Available online at www.sciencedirect.com
ScienceDirect
Journal of Empirical Finance 15 (2008) 778 – 788
Journal of EMPIRICAL FINANCE
www.elsevier.com/locate/jempfin

Asset pricing models with errors-in-variables ☆

Benoît Carmichael [a], Alain Coën [b,*]

[a] Département d'économique, Université Laval, Québec, Canada G1V 0A6
[b] École des sciences de la gestion (ESG) Université du Québec à Montréal (UQÀM), Montréal, Canada H3C 4R2

Received 25 January 2007; received in revised form 21 December 2007; accepted 7 January 2008
Available online 18 January 2008

Abstract

This paper revisits Fama and French [Fama, Eugene F., French, Kenneth R., (1993) Common risk factors in the returns on stock and bonds. Journal of Financial Economics 33 (1), 3–56] and Carhart [Carhart, Mark M., 1997. On persistence in mutual fund performance. Journal of Finance 52 (1), 57–82] multifactor model taking into account the possibility of errors-in-variables. In their well known paper, Fama and French [Fama, Eugene F., French, Kenneth R., 1997. Industry costs of equity. Journal of Financial Economic 43 (2), 153–193] concluded that *estimates of the cost of equity for the three-factor model of FF* (*1993*) *were imprecise*. We argue that this imprecision is even more severe because of the pervasive effects of measurement errors. We propose Dagenais and Dagenais [Dagenais, Marcel G., Dagenais, Denyse L., 1997. Higher moment estimators for linear regression models with errors in the variables. Journal of Econometrics 76 (1–2), 193–221] higher moments estimator as a solution. Our results show that estimates of the cost of equity obtained with Dagenais and Dagenais estimator differ sharply from popular OLS estimates and shed a new light on performance attribution and abnormal performance (α). Adapting the Generalized Treynor Ratio, recently developed by Hübner [Hübner, Georges, 2005. The generalized treynor ratio. Review of Finance 9 (3), 415–435], we show that the performance of managed portfolios with multi-index models should be revisited in presence of errors-in-variables.

JEL classification: C19; C49; G12; G3
Keywords: Asset pricing models; Cost of equity; Errors-in-variables; Generalized Treynor Ratio; Higher moments; Instrumental variables; Measurement errors

1. Introduction

The identification and measurement of systematic risks affecting asset returns are important tasks in financial economics. An extensive theoretical and empirical literature gives some guidance on this matter. For instance, the Capital Asset Pricing Model (CAPM) identifies the market return has the only systematic risk relevant for asset pricing.

☆ The authors would like to thank Jean-Claude Cosset, Georges Dionne, Georges Hübner, Lynda Khalaf, Jean-François L'Her, François-Éric Racicot and an anonymous referee for helpful comments. The usual disclaimer applies.
* Corresponding author. Tel.: +1 514 987 3000 5680; fax: +1 514 987 0422.
E-mail addresses: benoit.carmichael@ecn.ulaval.ca (B. Carmichael), coen.alain@uqam.ca (A. Coën).
URL: http://www.ecn.ulaval.ca/benoit.carmichael (B. Carmichael).

0927-5398/$ - see front matter
doi:10.1016/j.jempfin.2008.01.002

The empirical literature initiated by Fama and French (1992) concludes however that a single market factor is insufficient to explain expected returns.[1] Additional risks related to firm size, book-to-market equity and momentum must be included to explain expected returns properly.[2] Fama and French model has been criticized because of its empirical motivations and associated lack of strong theoretical underpinnings.[3] Despite these criticisms, Fama and French empirical specification has become the norm in financial analysis and managers evaluate risk exposures mainly with β coefficients drawn from this model.[4] Practitioners often find very imprecise β coefficients.[5] Perhaps more importantly, because true risk factors are unobserved, β estimates suffer from errors-in-variables (EIV) complications. Although, this is not always fully appreciated, EIV stand as one of the pervasive problems in applied financial econometrics. Roll (1977) well known critic of the CAPM is a reminder that EIV should be endemic in linear regression models of asset returns. The market factor being at best measured with error, estimates of the market β certainly suffer from EIV bias. The same applies to more general models such as Fama and French (1993), Carhart (1997) and/or Chen et al. (1986).

EIV are problematic because they lead to biased and inconsistent ordinary least squares (OLS) estimators. Textbook treatments of measurement errors are almost exclusively cast in terms of a simple bivariate linear model. In this framework, measurement errors bias the slope coefficient toward zero, an effect labelled attenuation effect by Cragg (1994). They also induce a bias of the opposite sign on the intercept coefficient when the mean of the explanatory variable is positive. Cragg (1994) calls the latter a *contamination effect*.

The conclusions drawn from the bivariate model partly explain why EIV do not attract much attention. Since the *attenuation effect* does not change coefficient sign, EIV give rise at worst to conservative estimates. Moreover, because the attenuation effect is negatively related to the regression R^2, high R^2 are indicative of negligible bias. Cragg (1994, 1997) demonstrate that these conclusions do not however generalize to multiple regressions. In this context, the bias of any given parameter depends on its own error (the attenuation effect) but also on the errors in all other variables (the contamination effect).[6] General results on the magnitude and direction of bias are no longer available. Knowledge of the stochastic processes involved is required to infer the direction and importance of individual bias.[7]

Surprisingly, efforts are seldom made to reduce EIV bias in financial economics. There are exceptions. Fama and MacBeth (1973) reduce measurement errors by grouping equities in portfolios. Shanken (1992) adjusts standard errors to correct bias induced by EIV. Kandel and Stambaugh (1995) suggest using a generalized least squares method. Difficulty remains in the last case because the true covariance matrix of returns must be known or be consistently estimated.

In principle, EIV can be treated with instrumental variables (IV) if one can find instruments correlated with the true factors but unrelated to the measurement errors. See Fuller (1987), Bowden and Turkington (1984) and Aigner et al. (1984) among others.[8] In practice, IV methods are not widely used because finding valid instruments is difficult.[9]

Cragg (1994, 1997), Dagenais and Dagenais (1997), and Lewbel (2006) argued that estimators based on moments of order higher than two could, in some circumstances, help resolve EIV complications. The higher moment estimator proposed in Dagenais and Dagenais (1997), hereafter referenced as DDHME, appears to be particularly promising in financial economics since, by constructing instruments from the original data higher moments (skewness, kurtosis and over), it offers a practical solution to the problems raised by Pal (1980) and Klepper and Leamer (1984) among others.

DDHME is not applicable in all situations of EIV however. To secure identification and estimability, it requires non-Gaussianity of the *true* unobserved variables coupled with normality of the measurement errors. More importantly, DDHME exploits orthogonality conditions in terms of higher moments to obtain parameter estimates. These higher moments will invariably depend on unknown moments which have to be consistently estimated by corresponding

[1] Specifically, Fama and French (1992, 1993, 1995, 1996, 1997, 1998, 2002) and Carhart (1997).

