

Can exchange rate volatility explain persistence in the forward premium?

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Abstract

The persistence of the forward premium has been cited both as evidence of the failure of the unbiasedness hypothesis and as rationale for the forward premium anomaly. This paper examines the recent proposition that forward premium persistence can be explained solely by the conditional variance of the spot rate. We provide theoretical and empirical evidence to challenge this proposition. Our empirical results are shown to be robust to the presence of structural breaks. A corollary of the results is that the ‘true’ risk premium contains a long memory component. This is non-standard and has implications for the construction of rational expectations models of the foreign exchange market.

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1. Introduction

The issue of whether the foreign exchange market is efficient has produced a voluminous literature. Market efficiency is based on the principle that asset prices reflect all publicly available information (Fama, 1970). Under the joint assumptions of risk neutrality and rational expectations, the expected returns to speculative activity in an efficient market should be zero. Therefore, in a forward or futures market the current price of an asset for delivery at a specified date should be an unbiased predictor of the future spot rate. Interestingly, recent literature has found evidence of long memory (see Baillie and Bollerslev, 1994; Choi and Zivot, 2007) or unit root (see Kellard et al., 2001) behaviour in the forward premium, suggesting a rejection of this unbiasedness hypothesis. In a similar vein, Maynard and Phillips (2001) propose that the literature should subsequently explore why the forward premium might display such time series characteristics.

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Possible determinants of the time series behaviour of the forward premium include persistent inflation differentials (Roll and Yan, 2000) and peso problems (Evans and Lewis, 1995). Baillie and Bollerslev (2000) also show, under certain assumptions, that consumption CAPM (CCAPM) implies that the conditional variance of the spot rate (CVSR), possibly in combination with a ‘true’ risk premium (TRP)¹, provides an alternative rationale. Indeed, Baillie and Bollerslev (2000) exploit this possibility by simulating a model of the foreign exchange market where the assumed long memory behaviour of the conditional variance is inherited by the forward premium. Interestingly, the simulated results are broadly consistent with the empirical features of the forward premium puzzle. Subsequently, both Maynard and Phillips (2001) and Baillie et al. (2002) note, but do not test, that the forward premium and the conditional variance possess similar time series characteristics.

The motivation for this paper is to examine whether the time series properties of the forward premium are solely inherited from the conditional variance of the spot rate. To our knowledge this has not been hitherto attempted in the literature. Firstly, we present a brief proof to show that CCAPM does not imply a uni-causal link between the CVSR and the forward premium. Secondly, we provide empirical support for our theoretical conjecture. Employing daily data from eight major currencies, and in contrast to the supposition of Baillie and Bollerslev (2000), it is shown that the forward premium and the CVSR are not fractionally cointegrated. Our empirical results are shown to be robust to the presence of structural breaks in the forward premium and CVSR.

Finally, we discuss the main implications of our findings. Given that exchange rate volatility cannot uniquely explain the persistence of the forward premium, CCAPM posits the existence of a non-stationary and fractionally integrated TRP. This is an interesting notion given the general assumption of a stationary risk premium in the literature (see Engel, 1996) and suggests, most obviously, that rational expectations models of the foreign exchange market should be capable of producing long memory in the TRP. Presently, it should be noted that asset pricing models have been typically unable to provide risk premia with properties that generate the forward premium anomaly. The recent work by Maynard and Phillips (2001) provides theoretical evidence that the anomaly is in fact, a statistical artefact driven by long memory in the forward premium. Our results provide evidence that these two strands of literature may be joined. We posit that long memory in the TRP helps explain the time series behaviour of the forward premium, the subsequent forward premium anomaly and failure of the unbiasedness hypothesis.

The paper is divided into six sections. Section 2 describes the theoretical foundations and Section 3, the empirical methodology to be adopted. Section 4 describes the data and the results, whilst Section 5 provides a discussion. Finally, Section 6 concludes.

2. Theoretical foundations

Following Zivot (2000), the unbiasedness hypothesis under rational expectations and risk neutrality is given by

$$f_{t-1} = E_{t-1}(s_t) \quad (1)$$

where s_t and f_t are the natural logarithms of the spot (S_t) and forward (F_t) rates at time t and $E_t(\cdot)$ is the expectations operator conditional on information available at time t . Moreover, Eq. (1) is commonly expressed as the levels relationship

$$s_t = f_{t-1} + \varepsilon_t \quad (2)$$

where ε_t is a random, zero-mean variable. From (2) and considering that spot and forward prices are generally found to be non-stationary (see Meese and Singleton, 1982), a necessary condition for market efficiency is the existence of cointegration between spot and lagged forward rates. The cointegrating regression can be specified as

$$s_t = \beta_0 + \beta_1 f_{t-1} + u_t. \quad (3)$$

Clearly, the unbiasedness hypothesis requires $\beta_0 = 0$, $\beta_1 = 1$ and that u_t is not serially correlated. Tests for cointegration generally confirm that spot and lagged forward rates are cointegrated, often with the required coefficients (see Kellard et al., 2001).

¹ The true risk premium differs in definition from the standard or rational expectations risk premium. See Section 2 for more details.

An equivalent approach for assessing the unbiasedness hypothesis comes from noting that the residual term in (2) can be expressed as

$$\varepsilon_t = s_t - f_{t-1} = (s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) \quad (4)$$

Given the stationary behaviour of the spot return $(s_t - s_{t-1})$, the order of integration of the forecast error $(s_t - f_{t-1})$ is determined solely by the lagged forward premium $(f_{t-1} - s_{t-1})$. Thus Maynard and Phillips (2001) note that inclusion of the spot return introduces unnecessary noise that may cause finite sample bias. Additionally they suggest that this finite sample bias is of particular significance because, as reported by Newbold et al. (1998), the forward premium is so small in magnitude that the time series properties of the forecast error can be easily dominated by those of the much larger spot return. However, as with evidence from the forecast error, cointegration and unit root tests on the forward premium have provided conflicting results. Hai et al. (1997), Horvath and Watson (1995) and Barnhart and Szakmary (1991) reject the presence of a unit root in forward premia series. However, Crowder (1994, 1995), Kuersteiner (1996) and Kellard et al. (2001) provide evidence to the contrary.

Inconsistent evidence has led some to suggest that neither short memory nor unit root models are entirely appropriate to model the data. Specifically, Maynard and Phillips (2001) and Baillie and Bollerslev (1994) find that a fractionally integrated model fits the forward premium adequately and reason that this provides an explanation for the dichotomy in the literature. Of course, long memory or unit root behaviour in the forward premium imply persistence in the forecast error, allowing it to be predictable from past values. This provides a rejection of the unbiasedness hypothesis. Thus, Maynard and Phillips (2001) propose that the literature should subsequently explore why the forward premium might display such time series characteristics.

