

Modeling the Euro overnight rate[☆]

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Received 23 May 2006; received in revised form 13 October 2006; accepted 11 April 2007

Available online 4 May 2007

Abstract

This paper describes the evolution of the daily Euro overnight interest rate (EONIA) by using several models containing the jump component, such as a single-regime ARCH-Poisson–Gaussian process, with either a piecewise function or an autoregressive conditional specification (ARJI) for the jump intensity, and a two-regime-switching process with jumps and time-varying transition probabilities. To model the jump intensity, we include the following effects which are significant for the occurrence of jumps: (1) the *end of maintenance period effect* because of reserve requirements, (2) the *end of month effect*, also known as the *calendar day effect*, caused mainly by accounting adjustments and finally, (3) the *meeting effect* caused by the meetings of the Governing Council of the European Central Bank (ECB). These effects lead to better performance and several of them are also included for the behavior of the transition probabilities. Since the target of the ECB is to maintain the EONIA rate close to the policy rate, we model the conditional mean of the overnight rate series as a reversion process to this policy rate, distinguishing two alternative speeds of reversion, specifically, a different speed if EONIA is higher or lower than the policy rate. We also study the jumps of the EONIA rate around the ECB's meetings by using the ex-post probabilities of the ARJI model. Finally, we develop a volatility forecasting analysis to measure the performance of the different candidate models.

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JEL classification: C13; C22; E43; E52

Keywords: ARCH-Poisson–Gaussian; Regime switching; Mean reversion; Autoregressive conditional jump intensity; Maintenance period; Calendar day effect; ECB's meeting

[☆] We are grateful to Juan Ayuso, Sunil Mohanty, Eliseo Navarro, Alfonso Novales, Gabriel Pérez Quirós, Diego Rodríguez, Antonio Rubia and two anonymous referees for helpful suggestions, as well as seminar audiences at the European Central Bank (Frankfurt, 2006), Multinational Finance Society (Athens, 2005), the Meeting on Financial Mathematics (Verbania, 2005), the Meeting of Applied Economy (Murcia, 2005) and the Finance Forum (Madrid, 2005). Financial support from the Spanish Ministry of Education and Science through the grants SEJ 2005-09372 (León) and SEJ 2006-05051/ECON (Benito and Nave) is gratefully acknowledged. The contents of this paper are the sole responsibility of the authors.

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1. Introduction

The control of very short-term interest rates, specifically the interest rate of the interbank market for unsecured overnight loans – the EONIA¹ for the Eurozone – is the first step in the mechanism for monetary policy transmission and the basis on which it acts in order to influence the behavior of longer term interest rates. The importance of EONIA goes far beyond its usefulness to the ECB. It has grown into a very practical tool for the trading community in recent years as the underlying rate of numerous derivatives transactions. In the euro market, banks, pension funds, insurance companies, money market mutual funds and hedge funds make extensive use of overnight interest rate swaps² (OISs), which are overwhelmingly referenced to the EONIA,³ for hedging and speculating on short-term interest rate movements. Trading in EONIA swaps has become the largest OIS in the world. Another important EONIA contract is the one month EONIA indexed futures.⁴ The success of euro interest rate OTC derivatives reaches a total daily turnover of around €380 billion, making it larger than the US dollar segment with a daily turnover of around €290 billion (BIS, 2005).⁵

The subject of several papers in recent years is understanding the empirical characteristics of the EONIA rate and how effective monetary policy measures are in maintaining it close to the policy rate. However, the literature on this interest rate market has not yet covered many aspects studied in others. The analysis of Fed Funds⁶ in the United States is much wider than the study of EONIA in the Eurozone where most studies have concentrated on finding whether the instruments and procedures to implement the monetary policy have repercussion on the overnight rate. For example, there has been an analysis of the implications of the institutional details of the reserve market on the behavior of the EONIA and how this rate is affected by the liquidity management of the European Central Bank (ECB). In this regard, Hartmann et al. (2001) and Manna et al. (2001) concentrate on analysing the operational framework of ECB, while Bindseil and Seitz (2001) and Angeloni and Bisagni (2002) concentrate more on the consequences of the liquidity conditions. Prati et al. (2002) studies how the operational procedures and intervention forms of the central banks affect the characteristics and behavior of the one-day rate in the most industrialised countries (Eurozone and G7). Gaspar et al. (2001) draws up a model for the behavior of the overnight rate during the reserve maintenance period, mainly in order to estimate the impact of the announcement of monetary policy measures. Würtz (2003) develops a descriptive model aimed at an exhaustive list of all the characteristics and variables that may explain the behavior of the EONIA rate. Moschitz (2004a) models the problem of the intertemporal decision in the reserve market, both for the central bank and for commercial banks.