[2] One may also cited Chen et al. (1986) that model asset returns as a linear functions of various macroeconomic risk factors, inflation, economic growth, interest rates, etc.

[3] See Black (1993), and Kothari et al. (1995) among others.

[4] This linear approach has been largely extended in the Hedge Funds literature: see for example Agarwal and Naik (2004).

[5] Fama and French (1997) reports standard errors of more than 3.0% per year.

[6] Because of the contamination effect, only one variable measured with error is sufficient to produce inconsistent parameter estimates in multiple regressions.

[7] Dagenais and Dagenais (1997) argue that EIV also have more perverse effects on confidence intervals and on the sizes of the type I errors.

[8] Alternative approaches to the EIV problem may be mentioned: Frisch (1934), Klepper and Leamer (1984), Hausman and Watson (1985), and Leamer (1987) among others.

[9] See Pal (1980) and Klepper and Leamer (1984) for further details.

sampling moments. In practice, this means that data have to exhibit skewness and/or excess kurtosis for DDHME to alleviate EIV bias.

Since financial variables are often found to exhibit nonnormality, financial models offer a natural laboratory to experiment the usefulness of higher moment estimator such as DDHME. This paper seeks to undertake such an evaluation taking as reference the popular Fama and French (1993) and Carhart (1997) model. The paper is organized as follows. Section 2 discusses the estimation of the four-factor model with Dagenais and Dagenais (1997) estimator. Section 3 presents the data. Section 4 reports and compares DDHME and OLS parameter estimates of Fama and French model. Section 5 discusses the consequences of EIV for portfolio selection by reporting a performance measure of managed portfolios based on Treynor's type Ratio computed with and without EIV corrections. The conclusions are drawn in Section 6.

2. Fama and French asset pricing model with EIV

The empirical literature, initiated by Fama and French (1992), models excess returns R_t as a linear function of a market premium (Mkt_t), a size premium (Smb_t), a book-to-market premium (Hml_t) and a momentum premium (Umd_t) introduced by Carhart (1997).

$$R_t = \alpha + \beta_m \cdot \mathrm{Mkt}_t + \beta_s \cdot \mathrm{Smb}_t + \beta_h \cdot \mathrm{Hml}_t + \beta_u \cdot \mathrm{Umd}_t + \varepsilon_t \tag{1}$$

Contrary to the implicit assumption made in this literature, lets assume that true premiums are unobserved by analysts. Under this alternative assumption, Eq. (1) is obtained after substituting true by observed premiums. Because of this substitution, ε_t must be reinterpreted as a compound error that includes measurement errors in addition to residual idiosyncratic risk. Clearly, under this alternative assumption, the error ε_t is correlated with the observed premiums. This makes OLS parameter estimates inconsistent and it induces attenuation and contamination biases discussed above.

Since the risk premiums on the right hand side of Eq. (1) are known to have non Gaussian higher moments, Dagenais and Dagenais (1997) estimator could be used to obtain consistent estimates of α, β_m, β_s, β_h *and* β_u if one makes the assumption that non-Gaussianity is a property of the true factors but not of the measurement errors. Being a IV type estimator, DDHME is most easily computed with two steps artificial regressions. See Davidson and MacKinnon (1993) for a complete description. One begins by constructing estimates $\hat{w}_t^{\mathrm{Mkt}}$, $\hat{w}_t^{\mathrm{Smb}}$, $\hat{w}_t^{\mathrm{Hml}}$, $\hat{w}_t^{\mathrm{Umd}}$ of EIV on *true* premiums. These are constructed as the residuals of companion regressions having observed risk premiums as dependent variables and the instruments (higher moments of the original data) as regressors. The choice of instruments is discussed in greater details below. At the second step of the procedure, $\hat{w}_t^{\mathrm{Mkt}}$, $\hat{w}_t^{\mathrm{Smb}}$, $\hat{w}_t^{\mathrm{Hml}}$, $\hat{w}_t^{\mathrm{Umd}}$ are introduced as additional regressors in Eq. (1). This yields the artificial regression:

$$R_t = \alpha + \beta_m \cdot \mathrm{Mkt}_t + \beta_s \cdot \mathrm{Smb}_t + \beta_h \cdot \mathrm{Hml}_t + \beta_u \cdot \mathrm{Umd}_t + \psi_m \cdot \hat{w}_t^{\mathrm{Mkt}} + \psi_s \cdot \hat{w}_t^{\mathrm{Smb}} + \psi_h \cdot \hat{w}_t^{\mathrm{Hml}} + \psi_u \cdot \hat{w}_t^{\mathrm{Umd}} + e_t \tag{2}$$

whose parameters can then be estimated by OLS. This procedure has two advantages. First, the null hypothesis (H_0: $\psi_k=0$, for $k=m$, s, h and u) of no EIV can be tested directly with a Durbin–Wu–Hausman (DWH) test that is asymptotically distributed as a χ^2 with 4° of freedom. Second, the $\widetilde{\beta}_m$, $\widetilde{\beta}_s$, $\widetilde{\beta}_h$ and $\widetilde{\beta}_u$ estimates drawn from Eq. (2) are consistent estimates of the true values even if the DWH test rejects the null of no EIV.[10] An Appendix at the end of the paper gives more details on the econometric procedures used to estimate and test Eq. (2).[11]

The selection of instruments is always an important step in the implementation of an IV estimator. Dagenais and Dagenais (1997) suggests various combinations of seven categories of instruments. These are: $z_1=x*x$, $z_2=x*y$, $z_3=y*y$, $z_4=x*x*x-3x\,[E\,(x'x/N)*I]$, $z_5=x*x*y-2x\,[E\,(x'y/N)*I]-y\,\{\iota'\,[E\,(x'x/N)*I]\}$, $z_6=x*y*y-x\,[E\,(y'y/N)]-2y\,[E\,(y'x/N)]$ and $z_7=y*y*y-3y\,[E\,(y'y/N)]$, where the symbol * is the Hadamard element-by-element matrix

[10] Equivalently, the DWH test can be based on an artificial regression having the predicted premiums of the first stage regressions as additional regressors in Eq. (2) instead of the $\hat{w}$ variables. We thank an anonymous referee for this remark. See chapter 7 of Davidson and MacKinnon (1993) for more details.

[11] Although parameters of Eq. (2) are estimated consistently by OLS, researchers should be aware that standard commercial software will generally under report the model's error variance. Because of this, printed covariance matrix will need to be corrected manually. The Appendix describes how to compute a valid covariance matrix for the parameters of Eq. (2).

multiplication operator, N is the number of observations, x is a $N\times 4$ matrix holding premiums Mkt, Smb, Hml and Umd expressed in mean deviation form and y is a vector holding the excess return R, also in mean deviation form.

