0.462	0.253	0.256	0.269	0.264	0.564	0.564
ADF unit root	−3.57	−3.78	−6.36	−6.09	−6.27	−6.05	−3.53	−3.46

This table presents the mean, median, standard deviation, maximum, minimum, skewness, kurtosis, autocorrelation of order j, AC(j), and the Augmented Dickey–Fuller (ADF) unit root test statistics for the average idiosyncratic volatility measures. All measures are monthly value-weighted average variances multiplied by 1200, and calculated using the CLMX and new methodology decomposing total risk into firm, industry, and market variance.

calculated as the square of the value-weighted average idiosyncratic standard deviation of all industries in the market less the variance of the excess market return.

4. Empirical results

4.1. Descriptive statistics for the aggregate idiosyncratic volatility measures

Table 1 shows the descriptive statistics for the aggregate firm-level idiosyncratic volatility measures obtained from the CLMX and new methodology decomposing total risk into firm, industry, and market variance. The results are presented for the annualized value-weighted average idiosyncratic variances of stocks traded on the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq.[10]

Panel A of Table 1 shows that for the sample period of CLMX (1962:07–1997:12) the sample mean of the aggregate idiosyncratic volatility of the NYSE/AMEX/Nasdaq stocks is about 6.44% for CLMX and 4.85% for the new methodology. That is, the value-weighted average idiosyncratic variance of stocks obtained from the CLMX methodology is on average 33% greater than the aggregate idiosyncratic volatility measure obtained from the new methodology. This level-difference is higher for the Nasdaq stocks and lower for the NYSE/AMEX and NYSE stocks. More specifically, the aggregate idiosyncratic volatility measure of CLMX is on average 46% greater than the new idiosyncratic volatility measure for the Nasdaq stocks. For the NYSE and NYSE/AMEX sample, the idiosyncratic volatility measure of CLMX is about 27% to 30% greater than the new volatility measure.

Panel A shows that the maximum and minimum values of the idiosyncratic variance measures obtained from the CLMX methodology are greater than those of the new methodology. The differences in maximum and minimum values are considerably high for the Nasdaq stocks: The maximum value of the average idiosyncratic variance of the Nasdaq stocks equals 64.54% for the CLMX and 46.32% for the new methodology.

To check whether the level-difference is because of some extreme observations in our sample, we also calculate the median values of the aggregate idiosyncratic volatility measures. The median is a robust measure of the center of the distribution and is less

[10] At an earlier stage of the study, we present results from the aggregate idiosyncratic volatility measures obtained from decomposing total risk into firm and market variance. Since the qualitative results turn out to be very similar to those reported in our tables, we choose not to discuss them in the paper. They are available from the authors upon request.

Table 2
Differences in idiosyncratic risk measures: CLMX versus new methodology

	NYSE/AMEX/NASDAQ	NYSE/AMEX	NYSE	NASDAQ
Panel A. Sample period of CLMX (1962:07–1997:12)				
Mean difference	1.79	1.30	1.08	4.59
Standard *t*	(45.45)	(48.15)	(43.03)	(37.61)
Newey–West *t*	[23.47]	[29.77]	[26.59]	[17.60]
Median difference	1.54	1.16	0.97	4.94
Kruskal–Wallis	(76.92)	(84.30)	(48.85)	(36.51)
p-value	[0.00]	[0.00]	[0.00]	[0.00]
Variance difference	3.80	2.09	1.82	29.21
Brown–Forsythe	(15.25)	(5.78)	(6.17)	(17.76)
p-value	[0.0001]	[0.0164]	[0.0132]	[0.0000]
Panel B. Extended sample period (1962:07–2005:12)				
Mean difference	1.87	1.25	1.06	5.50
Standard *t*	(28.55)	(45.50)	(39.67)	(41.66)
Newey–West *t*	[12.69]	[27.33]	[23.32]	[21.21]
Median difference	1.63	1.14	0.93	5.13
Kruskal–Wallis	(92.09)	(96.80)	(75.85)	(75.12)
p-value	[0.00]	[0.00]	[0.00]	[0.00]
Variance difference	12.80	3.06	2.95	62.46
Brown–Forsythe	(10.28)	(4.73)	(4.50)	(4.41)
p-value	[0.0014]	[0.0299]	[0.0341]	[0.0363]

This table presents statistics for the differences in aggregate idiosyncratic volatility measures obtained from CLMX and new methodology decomposing total risk into firm, industry, and market variance. All measures are monthly value-weighted average variances multiplied by 1200. The first row reports the differences in means, the standard *t*-statistics in parenthesis, and the Newey–West adjusted *t*-statistics in square brackets. The second row presents the differences in medians, the Kruskal–Wallis test statistics in parenthesis, and the *p*-values in square brackets. The third row displays the differences in variances, the Brown–Forsythe test statistics in parenthesis, and the *p*-values in square brackets.

sensitive to outliers than the mean. Panel A of Table 1 shows that based on the median values the average idiosyncratic variance of CLMX is 32%, 29%, 26%, and 54% greater than the new idiosyncratic volatility measure for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, respectively. These results indicate that the level-differences measured by the sample mean and median values yield very similar conclusions.

Panel A also indicates that the value-weighted average idiosyncratic variance of CLMX is more volatile than the new idiosyncratic variance measure. For the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, the standard deviation of the idiosyncratic volatility measures of CLMX is 35%, 28%, 29%, and 32% greater than the standard deviation of the new volatility measures, respectively.

Panel A shows that the distributions of aggregate idiosyncratic volatility measures obtained from both approaches are skewed to the right and have fatter tails than the normal distribution.[11] The skewness and kurtosis statistics are very similar from both approaches.

Another point is that the aggregate idiosyncratic variances from both approaches are highly persistent, as shown by the autocorrelation coefficients. Similar to the skewness and kurtosis statistics, there is almost no difference in the degree of serial correlations of idiosyncratic volatility measures obtained from the CLMX and new methodology. The AC(1) coefficients in Panel A are in the range of 0.53 to 0.57 for the NYSE/AMEX and NYSE stocks and about 0.83 for the Nasdaq stocks. Since the idiosyncratic volatility series exhibit strong serial correlation, one may think of the possibility that they contain unit roots. To check this, we use the augmented Dickey–Fuller (1979) tests that are based on regressions of the aggregate idiosyncratic volatility measures on a constant, linear trend, their lagged values, and the lagged difference terms.[12] As presented in Panel A, the hypothesis of a unit root is rejected for all the volatility series at the 1% level.