As Maynard (2003) comments, any rejection of unbiasedness may not be particularly problematical. For example, one would expect agents to evaluate returns in real as opposed to nominal terms. Under the previous assumptions of rational expectations and risk neutrality, expected real returns in the forward market must be zero

$$E_{t-1} \left\{ \frac{F_{t-1} - S_t}{P_t} \right\} = 0 \quad (5)$$

where P_t denotes the domestic price level. Assuming that exchange rates and prices are jointly lognormally distributed, then Eq. (5) in logarithmic form is

$$f_{t-1} = E_{t-1}(s_t) + \frac{1}{2} \text{var}_{t-1}(s_t) - \text{cov}_{t-1}(p_t, s_t) \quad (6)$$

where p_t is the natural logarithm of P_t . Note that even under the assumptions of rational expectations and risk neutrality, the RHS of (6) contains two conditional moment terms.² A risk-adjusted equivalent to (5) can be derived from the standard discrete consumption CAPM (CCAPM), where real returns are evaluated over current and future consumption streams of the representative agent

$$E_{t-1} \left\{ \left(\frac{F_{t-1} - S_t}{P_t} \right) \frac{U'(C_t)}{U'(C_{t-1})} \right\} = 0 \quad (7)$$

where the utility function $U(\cdot)$ is strictly increasing in consumption C . Under the assumptions of constant relative risk aversion (CRRA) and log normality, (7) can be expressed as

$$f_{t-1} = E_{t-1}(s_t) + \frac{1}{2} \text{var}_{t-1}(s_t) - \text{cov}_{t-1}(p_t, s_t) - \rho \text{cov}_{t-1}(s_t, c_t) \quad (8)$$

where c_t and ρ denote the logarithm of consumption and the degree of relative risk aversion respectively (see Obstfeld and Rogoff, 1996, p.592). Under rational expectations, Eq. (8) can be expressed as the levels relationship

$$s_t = f_{t-1} - \frac{1}{2} \text{var}_{t-1}(s_t) + \text{cov}_{t-1}(p_t, s_t) + \text{trp}_{t-1} + \varepsilon_t \quad (9)$$

² The conditional moment variables are more popularly known as Jensen Inequality terms.

where $\text{trp}_{t-1} = \rho \text{cov}_{t-1}(s_t, c_t)$, is a time dependent TRP.³ Subtracting s_{t-1} from both sides of (9), ignoring the covariance term due to very small size⁴ (see Engel, 1996) and rearranging leads to

$$(s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) = -\frac{1}{2} \text{var}_{t-1}(s_t) + \text{trp}_{t-1} + \varepsilon_t. \quad (10)$$

Given the short memory of the spot return process⁵ and assuming, as the literature often does, a similar process for the TRP, it is therefore possible that the time series properties of the forward premium are inherited solely from the CVSR. As recent literature has suggested the forward premium and conditional variance are fractionally integrated⁶, (10) implies the stationary cointegrating vector $-(f_{t-1} - s_{t-1}) + \frac{1}{2} \text{var}_{t-1}(s_t)$. In other words, the forward premium will be fractionally cointegrated with the conditional variance of the spot rate. Exploiting this possibility, Baillie and Bollerslev (2000) simulate (10), adopting a fractionally integrated ($d=0.75$) GARCH model of $\text{var}_{t-1}(s_t)$. As noted earlier, the simulated results were broadly consistent with the empirical features of the forward premium puzzle.

Baillie and Bollerslev (2000), Maynard and Phillips (2001) and Baillie et al. (2002) have all used (9) and (10) to suggest that the CVSR may explain the behaviour of the forward premium. However, it is important to stress that (9) is derived from the Euler equation for a *home* household's allocation problem. There is, of course, an equivalent Euler equation for *foreign* households, which implies

$$s_t = f_{t-1} + \frac{1}{2} \text{var}_{t-1}(s_t) + \text{cov}_{t-1}(p_t^*, s_t) - \text{trp}_{t-1}^* - \varepsilon_t^*. \quad (11)$$

where * indicates foreign counterpart variables. Subtracting s_{t-1} from both sides of (11), ignoring the covariance term due to very small size and rearranging leads to

$$(s_t - s_{t-1}) - (f_{t-1} - s_{t-1}) = \frac{1}{2} \text{var}_{t-1}(s_t) - \text{trp}_{t-1}^* - \varepsilon_t^*. \quad (12)$$

Again, assuming a stationary spot return and risk premium, (12) implies the stationary cointegrating vector $-(f_{t-1} - s_{t-1}) - \frac{1}{2} \text{var}_{t-1}(s_t)$. However, $-(f_{t-1} - s_{t-1}) + \frac{1}{2} \text{var}_{t-1}(s_t)$ is implied by (10). Clearly, both cannot be stationary and there must be a fractionally integrated risk premium to balance the model.⁷ The following simple proposition can now be stated.

Proposition. *Given the validity of Eq. (10) and its foreign counterpart (12), and allowing for a stationary spot return, then a fractionally integrated true risk premium must exist.*

An empirical corollary of the above proposition is that the forward premium will *not* be fractionally cointegrated with the CVSR as a bi-variate pair.

3. Empirical methodology

To test whether the forward premium and the CVSR are *or are not* fractionally cointegrated, we use both the Hassler et al. (2006) semi-parametric methodology and a conventional parametric approach. However, it is important to note that recent literature has suggested that occasional structural breaks may induce a spurious long memory effect in time series processes (see Granger and Hyung, 2004; Diebold and Inoue, 2001). In particular, Choi and Zivot (2007), after testing for long memory in the forward premium, use the Bai and Perron (1998, in press) procedure in an attempt to identify possible structural breaks. After any required demeaning, they repeat the long memory tests on the forward premium. We propose to follow a similar methodology and the test statistics to be applied are described in more detail below.

³ In the literature the term risk premium (rp_{t-1}) is often defined as $\text{rp}_{t-1} = f_{t-1} - E_{t-1}(s_t) = \frac{1}{2} \text{var}_{t-1}(s_t) - \text{cov}_{t-1}(p_t, s_t) - \rho \text{cov}_{t-1}(s_t, c_t)$. Engel (1996) refers to this as the rational expectations risk premium.

⁴ Additionally, Baillie et al. (2002) note that, 'In a low inflation environment, it seems reasonable to omit the $\text{cov}_{t-1}(p_t, s_t)$ on the grounds that it is negligible'.

⁵ Engel (1996) notes that numerous studies have established that the spot return is $I(0)$.