Section 4 below reports parameter estimates based on a subset of these instruments. This subset includes categories z_1 and z_4 as well as a constant. Four considerations motivate this selection. First, it takes into account two conflicting objectives. On the one hand, asymptotic efficiency requires using as many instruments as possible if the number of available observations is large.[12] This militates in favor of using all seven categories defined above. On the other hand, in finite-sample, increasing the number of instruments tends to make the finite-sample bias of IV estimators more important. On this account, a smaller set of instruments should be used instead. Second, the neglected categories z_2, z_3, z_5, z_6 and z_7 all involve moments or cross-moments of the asset return R_t. Because of this, they are likely to be more weakly correlated with the instrumented risk premiums. Adding weak instruments generally affects estimators adversely (Bowden and Turkington, 1984). Third, the *Monte Carlo* experiments of Dagenais and Dagenais (1997) revealed that the root mean squared errors of parameter estimates are smaller with this particular choice of instruments than with the other combinations, including the full set of instruments. Finally, DDHME is a linear combination of Durbin (1954) and Pal (1980) higher moments estimators with this particular set of instruments.

Instrumental variables introduce overidentifying restrictions when there are more instruments than parameters to estimate. These restrictions can be tested in several different ways. Below, we follow the procedure suggested in Davidson and MacKinnon (1993). The overidentifying restrictions (OR) are tested by comparing N times the uncentered R^2 from a regression of the IV residuals on the instruments to the χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions. This evaluates the joint null hypothesis that the Fama and French model is correctly specified and that the instruments used are valid. Since we use higher moments as instruments to estimate a linear asset pricing model, a significant OR statistic may be interpreted, to a certain extent, as a test for left out nonlinearities in the risk factor structure. These could, for instance, result from loss aversion.[13] Recall that Fama and French factors are constructed as long minus short portfolios. As such, they resemble portfolios of options which are known to encompass nonlinearities. The DDHME used is a linear combination of Durbin (1954) and Pal (1980) estimators. Thus, it uses third and fourth moments to account for possible nonlinearities in the risk factor.[14]

Summing up, the preceding discussion suggests the following strategy while estimating an asset pricing model such as Eq. (1). First, test whether risk factors exhibit skewness and excess kurtosis. Second, if risk factors are found non-Gaussian, compute Dagenais and Dagenais estimator. Third, test the null hypothesis of no EIV with a DWH test. If H_0 cannot be rejected, use OLS, otherwise use DDHME.

Before moving on to the results section, it is important to stress that Eq. (2) is not an alternative to Fama and French asset pricing model. The variables $\hat{w}_t^{\mathrm{Mkt}}$, $\hat{w}_t^{\mathrm{Smb}}$, $\hat{w}_t^{\mathrm{Hml}}$, $\hat{w}_t^{\mathrm{Umd}}$ in Eq. (2) are not new factors with associated risk premiums. Instead, they are estimates of the measurement errors on observed risk premiums. In the absence of EIV, the estimated $\hat{w}_t^k$, would not be related to R_t and estimated ψ_k coefficients would be insignificant. The purpose of these "artificial" variables is solely to estimate consistently the coefficients α, β_m, β_s, β_h and β_u of Fama and French's model.

3. The data

We use Fama and French (1992) monthly returns on twenty-five value-weighted portfolios constructed by the intersection of five market equity (ME) portfolios and five book equity portfolios (BE/ME). The dataset was taken directly from Kenneth French's website.[15] The sample used ranges from January 1963 to February 2006. This yields 518 usable monthly observations. As mentioned in Fama and French (1992), "The start date reflects the fact that book value of common equity, is not generally available prior to 1962. More important COMPUSTAT data for earlier years have a serious selection bias; the pre-1962 are titled toward big historically successful firms".[16]

As a preliminary step, we performed various normality tests (Bera-Jarque, Lilliefors, Cramer-von Mises, Watson, and Anderson-Darling) on the data to confirm the presence of skewness and excess kurtosis.[17] The results strongly

[12] Asymptotic efficiency also required to use an optimal weighting matrix.

[13] We would like to thank an anonymous referee for this point.

[14] Kraus and Litzenberger (1976) introduce a premium for skewness similar to instruments z_1. Nevertheless, we may mention that instruments z_4 are different from the premium introduced by the financial literature to take into account kurtosis.

[15] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data library.html.

[16] See Fama and French (1992) pp. 429.

[17] Detailed results are available upon request.

reject normality. This feature makes Dagenais and Dagenais (1997)'s solution to the EIV problems potentially useful in studies based on Fama and French's dataset. This is the subject of the next section.

4. The results

Tables 1-A–1-E reports the parameter estimates of Fama and French's model with (DDHME) and without (OLS) corrections for the presence of EIV. The results are presented in five panels grouping securities by market equity (ME) going from Small size (panel A) to Big size (panel E). As mentioned above, the instruments used to compute DDHME parameter estimates are z_1, z_4 and a constant, forming a group of nine different instruments.[18]

The DWH tests reported in panels 1–A to 1–E reject, at the 10% level, the null hypothesis of no EIV for twenty-four asset returns (twenty-one cases at the 1% level) studied. Individual *Student-t* tests reject the null of no EIV for at least one factor in all twenty-five portfolios. In twenty cases, one or two factor loadings, *Smb* and/or *Hml*, introduced by Fama and French (1992) induce statistically significant $\widetilde{\psi}_k$ coefficient. In nineteen cases the adjustment variable related to the book-to-market equity, *Hml*, is statistically significant at the 10% level (respectively seventeen, and fifteen cases at the 5%, and 1% levels), and respectively in fifteen, six and four cases for *Mkt*, *Umd*, and *Smb*. In four cases, the hypothesis of no EIV is individually rejected for at least three factors. In fifteen cases, the no EIV hypotheses are detected for at least two factors.

The OR statistics reported in the panels of Tables 1-A–1-E test the overidentifying restrictions. Under the null hypothesis that the model is correctly specified and the instruments are valid, this statistic is asymptotically distributed as a χ^2 with four degrees of freedom.[19] For twenty-three portfolios out of twenty- five, the criterion function is small and we cannot reject the null at 1% level. The null is also not rejected in sixteen out of twenty-five cases at a 5% level. In the cases of rejection, the R^2 are always small. The highest value being 0.036 only. This suggests that left out nonlinearities are not quantitatively important. We globally interpret the OR tests as not rejecting the four factors model and our choice of instruments.