As presented in Panel B of Table 1, the results for the extended sample of 1962:07–2005:12 are very similar to those reported in Panel A. Based on the mean and median values in Panel B, the value-weighted average idiosyncratic variance of CLMX is on average 32%, 26%, 23%, and 36% greater than the new measure of idiosyncratic volatility for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, respectively. Similar to our previous findings for the shorter sample, the maximum and minimum values of the aggregate idiosyncratic variance measures obtained from the CLMX methodology are greater than those of the new methodology. These differences are more pronounced for the Nasdaq stocks.

Panel B also shows that the average idiosyncratic variance measures of CLMX are more volatile than the new idiosyncratic variance measures. Similar to our findings for the shorter sample, there is almost no difference in the skewness, kurtosis, and autocorrelation statistics of the idiosyncratic volatility measures obtained from the CLMX and new methodology for the extended

[11] Andersen et al. (2001) find statistically significant skewness and kurtosis (i.e. non-normality) in daily realized variance series. They indicate that the square-root and log transformation of realized variances makes the distribution of realized volatility measures closer to normal distribution.

[12] The number of lagged differences to be included is determined by the standard t-test of significance on the last lagged difference term and by the Akaike Information and Schwarz Bayesian criterion.

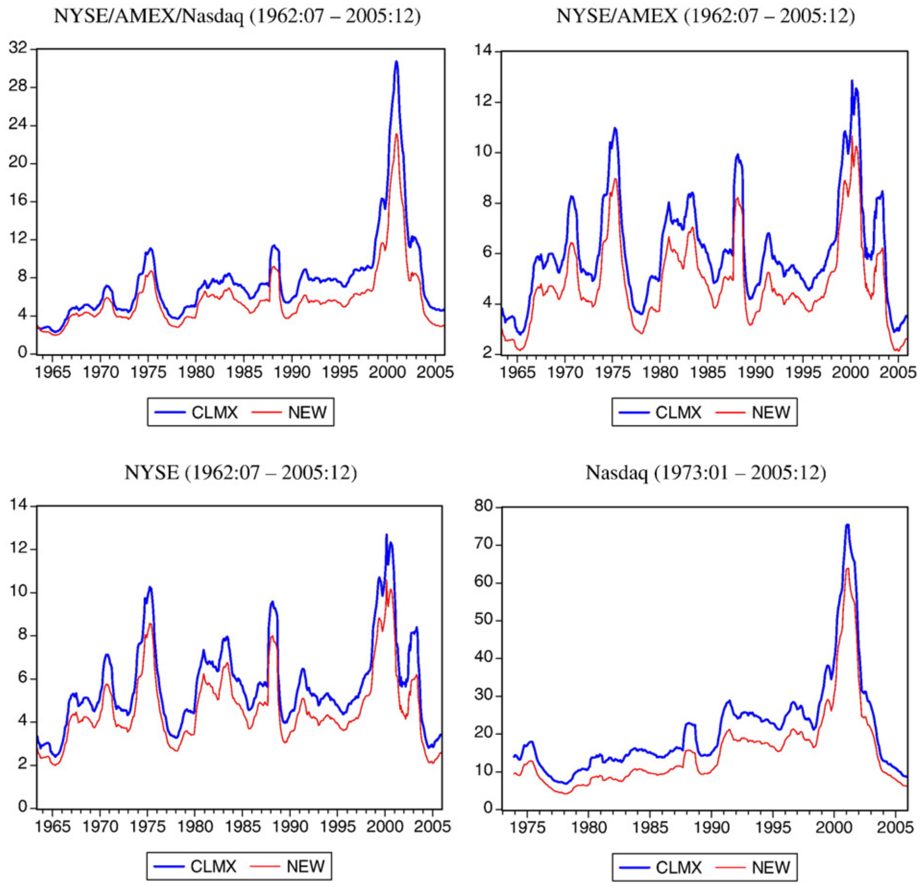

Fig. 1. Displays the 12-month moving averages of the aggregate idiosyncratic variance measures obtained from the CLMX and new methodology decomposing total risk into firm, industry, and market variance. The figures are plotted for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks for the extended sample (1962:07–2005:12). The plotted series is a moving average of order 12 (backward).

sample. The volatility series exhibit fairly high serial correlation, but there is no evidence of a unit root in any of the average idiosyncratic volatility measures based on the ADF test statistics.

4.2. Testing mean, median, and variance differences in idiosyncratic volatility measures

Since we do not detect a unit root in the value-weighted average idiosyncratic variances, we proceed to analyze the volatility series in levels rather than first differences. Table 2 presents statistics for the differences in aggregate idiosyncratic variance measures obtained from the CLMX and new methodology decomposing total risk into firm, industry, and market variance. All measures are the annualized value-weighted average variances. The first row of Table 2 reports the differences in means, the *t*-statistics in parenthesis, and the Newey–West (1987) adjusted *t*-statistics in square brackets.[13] Panel A of Table 2 shows that for the sample period of 1962:07–1997:12, the mean differences of aggregate idiosyncratic volatility measures obtained from the CLMX and new methodology are 1.79, 1.30, 1.08, and 4.59 for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, respectively. Based on the *t*-statistics, the means are statistically different from each other. As presented in Panel B, the mean differences are slightly larger for the extended sample period. They are in the range of 1.06 to 5.50, and statistically significant based on the *t*-statistics.

The second row of Table 2 presents the differences in medians, the Kruskal–Wallis (1952) test statistics in parenthesis, and the *p*-values in square brackets.[14] Panel A shows that for the shorter sample period, the median differences of the average idiosyncratic volatility measures obtained from the CLMX and new methodology are 1.54, 1.16, 0.97, and 4.94 for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, respectively. Based on the Kruskal–Wallis test statistics, the medians are statistically different

[13] We use five lags when computing the Newey–West (1987) corrected standard errors.

[14] At an earlier stage of the study, we also used the Mann–Whitney and van der Waerden median equality tests. Since the *p*-values from these two test statistics are found to be almost the same as those shown in Table 2, we do not report them. They are available upon request.