⁶ There are several other explanations for the persistence in exchange rate volatility and the forward premium aside from fractionally integrated long memory; for example, mean-breaks (see Sakoulis and Zivot, 2004), non-linear behaviour (see Samo et al., 2006) or local-to-unity processes (see Liu and Maynard, 2005). Moreover, there appears to be little agreement in the literature as to which might be the most appropriate. However, fractionally integrated long memory is perhaps the most commonly applied recent explanation for both variables (see Baillie et al., 2002).

⁷ We thank Charles Engel for this point.

(i) *Testing for long memory*

The introduction of the autoregressive fractionally integrated moving average (ARFIMA) model by Granger and Joyeux (1980) and Hosking (1981) allows the modelling of persistence or long memory where $0 < d < 1$. A time series y_t follows an ARFIMA (p, d, q) process if

$$\Phi(L)(1-L)^d y_t = \mu + \Theta(L)\varepsilon_t, \varepsilon_t \sim iid(0, \sigma^2) \quad (13)$$

where $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \dots - \theta_q L^q$. Such models may be better able to describe the long-run behaviour of certain variables. For example, when $0 < d < 1/2$, y_t is stationary but contains long memory, possessing shocks that disappear hyperbolically not geometrically. Contrastingly, for $1/2 < d < 1$, the relevant series is non-stationary, the unconditional variance growing at a more gradual rate than when $d = 1$, but mean reverting.

The memory parameter d can be estimated by a number of different techniques. The most popular, due to its semi-parametric nature, is the log periodogram estimator (Geweke and Porter-Hudak, 1983; Robinson, 1995a) henceforth known as the GPH statistic. This involves the least squares regression

$$\log I(\lambda_j) = \beta_0 - d \log \{4 \sin^2(\lambda_j/2)\} + u_j, j = l+1, l+2, \dots, m \quad (14)$$

where $I(\lambda_j)$ is the sample spectral density of y_t evaluated at the $\lambda_j = 2\pi j/T$ frequencies, T is the number of observations and m is small compared to T . Inter alia, Pynnönen and Knif (1998) and Hassler et al. (2006), note that the least-squares estimate of d can be used in conjunction with standard t -statistics. For the stationary range, $-1/2 < d < 1/2$, Robinson (1995a) demonstrated that the GPH estimate is consistent and asymptotically normally distributed. Additionally, Velasco (1999) shows that when the data are differenced, the estimator is consistent for $1/2 < d < 2$ and asymptotically normally distributed for $1/2 < d < 7/4$.

Fractional cointegration can be defined by supposing y_t and x_t are both $I(d)$, where d is not necessarily an integer and the residuals, $u_t = y_t - \beta x_t$, are $I(\delta = d - b)$. When $b < d$, where b is also not necessarily an integer, series are fractionally cointegrated. Testing for fractional cointegration can be accomplished using a multi-step methodology (see Hassler et al., 2006) where (i) the order of integration of the constituent series are estimated and tested for equality and (ii) the long-run equilibrium relationship is estimated⁸ and the residuals are examined for long-memory. Alternative methodologies include the joint estimation of memory parameters of the constituent series, the cointegrating residuals and the equilibrium relationship (see Velasco, 2003) or the use of bootstrap methods (see Davidson, 2005).

A frequently used approach is to adopt a multi-step methodology where the concluding step estimates the GPH statistic, δ , for the least squares residual of the equilibrium relationship (see Dittman, 2001). Inter alia, Tse et al. (1999) experimentally noted that t -statistics associated with δ might not be normally distributed. Hassler et al. (2006) demonstrate that δ has a limiting normal distribution provided the very first harmonic frequencies are trimmed. Specifically, this entails setting $l > 0$ in (14). Of course, given asymptotically normal estimators, standard inference procedures can be legitimately applied.

Agiakloglou, Newbold and Wohar (1992) note that GPH type estimation may suffer from finite sample bias in the presence of strongly autoregressive short memory. Thus, for comparative purposes, we also compute ARFIMA (p, d, q) models by exact maximum likelihood (ML).⁹ These estimates are less robust in a large sample but explicitly modelling the autoregressive and moving average terms in (13), less susceptible to finite sample bias.

(ii) *Testing for structural breaks*

A commonly used procedure to identify multiple structural breaks is due to Bai and Perron (1998, 2003a, 2003b, in press). They consider the m -breaks in mean model below

$$y_t = \beta_j + \varepsilon_t \quad (15)$$

where $j = 1, \dots, m+1$ and β_j is the mean level of y_t in the j th regime. Additionally, the m -partition $(T_1, \dots, T_m)$ are the breakpoints for the different regimes and conventionally, $T_0 = 0$ and $T_{m+1} = T$. As Choi and Zivot (2007) discuss, such

⁸ The long-run equilibrium relationship itself could be approximated by OLS, a fractional version of the Fully Modified method suggested by Kim and Phillips (2001), Gaussian semi-parametric estimation developed by Velasco (2003) or narrow band spectral estimates (see Robinson and Marinucci, 1998).

⁹ Applied to the first differenced series to satisfy the stationarity/invertibility condition $-0.5 < d < 0.5$ and again the resulting estimate of d was then increased by 1. Following Davidson (2005), the model order was chosen by minimizing the SBC.

breaks in the mean can be usefully interpreted as the direct impact of an economic shock. To estimate the breakpoints, the following objective function is employed

$$(\hat{T}_1, \dots, \hat{T}_m) = \arg \min_{T_1, \dots, T_m} S_T(T_1, \dots, T_m) \quad (16)$$

where for each m -partition $T_1, \dots, T_m$, the least squares estimates of β_j are generated by minimising the sum of the squared residuals

$$S_T(T_1, \dots, T_m) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (y_t - \beta_j)^2. \quad (17)$$

In other words, the breakpoint estimators correspond to the global minimum of the sum of the squares objective function. To solve the minimization problem in (16), Bai and Perron (in press) suggest the use of a specific dynamic programming algorithm. Clearly, after estimating the breakpoints, it is easy to obtain the counterpart least-squares regression parameter estimates $\hat{\beta}(\hat{T}_1, \dots, \hat{T}_m)$.

Bai and Perron (1998) propose a group of test statistics to choose the number of mean breaks (m). To begin, let $\text{SupF}_T(b)$ be the F -statistic for testing the null hypothesis of no structural breaks ($m=0$) against the alternative that there are breaks ($m=b$). Subsequently, two “double maximum” statistics can be developed, both testing the null hypothesis of no structural breaks versus the alternative of an unknown number of breaks (where M is an upper bound). First, $\text{UDmax} = \max_{1 \leq m \leq M} \text{SupF}_T(m)$ and second, WDmax , which applies differing weights to the individual $\text{SupF}_T(b)$ so that the marginal p -values are equal across values of m . Lastly, Bai and Perron (1998) define $\text{Sup}_T(m+1|m)$ to test the null hypothesis of m breaks against the alternative of $m+1$ and derive critical values for each test statistic.

