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A Bayesian view of temporary components in asset prices ☆

Bjørn Eraker

Department of Economics, Duke University

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Abstract

This paper studies models in which the a stock price contains a random walk and a stationary component, as in Fama and French [Fama, Eugene F., and Kenneth R. French, 1988, Permanent and Temporary Components of Stock Returns, Journal of Political Economy 96, 246–273.] and Poterba and Summers [Poterba, James, and Lawrence Summers, 1988, Mean Reversion in Stock Prices: Evidence and Implications, Journal of Financial Economics 22, 27–59.]. We extend this model to allow for two latent factors which generate short term and long term autocorrelations, respectively. To facilitate econometric identification, we assume that these factors are common across multiple asset returns, and we estimate the factor loadings. In an application to size and book/market sorted portfolios, we find the short term factor economically and statistically insignificant. Estimates of parameters relating to the long range component suggest that portfolios of small firm stock display about three times the amount of mean reversion than for large firm stocks. Overall, the evidence suggests that mean reversion is largely a small firm phenomenon. The evidence is consistent with dynamic equilibrium models in which asset prices co-integrate with aggregate consumption or dividends.

JEL classification: G12; G14; C11; C32
Keywords: Predictability; Long run reversal; Kalman filter

1. Introduction

Few topics in finance have been debated as heavily as the random walk hypothesis of security prices. There is no doubt that a large fraction of the daily or annual stock market return is unpredictable, as the random walk hypothesis would suggest. While it is understood that asset returns contain a very small predictable component at best, interest in the issue persists because even small amounts of predictability could prove valuable to asset managers.

In the debate over predictability, it is widely recognized that there exists time series such as price-multiples, interest rates and other macro variables that correlate significantly with future asset returns using standard methods of statistical inference. The results are still controversial because “standard methods of inference” do not adjust for the fact that these variables are selected out of a larger set of variables which leads to an overstated level of statistical significance (“data-snooping bias”). A second strand of the literature focuses on serial correlations. Results presented in Fama and French

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E-mail address: bjorn.eraker@duke.edu.

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(1988) in support of the predictability hypothesis, showing seemingly significant long run autocorrelations consistent with a model in which stock prices contain a random walk, and a stationary component. This evidence is widely contested. Richardson (1993) shows that the Fama–French analysis understate the standard errors in long term autocorrelations. Also, the infamous U shaped pattern in the autocorrelation function disappears once the highly volatile 1926 to 1940 period is removed from the data set. Fama (1991) concludes that the tests based upon the autocorrelation function have low power due to the effectively small sample sizes for long horizon correlations.

The lack of power in autocorrelation based tests poses a challenge which cannot be resolved unless the econometrician is willing to impose parametric assumptions about the functional form of the autocorrelation function. This can be accomplished by assuming a particular form of the dynamic model that generates asset returns. For example, in a model in which prices and dividends are cointegrated, as is the case in many dynamic equilibrium models, stock prices follow the stationary component model studied by Fama and French (1988) and Poterba and Summers (1988). The model (henceforth the Shiller–Summers model as coined in Fama (1991)) has only four parameters which jointly describe the autocorrelation function.

A generalized version of the Shiller–Summers model is estimated in Lamoureux and Zhou (1996). Their paper studies a model in which the stationary price component follows an AR(4) process. In a Bayesian analysis of the structural model, Lamoureux and Zhou conclude that the data contain little or no evidence of a predictable component. Their analysis also reveal that the data are very uninformative about the structural parameters. Many of the autoregressive coefficients that drive the stationary price component have posterior standard deviations which are almost indistinguishable from the prior standard deviations.

In this paper we study a generalized version of the Shiller–Summers model. We generalize the model in two important ways. First, the price process has three components: A permanent (random walk) shock, a long run mean reverting component and a short term predictable component. Second, the three factor structure makes the model very hard to identify from univariate time series data. To overcome the identification problem, we propose a model structure in which asset returns are generated in a linear factor form: the returns on all assets are linear functions of a short term AR(1) process and a long range differenced AR(1) process, as well as asset specific shocks. The coefficients relating the individual asset returns to the common factors are estimated jointly with the parameters governing the dynamics of the state variables. The class of models considered here are perfectly consistent with dynamic equilibrium models in which the time-variation in which the price responds negatively to a temporary increase in risks.

The hypothesis that prices contain a permanent, random walk component and a mean reverting component is perfectly consistent with equilibrium models. In particular numerous models produce equilibrium price/dividend dynamics of the form $P_t/D_t = G(F_t)$ where G is some nonlinear function of a set of state variables, F_t. The state variables are typically related to time varying risks. For example, a model with time-varying volatility of the intertemporal marginal rate of substitution will generate this type of dynamics with F_t equal to the volatility factor. Under additional assumptions, $G()$ can be made to be exponential affine or approximately exponential affine. This is the case in Campbell and Shiller (1987, 1988), Bansal and Yaron (1997), Bansal, Dittmar, and Lundblad (2005), Eraker (2006), among others. For these models, the equilibrium log price equals the log dividend plus the stationary component F_t and thus produce equilibrium price dynamics which is of the form considered in this paper. The presence of a predictable component of asset returns is thus perfectly consistent with equilibrium pricing. In fact, if stock prices exhibit pure random walk they would not be consistent with the dynamic equilibrium models in the above referenced papers.

A summary of the results obtained in this paper are as follows. First, we fit the Shiller–Summers model to size and book sorted portfolios using univariate specification. The parameters are quite well identified using the uniform prior distribution. Using the univariate model specification, we find moderate amounts of predictability. The predictable component is not deemed statistically significant. The lack of precision in parameter estimates, and thus also the latent extracted factor, lead to a significant posterior probability mass on the possibility that returns contain essentially only a random walk component. In moving to our multivariate generalization of the Shiller–Summers model, we obtain much sharper estimates of the parameters of interest. The results show some interesting patterns. Predictability is statistically significant at the three to ten year horizon for portfolios of small firms. Large firm portfolios, on the other hand, all contain insignificant amounts of predictability. The “factor loadings” increase almost uniformly along the size dimension.