¹ The EONIA (Euro OverNight Index Average) is published by the European Banking Federation as the balanced average of all overnight unsecured lending transactions between the most active credit institutions in the Eurozone money markets.

² In this contract, one side pays a fixed interest rate for the duration of the contract while the other party pays an average of the EONIA rate realized over the same period.

³ Another overnight rate is the overnight Euro LIBOR (London Interbank Offer Rate denominated in euros) which has a strong correlation with the EONIA, specifically a value of 0.99 for the period from January 2001 to December 2005. The daily mean and volatility are 2.841 (2.823) and 0.951 (0.953) respectively for the overnight Euro LIBOR (EONIA). Note that the Euro LIBOR exists mainly for continuity purposes in swap contracts dating back to pre-euro times and is not very commonly used. See <http://www.bba.org.uk>.

⁴ See <http://www.euronext.com>.

⁵ See BIS Triennial Central Bank Survey (March, 2005).

⁶ See Rudebusch (1995), Hamilton (1996), Balduzzi et al. (1997, 1998), Bartolini et al. (2002), Hamilton and Jordá (2002), Das (2002), Farnsworth and Bass (2003), Andersen et al. (2004) and Piazzesi (2005).

Pérez Quirós and Rodríguez Mendizábal (2006) test whether there are statistical differences in the behavior of the daily rate before and after the European Monetary Union (EMU), presenting a model for liquidity shocks focused from the demand side. Other studies by Moschitz (2004b), Blanco (2005) and Cassola and Morana (2003, 2004), among others, focus on understanding the volatility transmission from the EONIA rate along the yield curve.

Many of these authors reach common conclusions with respect to the behavior of the overnight rate. They unanimously agree that the martingale hypothesis – past observations should have no predictive content – does not hold. One widely tested empirical characteristic is the existence of predictable patterns in the behavior of the EONIA rate. The most obvious examples of non-compliance with the hypothesis come from the *end of maintenance period effect* and the *calendar effects* (end of month, quarter, half-year and year). The former is a consequence of averaging provisions to comply with reserve requirements, while the calendar effects may be attributed to window-dressing activities performed by banks on their balances when they are presented. Together with these effects, the periodical meetings that the Governing Council of the ECB holds constitute the main causes of the jumps observed in the dynamics of the EONIA rate.

In this context, our aim is to tackle the evolution of the EONIA rate through a jump-diffusion model which covers these three effects. As a starting model, we will use the Poisson–Gaussian process with ARCH conditional variance implemented by Das (2002) for modeling the Fed Funds rate. This model, in addition to properly capturing the empirical characteristics observed in the EONIA series, finds the jump intensity caused by each of the three effects studied. Likewise, and given that the aim of the ECB is to maintain the EONIA rate close to the monetary policy rate, we will model the conditional mean of this overnight interest rate as a reversion process to this key rate, distinguishing two alternative speeds of reversion, concretely, a different speed if EONIA is higher or lower than the policy rate.⁷ An extension of the above model is also implemented here, specifically the autoregressive conditional jump intensity model by Chan and Maheu (2002). This model will also be used to analyse the behavior of jumps around those ECB's meetings where the policy rate is modified. This study will be realized through the ex-post probabilities implicit in this model. Finally, we develop a two-regime-switching model with time-varying transition probabilities and each regime is characterised by an ARCH-Poisson–Gaussian process. The regime-switching model outperforms simple single-regime models for interest rates as many empirical applications, such as Gray (1996), confirm.