As far as estimation strategy is concerned, after several Monte Carlo simulations, Bai and Perron (in press) recommend the following approach. Examine the double maximum statistics to determine if any structural breaks are present. Next, if the double maximum statistics are significant, examine the $\text{SupF}_T(m+1|m)$ statistics to decide on the number of breaks, selecting that which rejects the largest value of m .

A useful characteristic of the Bai and Perron (1998, 2003a) method is that test statistics can be calculated under reasonably general specifications. In particular, specifications can allow for autocorrelation and heteroscedasticity in the regression model residuals, in addition to different moment matrices for the regressors in the different regimes. To allow for all these features, we adopt the most general Bai and Perron (1998, 2003a) specification.¹⁰

4. Data and results

Daily time series of spot rates, one-month interest rate differentials¹¹ and CVSRs were constructed for the period January 1991 to October 2005.¹² The data for spot exchange rates and eurocurrency rates¹³ were obtained from Datastream and calculated as the closing (London time) average of bid and ask quotes for eight currencies: US Dollar/Sterling, Yen/US Dollar, Deutschmark/US Dollar, Deutschmark/Sterling, Deutschmark/Yen, Euro/Yen, Euro/Sterling and Euro/US Dollar.¹⁴ Finally, the CVSR is proxied by the square of ‘traded’ implied volatilities which measure the market’s expectations about the future volatility of the spot exchange rate.¹⁵

¹⁰ Specifically, using the notation of Bai and Perron (in press), we set $\text{cor_u}=1$, $\text{het_u}=1$ and $\pi=0.15$. Following Choi and Zivot (2007), we set $M=5$. Note that the Bai and Perron (1998, 2003a,b) statistics are computed using the GAUSS program available from Pierre Perron’s home page at <http://econ.bu.edu/perron/>.

¹¹ Under covered interest parity (CIP) the forward premium is equal to the interest rate differential. Maynard and Phillips (2001) demonstrate that the differential is a much cleaner series than the forward premium and thus is preferred for use in empirical analysis.

¹² The choice of start date was governed by the availability of implied volatility data.

¹³ Eurocurrency rates are annualized rates so that a quoted interest rate of 5% typically translates to a thirty-day rate of 0.05(30/360). The calculations assume that annualized rates for the Dollar, Euro, Mark and Yen refer to a 360-day year, whereas annualized rates for Sterling refer to a 365-day year.

¹⁴ Specifically, for the US Dollar/Sterling and Yen/US Dollar, the data run from 2nd January 1991 to 20th October 2005 and total 3792 observations. Due to the introduction of the Euro, the Deutschmark/US dollar, Deutschmark/Yen and Deutschmark/Sterling series run from 2nd January 1991 to 31st December 1998 and total 2023 observations. The Euro/Yen, Euro/Sterling and Euro/US Dollar series run from the 4th January 1999 to 20th October 2005 and total 1769 observations.

¹⁵ Implied volatilities are also annualized rates so that a quoted volatility of 5% typically translates to a monthly variance rate of $(0.05^2)(21/252)$. The calculations assume that annualized rates refer to a 252 trading day year.

Table 1
GPH/ARFIMA tests for the forward premium

	d_{GPH}	$\tau_{d=1}$	d_{ML}	$\tau_{d=1}$	(p, q)
DM/US\$	0.970 (0.039)	-0.769	0.982 (0.021)	-0.857	(1,0)
US\$/UK£	0.981 (0.030)	-0.633	0.996 (0.021)	-0.190	(0,1)
Yen/US\$	0.920 (0.030)	-2.667	1.040 (0.035)	1.143	(0,2)
DM/Yen	0.849 (0.039)	-3.872	0.993 (0.031)	-0.226	(3,0)
DM/UK£	0.882 (0.039)	-3.026	0.934 (0.020)	-3.300	(1,0)
Euro/Yen	0.983 (0.041)	-0.415	0.958 (0.019)	-2.211	(0,0)
Euro/UK£	0.965 (0.041)	-0.854	0.961 (0.019)	-2.053	(0,0)
Euro/US\$	0.996 (0.041)	-0.098	0.967 (0.019)	-1.737	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

Table 2
GPH/ARFIMA tests for the CVSR

	d_{GPH}	$\tau_{d=1}$	d_{ML}	$\tau_{d=1}$	(p, q)
DM/US\$	0.832 (0.039)	-4.308	0.789 (0.034)	-6.206	(1,0)
US\$/UK£	0.763 (0.030)	-7.900	0.772 (0.025)	-9.120	(1,0)
Yen/US\$	0.650 (0.030)	-11.67	0.747 (0.029)	-8.724	(3,0)
DM/Yen	0.692 (0.039)	-7.897	0.686 (0.027)	-11.63	(0,2)
DM/UK£	0.853 (0.039)	-3.769	0.883 (0.019)	-6.158	(0,0)
Euro/Yen	0.937 (0.041)	-1.537	0.840 (0.037)	-4.324	(0,2)
Euro/UK£	0.900 (0.041)	-2.439	0.897 (0.020)	-5.150	(0,0)
Euro/US\$	0.879 (0.041)	-2.951	0.908 (0.020)	-4.600	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

Table 3
GPH/ARFIMA tests for the spot return

	d_{GPH}	$\tau_{d=0}$	d_{ML}	$\tau_{d=0}$	(p, q)
DM/US\$	0.045 (0.039)	1.154	0.007 (0.018)	0.389	(0,0)
US\$/UK£	0.029 (0.030)	0.967	0.036 (0.013)	2.769	(0,0)
Yen/US\$	0.014 (0.030)	0.467	0.009 (0.013)	0.692	(0,0)
DM/Yen	0.002 (0.039)	0.051	0.036 (0.018)	2.000	(0,0)
DM/UK£	0.004 (0.039)	0.103	0.00009 (0.017)	0.005	(0,0)
Euro/Yen	0.032 (0.041)	0.780	-0.030 (0.028)	-1.071	(0,1)
Euro/UK£	-0.022 (0.041)	-0.537	-0.006 (0.020)	-0.300	(0,0)
Euro/US\$	-0.014 (0.041)	-0.341	0.012 (0.018)	0.667	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}})/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

The more common approach in the literature (see, inter alia, [Baillie and Bollerslev, 2000](#)) has been to generate the CVSRs using a GARCH-type process. This approach has a number of disadvantages including that recent work has demonstrated that implied volatility outperforms GARCH models in forecasting future currency volatility (see, for example, [Jorion, 1995](#); [Dunis et al., 2000](#); [Dunis and Huang, 2002](#)). However, as currency volatility has now become a traded quantity in financial markets, it is therefore directly observable on the marketplace. The data used are at-the-money, one-month forward, market quoted volatilities at close of business in London, obtained from brokers by Reuters. The databank is maintained by CIBEF at Liverpool John Moores University. Since these data are directly quoted from brokers, they avoid the potential biases associated with the backing out of implied volatilities from a specific option-pricing model.¹⁶

¹⁶ This traded volatility data has also been employed in another recent study by [Sarantis \(2006\)](#).