Our multivariate model specification also contain a common factor designed to capture predictable variation in returns at short horizons. Specifically, the “short term factor” is an AR(1) process that enter linearly in the return generating process. This is consistent with the equilibrium specifications in Bekaert and Harvey (1995), Brandt and

Kang (2004), and Johannes, Polson, and Stroud (2004), among others. Our analysis does not find that this short term factor generates significant predictability in returns: The autoregressive coefficient that determine the amount of predictability has most of its mass near zero. Interestingly, the factor loadings associated with this short term component are widely dispersed along the book/market dimension consistent. This is consistent with permanent differences in risks across book/market assets.

The remainder of the paper is organized as follows. Section 2 discusses the model setup, introduces a generalization of the basic two component model and discusses estimation. Section 3 presents the empirical evidence and Section 4 concludes.

2. Model

2.1. Basic model

Let p_t be the log value of a portfolio in which dividend payments are re-invested such that $r_t = p_t - p_{t-1}$ is the log-return. The model studied in Fama and French (1988), Poterba and Summers (1988) is

$$p_t = q_t + z_t, \tag{1}$$

$$q_t = q_{t-1} + \mu + \sigma_\eta \eta_t, \tag{2}$$

$$z_t = \phi z_{t-1} + \sigma_\epsilon \epsilon_t. \tag{3}$$

Likelihood inference for this model is possible under distributional assumptions $\eta_t, \epsilon_t \sim iid.N\,(0, 1)$. The properties of the model are well documented, but we repeat some basic features here for completeness. The long run expected rate of return is μ. The process z_t is unobserved by the econometrician. The expected T period return at time t conditional upon the value of z_t, is $E_t(p_{t+T} - p_t) = E_t(z_{t+T} - z_t) + \mu T = (\phi^T - 1)z_t + \mu T$.

Thus, we have that the expected rate of return is less than the unconditional expected return μ whenever $z_t > 0$, and vise versa. In other words, when the value of the unobserved component z_t exceeds its average long term value of zero, the expected rate of return drops below its long term mean μ. The speed of mean reversion in the stationary component, ϕ, determines the shape of the autocorrelation function. For small values of ϕ, the model displays short term negative autocorrelation, while values of ϕ close to unity lead to long term reversals. A unit value of ϕ implies that the prices follow a random walk.[1]

The properties of the Shiller–Summers model are well documented. Fama and French (1988) derive the autocorrelation function. In the following, we provide some additional insights into the dynamic behavior of the model. Define

$$R^2(T) = \frac{\mathrm{Var}(E_t(p_{t+T} - p_t))}{\mathrm{Var}(p_{t+T} - p_t)}$$

to be the fraction of variance of the expected rate of return to the total variance. The definition follows the standard definition of an R-square from linear regression. We can show that

$$R^2(T) = \frac{(\phi^T - 1)^2}{\phi^{4T} - 3\phi^{2T} + 2 + \frac{\sigma_\eta^2}{\sigma_\epsilon^2} T(1 - \phi^2)}. \tag{4}$$

Fig. 1 illustrates how $R^2(T)$ changes with different values of the key parameters ϕ and σ_ϵ. The left plot keeps $\sigma_\epsilon = \sigma_\eta = 0.2$ (fixed) and illustrates that the R^2 is increasing and/or decreasing over various values of ϕ. A value of ϕ close to zero always implies more predictability at the short horizon. The R^2 approaches zero as ϕ approaches unity —

[1] In this case, we cannot separately identify σ_η and σ_ϵ. That is, the likelihood function has a ridge at $\sigma_\eta^2 + \sigma_\epsilon^2 = \hat{\sigma}^3$ where the latter is the ML estimator for σ^2 under the assumption $r_t \sim N(\mu, \sigma^2)$. Notice that even though Maximum likelihood estimators for the two variance components are not well defined when $\phi = 1$, Bayesian posterior distributions are well defined because the posterior distribution of $\sigma_\eta^2 + \sigma_\epsilon^2$ is proper on $\mathbb{R}_+^2$.

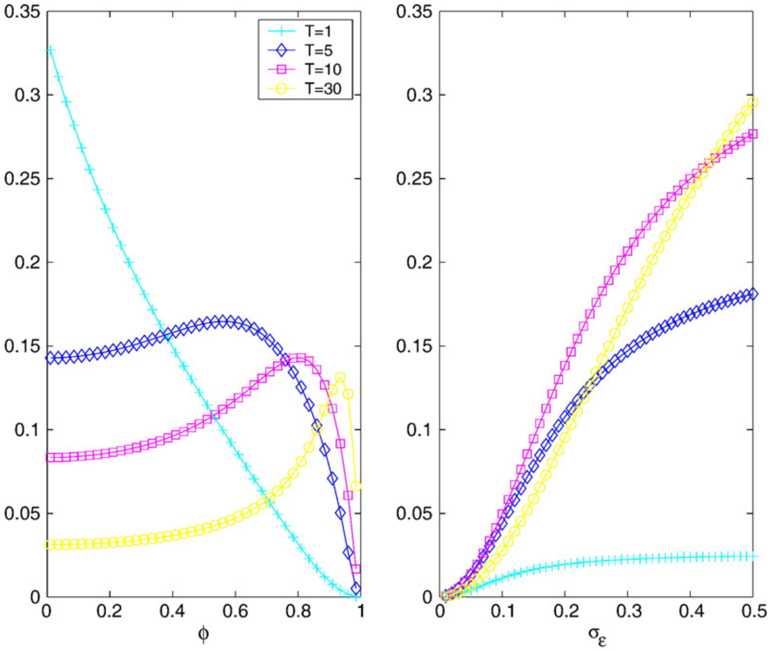

Fig. 1. The figure shows $R^2(T)$'s as functions of ϕ (left) and σ_ϵ for various forecast horizons.

the case in which returns are independent. Note that the R^2's may be large at longer forecasting horizons even when ϕ is fairly close to unity. This is true for relatively large values of σ_ϵ. The right plot in Fig. 1 illustrates that the R^2 approaches zero as σ_ϵ approaches zero. Thus, even with the prior on $p(\phi) \sim U(-1, 1)$ which necessarily leads to an estimate of ϕ strictly less than unity, a low value of σ_ϵ will imply little or no predictability.

2.2. Multivariate generalization

The development of a multivariate generalization of the Campbell–Shiller model is motivated by the lack of conclusive empirical evidence from the univariate model. Estimates of the posterior distributions of the relevant model parameters ϕ, σ_ϵ and σ_η reported in the next section are sufficiently dispersed that when mapped into the corresponding posterior for R^2, it becomes clear that the posteriors for the R^2's place imply a substantial probability of no predictability.