In our sample, *Hml* and *Mkt* appear to be most contaminated by EIV with respectively nineteen and fifteen significant ψ_k coefficients at a 10% significance level. For *Mkt*, this may be an illustration of the conceptual difficulty of measuring true market risk premium. Even though point estimates of β_h obtained with DDHME often differ substantially from OLS, the ranking of Fama and French twenty-five assets between value stocks (positive coefficients) and growth stocks (negative coefficients) is nevertheless preserved. We also observed that all significant $\widetilde{\psi}_h$ coefficients are negative. This suggests that β_h coefficients computed with OLS are prone to attenuation bias. *Smb* and *Umd* have a smaller number of individually significant ψ_k coefficients. As such, they appear to be a less important source of EIV.

Taking account of EIV may modify decisions in portfolio management. Our results suggest that this will particularly be the case for decisions based on abnormal performance. OLS and DDHME estimates of Jensen's α differ substantially. In nine cases, the α estimate even changes sign. Recall that OLS estimate of α will tend to be positive when attenuation biases are prevalent since mean risk premiums are positive in our sample.

Tables 1-A–1-E do not invalidate Fama and French empirical asset pricing model. It confirms the relationship between asset returns and Smb, Hml, and Umd even though factors are found to be affected by EIV. This leads us to the conclusion that EIV in asset pricing models are more than a theoretical curiosity. Researchers and portfolio analysts alike should be aware that EIV can be widespread and quantitatively important. For this reason, in empirical applications, we recommend a prudent rule: the systematic use of higher moment estimators to detect possible EIV, and then to correct estimates before drawing any conclusions in portfolio management.

The evidence of EIV in measured risk premiums calls seriously into question the accuracy and usefulness of performance measures for portfolio management based on OLS. This question is taken next.

5. Performance measures for managed portfolios revisited

Following Jensen (1967), an important literature has been devoted to measure the performance of managed portfolios. In the Sharpe (1966) ratio and the Treynor (1965) and Black (1993) ratios, derived from the Capital Market Line, the level

[18] We have experimented with other combinations of instruments. These experiments yielded qualitatively the same conclusion. Results are available upon request. Instrument lists that exclude both z_1 and z_4 appear to suffer from a weak instruments problem however. Specifically, parameter estimates are insignificant and widely differ from the OLS. We even observed a negative market β_m in many instances.

[19] Nine instruments are used to obtain the estimates $\widetilde{\alpha}$, $\widetilde{\beta}_m$, $\widetilde{\beta}s$, $\widetilde{\beta}_h$ and $\widetilde{\beta}_u$.

Table 1-A
Higher moments estimators of the four factor model: 25 stock portfolios formed on size and book-to-market equity: Market Equity (ME) group Small

BE/ME		$\bar{R}^2$	DW	GTR	DWH	OR	α	β_m	β_s	β_h	β_u	ψ_m	ψ_s	ψ_h	ψ_u
Low ranking: 1	DDHME	0.92	1.91	−0.32	14.13	10.87	−0.176	1.045	1.563	−0.425	−0.276	0.038	−0.223	0.135	0.285
					0.01	0.03	(−0.86)	(9.39)	(8.96)	(−1.52)	(−3.24)	(0.33)	(−1.33)	(0.49)	(3.13)
	OLS	0.92	1.93	−0.47			−0.378	1.074	1.374	−0.327	−0.048				
							(−3.19)	(37.81)	(29.99)	(−6.15)	(−0.96)				
Ranking: 2	DDHME	0.95	2.10	0.09	47.63	9.51	0.075	0.787	1.670	0.015	−0.073	0.198	−0.424	0.074	0.064
					0.00	0.05	(0.67)	(15.35)	(20.25)	(0.12)	(−1.98)	(3.24)	(−4.46)	(0.59)	(1.38)
	OLS	0.94	2.16	0.01			0.013	0.963	1.318	0.053	−0.011				
							(0.16)	(46.26)	(31.54)	(1.3)	(−0.36)				
Ranking: 3	DDHME	0.95	2.03	−0.10	12.68	10.42	−0.114	0.878	1.369	0.580	−0.055	0.051	−0.293	−0.316	0.070
					0.01	0.03	(−1.14)	(13.17)	(12.69)	(5.31)	(−1.)	(0.76)	(−2.82)	(−2.98)	(1.16)
	OLS	0.95	2.02	0.02			0.020	0.921	1.100	0.289	0.002				
							(0.32)	(57.36)	(41.21)	(9.12)	(0.08)				
Ranking: 4	DDHME	0.95	2.00	0.02	15.81	17.62	0.023	0.980	1.171	0.792	−0.074	−0.089	−0.136	−0.378	0.108
					0.00	0.00	(0.21)	(15.16)	(11.48)	(7.37)	(−1.22)	(−1.35)	(−1.4)	(−3.68)	(1.72)
	OLS	0.94	1.99	0.170			0.184	0.898	1.029	0.454	0.008				
							(2.94)	(47.55)	(34.63)	(14.51)	(0.35)				
High ranking: 5	DDHME	0.95	1.94	0.13	36.46	12.38	0.159	1.091	1.048	0.854	−0.155	−0.131	0.065	−0.208	0.157
					0.00	0.01	(1.58)	(31.25)	(13.71)	(8.29)	(−3.02)	(−3.17)	(0.82)	(−1.97)	(2.81)
	OLS	0.95	1.99	0.16			0.192	0.971	1.083	0.675	−0.039				
							(2.86)	(52.18)	(33.95)	(19.76)	(−1.65)				

Sample period: 1963:01 to 2006:02. DW, DWH and OR refer to the Durbin–Watson, Durbin–Wu–Hausman and Overidentifying Restrictions tests statistics. GTR is the Generalized Treynor Ratio. Numbers in parentheses are Student's *t*-statistics. Numbers below DWH and OR statistics are the marginal significance level of each statistic. All DDHME and OLS statistics are computed with White (1980) H_0 heteroskedasticity-consistent covariance matrix estimate HCCME.

of risk is measured with the standard deviation of portfolios returns. Three usual metrics are derived from the Security Market Line. Treynor (1965) introduced a measure defined as the alpha divided by the portfolio beta. Another popular measure is the Information ratio defined as the alpha divided by the standard deviation of the portfolio residual returns.