Table 4

Wald tests for the equality of the GPH estimates for the forward premium and the CVSR

DM/US\$	6.223 [0.013]
US\$/UK£	25.203 [0.000]
Yen/US\$	38.786 [0.000]
DM/Yen	8.460 [0.004]
DM/UK£	0.267 [0.605]
Euro/Yen	0.661 [0.416]
Euro/UK£	1.276 [0.259]
Euro/US\$	3.970 [0.046]

Note: the Wald statistic has a $\chi^2(1)$ distribution. The figures in square brackets are p -values.

Tables 1 and 2 report the GPH statistics¹⁷ for the forward premium and CVSR respectively, estimated using differenced data¹⁸ and Ox version 4.10 (see Doornik, 1999). GPH statistics for the spot return were estimated using levels data and are shown in Table 3.

Table 1 contains some interesting results. Firstly, the GPH point estimates of fractional differencing in the forward premia are spread over the range 0.849 to 0.996. These point estimates are similar to Maynard and Phillips (2001) but much higher than those of Baillie and Bollerslev (1994), whose values range from 0.45 to 0.77. Despite the closeness of their point estimates to unity, Maynard and Phillips (2001) concur with Baillie and Bollerslev that the forward premium is a fractionally integrated series. Table 1 indicates that when the standard errors of the GPH point estimates are considered this conclusion is conditional on the currency examined. Specifically, three out of eight series reject the null of a unit root. Furthermore, it should be noted that the point estimates from the estimated ARFIMA models typically lie slightly closer to unity than their semi-parametric counterparts.

Table 2 shows the GPH point estimates for the CVSR range from 0.650 to 0.937. Generally speaking, these values are not untypical when compared with those previously noted (see Baillie et al., 1996). Tests for $d=1$ show that, in particular, the traded volatility series are fractionally integrated with $0.5 < d < 1$. Similar conclusions can be drawn from the estimated ARFIMA models. Finally, Table 3 confirms previous literature, both the GPH and ARFIMA tests finding the spot return¹⁹ is $I(0)$.

It is interesting to note that the GPH point estimates are closer to unity for the forward premium than for the CVSR for all currencies. To examine this in more detail we test that the fractional orders of the constituent variables are equal by applying the homogenous restriction

$$H_0 : PD = 0 \quad (18)$$

where $D = \begin{bmatrix} d_y \\ d_x \end{bmatrix}$ and $P = [1 \ 1]$. Robinson (1995a) noted the relevant Wald test statistic could be expressed as

$$\hat{D}' P' \left[(0, P) \left\{ (Z' Z)^{-1} \otimes \Omega \right\} (0, P)' \right]^{-1} P \hat{D} \quad (19)$$

where Ω is residual variance–covariance matrix from (14), $Z = [Z_{t+1} \dots Z_m]'$ and $Z_j = [1, \log\{4\sin^2(\lambda_j/2)\}]'$. Table 4 contains the Wald test results.

For the Mark/US Dollar, US Dollar/Sterling, Yen/US Dollar, Mark/Yen and the Euro/US Dollar, the test indicates different fractional orders for the two variables of interest. Only for the Mark/Sterling, Euro/Yen and Euro/Sterling, does the Wald test assert equivalence. Theoretically, for currencies with unequal orders of integration, fractional cointegration cannot hold. However, for the reasons discussed in the section on methodology, it can be difficult to

¹⁷ Note that the GPH statistic was estimated at $m = T^{0.75}$ following Maynard and Phillips (2001). The estimated standard error of d is that derived by Geweke and Porter-Hudak (1983) and shown in Eq. (4) of Hassler et al. (2006), who show it to be more appropriate than the conventional and Robinson (1995a, 1995b) alternatives.

¹⁸ The resulting estimate of d was then increased by 1. Also note that in (14) l is set equal to zero, indicating no trimming of the harmonic frequencies.

¹⁹ Following Maynard and Phillips (2001), Table 3 shows the daily spot return. To check whether the overlapping data problem will generate upward bias in d , the one period spot return using monthly spaced data was also estimated. These generated results similar to those in Table 3 (available on request).

Table 5

Hassler *et al.*/ARFIMA tests for the cointegrating vector

	d_{GPH}	$\tau_{d=0.5}$	d_{ML}	$\tau_{d=0.5}$	(p, q)
DM/US\$	0.852 (0.040)	8.80	0.789 (0.034)	8.500	(1,0)
US\$/UK£	0.795 (0.031)	9.516	0.802 (0.019)	15.89	(0,1)
Yen/US\$	0.643 (0.031)	4.613	0.748 (0.029)	8.552	(3,0)
DM/Yen	0.683 (0.040)	4.58	0.702 (0.031)	6.516	(0,2)
DM/UK£	0.872 (0.040)	9.30	0.883 (0.019)	20.16	(0,0)
Euro/Yen	0.912 (0.043)	9.58	0.834 (0.037)	9.027	(0,2)
Euro/UK£	0.963 (0.043)	10.77	0.901 (0.020)	20.05	(0,0)
Euro/US\$	0.900 (0.043)	9.302	0.909 (0.020)	20.45	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1/2)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1/2)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

Table 6

Bai and Perron statistics for tests of multiple structural breaks in the forward premium

	UDmax ^a	WDmax(5%) ^b	$F(1 0)$ ^c	$F(2 1)$ ^d	$F(3 2)$ ^e	$F(4 3)$ ^f	$F(5 4)$ ^g
DM/US\$	2109.21***	4628.20**	262.84***	15.57***	13.39**	12.70**	3.34
US\$/UK£	134.94***	194.27**	80.33***	24.94***	11.50**	0.90	0.29
Yen/US\$	2276.85***	3914.90**	374.01***	74.18***	3.48	11.47*	—
DM/Yen	204.78***	341.60**	16.83***	43.66***	2.67	1.15	92.38***
DM/UK£	68.31***	127.06**	11.85**	62.92***	35.19**	1.34	—
Euro/Yen	1211.08***	1439.20**	74.11***	187.40***	187.40***	15.27***	29.09***
Euro/UK£	416.90***	600.17**	37.21***	21.78***	7.32	9.32	1.62
Euro/US\$	120.55***	264.54**	14.67***	16.47***	83.90***	20.37***	20.37***

***Significant at the 1% level; **significant at the 5% level; *significant at the 1% level.

^a 10, 5 and 1% critical values are 7.46, 8.88 and 12.37, respectively.