In the following we consider a model that maintains the basic structure of the Campbell–Shiller model, but is significantly more parsimonious. This model is

$$r_t = \beta \Delta z_t + \gamma f_t + \mu + \eta_t, \qquad \eta_t \sim N(0, \Omega), \tag{5}$$

$$z_t = \phi z_{t-1} + \sigma_\epsilon \epsilon_t, \qquad \epsilon_t \sim N(0, 1), \tag{6}$$

$$f_t = \kappa f_{t-1} + \sigma_f e_t, \qquad e_t \sim N(0, 1), \tag{7}$$

where r_t is a vector of N returns. β, γ are length N vectors containing "factor loadings" associated with the latent factors z_t and f_t.

The covariance matrix Ω of asset specific shocks to returns is modelled as

$$\Omega = DRD$$

where D is matrix with diagonal element i, i given by σ_i, and hence, D contains asset specific standard deviations. R is a correlation matrix. Modelling the correlation matrix R constitutes a significant challenge because the number of parameters needed to model pairwise, unrestricted correlations grow quadratically in the number of assets. For example, for the twenty five Fama–French five by five size/book sorted portfolios we would need three hundred correlation coefficients. This would likely be an unfeasible modelling task. Lack of parsimony in the modelling of Ω would aversely affect the identification of the structural parameters. For this reason, we simply assume that the correlation between all shocks to the assets is the same, determined by a parameter ρ which is to be estimated.

The multivariate model differs from the univariate model(s) in several important ways. First, and most important, it restricts the latent z_t factor to be the same across portfolios. Second, it has a second factor f_t which potentially pics up short term predictable variation. This constitutes a major difference from the basic Shiller–Summers model. The model still maintains the basic feature of the Shiller–Summers model in that prices contain a stationary, predictable component as well as a random walk component. Third, the factor loadings differ across portfolios. This generates higher levels of predictability for assets with high β's. On the other hand, it does restrict the ϕ's to be identical across those portfolios. Finally, this model allows for the contemporaneous correlation structure in the exogenous shocks, í$_t$.

We can derive R^2, as defined above. Some algebra produces

$$R^2(T) = \frac{\gamma^2 V_e + \beta^2 Q_e}{\gamma^2 V_T + \beta^2 Q_T + T\sigma_\eta^2} \tag{8}$$

$$Q_e = \frac{(\phi^{2T}-1)^2}{(1-\phi^2)}\sigma_\epsilon^2 \tag{9}$$

$$Q_T = (\phi^{4T} - 3\phi^{2T} + 2)\frac{\sigma_\epsilon^2}{1-\phi^2} \tag{10}$$

$$V_e = \frac{\sigma_f^2\kappa^2(1-\kappa^{2T})}{(1-\kappa^2)^2} \tag{11}$$

$$V_T = \frac{\sigma_f^2(\kappa^2 T - 2\kappa^{T+2} + \kappa^{2(1+T)} + \kappa^2 - 2\kappa^{T+1} + 2\kappa - T)}{(\kappa+1)(\kappa-1)^3}. \tag{12}$$

The expression for the R^2 consists for four parts where Q_e and Q_T measure the expected and total variation in the z_t component, respectively. Similarly, V_e and V_T measure the expected, total variation in f_t. The asset specific parameters γ and β measure the sensitivity of a particular asset to the two predictable components. When these coefficients are zero, there is no predictability and $R(T)=0$ for all T.

2.3. Estimation

Estimation of the models considered here is straightforward. Indeed, the simple linear form leads to an easy adaptation of Kalman filtering techniques, as discussed below. This is one of the main advantages of the model setup here, as opposed to, say, a generalized model which allows for stochastic volatility. Stochastic volatility models require numerical integration techniques which are computationally costly.

A standard multivariate unobserved components model take on the form

$$y_t = h' x_t + e_t \tag{13}$$

$$x_t = F x_{t-1} + u_t \tag{14}$$

where h, F and Cov$(e_t)=R$ and Cov$(u_t)=gg'=G$ are parameters. To adapt our model to this form, we use the following variable definitions

$$x_t = (z_{t-1}, z_t, f_t), \qquad F = (\beta, -\beta, \gamma),$$

$$h = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & \kappa \end{pmatrix}, \qquad g = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\epsilon & 0 \\ 0 & 0 & \sigma_f \end{pmatrix}.$$

The Kalman filtering recursions are as follows

$$\hat{\hat{x}}_t = Fx_{\hat{t}-1}, \hat{\hat{P}}_t = F'P_{\hat{t}-1}F + gg', C_t = h'\hat{\hat{P}}h + R, \hat{e}_t = y_t - \mu - h'\hat{\hat{x}}_t, K = \hat{\hat{P}}_t h(h'\hat{\hat{P}}_t h + R)^{-1}, \hat{x}_t = \hat{\hat{x}}_t + K\hat{e}_t, \hat{P}_t = \hat{\hat{P}}_t - Kh'\hat{\hat{P}}_t.$$

The likelihood function can be computed $l = \sum_t -.5\log \mid C_t \mid -.5\hat{e}_t{}'C_t^{-1}\hat{e}_t$. We employ a Metropolis Hastings sampler to collect random draws from the posterior distributions under a uniform prior on the parameter space.

2.3.1. Identification

The multivariate model specification in Eq. (5) is, as written, not identified. To see why this is the case, suppose we change σ_ϵ^2 by a factor of two as well as multiplying β by one half. The likelihood function will remain unchanged as a result of having re-scaled the parameters. Thus, we really cannot separately identify all the factor loadings at the same time as the factor variances. Also note that this re-scaling of the parameters leave the R^2 in Eq. (8) unchanged since β^2 and σ_ϵ^2 enter as a product in Eq. (8).