Table 1-B
Higher moments estimators and the four-factor model: 25 stock portfolios formed on size and book-to-market equity: Market Equity (ME) group 2

BE/ME		$\bar{R}^2$	DW	GTR	DWH	OR	α	β_m	β_s	β_h	β_u	ψ_m	ψ_s	ψ_h	ψ_u
Low ranking: 1	DDHME	0.95	2.02	−0.32	9.61	8.70	−0.196	1.005	1.111	−0.214	−0.074	0.125	−0.123	−0.232	0.034
					0.050	0.070	(−1.99)	(25.68)	(14.14)	(−1.9)	(−1.77)	(2.75)	(−1.64)	(−2.1)	(0.82)
	OLS	0.95	2.01	−0.21			−0.143	1.114	0.983	−0.416	−0.051				
							(−2.01)	(61.14)	(31.69)	(−11.03)	(−1.9)				
Ranking: 2	DDHME	0.95	2.05	−0.36	43.03	2.39	−0.390	1.116	0.971	0.675	−0.031	−0.102	−0.077	−0.595	−0.033
					0.00	0.66	(−3.96)	(20.75)	(11.7)	(6.72)	(−0.75)	(−1.76)	(−0.99)	(−6.04)	(−0.72)
	OLS	0.94	1.94	−0.06			−0.056	1.024	0.868	0.157	−0.063				
							(−0.82)	(55.96)	(29.42)	(3.95)	(−2.62)				
Ranking: 3	DDHME	0.94	2.01	−0.15	61.37	3.380	−0.167	1.083	0.721	0.844	0.005	−0.118	0.088	−0.516	−0.025
					0.00	0.50	(−1.95)	(25.07)	(13.96)	(10.25)	(0.19)	(−2.42)	(1.63)	(−6.16)	(−0.71)
	OLS	0.93	1.98	0.10			0.100	0.977	0.758	0.406	−0.026				
							(1.58)	(53.87)	(22.38)	(10.71)	(−1.01)				
Ranking: 4	DDHME	0.94	1.91	−0.07	21.60	1.470	−0.077	1.016	0.734	0.856	0.016	−0.040	−0.003	−0.321	−0.023
					0.00	0.83	(−0.88)	(20.31)	(10.34)	(8.59)	(0.59)	(−0.78)	(−0.04)	(−3.27)	(−0.74)
	OLS	0.94	1.89	0.08			0.092	0.980	0.710	0.581	−0.007				
							(1.56)	(65.44)	(32.38)	(20.34)	(−0.35)				
High Ranking: 5	DDHME	0.95	1.94	−0.04	9.59	1.890	−0.040	1.082	0.854	0.902	−0.036	0.001	0.003	−0.148	0.039
					0.05	0.76	(−0.46)	(21.82)	(17.71)	(12.1)	(−1.63)	(0.02)	(0.06)	(−1.88)	(1.41)
	OLS	0.95	1.94	0.00			−0.002	1.081	0.842	0.776	−0.009				
							(−0.03)	(61.63)	(37.74)	(28.12)	(−0.57)				

Sample period: 1963:01 to 2006:02. DW, DWH and OR refer to the Durbin–Watson, Durbin–Wu–Hausman and Overidentifying Restrictions tests statistics. GTR is the Generalized Treynor Ratio. Numbers in parentheses are Student's *t*-statistics. Numbers below DWH and OR statistics are the marginal significance level of each statistic. All DDHME and OLS statistics are computed with White (1980) H_0 heteroskedasticity-consistent covariance matrix estimate HCCME.

Table 1-C
Higher moments estimators and the four-factor model: 25 stock portfolios formed on size and book-to-market equity: Market Equity (ME) group 3

BE/ME		$\bar{R}^2$	DW	GTR	DWH	OR	α	β_m	β_s	β_h	β_u	ψ_m	ψ_s	ψ_h	ψ_u
Low ranking:	DDHME	0.95	1.99	0.08	14.27	8.46	0.033	0.936	0.807	−0.459	−0.068	0.159	−0.084	−0.014	0.035
1					0.01	0.08	(0.38)	(22.58)	(14.42)	(−5.16)	(−2.31)	(3.43)	(−1.33)	(−0.15)	(0.94)
	OLS	0.95	2.02	−0.05			−0.030	1.076	0.725	−0.470	−0.042				
							(−0.41)	(54.95)	(25.11)	(−14.78)	(−2.05)				
Ranking: 2	DDHME	0.92	1.96	−0.26	31.14	1.07	−0.277	1.223	0.483	0.694	0.001	−0.186	0.077	−0.562	−0.035
					0.00	0.90	(−2.65)	(23.07)	(5.33)	(5.88)	(0.03)	(−3.11)	(0.78)	(−4.73)	(−0.8)
	OLS	0.91	1.94	0.05			0.046	1.056	0.513	0.213	−0.036				
							(0.59)	(47.65)	(12.81)	(4.87)	(−1.15)				
Ranking: 3	DDHME	0.91	1.97	−0.32	48.71	7.55	−0.380	1.142	0.417	1.005	0.025	−0.149	0.056	−0.606	−0.088
					0.00	0.11	(−3.59)	(16.82)	(4.74)	(9.31)	(0.68)	(−2.1)	(0.64)	(−5.64)	(−1.84)
	OLS	0.90	2.06	−0.02			−0.017	1.008	0.427	0.487	−0.053				
							(−0.24)	(45.85)	(10.96)	(11.74)	(−1.85)				
Ranking: 4	DDHME	0.91	1.90	−0.17	46.45	2.43	−0.180	1.050	0.226	0.916	0.079	−0.052	0.231	−0.328	−0.120
					0.00	0.66	(−1.79)	(21.24)	(3.76)	(10.31)	(1.89)	(−0.99)	(3.47)	(−3.51)	(−2.36)
	OLS	0.90	1.97	0.01			0.006	1.004	0.396	0.653	−0.027				
							(0.08)	(58.94)	(13.64)	(16.74)	(−0.84)				
High ranking:	DDHME	0.91	1.99	−0.12	27.42	8.17	−0.147	1.312	0.466	1.178	−0.095	−0.234	0.103	−0.415	0.041
5					0.00	0.09	(−1.27)	(21.55)	(5.93)	(10.79)	(−2.09)	(−3.31)	(1.27)	(−3.81)	(0.72)
	OLS	0.90	1.99	0.06			0.080	1.101	0.527	0.822	−0.070				
							(1.00)	(45.72)	(12.97)	(21.08)	(−2.86)				

Sample period: 1963:01 to 2006:02. DW, DWH and OR refer to the Durbin–Watson, Durbin–Wu–Hausman and Overidentifying Restrictions tests statistics. GTR is the Generalized Treynor Ratio. Numbers in parentheses are Student's t-statistics. Numbers below DWH and OR statistics are the marginal significance level of each statistic. All DDHME and OLS statistics are computed with White (1980) H_0 heteroskedasticity-consistent covariance matrix estimate HCCME.