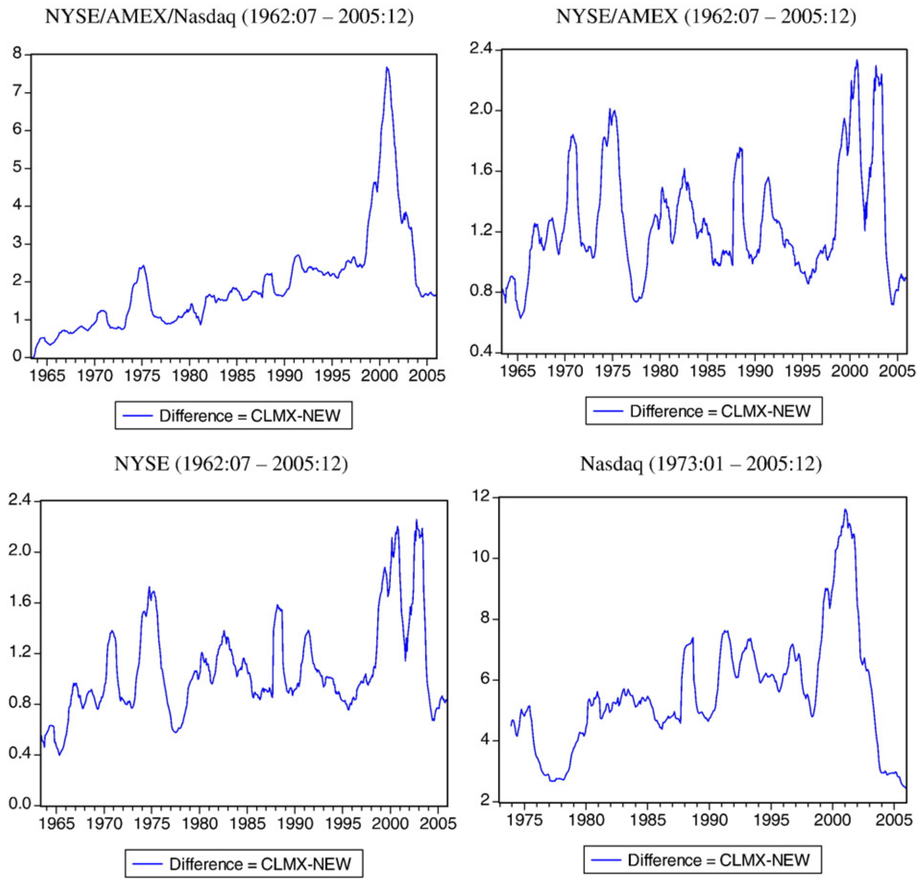

Fig. 2. Displays the differences between the 12-month moving averages of the aggregate idiosyncratic variances of CLMX and new methodology decomposing total risk into firm, industry, and market variance. The figures are plotted for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks for the extended sample (1962:07–2005:12). The plotted series is a moving average of order 12 (backward).

from each other. For the extended sample period, the median differences shown in Panel B are statistically significant and in the range of 0.93 to 5.13.

The third row of Table 2 displays the differences in variances, the Brown–Forsythe (1974) test statistics in parenthesis, and the *p*-values in square brackets.[15] Panel A shows that for the sample period of 1962:07–1997:12, the variance differences of the average idiosyncratic variance measures obtained from the CLMX and new methodology are 3.80, 2.09, 1.82, and 29.21 for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, respectively, and they are statistically different from each other based on the Brown–Forsythe test statistics. The differences in variances are more pronounced for the Nasdaq stocks. The results in Panel B of Table 2 indicate that the variance differences are much larger for the extended sample period. They are in the range of 2.95 to 62.46, and statistically significant.

4.3. Graphical analysis of idiosyncratic volatility measures

CLMX examine the market volatility (MKT), industry-level idiosyncratic volatility (IND), and firm-level idiosyncratic volatility (FIRM) which are estimated monthly using the daily data from 1962 to 1997. Their main empirical findings can be summarized as follows: (1) There is no trend in MKT and IND; (2) FIRM is on average much higher than MKT and IND, which implies that firm-specific volatility is the largest component of the total volatility of an average firm; (3) there is a significant upward trend in FIRM; and (4) NBER-dated recessions and the crash in October 1987 has a significant impact on FIRM.

Their results provide strong evidence that the stock market has become more volatile over the sample period of 1962–1997 but on a firm level instead of a market or industry level. Therefore, we do not estimate and analyze the market and industry variances, which have no significant trend and have been fairly stable. Instead, we focus on the firm-level average idiosyncratic volatility measured by the CLMX and new methodology.

[15] Identical inferences come from other versions of tests for equality of variances. Since the *p*-values from the standard *F*, Siegel–Tukey, Bartlett, and Levene variance equality test statistics turn out to be almost the same as those presented in Table 2, we choose not to report them. They are available upon request.

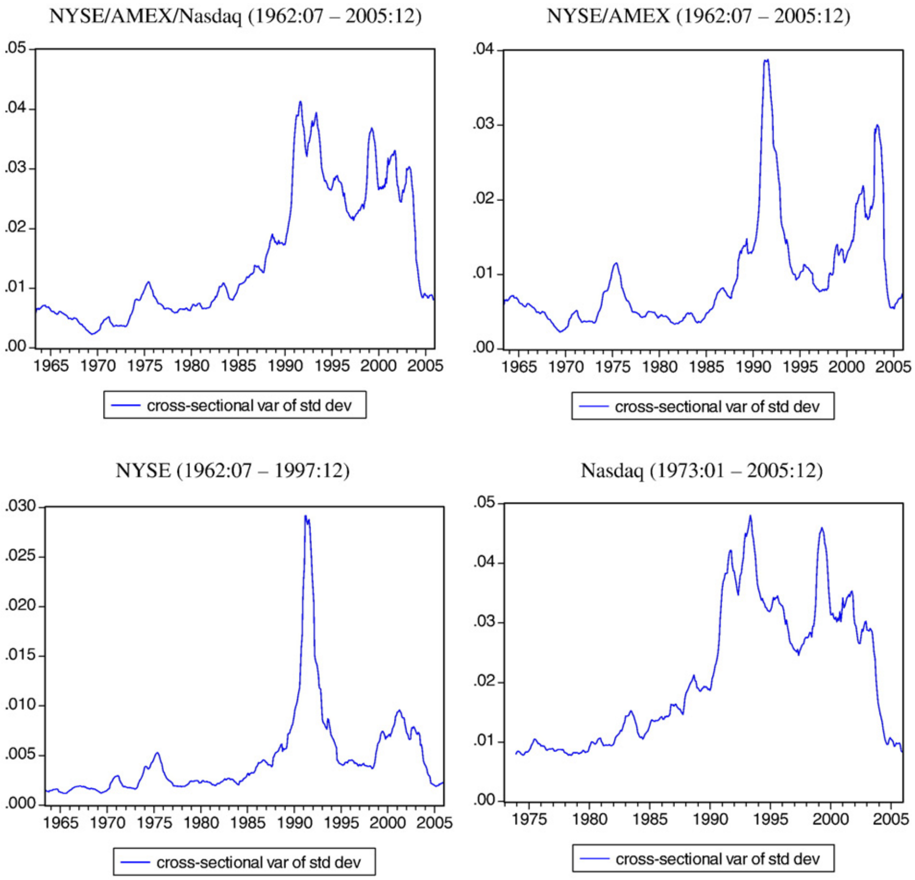

Fig. 3. Displays the cross-sectional dispersion of stock return volatilities measured by the cross-sectional variance of the standard deviation of individual stock returns. The 12-month moving averages of the cross-sectional variances are presented below. The figures are plotted for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks for the extended sample (1962:07–2005:12). The plotted series is a moving average of order 12 (backward).