Table 1
Parameter estimates for the univariate model

	Low	2	3	4	High
ϕ					
Small	0.84	0.86	0.88	0.89	0.89
	(0.11)	(0.09)	(0.09)	(0.09)	(0.09)
2	0.91	0.94	0.94	0.93	0.90
	(0.07)	(0.06)	(0.05)	(0.05)	(0.07)
3	0.93	0.95	0.95	0.94	0.91
	(0.05)	(0.04)	(0.04)	(0.04)	(0.06)
4	0.95	0.95	0.94	0.95	0.92
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)
Large	0.95	0.96	0.94	0.93	
	(0.04)	(0.03)	(0.04)	(0.05)	
σ_η					
Small	0.22	0.21	0.17	0.16	0.16
	(0.10)	(0.09)	(0.09)	(0.08)	(0.08)
2	0.16	0.14	0.13	0.14	0.16
	(0.08)	(0.06)	(0.06)	(0.07)	(0.07)
3	0.14	0.12	0.11	0.12	0.16
	(0.07)	(0.05)	(0.05)	(0.05)	(0.07)
4	0.10	0.10	0.11	0.12	0.16
	(0.04)	(0.04)	(0.05)	(0.06)	(0.07)
Large	0.07	0.06	0.09	0.11	
	(0.03)	(0.03)	(0.04)	(0.05)	
σ_ϵ					
Small	0.20	0.16	0.18	0.17	0.17
	(0.11)	(0.09)	(0.09)	(0.09)	(0.09)
2	0.16	0.14	0.13	0.15	0.14
	(0.08)	(0.07)	(0.06)	(0.07)	(0.07)
3	0.15	0.12	0.11	0.11	0.15
	(0.07)	(0.06)	(0.06)	(0.06)	(0.08)
4	0.09	0.09	0.10	0.12	0.15
	(0.05)	(0.05)	(0.05)	(0.06)	(0.08)
Large	0.06	0.05	0.08	0.10	
	(0.04)	(0.03)	(0.05)	(0.05)	

The table reports posterior means and standard deviations (in parenthesis), for parameters in the latent fundamentals model for Fama–French 5×5 size/book sorted portfolios. Results are based on annual portfolio returns from 1927–2004.

To facilitate estimation, therefore, we need to impose parametric constraints. There are two options. We can constrain the β's and γ's in some way, for example by setting $\sum_i^N \beta_i = 1$ and $\sum_i^N \gamma_i = 1$ across the N assets. This leaves σ_ϵ and σ_f as free parameters. A second possibility is to set σ_f and σ_ϵ at some fixed value. We choose the restrictions $\sigma_f = 1$ and $\sigma_\epsilon = \sqrt{(1-\phi^2)/(2-2\phi)}$ which will give $\mathrm{Var}(z_t) = 1$ unconditionally. Note that these restrictions do not in any way alter the possible set of posterior distributions for R^2's since the re-scaling between γ/σ_f and β/σ_ϵ is arbitrary. The parametric constraints only apply to the multivariate model in Eq. (5) and not the univariate Campbell–Shiller model.

Since our analysis is Bayesian, it requires the specification of a prior distribution on the parameter space. We specify a uniform prior on the parameter-space defined in the following way. The autoregressive parameters ϕ and κ are restricted to lie $(-1, 1)$. Bounding the parameters away from the non-stationarity region ensures that the likelihood function, and thus the posterior, is well defined. All variance parameters have improper uniform priors on $(0, \infty)$, ensuring positivity, while the remaining parameters, β, γ and μ have improper, uninformative, uniform priors on $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$.

3. Empirical results

Table 1 reports posterior means and standard deviations for the univariate model in Eqs. (1)–(3). As can be seen from the table, estimates of the mean reversion coefficients, ϕ, range from 0.84 to 0.96 across portfolios. Estimates of σ_ϵ range from 0.05 to 0.22 while σ_η estimates range from 0.06 to 0.2. It is interesting to note that all parameter estimates appear to increase or decrease almost uniformly in the size dimension. The amount of total variation, and also predictable variation, thus appear to vary strongly across portfolios of different capitalization.

Table 2 shows that R^2's for the univariate model are virtually zero at the short forecasting horizon, and in the 3–9% range at the 10 year horizon. There is a fair amount of posterior variability in all estimated parameters. For example, the posterior standard deviation of the variation in the random walk component, σ_η, is typically a little less than half that of the posterior mean. This translates into a significant posterior uncertainty about what fraction of the variation in stock returns can be attributed to predictable variation. Underscoring this point, Fig. 2 depicts the posterior 2.5, 50 and 97.5 percentiles of the $R^2(T)$'s. To conserve space, the plot shows results for the lowest, medium, and largest size and book deciles. The results are representative for the omitted portfolios' R^2. As can be seen the line depicting the 2.5% lower posterior percentile of the R^2 distribution appears visually indistinguishable from the horizontal axis in the plots. This indicates that there well might be an insignificant amount of predictable variation in the data. On the

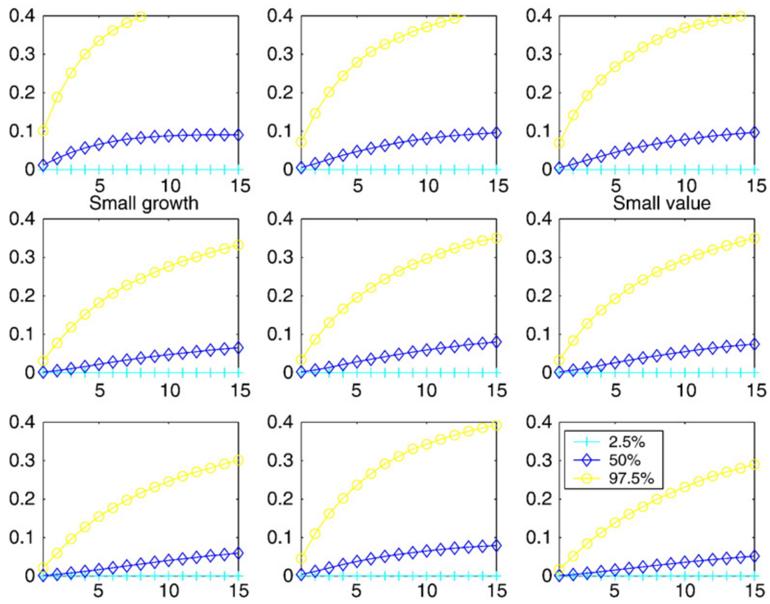

Fig. 2. Pointwise posterior quantiles of $R^2(T)$. Univariate model.