In short, we show that jumps contribute to a considerable improvement to modeling the EONIA rate mainly due to the seasonal components inherent in this series such as the end of the maintenance period and the calendar effects already mentioned. This jump evidence is also important when we model derivatives traded on EONIA, as the underlying rate, since it would lead to a fair valuation instead of mispricing. A jump-diffusion process is also found in the study by Guan et al. (2005) for the LIBOR rates providing superior results for both in-sample and out-

⁷ Modeling the policy rate is beyond the scope of this paper. For an endogenous behavior of this key rate, see Piazzesi (2005) and Athanasopoulou (2006) for the high-frequency policy rule of Fed and ECB respectively. In these studies, the key rate is modeled as a pure Poisson process where the intensity parameter depends on a set of state variables. The parameters are obtained through bond yields by simulated maximum likelihood. There is another study by Dewachter et al. (2006) in which the key rate depends on both observable macroeconomic factors (output gap and inflation) and latent factors with a clear macroeconomic interpretation. They use both US bond yields and the above observable factors to estimate the parameters by Kalman filtering.

of-sample performance when compared to the pure diffusion counterpart for pricing American options with Eurodollar futures contracts as the underlying asset.⁸

The rest of this paper is divided into eight sections. The next section describes the effects mentioned above. Section three analyses the EONIA rate data corresponding to the period under analysis. Each section from four to six is divided into two subsections: a first part containing a specific model and a second part presenting its empirical results. Section seven shows the volatility forecasting analysis for the different models and finally, Section eight concludes.

2. Stylized facts of EONIA behavior

In order to meet its aims, including the control of very short-term interest rates, the Eurosystem uses a series of instruments and procedures which make up its operational framework: it carries out *open market operations*, it offers *standing facilities* and it demands that credit institutions hold *minimum reserves*.

The most important *open market operations* are the *main refinancing operations* (MRO), which are liquidity-providing transactions that are conducted on a weekly basis. These operations are executed by the national central banks on the basis of standard tenders and play a pivotal role in pursuing the purposes of the Eurosystem's open market operations. They provide the bulk of refinancing to the financial sector. The Eurosystem also offers *standing facilities* to its counterpart institutions, who may recur on their own initiative to the *marginal lending facility* (overnight liquidity from national central banks at a pre-specified interest rate against eligible assets); and the *deposit facility* (overnight deposits with national central banks at a pre-specified interest rate). The interest rates applied to these facilities are, in general, unfavourable regarding the market interest rates. They determine the fluctuation band of the EONIA rate with the aim of reducing volatility. In Fig. 1, together with the EONIA rate, we can see the evolution of the interest rates on the MROs, marginal lending facility and deposit facility. The ECB considers these three rates as key rates. The interest rate from these operations represents the guideline of the monetary policy. For the sake of clarity, in the remainder of the paper we use either the MRO⁹ rate or policy rate interchangeably as the ECB's policy rate.

The ECB also requires credit institutions to hold *minimum reserves* on accounts with the national central banks. The compliance with such reserve requirements may lead to a mechanism of averages, that is, the *minimum reserves* are calculated as an averaging provision for the whole month covered by the maintenance period, not for each day. The maintenance period begins on the 24th calendar day of each month and ends on the 23rd calendar day of the following month.¹⁰ The ECB uses this average mechanism to stabilize interest rates. Credit institutions may soften daily liquidity fluctuations, as the transitory reserve imbalances may be compensated with imbalances in the other direction within the same maintenance period. This averaging provision

⁸ The LIBOR term structure is inferred from the Eurodollar futures contracts. The methodology used in Guan et al. (2005) to obtain the parameters by calibration – i.e. minimizing the mean squared percentage errors between observed market and theoretical option prices – would be appropriate, for instance, in case of trading with options on the EONIA.

⁹ Specifically, the policy rate is assumed to be the fixed rate (until June 27, 2000) and the minimum bid rate (since June 28, 2000) at which the ECB conducts the MROs.