Fig. 1 plots the 12-month moving averages of the volatility measures obtained from the CLMX and new methodology decomposing total risk into firm, industry, and market variance. The figures are plotted for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks, and for the extended sample (1962:07–2005:12). The original monthly variances look very noisy since the idiosyncratic volatility of stock returns fluctuates through time. Fig. 1 displays their smoothed version. The original volatility series are multiplied by 1200 to report the annualized measures.

The first notable point in Fig. 1 is that the average firm-level idiosyncratic volatility obtained from CLMX is always higher through time than the new idiosyncratic volatility measure for both the shorter and extended sample periods. This level difference in the idiosyncratic variance measures is clearly observed for the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq stocks. The difference is more pronounced for the Nasdaq stocks and seems even more striking for the shorter sample. However, we should note that since the volatility of stock returns is considerably high in the year 2000 and 2001, the 12-month moving averages are substantially higher from January 2000 to December 2001, and show a huge spike in January 2001. Therefore, the level difference does not seem to be as striking for the sample period of 1962:07–2005:12.

The second notable point is that confirming the findings of CLMX Fig. 1 demonstrates a significant upward trend in the average idiosyncratic volatility of the NYSE/AMEX/Nasdaq stocks for both the shorter and the extended sample periods. However, the trend in the new idiosyncratic volatility measure is not as strong as in the idiosyncratic volatility measure of CLMX. As will be discussed in more detail, the slope on the linear trend is always higher in the firm-level volatility measure of CLMX as compared to the volatility measure obtained from the new methodology.

The third notable point in Fig. 1 is that there is a strong upward trend in the average idiosyncratic volatility of the Nasdaq stocks, but the plots for the NYSE/AMEX and NYSE stocks do not exhibit a significant upward slope especially for the sample period of 1962–1997. This indicates that the upward trend in the idiosyncratic volatility of the NYSE/AMEX/Nasdaq stocks can mainly be due to the volatility of the Nasdaq stocks.[16]

[16] This is consistent with Campbell et al. (2001) who provide evidence that the trend is weaker for large firms, which is related to the result presented in the paper that the trend is not as strong for the NYSE stocks.

Table 3
Linear trends and test statistics based on CLMX and new methodology

	NYSE/AMEX/NASDAQ	NYSE/AMEX	NYSE	NASDAQ
Panel A. Sample period of CLMX (1962:07–1997:12)				
CLMX				
$\beta_2^{OLS} \times 10^5$	0.9654	0.1869	0.2970	6.0046
$\beta_2^{partial-sum} \times 10^5$	1.0049	0.3061	0.4067	6.3383
$t-\mathrm{PS}_T^1$ (90%)	3.9036	0.9516	1.3981	5.1773
$t-\mathrm{PS}_T^1$ (95%)	3.9720	0.9903	1.4497	5.7991
$t-\mathrm{PS}_T^1$ (99%)	4.2236	1.1398	1.6497	8.6604
New method				
$\beta_2^{OLS} \times 10^5$	0.5652	0.1680	0.2285	4.6043
$\beta_2^{partial-sum} \times 10^5$	0.6064	0.2652	0.3153	4.7545
$t-\mathrm{PS}_T^1$ (90%)	2.7248	1.0152	1.3387	5.0138
$t-\mathrm{PS}_T^1$ (95%)	2.7930	1.0618	1.3941	5.6428
$t-\mathrm{PS}_T^1$ (99%)	3.0484	1.2445	1.6093	8.5705
Panel B. Extended sample period (1962:07–2005:12)				
CLMX				
$\beta_2^{OLS} \times 10^5$	1.3577	0.1878	0.2956	4.7264
$\beta_2^{partial-sum} \times 10^5$	1.4396	0.2892	0.4000	6.3590
$t-\mathrm{PS}_T^1$ (90%)	4.1944	1.3589	2.0047	5.5059
$t-\mathrm{PS}_T^1$ (95%)	4.8433	1.4422	2.1269	6.7693
$t-\mathrm{PS}_T^1$ (99%)	8.0546	1.7798	2.6221	14.053
New method				
$\beta_2^{OLS} \times 10^5$	0.8321	0.1477	0.2082	4.1919
$\beta_2^{partial-sum} \times 10^5$	0.8986	0.2493	0.3109	5.4219
$t-\mathrm{PS}_T^1$ (90%)	3.4140	1.4015	1.8634	5.1573
$t-\mathrm{PS}_T^1$ (95%)	3.8762	1.4946	1.9850	6.3490
$t-\mathrm{PS}_T^1$ (99%)	6.0732	1.8764	2.4825	13.242

This table presents the results of linear trend regressions for the aggregate idiosyncratic volatility measures. All measures are monthly value-weighted average variances, and calculated using the CLMX and new methodology decomposing total risk into firm, industry, and market variance. The results are reported for the sample period of CLMX (1962:07–1997:12) and for the extended sample (1962:07–2005:12). The critical values for Vogelsang $t-\mathrm{PS}_T^1$ statistic at the 10%, 5%, and 1% level of significance are 1.331, 1.720, and 2.647, respectively.

4.4. Investigating the differences in idiosyncratic volatility measures

Fig. 2 displays the differences between the 12-month moving averages of the aggregate idiosyncratic variances of CLMX and new methodology decomposing total risk into firm, industry, and market variance. The first striking feature in Fig. 2 is that the difference between the average idiosyncratic volatility measures is not only positive through time, but has also a strong upward trend. This level and trend difference is clearly observed for both the shorter and extended sample periods, and for the NYSE/AMEX/Nasdaq, NYSE, and Nasdaq stocks. The only exception is the NYSE/AMEX sample for the period of 1962–1997, where we do not observe a significant upward slope in the idiosyncratic volatility difference.

Fig. 2 makes it clear why the upward trend in the new idiosyncratic volatility measure is not as strong as in the idiosyncratic volatility measure of CLMX. In other words, the slope on the linear time trend is always higher in the firm-level volatility measure of CLMX as compared to the volatility measure obtained from the new methodology.

The second important point in Fig. 2 is that the differences are more pronounced for the Nasdaq stocks. In other words, there is a very large and significant time trend in the idiosyncratic volatility difference for the Nasdaq sample. This provides supporting evidence for our previous finding that the Nasdaq stocks may have significant impact on the idiosyncratic volatility trend of the NYSE/AMEX/Nasdaq stocks.

The descriptive statistics in Table 1, the test statistics for the mean, median, and variance differences in Table 2, and the graphical analyses in Figs. 1 and 2 provide strong empirical evidence that there are significant level and trend differences between the average idiosyncratic volatility measures of CLMX and new methodology.