Table 2
Univariate model R^2

	Low	2	3	4	High
1 year					
Small	0.01	0.01	0.01	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
Large	0.00	0.00	0.00	0.00	
5 year					
Small	0.07	0.04	0.05	0.04	0.04
2	0.04	0.02	0.02	0.03	0.03
3	0.03	0.02	0.02	0.02	0.04
4	0.02	0.02	0.02	0.02	0.03
Large	0.02	0.01	0.02	0.02	
10 year					
Small	0.09	0.06	0.08	0.07	0.08
2	0.07	0.05	0.04	0.06	0.06
3	0.05	0.05	0.04	0.05	0.07
4	0.03	0.04	0.04	0.04	0.05
Large	0.03	0.03	0.04	0.05	

other hand, the upper 97.5 percentile of the distributions in some cases (small portfolios) exceed 40% indicating that there indeed *could* be a very significant amount of predictable variation in the returns. We conclude, therefore, that posterior distributions obtained for the respective parameters in the univariate Shiller–Summers model are not informative enough to render a sharp conclusion about the amount of predictable variation in annual stock returns.

In assessing the reason why our analysis fail to detect significant amounts of predictability, Fig. 3 depicts empirical histograms of the posterior parameter draws for the middle sized/BM portfolio. The others are have identically distributed parameters. The posteriors histograms indicate that ϕ has significant posterior mass near unity, and σ_ϵ has significant mass near zero. Both observations indicate predictable variation is statistically insignificant. On the other

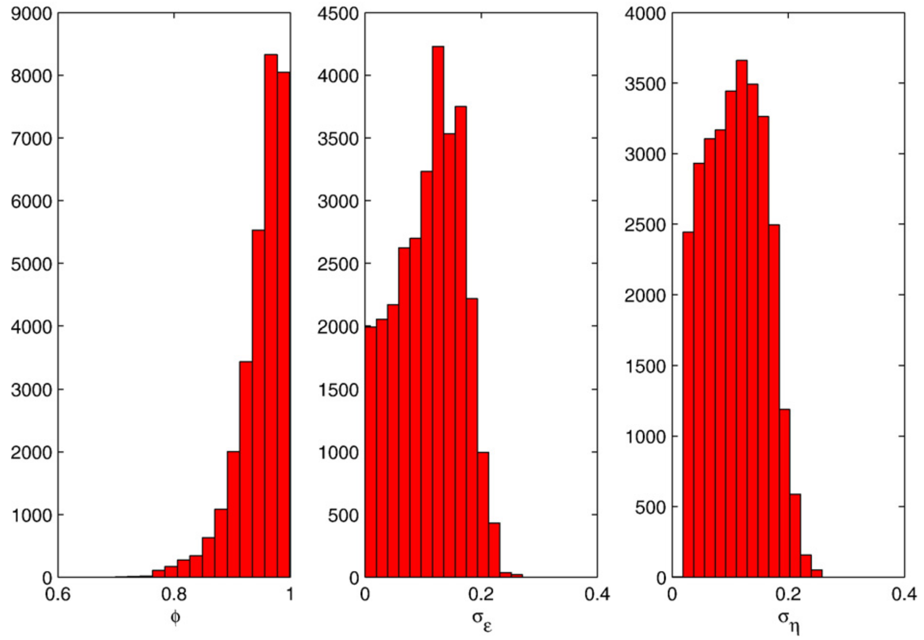

Fig. 3. Posterior histograms for univariate model parameters.

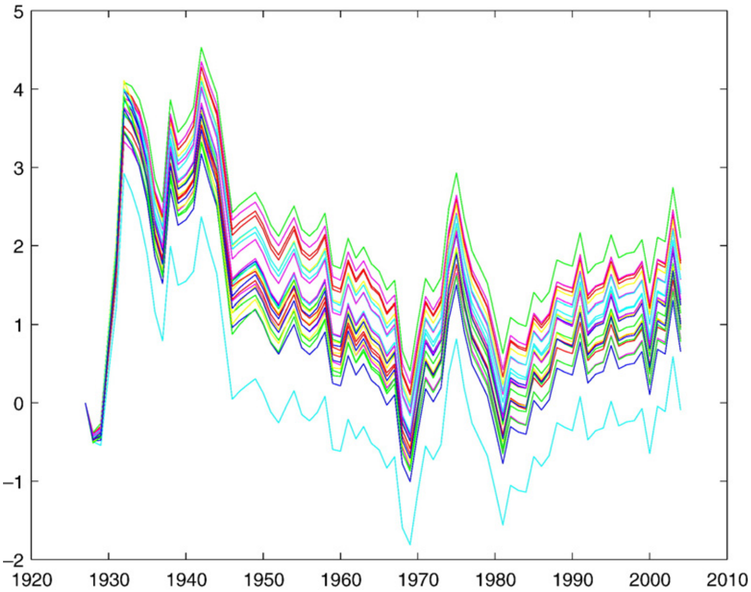

Fig. 4. Latent state-variables z_t from univariate model, standardized by its sample standard deviation.

hand, σ_η also has mass near zero. Since all variation in the stock price is forecastable when σ_η is zero, the estimated model parameters also place significant posterior mass on the possibility of a large predictable component.

The multivariate generalization of the Shiller–Summers model studied below is based on the assumption that prices contain a single, common predictable component. The implication of this modelling assumption is that when we estimate a similar model, such as the univariate Shiller–Summers model, the extracted latent factor z_t should look similar across the respective assets. Fig. 4 depicts the filtered state variables from the respective univariate models. As

Table 3
Multivariate model parameter estimates

	ϕ		κ	ρ	
	0.838 (0.088)		−0.074 (0.167)	0.899 (0.030)	
	Low	2	3	4	High
β					
Small	0.273	0.315	0.311	0.319	0.317
2	0.245	0.230	0.240	0.255	0.276
3	0.204	0.201	0.201	0.210	0.250
4	0.124	0.167	0.172	0.190	0.204
Large	0.076	0.084	0.093	0.127	
γ					
Small	−0.040	0.000	0.018	0.050	0.054
2	−0.068	0.005	0.035	0.074	0.087
3	−0.069	0.021	0.053	0.079	0.146
4	−0.040	0.020	0.080	0.106	0.146
Large	−0.005	0.049	0.097	0.145	
μ					
Small	−0.013	0.056	0.099	0.130	0.141
2	0.048	0.101	0.119	0.122	0.125
3	0.069	0.104	0.112	0.117	0.112
4	0.081	0.090	0.107	0.110	0.105
Large	0.078	0.080	0.086	0.087	

Table 4
Multivariate model R^2

	Low	2	3	4	High
1 year					
Small	0.06	0.11	0.12	0.13	0.13
2	0.11	0.11	0.11	0.10	0.10
3	0.09	0.09	0.09	0.09	0.07
4	0.05	0.07	0.06	0.06	0.05
Large	0.02	0.02	0.02	0.03	
5 year					
Small	0.10	0.18	0.20	0.21	0.20
2	0.18	0.17	0.18	0.16	0.16
3	0.15	0.14	0.14	0.13	0.10
4	0.08	0.12	0.09	0.09	0.06
Large	0.03	0.03	0.02	0.03	
10 year					
Small	0.08	0.15	0.17	0.18	0.18
2	0.15	0.14	0.15	0.13	0.13
3	0.12	0.11	0.11	0.11	0.08
4	0.06	0.09	0.07	0.07	0.05
Large	0.02	0.02	0.01	0.02	

seen, the filtered state-variables are strongly correlated, and the average correlation between the series is 0.98. This suggests that very little information is lost by constraining the latent factor to be a single common factor as is the case in the multivariate model. We turn to this model next.