^b Critical value is 9.91.

^c 10, 5 and 1% critical values are 7.04, 8.58 and 12.29, respectively.

^d 10, 5 and 1% critical values are 8.51, 10.13 and 13.89, respectively.

^e 10, 5 and 1% critical values are 9.41, 11.14 and 14.80, respectively.

^f 10, 5 and 1% critical values are 10.04, 11.83 and 15.28, respectively.

^g 10, 5 and 1% critical values are 10.58, 12.25 and 15.76, respectively.

estimate the d parameter with precision. Therefore we will adopt a cautious approach and examine the fractional differencing parameter of the possible cointegrating relationship²⁰ for all eight currencies. Table 5 shows the estimation of the memory parameter δ for the possible cointegrating vectors using (i) the Hassler *et al.* (2006) methodology with $l=1$ in (14) and (ii) the conventional ARFIMA methodology discussed previously. Interestingly, the point estimate of δ is almost exactly the same as the fractional parameter d of constituent series. So far, the results provide no evidence of fractional cointegration between the forward premium and the CVSR for any of the eight exchange rates. Tables 6 and 7 present the Bai and Perron (1998, 2003a) statistics for tests of multiple structural breaks in the forward premium and CVSR respectively. For every series, the UDmax and WDmax statistics indicate the presence of mean breaks. For the forward premium, the $\text{SupF}_{T(m+1|m)}$ statistics suggest a selection of between 2 and 5 breaks, depending on the currency; for the CVSR the range is lower, the $\text{SupF}_{T(m+1|m)}$ statistics suggesting the presence of 1 to 3 breaks.²¹

²⁰ The cointegrating vector is estimated by OLS. Ng and Perron (1997) examine the normalization issue in two-variable models. They demonstrate that the least squares estimator may possess poor finite sample properties when normalized in one direction but can be well behaved when normalized in the other. As a practical suggestion, they advise using as regressand the variable that is less integrated. Thus, we use conditional variance of the spot rate as the dependent variable.

²¹ Choi and Zivot (2007) also find evidence of multiple structural breaks in the forward premium. We are not aware of any other study that has attempted to detect breaks in traded implied volatility.

Table 7

Bai and Perron statistics for tests of multiple structural breaks in the CVSR

	UDmax ^a	WDmax(5%) ^b	$F(1 0)$ ^c	$F(2 1)$ ^d	$F(3 2)$ ^e	$F(4 3)$ ^f	$F(5 4)$ ^g
DM/US\$	23.81***	31.61**	23.81***	6.89	6.93	8.93	1.41
US\$/UK£	48.41***	48.41**	48.41***	1.81	1.24	0.47	1.34
Yen/US\$	14.27***	18.51**	10.56**	14.94***	7.30	7.30	7.30
DM/Yen	13.88***	18.58**	13.44***	12.40**	11.22**	1.62	-
DM/UK£	69.34***	82.41**	44.55***	4.53	7.30	2.32	-
Euro/Yen	77.48***	88.56**	77.65***	7.51	7.56	3.92	5.59
Euro/UK£	97.38***	151.54**	16.43***	11.31**	14.82***	1.42	-
Euro/US\$	35.95***	52.55**	6.84	27.19***	14.78***	2.36	-

***Significant at the 1% level; **significant at the 5% level; *significant at the 1% level.

^a 10, 5 and 1% critical values are 7.46, 8.88 and 12.37, respectively.^b Critical value is 9.91.^c 10, 5 and 1% critical values are 7.04, 8.58 and 12.29, respectively.^d 10, 5 and 1% critical values are 8.51, 10.13 and 13.89, respectively.^e 10, 5 and 1% critical values are 9.41, 11.14 and 14.80, respectively.^f 10, 5 and 1% critical values are 10.04, 11.83 and 15.28, respectively.^g 10, 5 and 1% critical values are 10.58, 12.25 and 15.76, respectively.

As noted earlier, the presence of structural breaks may cause the detection of spurious long memory. Following the methodology of Choi and Zivot (2007), we demean each series using the estimates $\hat{\beta}_j$ from the least squares regression of Eq. (15), conditional on the estimated break points $(\hat{T}_1, \dots, \hat{T}_m)$. Both the estimated coefficients and break points for each series are shown in Tables 8 and 9.

We then repeat the tests for long memory. Results for the demeaned forward premium and CVSR are given in Tables 10 and 11. The demeaned results, taken as a whole, show little difference from their original counterparts. The GPH point estimates for the forward premium are now spread over the slightly lower range of 0.740 to 0.985. Notably, the corresponding ARFIMA estimates are now spread over a similar range of 0.811 to 0.966. Interestingly, there is now more evidence for long memory in the forward premium; the GPH statistics showing half the exchange rates reject the null of a unit root, the ARFIMA results showing seven out of eight series reject the null of a unit root. GPH and ARFIMA point estimates for the CVSR have likewise decreased marginally. In summary, fractional differencing

Table 8

Bai and Perron regime means and end dates for the forward premium

	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5	Regime 6
DM/US\$	0.296 (0.026)	0.458 (0.022)	0.182 (0.007)	-0.101 (0.022)	-0.179 (0.004)	
End date	3/11/92	7/16/93	9/26/94	12/6/95		
US\$/UK£	-0.457 (0.026)	-0.096 (0.025)	-0.002 (0.013)	-0.189 (0.002)		
End date	3/29/93	2/4/99	9/14/01			
Yen/US\$	0.054 (0.030)	-0.402 (0.025)	-0.153 (0.014)			
End date	4/18/94	9/14/01				
DM/Yen	0.165 (0.008)	0.414 (0.007)	0.304 (0.020)	0.258 (0.019)	0.225 (0.002)	0.254 (0.002)
End date	3/11/92	6/3/93	8/12/94	1/30/96	10/9/97	
DM/UK£	-0.182 (0.048)	0.050 (0.033)	-0.202 (0.017)	-0.308 (0.006)		
End date	3/20/92	1/18/95	5/1/97			
Euro/Yen	0.229 (0.007)	0.351 (0.009)	-0.005 (0.0004)	0.00003 (0.0004)	0.004 (0.0003)	0.001 (0.0003)
End date	3/16/92	4/1/93	10/28/94	11/16/95	12/5/96	
Euro/UK£	-0.206 (0.004)	-0.112 (0.020)	-0.351 (0.017)			
End date	3/16/92	4/1/93				
Euro/US\$	-0.201 (0.014)	-0.149 (0.019)	-0.252 (0.029)	-0.134 (0.005)	-0.100 (0.005)	-0.245 (0.004)
End date	3/16/92	4/1/93	4/21/94	5/10/95	12/9/96	

Note: The first number in each cell is the estimated mean for the regime; standard errors are reported in parentheses. The end date for each regime is shown below the estimated mean.