One can easily show that the differences in the idiosyncratic volatility measures will be more significant when the cross-sectional dispersion in the standard deviation of individual stocks is larger. If there is an increase in the cross-sectional dispersion of the volatility of stock returns through time we expect an upward trend in the differences of the two aggregate idiosyncratic volatility measures. To measure the cross-sectional dispersion of total risk measures for each month from July 1962 to December 2005, we calculate the cross-sectional variance of the monthly standard deviation of stock returns.[17]

[17] At an earlier stage of the study, we measure the cross-sectional dispersion with the cross-sectional variance (and standard deviation) of stock return variances and standard deviations. Since the level and trend in the cross-sectional dispersion of stock return volatilities are found to be very similar when measured by the cross-sectional variance or standard deviation we do not present them in the paper. They are available upon request.

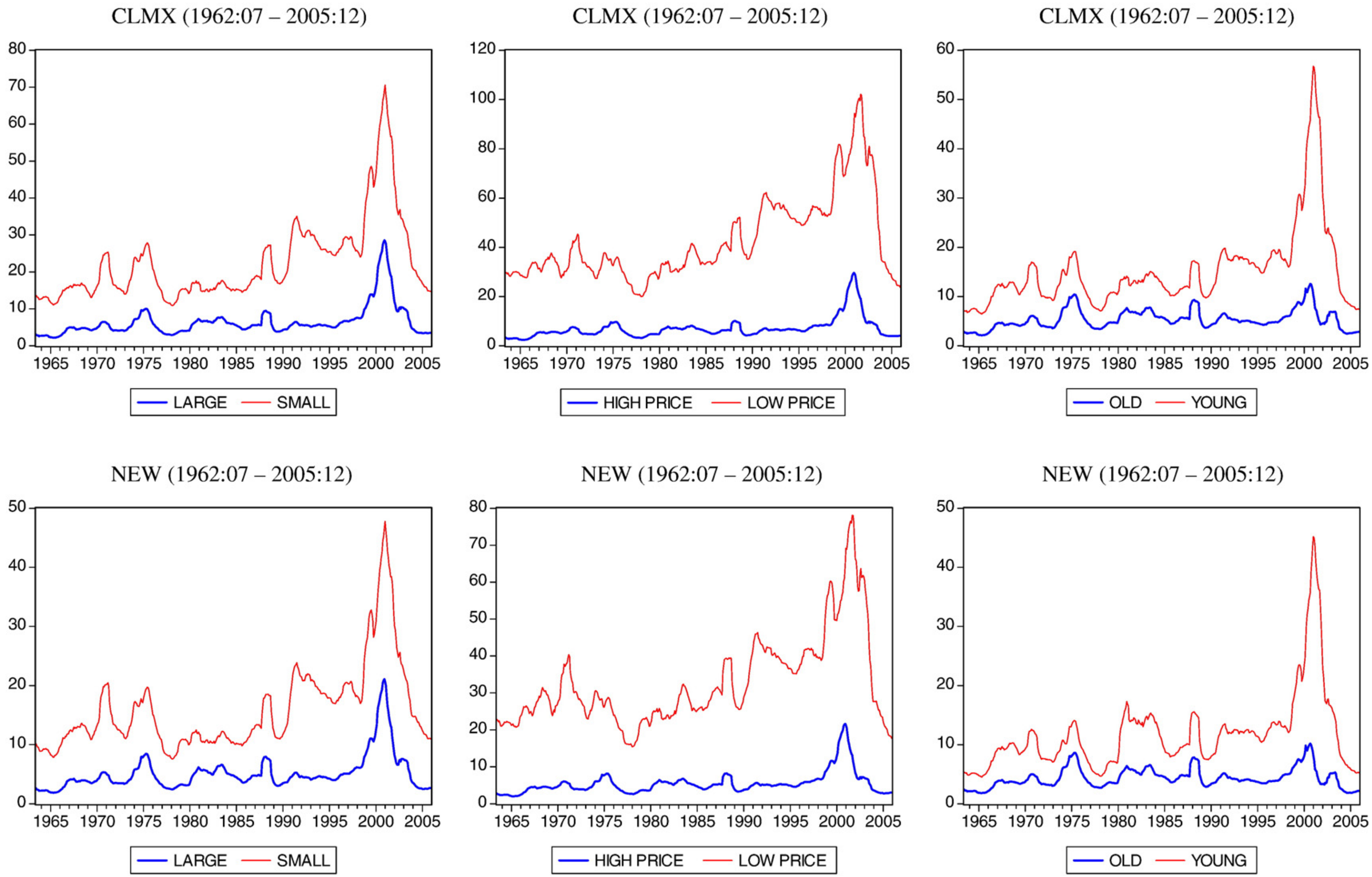

Fig. 4. Displays the 12-month moving averages of the aggregate idiosyncratic variance measures obtained from the CLMX and new methodology decomposing total risk into firm, industry, and market variance for the extended sample period (1962:07–2005:12). The figures are plotted for the Large/Small, High-Priced/Low-Priced, and Old/Young firms. The plotted series is a moving average of order 12 (backward).

Fig. 3 presents the 12-month moving averages of the cross-sectional variance of monthly stock return volatilities. A notable point in Fig. 3 is that there is a significant time trend in the cross-sectional dispersion of the standard deviation of the NYSE/AMEX/Nasdaq and Nasdaq stocks for both sample periods. However, the plots for the NYSE/AMEX and NYSE sample do not exhibit such a strong upward slope. These theoretical and empirical results explain why there is a strong level and trend difference between the average idiosyncratic volatility of the CLMX and new methodology for the NYSE/AMEX/Nasdaq and Nasdaq stocks, but rather weak difference for the NYSE/AMEX and NYSE sample.

4.5. Testing the presence of a linear trend in idiosyncratic volatility measures

Since all idiosyncratic volatility series considered in the paper are highly persistent, standard trend tests do not provide a sound statistical framework. Following CLMX and Xu and Malkiel (2003), we use the procedure developed by Vogelsang (1998) to test the presence of a deterministic linear time trend in the aggregate idiosyncratic volatility measures. Vogelsang introduces a Wald-type test that is robust to various forms of serial correlation. The test is based on the following model:

$$\begin{aligned} \upsilon_t &= \beta_1 + \beta_2 t + \rho\upsilon_{t-1} + u_t \\ u_t &= \alpha u_{t-1} + d(L)e_t, \end{aligned} \tag{30}$$

where υ_t is the value-weighted firm-level idiosyncratic variance measure, β_1 is the intercept, β_2 is the linear trend coefficient, and ρ captures the relation between the current idiosyncratic volatility and its own first lag. The error term u_t depends on its own first lag and on an infinite moving average of the white-noise innovation e_t through coefficients $d(L)=\sum_{i=0}^{\infty} d_i L^i$, where L is the lag operator.