3.1. Multivariate model results

Table 3 reports the posterior estimates of the parameters in the multivariate model, Eqs. (5)–(7). The results are based on the Fama–French five-by-five size and book/market sorted portfolios.[2] The results show several interesting features. First, note that the mean of the posterior for ϕ is much smaller than for the univariate models, indicating that the mean reversion is quicker in the multivariate data-set. The short term factor f_t has an autocorrelation of $\kappa=-.074$ with posterior standard deviation 0.167. Thus, the posterior distribution of κ is close to zero, indicating that this factor is picking up almost no predictable variation in the returns. Thus, we conclude that almost all of the predictable variation is found to come from the long-term factor z_t.

Table 3 shows some interesting differences in the predictable return components along the size and book/market dimensions. The factor sensitivities, β, associated with the long-term factor z_t, are strongly decreasing with size. Small firm portfolios have β's roughly three times that of large firm portfolios. This would indicate that the predictable variation in small firm portfolios is much more pronounced than in large firm portfolios. The table also shows that posterior means for β increase along the value/growth dimension, as portfolios of high book/market (value) firms generally have higher β's. Overall, therefore, we expect predictability to be stronger in small, value portfolios.

Table 4 illustrates that this is the case by tabulating the posterior median R^2 as computed using Eq. (8). This table reveals that R^2's are generally much larger for the small firm portfolios than for large, and generally somewhat larger for high B/M portfolios. Table 4 illustrates that small, value portfolios have R^2's that peak at around 20% at the three to seven year forecasting horizon. We find that large firms have negligible R^2's at almost all forecasting horizons, suggesting that long run reversal, if any, is largely a small firm phenomena.

Fig. 5 depicts the 2.5, 50 and 97.5 posterior percentiles of the R^2's. Again, the figure shows the low, middle and high size and book portfolios for brevity. The results are indicative of the remaining portfolios, except for the smallest size and book portfolio in upper, left-most corner of the graph which shows much less predictability than the other small size portfolios. The figure confirms the conclusion that predictability is minimal in large firm portfolios, as the lower

[2] We exclude the 25th portfolio containing the largest firms with the highest book/market values due to missing observations.

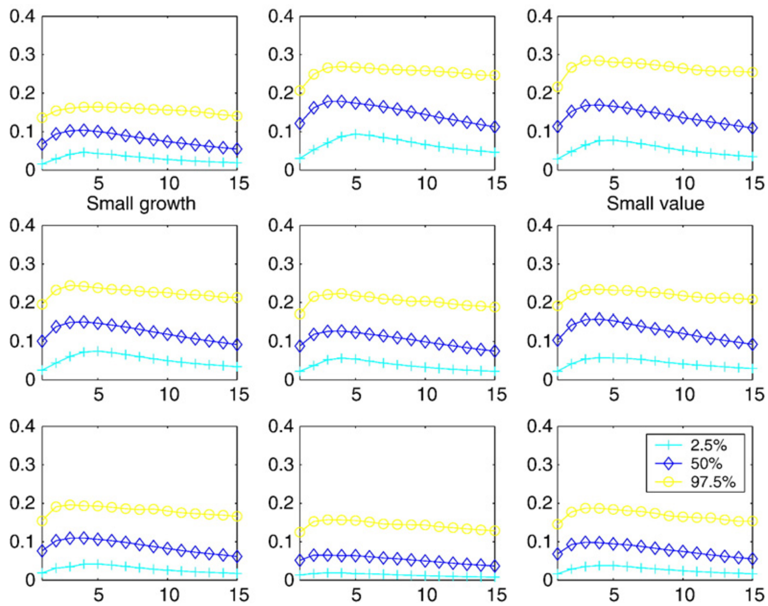

Fig. 5. Pointwise posterior quantiles of $R^2(T)$. Multivariate model.

2.5% percentile is visually indistinguishable from the horizontal axis. In most cases also the upper 97.5 percentiles peak at values less than 15%, suggesting that long term mean reversion is an unlikely occurrence in large firm stock. For small value firms, the situation is different. These portfolios have point-wise 2.5 percentiles that peak at values in the 7–8% range. We interpret this as statistically significant.[3] Fama and French (1988) find that small firm portfolios have larger long run autocorrelations, however, the difference between large and small firms is less substantial than results reported in Table 4 and Fig. 5.

The "short-term factor," f_t, does not contribute a significant amount of predictable variation in the observed returns. This is evidenced by the fact that κ has a significant posterior mass both above and below zero. When $\kappa=0$, f_t is an i.i.d. process which again implies that f_t does not contribute to explaining any predictable variation in the observed returns.

What explains the difference in the R^2's from the univariate models to the multivariate one? Mechanically, the fact that the R^2 is higher in the multivariate model is due to the fact that ϕ is estimated to be lower than in the univariate model. In simulation experiments (not reported), the univariate model was fitted to data generated from the multivariate model. This provided estimates of the individual portfolio's ϕ's which typically were closer to unity than the true values. This suggests that the univariate model may put too much posterior mass on the possibility that $\phi=1$ (no predictability) when the model is (mildly) misspecified.

A second factor which contributes to the apparent significance of the R^2's for the multivariate model is the fact that this model is easier to identify from data. The multivariate model is much more parsimonious than the univariate ones because the identification of the $T=78$ latent data for z_t is done through the 24 cross-sectional portfolio return observations for the multivariate model, and only a single return observation pr. latent data-point in the univariate model. This translate into significantly higher estimation risk in the univariate models compared to the multivariate one. This is reflected in the high dispersion in the univariate R^2's shown in Fig. 2 relative to the multivariate R^2's in Fig. 5.