Jobson and Korbie (1984), Connor and Korajczyk (1986) and Sharpe (1992, 1994) have developed multi-factor performance measures. Recently, Hübner (2005) has extended to a multi-index framework the Treynor (1965) performance measure introducing the so-called Generalized Treynor Ratio (GTR) defined as "abnormal return of a

Table 1-D
Higher moments estimators and the four-factor model: 25 stock portfolios formed on size and book-to-market equity: Market Equity (ME) group 4

BE/ME		$\bar{R}^2$	DW	GTR	DWH	OR	α	β_m	β_s	β_h	β_u	ψ_m	ψ_s	ψ_h	ψ_u
Low ranking:	DDHME	0.94	1.89	0.53	6.40	11.18	0.206	0.871	0.506	−0.524	0.031	0.207	−0.168	0.098	−0.032
1					0.17	0.02	(1.99)	(10.61)	(3.75)	(−4.64)	(0.55)	(2.44)	(−1.21)	(0.88)	(−0.53)
	OLS	0.94	1.95	0.23			0.135	1.056	0.367	−0.444	0.010				
							(1.87)	(50.41)	(9.03)	(−12.34)	(0.36)				
Ranking: 2	DDHME	0.90	1.80	−0.44	39.52	13.03	−0.530	1.329	0.170	0.747	0.035	−0.272	0.073	−0.596	−0.115
					0.00	0.01	(−4.16)	(19.87)	(1.62)	(5.44)	(0.75)	(−3.93)	(0.7)	(−4.49)	(−2.21)
	OLS	0.89	1.87	−0.12			−0.107	1.087	0.205	0.233	−0.060				
							(−1.33)	(42.22)	(5.15)	(5.04)	(−2.)				
Ranking: 3	DDHME	0.89	1.89	−0.26	31.18	4.33	−0.327	1.218	0.211	1.013	−0.011	−0.163	−0.009	−0.608	−0.046
					0.00	0.36	(−2.89)	(14.33)	(1.63)	(8.61)	(−0.25)	(−1.89)	(−0.07)	(−5.27)	(−0.87)
	OLS	0.88	1.91	0.03			0.031	1.071	0.167	0.487	−0.054				
							(0.4)	(46.2)	(4.09)	(11.49)	(−1.86)				
Ranking: 4	DDHME	0.90	1.94	0.02	20.11	5.470	0.016	0.837	0.345	0.793	0.033	0.228	−0.138	−0.204	−0.081
					0.00	0.24	(0.13)	(10.03)	(2.68)	(7.05)	(0.51)	(2.7)	(−1.06)	(−1.8)	(−1.09)
	OLS	0.89	1.99	0.08			0.089	1.040	0.207	0.620	−0.033				
							(1.11)	(44.43)	(6.7)	(16.45)	(−1.12)				
High ranking:	DDHME	0.87	2.09	−0.10	30.60	1.660	−0.120	1.172	0.198	1.056	−0.073	−0.013	0.086	−0.286	0.045
5					0.00	0.80	(−0.91)	(17.55)	(2.7)	(11.31)	(−1.21)	(−0.19)	(1.05)	(−2.86)	(0.67)
	OLS	0.86	2.02	−0.03			−0.038	1.158	0.242	0.818	−0.048				
							(−0.4)	(45.98)	(6.48)	(21.37)	(−1.58)				

Sample period: 1963:01 to 2006:02. DW, DWH and OR refer to the Durbin–Watson, Durbin–Wu–Hausman and Overidentifying Restrictions tests statistics. GTR is the Generalized Treynor Ratio. Numbers in parentheses are Student's t-statistics. Numbers below DWH and OR statistics are the marginal significance level of each statistic. All DDHME and OLS statistics are computed with White (1980) H_0 heteroskedasticity-consistent covariance matrix estimate HCCME.

Table 1-E
Higher moments estimators and the four-factor model: 25 stock portfolios formed on size and book-to-market equity: Market Equity (ME) group Big

BE/ME		$\bar{R}^2$	DW	GTR	DWH	OR	α	β_m	β_s	β_h	β_u	ψ_m	ψ_s	ψ_h	ψ_u
Low ranking:	DDHME	0.94	1.88	0.08	28.67	9.91	0.044	0.980	−0.209	−0.088	0.001	−0.020	−0.033	−0.348	−0.004
1					0.00	0.04	(0.49)	(20.21)	(−3.25)	(−1.06)	(0.04)	(−0.42)	(−0.51)	(−4.05)	(−0.15)
	OLS	0.93	1.88	0.54			0.211	0.961	−0.263	−0.387	−0.008				
							(3.65)	(56.58)	(−11.77)	(−14.06)	(−0.43)				
Ranking: 2	DDHME	0.90	2.03	−0.20	19.52	3.73	−0.199	1.108	−0.115	0.478	−0.037	−0.079	−0.107	−0.388	0.037
					0.00	0.44	(−2.17)	(23.57)	(−1.55)	(4.25)	(−0.83)	(−1.54)	(−1.49)	(−3.62)	(0.78)
	OLS	0.90	1.94	0.00			0.000	1.036	−0.231	0.136	−0.011				
							(0.)	(55.74)	(−8.83)	(3.67)	(−0.44)				
Ranking: 3	DDHME	0.85	1.87	−0.15	12.69	18.66	−0.140	1.150	−0.187	0.453	−0.076	−0.182	−0.043	−0.159	0.114
					0.01	0.00	(−0.98)	(16.67)	(−1.47)	(2.43)	(−1.02)	(−2.7)	(−0.36)	(−0.87)	(1.51)
	OLS	0.85	1.84	−0.06			−0.057	0.986	−0.231	0.305	0.013				
							(−0.74)	(43.73)	(−7.05)	(7.87)	(0.38)				
Ranking: 4	DDHME	0.89	2.09	−0.15	22.46	5.49	−0.171	0.939	−0.146	0.883	−0.053	0.062	−0.047	−0.327	0.011
					0.00	0.24	(−1.8)	(19.63)	(−2.03)	(9.03)	(−2.83)	(1.22)	(−0.62)	(−3.29)	(0.38)
	OLS	0.88	2.09	−0.04			−0.053	0.992	−0.215	0.604	−0.050				
							(−0.82)	(59.42)	(−9.54)	(18.28)	(−2.6)				
High ranking:	DDHME	0.81	2.03	−0.20	8.71	13.73	−0.269	0.909	0.126	0.976	−0.020	0.175	−0.237	−0.215	−0.036
5					0.07	0.01	(−1.9)	(14.58)	(1.13)	(5.99)	(−0.42)	(2.42)	(−2.)	(−1.33)	(−0.59)
	OLS	0.80	2.04	−0.12			−0.176	1.064	−0.092	0.782	−0.048				
							(−1.76)	(35.97)	(−2.01)	(14.64)	(−1.5)				

Sample period: 1963:01 to 2006:02. DW, DWH and OR refer to the Durbin–Watson, Durbin–Wu–Hausman and Overidentifying Restrictions tests statistics. GTR is the Generalized Treynor Ratio. Numbers in parentheses are Student's *t*-statistics. Numbers below DWH and OR statistics are the marginal significance level of each statistic. All DDHME and OLS statistics are computed with White (1980) H_0 heteroskedasticity-consistent covariance matrix estimate HCCME.

portfolio per unit premium-weighted average systematic risk normalized by the premium weighted average systematic risk of the benchmark ".[20] This ratio may be described by the following equation:

$$\mathrm{GTR}_i = \alpha^i \cdot \frac{\sum_{k=1}^{K} \beta_{bk}\overline{F}_k}{\sum_{k=1}^{K} \beta_{ik}\overline{F}_k} \tag{3}$$

where K denotes the number of risk premiums. β_{ik} and $\overline{F}_k$ are respectively risk loadings and average returns for the premiums k. The subscript b stands for the benchmark portfolio. Hübner (2005) demonstrates that the GTR is better to rank portfolios than the Jensen's α or the information ratio. Although this ratio is known to be robust to change in asset pricing models, it may nevertheless be very sensitive to measurement errors. To illustrate this point and highlight the impact of EIV, Tables 1-A–1-E reports GTR computed with DDHME and OLS.