Vogelsang (1998) provides a detailed discussion on the power of the PS_T^i test statistics which are robust to both $I(0)$ and $I(1)$ errors. He indicates that PS_T^1 or its t-statistics version denoted by $t-\mathrm{PS}_T^1$ is more appropriate if the series follow an $I(0)$ process.

Since the ADF statistics presented in Tables 1 and 2 indicate strong rejection of a unit root in all volatility series we use the $t-\mathrm{PS}_T^1$ statistic to obtain the best power. Following CLMX and Xu and Malkiel (2003), we use the simple linear trend model, $y_t=\beta_1+\beta_2^t+u_t$, and the partial sum transformation of Vogelsang, $z_t=\beta_1^t+\beta_2[0.5(t^2+t)]+S_t$, to test the presence of a linear time trend in idiosyncratic volatility measures. The null hypothesis is that $\beta_2 \geq 0$.

Table 3 presents the results of linear trend regressions for the idiosyncratic volatility measures. All measures are monthly value-weighted average variances, and calculated using the CLMX and new methodology decomposing total risk into firm and market variance. Panel A of Table 3 shows statistical results for the sample period of CLMX and confirms the visual evidence from Figs. 1 and 2.

As shown in Panel A, the average idiosyncratic volatility of the NYSE/AMEX and NYSE stocks have small positive and statistically insignificant or marginally significant trend coefficients whereas the trend in the idiosyncratic volatility of the NYSE/AMEX/Nasdaq and Nasdaq stocks is much larger and significant at the 1% level. Specifically, the trend coefficients (β_2^{OLS} and $\beta_2^{partial-sum}$) from the

Table 4
Differences in idiosyncratic risk measures after controlling for size, price, and age

	Large	Small	Price > $10	Price < $10	Age > median	Age < median
Panel A. Sample period of CLMX (1962:07–1997:12)						
Mean difference	0.95	5.49	1.22	8.72	0.95	2.89
Standard t	(36.97)	(44.64)	(40.23)	(42.87)	(39.22)	(28.13)
Newey–West t	[22.62]	[20.98]	[23.78]	[19.68]	[22.81]	[12.75]
Median difference	0.91	4.85	1.20	7.82	0.92	2.54
Kruskal–Wallis	(60.62)	(158.16)	(87.64)	(113.97)	(70.10)	(96.10)
p-value	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Variance difference	2.18	28.86	2.56	76.01	2.04	4.38
Brown–Forsythe	(4.82)	(24.75)	(7.14)	(21.96)	(3.77)	(4.45)
p-value	[0.0283]	[0.0000]	[0.0077]	[0.0000]	[0.0526]	[0.0352]
Panel B. Extended sample period (1962:07–2005:12)						
Mean difference	1.29	6.55	1.55	9.83	1.00	3.40
Standard t	(21.15)	(33.43)	(24.47)	(37.46)	(39.02)	(28.14)
Newey–West t	[9.86]	[15.13]	[11.48]	[17.51]	[22.15]	[12.87]
Median difference	0.94	4.99	1.23	7.71	0.91	2.52
Kruskal–Wallis	(55.34)	(151.40)	(80.16)	(101.85)	(67.12)	(85.90)
p-value	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Variance difference	11.28	91.19	11.85	183.64	2.73	33.81
Brown–Forsythe	(5.71)	(20.69)	(6.57)	(17.16)	(4.86)	(4.34)
p-value	[0.0170]	[0.0000]	[0.0105]	[0.0000]	[0.0277]	[0.0376]

This table presents statistics for the differences in aggregate idiosyncratic volatility measures obtained from CLMX and new methodology decomposing total risk into firm, industry, and market variance. All measures are monthly value-weighted average variances multiplied by 1200. The first row reports the differences in means, the standard t-statistics in parenthesis, and the Newey–West adjusted t-statistics in square brackets. The second row presents the differences in medians, the Kruskal–Wallis test statistics in parenthesis, and the p-values in square brackets. The third row displays the differences in variances, the Brown–Forsythe test statistics in parenthesis, and the p-values in square brackets.

Table 5
Linear trends and test statistics after controlling for size, price, and age

	Large	Small	Price>$10	Price<$10	Age>median	Age<median
Panel A. Sample period of CLMX (1962:07–1997:12)						
CLMX						
$\beta_2^{OLS}\times 10^5$	0.4888	2.6491	0.6002	5.2816	0.3427	1.5545
$\beta_2^{partial-sum}\times 10^5$	0.5242	2.0285	0.5766	4.1390	0.5247	1.2682
$t-\mathrm{PS}_T^1$ (90%)	1.9260	2.0422	2.3392	2.1176	1.4569	2.7820
$t-\mathrm{PS}_T^1$ (95%)	1.9667	2.1903	2.3957	2.3026	1.5051	2.8623
$t-\mathrm{PS}_T^1$ (99%)	2.1177	2.8055	2.6067	3.0963	1.6889	3.1651
New method						
$\beta_2^{OLS}\times 10^5$	0.3670	1.5283	0.4254	3.2298	0.2601	1.1531
$\beta_2^{partial-sum}\times 10^5$	0.4137	0.9940	0.4341	2.3402	0.4049	1.1853
$t-\mathrm{PS}_T^1$ (90%)	1.7801	1.3328	2.0614	1.6494	1.3849	3.1947
$t-\mathrm{PS}_T^1$ (95%)	1.8209	1.4615	2.1142	1.7898	1.4304	3.2707
$t-\mathrm{PS}_T^1$ (99%)	1.9727	2.0247	2.3121	2.3892	1.6035	3.5542
Panel B. Extended sample period (1962:07–2005:12)						
CLMX						
$\beta_2^{OLS}\times 10^5$	1.0082	3.5856	1.0368	5.8062	0.2599	2.2356
$\beta_2^{partial-sum}\times 10^5$	1.0042	3.6911	1.0701	6.5024	0.4233	2.3581
$t-\mathrm{PS}_T^1$ (90%)	2.7694	3.3624	2.9484	3.9824	1.8231	3.0922
$t-\mathrm{PS}_T^1$ (95%)	3.2063	3.9606	3.4042	4.7777	1.9328	3.5506
$t-\mathrm{PS}_T^1$ (99%)	5.3823	7.0671	5.6589	9.0953	2.3767	5.7887
New method						
$\beta_2^{OLS}\times 10^5$	0.6971	2.2272	0.7101	3.9339	0.1796	1.6565
$\beta_2^{partial-sum}\times 10^5$	0.7268	2.2132	0.7586	4.2215	0.3226	1.8135
$t-\mathrm{PS}_T^1$ (90%)	2.8150	2.7554	2.9637	3.3199	1.7014	3.4428
$t-\mathrm{PS}_T^1$ (95%)	3.2299	3.2474	3.4073	3.9540	1.8116	3.8899
$t-\mathrm{PS}_T^1$ (99%)	5.2519	5.8056	5.5796	7.3363	2.2477	5.9903

This table presents the results of linear trend regressions for the aggregate idiosyncratic volatility measures. All measures are monthly value-weighted average variances, and calculated using the CLMX and new methodology decomposing total risk into firm, industry, and market variance. The results are reported for the sample period of CLMX (1962:07–1997:12) and for the extended sample (1962:07–2005:12). The critical values for Vogelsang $t-\mathrm{PS}_T^1$ statistic at the 10%, 5%, and 1% level of significance are 1.331, 1.720, and 2.647, respectively.