We have concluded that κ is "insignificant," and the factor f_t mimics a random walk. It is interesting in this context that the factor loading relating to f_t, γ, are almost monotonically increasing in the book/market deciles. For the lowest B/M portfolios, the γ's are negative near $-.05$, while the high B/M portfolios have γ's ranging from 0.054 to 0.146. It is reasonable to interpret the differences in the γ's as evidence of a permanent difference in covariance structure, and hence risk, between portfolios of different characteristics.

[3] The posterior probability that $R^2<\epsilon$ for any ϵ less than a particular portfolios 2.5 percentile R^2 is less than 2.5%.

Are the results in Table 3 consistent with financial market equilibrium, or are they more likely the outcome of over-reaction, or other behavioral explanations? It is convenient to attribute the results to over-reaction, as small firm stock may be more prone to over-reaction than large stock closely monitored by analysts. Of course, the Shiller–Summers model was initially formulated to assess the possibility of fads. There is, however, nothing than in principle excludes the possibility that these results are the outcome of perfectly rational equilibrium asset pricing. Indeed, Bansal, Kiku and Yaron demonstrate that long-run-risk models can generate significant predictability patterns at long horizons with R^2's ranging from 36% to 59% at long (ten year) horizons. Predictability as an equilibrium outcome is not limited to their model. Rather, it is an inherent consequence of any model in which stock prices obtain as present values of dividends and the price dividend ratios are not constant.

In considering possible equilibrium explanations to the results in Table 3, we may interpret the factors z_t and f_t as systematic risk factors. Since the persistence in f_t is basically zero, it is natural to interpret this as a factor which generates a constant risk premium across the book/market dimension and carries a constant risk premium. The second factor, z_t, generates time-varying expected returns which primarily affect small cap stocks. As such, an equilibrium explanation of the findings in Table 3 would likely need to have specific risks affecting operational risks of firms with small cash flows. This can be accomplished in models in which firms' birth and death is explicitly modelled, because such models allow firms to enter the capital market when their cash flows are small, and the firm is relatively more risky, thereby creating a correlation between size and risk. Models which incorporate this idea include those of Berk et al. (1999) and Gomes et al. (2003). It is also possible that risk premium related micro-market factors such liquidity may explain the findings. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) develop equilibrium models and show that size sorted portfolios have factor loadings that decease uniformly with size. An interesting point to note here is that the liquidity factor constructed by Pastor and Stambaugh has a correlation of −0.45 with the latent z factor. This suggests that long run predictability and liquidity is a potentially interesting avenue for future research.

3.2. Robustness

It is well known that the infamous U shaped autocorrelation function found empirically by Fama and French (1988) is not robust across all sampling periods. In particular, Fama and French themselves point out that the pattern is significantly less pronounced in post war data. It is widely believed that the highly volatility depression era is not

Table 5
Parameter estimates using post-war data

	ϕ	κ		ρ	
	0.917 (0.057)	−0.183 (0.184)		0.915 (0.015)	
	Low	2	3	4	High
β					
Small	0.259	0.269	0.242	0.234	0.251
2	0.204	0.186	0.189	0.191	0.194
3	0.142	0.154	0.149	0.163	0.192
4	0.095	0.109	0.120	0.131	0.145
Large	0.023	0.044	0.044	0.069	
γ					
Small	−0.150	−0.044	−0.010	0.030	0.035
2	−0.119	−0.029	0.013	0.057	0.061
3	−0.124	−0.006	0.039	0.067	0.083
4	−0.110	0.006	0.041	0.066	0.062
Large	−0.067	−0.006	0.004	0.068	
μ					
Small	0.052	0.116	0.129	0.154	0.169
2	0.073	0.118	0.142	0.147	0.164
3	0.091	0.127	0.129	0.146	0.154
4	0.104	0.112	0.140	0.138	0.147
Large	0.105	0.109	0.125	0.124	

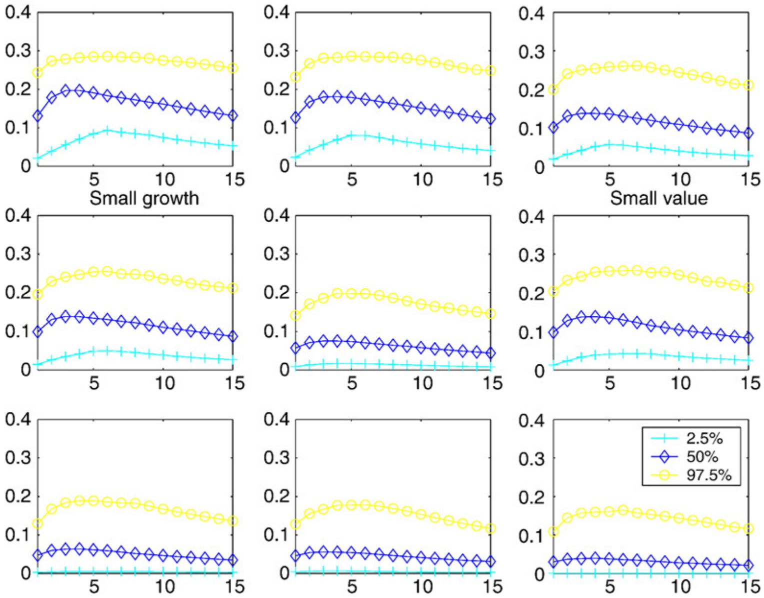

Fig. 6. Posterior $R^2(T)$ quantiles for the multivariate model using post-war data.

representative for the subsequent sample. It is also easy to criticize the model presented here as it does not account for stochastic, time-varying volatility. Volatility is particularly high prior to the second world war. Table 5 reports parameter estimates computed over the 1946 to 2003 period. The table reveals that the estimated parameters are quite similar to those in Table 3. Fig. 6 shows that small firm R^2 are significant and median posterior R^2 peak at around 20% for small value portfolios at the ten year horizon. The lower 2.5 percentiles peak at around 10% for these portfolios. Overall the R^2's increase relative to the ones obtained using the whole sample. The estimates seem to imply somewhat larger importance of the "short term factor" f_t, reflected by a larger posterior mean of κ in absolute value. Overall, the evidence in Table 5 and Fig. 6 is largely consistent with those obtained in the full sample in Table 3.