In presence of EIV, one should expect GTR computed with and without corrections to differ. To give an illustration of the practical application of the DDHME in portfolio management, we rank the twenty-five portfolios in five categories by size as reported in Tables 1-A–1-E (from Small size to Big size). For each category, we compute a benchmark index. This benchmark index is a composite index computed by taking the equally weighted average monthly returns of each portfolio in each category.[21] To correct for EIV for the five computed indices, we run Eq. (2).[22] Then, we compute for each twenty-five portfolios a measure of GTR corrected for EIV. The results are reported in Tables 1-A–1-E in the column labelled GTR. As anticipated, our results high- light sharp contrasts between GTR estimated by OLS and GTR corrected for EIV using DDHME. This instrumental variable technique sheds a new light on the ranking of portfolios. We can rank the portfolios for each category from the worst to the best performance

[20] Definition given by Hübner (2005), p. 416.
[21] This methodology is coherent with the literature. See for example Hübner (2005) for more details.
[22] As expected, EIV are detected and we correct for the problem using the DDHME. Detailed results are available from authors upon request.

measure. If we compare the ranking from OLS with the one with DDHME, we note significant differences. For categories one to five respectively, the ranking of portfolios are {1, 2, 3, 5, 4}, {1, 2, 5, 4, 3}, {1, 3, 4, 2, 5}, {2, 5, 3, 4, 1}, and {5, 3, 4, 2, 1} with OLS compared to {1, 3, 4, 2, 5}, {2, 1, 3, 4, 5}, {3, 2, 4, 5, 1}, {2, 3, 5, 4, 1}, and {5, 2, 4, 3, 1} with DDHME. Measurement errors can therefore alter seriously the ranking established by portfolio managers ignoring the problem of EIV.

6. Conclusions

The objective of this paper was to revisit an often forgotten and neglected puzzle in financial economics, namely, the consequences of errors-in-variables for parameter estimates of linear asset pricing models and performance measures based on these models. The paper suggests that Dagenais and Dagenais (1997) instrumental variables estimator, based on sample moments of order higher than two, could be appropriate for linear asset pricing models. To illustrate this point, the paper reports estimation results of the popular Fama and French (1992, 1993) and Carhart (1997) four-factor model. The dataset is taken directly from Kenneth French's website and covers the excess returns of twenty-five value-weighted portfolios. The results show that Dagenais and Dagenais estimator detects systematically the presence of significant and quantitatively important errors-in-variables for all four factors considered. This casts serious doubts on the consistency and unbiasness of parameter estimates of linear asset pricing models obtained with OLS.

Our results shed a new light on α and β used in financial economics and by the financial industry. The message for investors and speculators is clear; in presence of errors-in-variables returns are more abnormal than they may think. The quest of *the holy grail of active investment* (Leibowitz (2005)) may be full of traps. As illustrated by the Generalized Treynor Ratio, the ranking of portfolios can be seriously altered when errors-in-variables problems are treated with DDHME. Measurement errors problems are not purely theoretical. They have practical and financial consequences. The message of the paper is not that the Fama and French model is dead but that it could be improved by the use of DDHME estimators.

Our approach could also be seen as a parsimonious alternative to asset pricing models with higher moments. It would be interesting to focus on the real sources of premiums related to skewness, coskewness and Kurtosis, actually so often plugged in linear asset pricing models. Their real utility may find justification in detection of presence of errors in variables. We leave this point for future research.

Appendix

This appendix gives more details on the econometric method used in Tables 1-A–1-E. The starting point is the matrix form of Eq. (1).

$$R = X \cdot \delta + \varepsilon, \qquad (\mathrm{A}-1)$$

where $X=(\iota\ F)$ is the factors matrix $F=(\mathrm{Mkt\ Smb\ Hml\ Umd})$ augmented with a N unit elements vector ι and $\delta=(\alpha\ \beta_m\ \beta_s\ \beta_h\ \beta_u)^T$.

Let $P_Z=Z\ (Z^T\ Z)^{-1}Z^T$ and $M_Z=I-Z\ (Z^T\ Z)^{-1}Z^T$ denote the matrix that project orthogonally onto and off the subspace spanned by the column of matrix Z. Following Davidson and MacKinnon (1993)'s suggestion, DDHME reported in Tables 1-A–1-E are obtained by applying OLS to the artificial regression (2) rewritten here in matrix form

$$R = X \cdot \delta + M_z F \cdot \psi + e, \qquad (\mathrm{A}-2)$$

where $M_Z F$ are the residuals of the first stage regressions of Fama and French factors on the instruments Z. The instruments used are enumerated in Section 2.

By the Frisch–Waugh–Lovell Theorem, OLS estimates of δ and ψ in Eq. (A-2) are

$$\tilde{\delta} = \left(X^T P_Z X\right)^{-1} X^T P_Z R \qquad (\mathrm{A}-3)$$

and

$$\tilde{\psi} = \left(F^T M_Z M_X M_Z F\right)^{-1} F^T M_Z M_X R \qquad (A-4)$$

Note that estimates (A-3) are numerically identical to IV estimates of δ in the original model (A-1). The Student's *t*-statistics reported in the tables are computed with White (1980) heteroskedasticity-consistent covariance matrix estimators (HCCME) $V(\tilde{\delta})$ and $V(\tilde{\psi})$. The exact formula used are

$$V\left(\tilde{\delta}\right) = \left(X^T P_Z X\right)^{-1}\left(X^T P_Z \tilde{\Omega} P_Z X\right)\left(X^T P_Z X\right)^{-1} \qquad (A-5)$$

and

$$V\left(\tilde{\psi}\right) = \left(F^T M_Z M_X M_Z F\right)^{-1}\left(F^T M_Z M_X \tilde{\Omega} M_X M_Z F\right)\left(F^T M_Z M_X M_Z F\right)^{-1}. \qquad (A-6)$$

Where $\tilde{\Omega}=\text{diag}\ (\tilde{\varepsilon}_1^2,\ldots,\tilde{\varepsilon}_N^2)$ and $\tilde{\varepsilon}_i=R_i-X_i\cdot\tilde{\delta}$ are the residuals of Eq. (A-1) computed with estimates (A-3). The HCCME used in Eqs. (A-5) and (A-6) are often denoted by HC_0.