CLMX methodology are in the range of 0.97 to 1.00 for the NYSE/AMEX/Nasdaq stocks and 6.00 to 6.34 for the Nasdaq stocks, whereas they are in the range of 0.19 to 0.31 for the NYSE/AMEX stocks and 0.30 to 0.41 for the NYSE stocks. The $t-\mathrm{PS}_T^1$ statistics of 4.2236 for the NYSE/AMEX/Nasdaq stocks and 8.6604 for the Nasdaq stocks indicate a strong idiosyncratic volatility trend, whereas the $t-\mathrm{PS}_T^1$ statistics of 0.9516 for the NYSE/AMEX stocks and 1.3981 for the NYSE stocks suggest a positive but insignificant or very weak upward slope in the average idiosyncratic volatility for the NYSE/AMEX and NYSE sample.

Panel A shows that the qualitative results from the new methodology are very similar to those obtained from the CLMX methodology. That is, there is an insignificant or weak trend in the average idiosyncratic volatility of the NYSE/AMEX and NYSE stocks whereas the trend in the volatility of the NYSE/AMEX/Nasdaq and Nasdaq stocks is much larger and highly significant.

Panel A also confirms our previous finding that there is a strong level and trend difference between the average idiosyncratic volatility of CLMX and the new methodology for the NYSE/AMEX/Nasdaq and Nasdaq stocks, but rather weak difference for the NYSE/AMEX and NYSE sample. As shown in Panel A, the trend coefficients from the CLMX methodology are in the range of 0.97 to 1.00 for the NYSE/AMEX/Nasdaq and 6.00 to 6.34 for the Nasdaq stocks, whereas the trend coefficients from the new methodology are in the range of 0.57 to 0.61 for the NYSE/AMEX/Nasdaq and 4.60 to 4.75 for the Nasdaq stocks. In other words, there is a significant difference between the magnitudes of the trend coefficients. However, this difference is smaller for the NYSE/AMEX and NYSE sample.

Panel B of Table 3 shows that the results for the extended sample period are very similar to those reported in Panel A. The trend coefficients obtained from the CLMX and new methodology are much smaller for the NYSE/AMEX and NYSE stocks than for the Nasdaq stocks, indicating a strong upward trend in the average idiosyncratic volatility of the Nasdaq stocks, but rather weak positive slope in the idiosyncratic volatility for the NYSE/AMEX and NYSE sample. Similar to our previous findings, the trend coefficients obtained from the CLMX methodology are greater than those calculating using the new methodology. This difference is more pronounced for the Nasdaq stocks, and much smaller for the NYSE/AMEX and NYSE sample. For example, the trend coefficients from the CLMX methodology are in the range of 4.73 to 6.36 for the Nasdaq stocks whereas the corresponding figures from the new methodology are about 4.19 to 5.42.

The main difference between Panel A and Panel B is regarding the significance of time trend in the average idiosyncratic volatility of the NYSE/AMEX and NYSE stocks. The $t-\mathrm{PS}_T^1$ statistics in Panel A suggest an insignificant or weak trend in the idiosyncratic volatility of the NYSE/AMEX and NYSE stocks for the sample period of CLMX. However, the $t-\mathrm{PS}_T^1$ statistics in Panel B provide evidence for a significant upward slope in the idiosyncratic volatility of the NYSE/AMEX and NYSE stocks for the extended sample period. As presented in Fig. 1, there is a substantial jump in the level of the aggregate idiosyncratic volatility measures between the years of 1999 and 2001. It is clear that these extreme observations for the extended sample period lead the trend coefficients to be larger and statistically significant for the NYSE/AMEX and NYSE sample.

5. Upward trend in idiosyncratic volatility: the effect of firm size, price, and age

Many recent papers examine the determinants of the time trend in aggregate idiosyncratic risk. Wei and Zhang (2006) find that fundamental factors, such as a decrease in corporate earnings and an increase in earnings volatility, account for the increase in idiosyncratic volatility. They also indicate that new firms are more volatile than old firms. Brown and Kapadia (2007) provide an alternative explanation that increasingly risky firms have listed publicly and hence the overall composition of publicly traded firms has changed significantly over the last 40 years. They show that the new listing effect explains the upward trend in idiosyncratic risk. Cao, Simin, and Zhao (in press) provide a theoretical relation between growth options available to managers and the idiosyncratic risk of equity. They present empirical evidence that both the level and volatility of corporate growth options are significantly related to idiosyncratic risk. Their results indicate that controlling for growth options eliminates or even reverses the upward trend in firm-level volatility. Brandt, Brav, and Graham (2005) show that the high and increasing idiosyncratic risk is concentrated in firms with low stock prices.

Following the aforementioned studies, we examine the significance of trend in idiosyncratic risk for a group of stocks formed based on firm size, price, and age. Specifically, we test the presence and significance of time trend in aggregate idiosyncratic volatility of large, small, high-priced, low-priced, young, and old firms.[18]

Our earlier findings from the NYSE and Nasdaq sample indicate that the level and the upward trend in idiosyncratic risk are stronger for the Nasdaq stocks as compared to the NYSE stocks. This motivated us to examine the effect of market capitalization on the time trend. For each month, all NYSE stocks on CRSP are sorted by firm size to determine the median market capitalization of NYSE stocks. Then, the NYSE/AMEX/Nasdaq stocks with higher than the median NYSE market cap are called “large” stocks, whereas the NYSE/AMEX/Nasdaq stocks with lower than the median NYSE market cap are called “small” stocks. We define “low-priced” stocks (“high-priced” stocks) as those with stock prices of less than \$10 (greater than \$10). For each month, the NYSE/AMEX/Nasdaq stocks are sorted by the CRSP listing date to determine the median firm age. Then, the stocks with higher than the median firm age are labeled as “old”, whereas those with lower than the median firm age are labeled as “young”.[19]

Fig. 4 displays the 12-month moving averages of the aggregate idiosyncratic variance measures obtained from the CLMX and new methodology for the Large/Small, High-Priced/Low-Priced, and Old/Young firms. The time-series plots for both the CLMX and new methodology clearly indicate that the level of average idiosyncratic risk as well as the upward trend are much higher for smaller, lower-priced, and younger firms. The figures also provide visual evidence for significant level and trend differences between the average idiosyncratic volatility measures of CLMX and the new methodology. For all groups of large, small, high-priced, low-priced, old, and young firms, the volatility measure of CLMX is greater and has a stronger upward trend than the new idiosyncratic volatility measure.