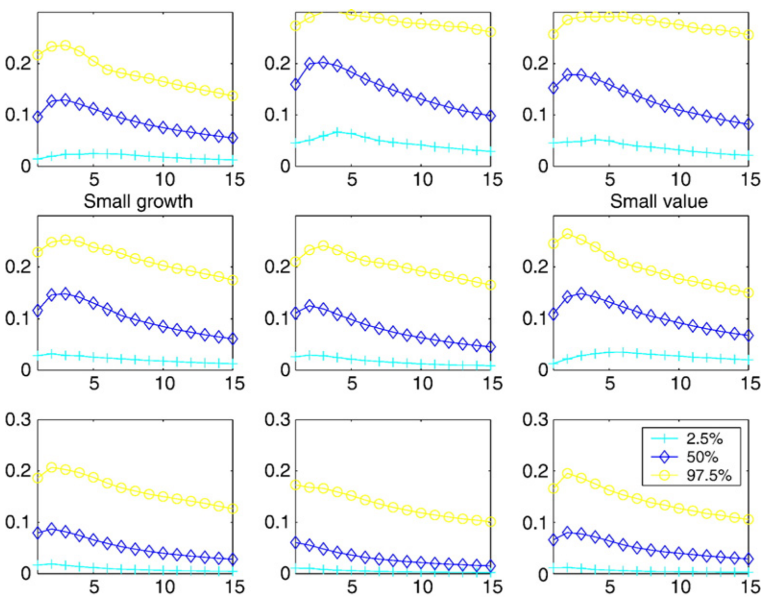

Fig. 7. Posterior $R^2(T)$ quantiles for the multivariate model using data standardized by GARCH(1,1) time-varying volatility.

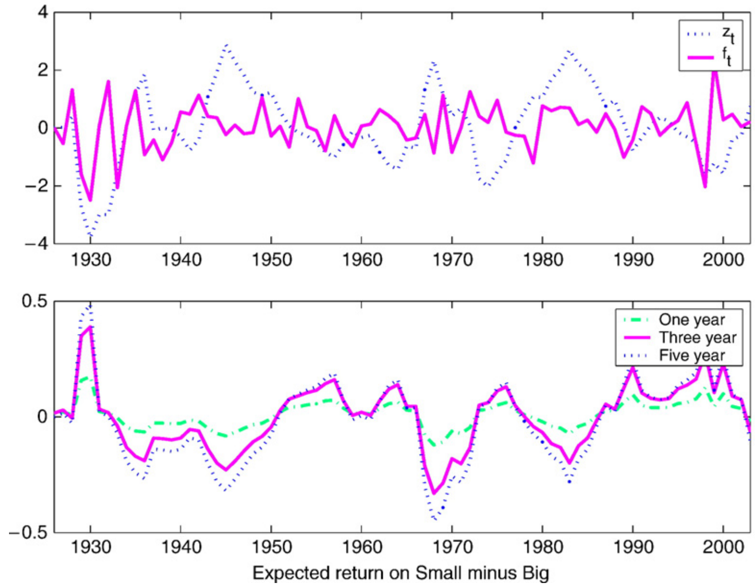

Fig. 8. Filtered factors (above) and expected rates of return on small minus big firm portfolios (below).

In order to further assess the effects of time-varying volatility on our conclusions, we ran an additional estimation under the assumption that the local volatility of returns are generated according to a GARCH(1,1) model. The standardized returns were then used to re-estimate the model parameters. The estimates were quite similar to those in Table 5 and thus not reported. Fig. 7 shows posterior R^2's resulting from this exercise. The evidence is still suggestive of significant amounts of predictability in small value stocks, although somewhat weaker than for the full sample.

3.3. Latent factors

A particular advantage of the Kalman filtering approach to estimate these models is that we recover filtered estimates of the unobserved state variables in the model. Fig. 8 plots the filtered values at the posterior mean of the parameters as well as the model implied expected rates of return for one, three and five year forecasting horizons. Most of the variation in expected rates of return is due to variation in z_t. This is visually evident in the figure. The model generates significant time-variation in expected rates of return. This is particularly true at long forecasting horizons. For example, at the end of 2003, the forecast is for small firms to beat large by 20% cumulative return for the 2004 to 2009 period, corresponding to about 4% annually.

Another interesting aspect of the models considered in this paper is that they provide a different mechanism to compute expected returns than autocorrelations. Forecasts based on autocorrelations are formed with a small number of observations preceding the date of the forecast. Within the models considered here expected returns are formed using filtered values of the unobserved state-variables which utilizes the whole sample prior to the date of the forecast. Lewellen (2001) shows that predictability is significant when running regressions of twelve to eighteen month returns onto one to five year lagged returns. The fact that the longer lag structure improves the forecasting performance is consistent with the models and results presented here.

4. Conclusion

This paper has studied dynamic latent component models of stock price data. We find that a significant fraction of variation in small firm portfolios is due to predictability. Conversely, large firm stock portfolios contain insignificant predictable components.

There are multiple ways in which the models considered here could be generalized. It is straightforward to include covariate information to increase the amount of predictability. While we have demonstrated that pre-filtering the data with time varying GARCH volatiltiy does not alter the conclusions, it is possible to include time-varying volatility as part of the model. This is a particularly relevant extension in the context of higher frequency returns data. Higher frequency data will not allow us to identify the long term factor, z_t, with much greater precision, however. Higher frequency data will on the other hand allow the identification of a short term factor such as f_t, assuming that such a factor is relevant. The addition of stochastic volatility is complicated in the high dimensional setting such as here because the likelihood function must be computed by numerical integration which is typically done though Monte-Carlo.

Our analysis complements the many recent studies of dynamic equilibrium models which generate time-variation in expected rates of return. In particular, our results are consistent with risk premia driven differences in conditional and unconditional expected rates of return across size and book/market assets. That said, this analysis is not a test of equilibrium and as such and the results may also be consistent with irrational pricing. The results indicate that an equilibrium alternative to the over-reactions hypothesis would need to focus specifically on companies with low capitalization, and whether these exhibit higher exposures to risk factors with time-varying premia, than do large cap companies. Liquidity is one risk factor which has been shown to predict expected returns on small cap stocks. It is left for future research to determine whether this, or other similar factors, explain time-variation in expected long run returns.

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