The DWH test reported is also a heteroskedasticity-robust test. Using White (1980) HCCME, the hypothesis $\tilde{\psi}=0$ yields the statistic

$$\text{DWH} = \tilde{\psi}^T \left(F^T M_Z M_X M_Z F\right)\left(F^T M_Z M_X \tilde{\Omega} M_X M_Z F\right)^{-1}\left(F^T M_Z M_X M_Z F\right)\tilde{\psi} \qquad (A-7)$$

which is, under H_0, asymptotically distributed as a χ_q^2 where $q=4$ is the number of elements in ψ.

References

Agarwal, Vikas, Naik, Narayan Y., 2004. Risks and portfolio decisions involving hedge funds. Review of Financial Studies 17 (1), 63–98.

Aigner, Dennis J., Hsiao, Cheng, Kapteyn, Arie, Wansbeek, Tom, 1984. Latent variable models in econometrics. Handbook of Econometrics, vol. 2. Elsevier.

Black, Fischer, 1993. Beta and return. Journal of Portfolio Management 20 (1), 8–18.

Bowden, Roger J., Turkington, Darrell A., 1984. Instrumental variables. Cambridge University Press.

Carhart, Mark M., 1997. On persistence in mutual fund performance. Journal of Finance 52 (1), 57–82.

Chen, Nai-Fu, Roll, Richard, Ross, Stephen A., 1986. Economic forces and the stock market. Journal of Business 59 (3), 383–403.

Connor, Gregory, Korajczyk, Robert A., 1986. Performance measurement with the arbitrage pricing theory: a new framework for analysis. Journal of Financial Economics 15 (3), 373–394 available at http://ideas.repec.org/a/eee/jfinec/v15y1986i3p373-394.html.

Cragg, John G., 1994. Making good inferences from bad data. Canadian Journal of Economics, 27 (4), 776–800.

Cragg, John G., 1997. Using higher moments to estimate the simple errors-in-variables model. RAND Journal of Economics 28 (0), S71–S91.

Dagenais, Marcel G., Dagenais, Denyse L., 1997. Higher moment estimators for linear regression models with errors in the variables. Journal of Econometrics 76 (1–2), 193–221.

Davidson, Russell, MacKinnon, James G., 1993. Estimation and Inference in Econometrics. Oxford University Press, New York.

Durbin, James, 1954. Errors in variables. International Statistical Review 22, 23–32.

Fama, Eugene F., MacBeth, James D., 1973. Risk, return, and equilibrium: empirical tests. Journal of Political Economy 81 (3), 607–636.

Fama, Eugene F., French, Kenneth R., 1992. The cross-section of expected stock returns. Journal of Finance 47 (2), 427–465.

Fama, Eugene F., French, Kenneth R., 1993. Common risk factors in the returns on stock and bonds. Journal of Financial Economics 33 (1), 3–56.

Fama, Eugene F., French, Kenneth R., 1995. Size and book-to-market factors in earnings and returns. Journal of Finance 50 (1), 131–155.

Fama, Eugene F., French, Kenneth R., 1996. Multifactor explanations of asset pricing anomalies. Journal of Finance 51 (1), 55–84.

Fama, Eugene F., French, Kenneth R., 1997. Industry costs of equity. Journal of Financial Economic 43 (2), 153–193.

Fama, Eugene F., French, Kenneth R., 1998. Value versus growth: the international evidence. Journal of Finance 53 (6), 1975–1999.

Fama, Eugene F., French, Kenneth R., 2002. The equity premium. Journal of Finance 57 (2), 637–659.

Frisch, Ragnar, 1934. Statistical confluence analysis by means of complete regression systems. Publication No. 5, University Institute of Economics, Oslo.

Fuller, Wayne A., 1987. Measurement error models. Wiley Series in Probability and Mathematical Statistics.

Hausman, Jerry A., Watson, Mark W., 1985. Error-in-variable and seasonal adjustment procedures. Journal of the American Statistical Association 80, 531–540.

Hübner, Georges, 2005. The generalized Treynor ratio. Review of Finance 9 (3), 415–435.

Jensen, Michael C., 1967. The performance of mutual funds in the period 1945–1964. Journal of Finance 23 (2), 389–416.

Jobson, J.D., Korbie, B.M., 1984. A note on the Jensen measure and marginal improvements in portfolio performance. Journal of Finance 39 (1), 245–251.

Kandel, Shmuel, Stambaugh, Robert F., 1995. Portfolio inefficiency and the cross-section of expected returns. Journal of Finance 50 (1), 157–184.
Klepper, Steven, Leamer, Edward E., 1984. Consistent sets of estimates for regressions with errors in all variables. Econometrica 52 (1), 163–183.
Kothari, S.P., Shanken, Jay, Sloan, Richard G., 1995. Another look at the cross-section of expected stock returns. Journal of Finance 50 (1), 185–224.
Kraus, Alan, Litzenberger, Robert. H., 1976. Skewness preference and the valuation of risk assets. Journal of Finance 31 (4), 1085–1100.
Leamer, Edward E., 1987. Errors in variables in linear systems. Econometrica 55 (4), 893–909.
Leibowitz, Martin A., 2005. Alpha hunters and beta grazers. Financial Analysts Journal 61 (5), 32–39.
Lewbel, Arthur, 2006. Using heteroskedasticity to identify and estimate mismeasured and endogenous regressor models. Boston College.
Pal, Manoranjan, 1980. Consistent moment estimators of regression coefficients in the presence of errors in variables. Journal of Econometrics 14 (3), 349–364.
Roll, Richard, 1977. A critique of the asset pricing theory's tests part I: On past and potential testability of the theory. Journal of Financial Economics 4 (2), 129–176.
Shanken, Jay, 1992. On the estimation of beta-pricing models. Review of Financial Studies 5 (1), 1–33.
Sharpe, William F., 1966. Mutual fund performance. Journal of Business 39 (1), 119–138.
Sharpe, William F., 1992. Asset allocation: management style and performance measurement. Journal of Portfolio Management 18, 7–19.
Sharpe, William F., 1994. The Sharpe ratio. Journal of Portfolio Management 21, 49–58.
Treynor, J.L., 1965. How to rate management investment funds. Harvard Business Review 43, 63–75.
White, Halbert, 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48 (4), 817–838.