Table 9

Bai and Perron regime means and end dates for the CVSR

	Regime 1	Regime 2	Regime 3	Regime 4
DM/US\$	0.132 (0.01)	0.075 (0.006)		
End date	1/4/96			
US\$/UK£	0.148 (0.012)	0.060 (0.003)		
End date	11/9/93			
Yen/US\$	0.097 (0.009)	0.183 (0.021)	0.078 (0.003)	
End date	12/11/97	7/14/00		
DM/Yen	0.093 (0.008)	0.137 (0.011)	0.073 (0.008)	0.186 (0.024)
End date	9/9/92	5/10/94	10/20/97	
DM/UK£	0.015 (0.001)	0.061 (0.007)		
End date	8/28/92			
Euro/Yen	0.182 (0.011)	0.076 (0.005)		
End date	7/21/93			
Euro/UK£	0.053 (0.003)	0.098 (0.005)	0.042 (0.004)	0.026 (0.001)
End date	2/6/92	3/23/93	9/6/96	
Euro/US\$	0.081 (0.005)	0.145 (0.007)	0.088 (0.005)	0.067 (0.002)
End date	2/3/92	5/20/93	9/6/96	

Note: The first number in each cell is the estimated mean for the regime; standard errors are reported in parentheses. The end date for each regime is shown below the estimated mean.

Table 10

GPH/ARFIMA tests for the demeaned forward premium

	d_{GPH}	$\tau_{d=1}$	d_{ML}	$\tau_{d=1}$	(p, q)
DM/US\$	0.952 (0.039)	-1.231	0.909 (0.017)	-5.353	(0,0)
US\$/UK£	0.963 (0.030)	-1.233	0.966 (0.023)	-1.478	(0,1)
Yen/US\$	0.953 (0.030)	-1.567	0.887 (0.012)	-9.417	(0,0)
DM/Yen	0.890 (0.039)	-2.821	0.811 (0.017)	-11.12	(0,0)
DM/UK£	0.926 (0.039)	-1.897	0.941 (0.025)	-2.360	(1,0)
Euro/Yen	0.740 (0.041)	-6.341	0.815 (0.020)	-9.250	(0,0)
Euro/UK£	0.929 (0.041)	-1.732	0.870 (0.018)	-7.223	(0,0)
Euro/US\$	0.985 (0.041)	-0.366	0.956 (0.019)	-2.316	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

Table 11

GPH/ARFIMA tests for the demeaned CVSR

	d_{GPH}	$\tau_{d=1}$	d_{ML}	$\tau_{d=1}$	(p, q)
DM/US\$	0.813 (0.039)	-4.795	0.786 (0.035)	-6.114	(1,0)
US\$/UK£	0.724 (0.030)	-9.200	0.763 (0.027)	-8.778	(1,0)
Yen/US\$	0.659 (0.030)	-11.36	0.760 (0.029)	-8.276	(3,0)
DM/Yen	0.660 (0.039)	-8.718	0.563 (0.035)	-12.49	(0,3)
DM/UK£	0.814 (0.039)	-4.769	0.661 (0.019)	-17.84	(0,3)
Euro/Yen	0.894 (0.041)	-2.585	0.742 (0.090)	-2.867	(2,0)
Euro/UK£	0.925 (0.041)	-1.829	0.861 (0.020)	-6.950	(0,0)
Euro/US\$	0.822 (0.041)	-4.341	0.903 (0.021)	-4.619	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

Table 12

Wald tests for the equality of the GPH estimates for the demeaned forward premium and the CVSR

DM/US\$	6.706 [0.01]
US\$/UK£	31.258 [0.000]
Yen/US\$	48.02 [0.000]
DM/Yen	15.74 [0.000]
DM/UK£	3.884 [0.049]
Euro/Yen	6.906 [0.009]
Euro/UK£	0.004 [0.947]
Euro/US\$	9.006 [0.003]

Note: the Wald statistic has a $\chi^2(1)$ distribution. The figures in square brackets are p -values.

Table 13

Hassler et al./ARFIMA tests for the cointegrating vector–demeaned series

	d_{GPH}	$\tau_{d=0.5}$	d_{ML}	$\tau_{d=0.5}$	(p, q)
DM/US\$	0.828 (0.040)	8.20	0.786 (0.035)	8.171	(1,0)
US\$/UK£	0.743 (0.031)	7.84	0.761 (0.027)	9.667	(1,0)
Yen/US\$	0.669 (0.031)	5.45	0.759 (0.029)	8.931	(3,0)
DM/Yen	0.664 (0.040)	4.10	0.564 (0.035)	1.829	(0,3)
DM/UK£	0.820 (0.040)	8.00	0.659 (0.030)	5.300	(0,3)
Euro/Yen	0.754 (0.043)	5.91	0.814 (0.020)	15.70	(0,0)
Euro/UK£	0.915 (0.043)	9.65	0.860 (0.020)	18.00	(0,0)
Euro/US\$	0.842 (0.043)	7.95	0.905 (0.021)	19.29	(0,0)

Note: numbers in parentheses next to the estimates for d_{GPH} and d_{ML} are standard errors (σ_d). Numbers in the third column represent the test statistic $(d_{\text{GPH}} - 1/2)/\sigma_d$. Numbers in the fifth column represent the test statistic $(d_{\text{ML}} - 1/2)/\sigma_d$. Finally, the sixth column shows the selected (p, q) order of the estimated ARFIMA.

estimates for all series are still comfortably in the non-stationary region. It would appear that the time series behaviour of the daily forward premium and CVSR can be typically characterised by both long memory and structural breaks.²²

Table 12 contains the Wald test results for equal memory in the demeaned forward premium and CVSR. Now, only for the Euro/Sterling is equivalence suggested. However, persisting with a cautious approach, Table 13 reports estimation of the cointegrating vector memory parameter δ using all the demeaned series. Unsurprisingly, given the small memory changes reported in Tables 10 and 11, the results again provide no evidence of fractional cointegration between the forward premium and the CVSR.