Table 4 presents statistics for the differences in idiosyncratic risk measures of CLMX and new methodology for large, small, high-priced, low-priced, old, and young firms. As shown in Panels A and B, the mean, median, and variance differences in the volatility measures of CLMX and new methodology are highly significant for all groups of stocks and for both sample periods. We should note that the level and trend differences are more pronounced for smaller, lower-priced, and younger firms.

Table 5 presents the Vogelsang test results for the significance of time trend in idiosyncratic volatility measures of large, small, high-priced, low-priced, old, and young firms. As shown in Panels A and B, the average idiosyncratic volatility of small, low-priced, and young firms have much bigger and highly significant trend coefficients than large, high-priced, and old firms. For the shorter sample period, the trend coefficients (β_2^{OLS} and $\beta_2^{\text{partial-sum}}$) from the CLMX methodology are in the range of [0.49;0.52], [0.58;0.60], and [0.34;0.52] for large, high-priced, and old firms, respectively. The corresponding trend coefficients are much higher for small, low-priced, and young firms, in the range of [2.03;2.65], [4.14;5.28], and [1.27;1.55], respectively. Similar effect of firm size, price, and age on the significance of trend in idiosyncratic volatility is obtained from the new methodology. For the sample period of 1962:07–1997:12, the trend coefficients from the new methodology are in the range of [0.37;0.41], [0.42;0.43], and [0.26;0.40] for large, high-priced, and old firms, respectively. The corresponding trend coefficients are much again higher for small, low-priced, and young firms, in the range of [0.99;1.53], [2.34;3.23], and [1.15;1.19], respectively.

As shown in Panel B of Table 5, the differences in the magnitudes of trend coefficients on the idiosyncratic volatility of large vs. small, high-priced vs. low-priced, and old vs. young firms are more pronounced for the extended sample period of 1962:07–2005:12. For example, the trend coefficients on the idiosyncratic volatility measures of CLMX are in the range of [1.00;1.01], [1.04;1.07], and [0.26;0.42] for large, high-priced, and old firms, respectively. The corresponding trend coefficients are much larger for small, low-priced, and young firms, in the range of [3.59;3.69], [5.81;6.50], and [2.24;2.36], respectively. These results indicate that for the extended sample period (i) the significance of trend in idiosyncratic volatility is about three times bigger for small stocks as compared to large stocks; (ii) the significance of trend is about six times bigger for low-priced stocks as compared to high-priced stocks; and (iii) the significance of trend is about six to ten times bigger for young stocks as compared to old stocks.

Confirming the findings of earlier studies, our results indicate that the upward trend in idiosyncratic volatility is stronger for smaller, lower-priced, and younger firms.

6. Conclusion

This paper introduces a new measure of aggregate idiosyncratic risk based on the mean-variance portfolio theory and the concept of gain from portfolio diversification. The crucial difference between the new and existing idiosyncratic volatility measures

[18] Bali and Cakici (2008) find that smaller, lower-priced, and less liquid stocks have higher idiosyncratic volatility.

[19] As discussed in Cao, Simin, and Zhao (in press), the value of equity for younger firms depends on more distant and hence uncertain future cash flows than those of older firms implying that a decrease in the average age of firms over time would contribute to increasing firm specific volatility.

is that the new measure does not depend on any parametric specifications of the return generating process such as the market model or the one-factor or three-factor capital asset pricing model. Therefore, with the new approach the estimation of industry- and firm-level beta coefficients or correlations can be avoided when constructing the average idiosyncratic volatility at the industry and firm level. This model-independent measure of idiosyncratic risk is defined as the difference between the variances of the non-diversified and fully diversified portfolios. Two versions of the new methodology are presented in the paper: one decomposing total risk into firm and market variance, and the other decomposing total risk into firm, industry, and market variance.

The idiosyncratic volatility measure proposed in the paper is compared with the idiosyncratic volatility measure of CLMX. Based on the mean and median values, the average idiosyncratic variances of stocks estimated with the CLMX methodology are greater than those calculated with the new methodology. The maximum and minimum values of the idiosyncratic variance measures of CLMX are also greater than those of the new methodology. The aggregate idiosyncratic variances of CLMX are more volatile than the new idiosyncratic variance measures. There is no significant difference in the skewness, kurtosis, autocorrelation, and ADF unit root test statistics of the volatility measures of CLMX and the new methodology. These results are robust across different groups of stocks traded on the NYSE/AMEX/Nasdaq, NYSE/AMEX, NYSE, and Nasdaq.

The statistical results and graphical analyses obtained from the existing and new approaches indicate that the average idiosyncratic volatility has trended upwards for the whole sample, but the trend is more pronounced for the Nasdaq stocks and relatively weaker for the NYSE/AMEX and NYSE sample. As discussed by CLMX, these results indicate that correlations among individual stocks and the explanatory power of the market model for a typical stock have decreased over time. The increased idiosyncratic volatility also implies that the number of stocks to achieve a given level of portfolio diversification has increased.

The empirical findings provide evidence that there are significant level and trend differences between the average idiosyncratic volatility measures of CLMX and the new methodology. The upward trend in the new idiosyncratic volatility measure is not as strong as in the volatility measure of CLMX. The level and trend differences are more noticeable for the Nasdaq stocks, but rather weak for the NYSE/AMEX and NYSE sample. These results hold for the sample period of CLMX and for the extended sample period of 1962:07–2005:12. We show both theoretically and empirically that the differences in the idiosyncratic volatility measures are more significant when the cross-sectional dispersion in the standard deviation of individual stocks is larger. Since there is an increase in the cross-sectional dispersion of the volatility of individual stocks through time, we find an upward trend in the differences of the two aggregate idiosyncratic volatility measures. A notable point is that there is a significant time trend in the cross-sectional dispersion of the monthly volatilities of Nasdaq stocks for both sample periods. However, the plots for the NYSE/AMEX and NYSE sample do not exhibit such a strong upward slope.

The paper also investigates the effect of firm size, price, and age on the significance of time trend in idiosyncratic volatility. Specifically, we test the presence and significance of time trend in aggregate idiosyncratic volatility of large, small, high-priced, low-priced, young, and old firms. For both the CLMX and new measures of idiosyncratic risk, the statistical results indicate that the upward trend is much stronger for smaller, lower-priced, and younger firms.

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