5. Discussion

The proposition given in Section 2 suggests that a fractionally integrated foreign exchange TRP exists. The empirical corollary being that the forward premium is not fractionally cointegrated with the CVSR as a bi-variate pair. The results in the previous section find exactly this and therefore give some credence to our proposition.

Given the common assumption of a stationary risk premium in the literature, evidence for a fractionally integrated TRP has a number of implications. For example, our results help shed more light on the failure of the forward rate unbiasedness hypothesis, over both the long and short-run. In the long run, we posit that it is the forward premium, the CVSR and the unobserved TRP that are fractionally cointegrated as a tri-variate group. In other words, it is a joint long memory component that is transferred to the forward premium. This component is highly correlated with the past and therefore partly forecastable.

²² Choi and Zivot (2007) examined the time series behaviour of 5 one-month forward premium series (Deutschmark, French Franc, Italian Lira, Canadian Dollar and British Pound, all relative to the US Dollar) using monthly data over the period 1976:1 to 1999:1. Similarly to the results reported above, they find that after accounting for breaks, long memory can still be detected in all series. However, the fractional differencing parameter moves from the non-stationary to the stationary region. This divergence in the results of the two studies may have occurred through the use of a different data period or frequency. Additionally, as noted earlier, we employ the 'cleaner' interest rate differential as a proxy for the forward premium.

In the short-run, the failure of the unbiasedness hypothesis is most often observed via the so-called forward premium puzzle. Given the short-run version of (3) below

$$s_t - s_{t-1} = \beta_0 + \beta_1(f_{t-1} - s_{t-1}) + u_t \quad (20)$$

the forward premium puzzle relates to the preponderance of negative coefficients for β_1 estimated in the literature (see Chinn and Meredith, 2004). A negative coefficient indicates that an expected appreciation in the exchange rate is typically followed by a depreciation and vice-versa. In other words, market forecasts of the spot return are not only sub-optimal, they are perverse.

There have been a number of attempts to explain the puzzle. For example, Engel (1996) surveys the literature and concentrates on time-varying rational expectations risk premia. On the other hand, Maynard and Phillips (2001) suggest that standard asymptotic theory is inappropriate in the presence of long memory in the forward premium. They derive non-standard limiting distributions for β_1 with long left tails. It is these long left tails that are posited as being responsible for the puzzle.

Our results provide further evidence that the economic and statistical explanations of the puzzle may be combined. The CCAPM implies that long memory in the TRP and CVSR explains analogous time series behaviour in the forward premium; the induced long memory in the forward premium then resulting in the long tailed distributions that help produce the large number of negative slope coefficients in (20).²³

Finally, it should be stressed that as the TRP is unobserved, our conclusions clearly follow from the adoption of a rational expectations approach.²⁴ If a rational expectations story is to be maintained, theoretical models of the TRP that assume rational expectations (see inter alia, Engel, 1999) should be capable of producing long memory.²⁵ This requires that one or more of the variables that explain the TRP should also contain long memory. Evans and Lewis (1995) have argued that because the variables that explain the risk premium are stationary, the risk premium is also stationary. However, as a first step, we suggest that the time series properties of these explanatory variables could be usefully re-examined for long memory. Clearly, any finding of long memory would provide further evidence as to the existence of the TRP.

As an example, one could consider a time-varying version of the constant risk premium model by Engel (1999). Using a sticky-price general equilibrium model where there is pricing to market²⁶ and a cash-in-advance constraint, it is shown that the TRP depends on the variance of the money growth.²⁷ In other words, it is implied that monetary variability causes the correlation between consumption and exchange rates. Hence, the variance of money growth will need to be fractionally integrated in order to generate a fractionally integrated risk premium. Certainly a prima facie case for this can be provided by previous empirical work examining money growth volatility. For example, Mehra (1989) noted that the assumption that money growth volatility is stationary is not supported by any evidence. Thornton (1995) applies conventional ADF tests to data from nine countries and results suggest some uncertainty as to whether the null hypothesis of a unit root can be rejected. This perhaps mimics the uncertainty in the forward premium stressed earlier. More recently, Radchenko (2003) uses a Bayesian framework to circumvent acknowledged uncertainty over the order of integration of money growth volatility. The above would suggest further empirical work regarding the time series properties of the variance of money growth²⁸ and the possibility of fractional cointegration between the forward premium and the variance of money growth.

²³ This explanation, comprising both economic and statistical rationale, is congruent with new work that suggests long memory behaviour in the forward premium may not be able to explain all the forward premium puzzle (see Maynard, 2006).

²⁴ The debate about the extent of applicability of the rational expectations assumption is still fiercely contested. For example, literature employing survey-based expectations has suggested that there may be departures from rational expectations (see, inter alia, Chinn and Frankel, 2002). Of course, there are a number of methodological critiques of the survey-based approach (see Bacchetta et al., 2006). In any case, for an overview of explanations for departures from rationality based on psychological behaviour see Hirscheleifer (2001).

²⁵ Several studies have attempted to produce models of the foreign exchange risk premia that account for other empirically inferred time series properties (for example, Bekaert et al., 1994; Backus et al., 1993).

²⁶ Producers set the price in the currency of the consumers. Therefore, the prices consumers face are invariant to the exchange rate.

²⁷ See Engel (1999) Eq. (23).

²⁸ Although beyond the scope of the present paper, the authors consider this an interesting line of future empirical inquiry. However, it should be noted that the lack of higher frequency money data suggests that the relevant models cannot easily be directly tested.

6. Conclusions

The extant foreign exchange literature has reported evidence of long memory behaviour in the forward premium of several currencies. This is important because the long memory component motivates a rejection of the unbiasedness hypothesis as well as providing a statistical rationale for the forward premium anomaly. Given this context, a key question is what causes the time series behaviour of the forward premium?

Adopting a rational expectations framework, this paper provides theoretical and empirical evidence that the fractionally integrated behaviour of the forward premium can be jointly explained by similar behaviour in the true risk premium (TRP) and the conditional variance of the spot rate. This provides a new channel for the risk premium to play a part in explaining the empirical biases typically found in foreign exchange markets. Specifically, the long memory in the TRP helps explain the analogous time series behaviour in the forward premium; the induced long memory in the forward premium then resulting in the failure of the unbiasedness hypothesis and contributing to the forward premium anomaly.

Finally, it is important to note that in this paper the TRP is unobserved and the above conclusions are dependent on the rational expectations framework. In particular, to provide further evidence for a rational expectations view of the foreign exchange market, similar long memory properties should be present in the explanatory variables indicated by models of the TRP. Further work along these lines is encouraged.

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