¹⁰ From March 8, 2004 the schedule for reserve maintenance periods always starts on the settlement day of the first MRO following the Governing Council meeting at which the monthly assessment of the monetary policy stance is pre-scheduled and will end on the day preceding the corresponding settlement day in the following month. Another change to the Eurosystem's monetary policy was shortening the maturity of the MROs from two weeks to one week. For more details, see ECB (2004). We do not study these effects due to the lack of enough data.

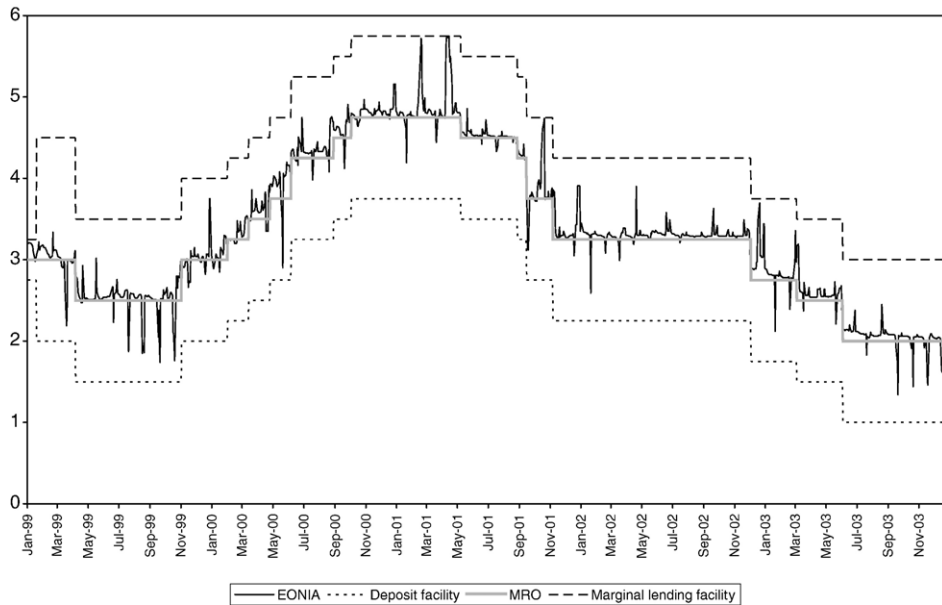


Fig. 1. Evolution of the following rates: EONIA, MRO, marginal lending facility and deposit facility for the period from 4 January 1999 to 31 December 2003. EONIA is the Euro Overnight Index Average, MRO is the ECB policy rate (fixed rate until June 27, 2000 and minimum bid rate for variable rate tenders since June 28, 2000), the *marginal lending facility* provides overnight liquidity from national central banks at a pre-specified interest rate and the *deposit facility* is the overnight deposit with national central banks at a pre-specified interest rate. Both facilities determine the fluctuation band of the EONIA rate.

allows a certain intertemporal arbitrage which should, in theory, guarantee equality throughout the maintenance period between the current level and the expected level of the shortest interest rate at the end of the maintenance period. However, towards the end of the maintenance period, and especially after the last MRO, the reserve must be adapted to comply with the required minimum average. It is not possible to either transfer a deficit or surplus liquidity into the future or turn to an MRO. This leads to the peaks observed in the series of the EONIA rate towards the end of the maintenance period. This is one of the effects to be analysed in this paper.

Another event that causes peaks in the EONIA series are the calendar effects due to the *window-dressing* activities by which banks adjust their balances at the end of the month, making the demand for one-day funds shoot up. As Moschitz (2004a) says, the central banks cannot do much to cancel these effects, as changes in the offer of reserves during these days would be interpreted as temporal changes and, therefore, would not affect the behavior of the interest rate.

Finally, we also study the effects from the meetings of the Governing Council of the ECB, after which changes of the rate of the MRO may occur. The expectation regarding changes in the monetary policy rates is one of the important variables in the behavior of the EONIA rate. One of the concerns of the central banks is to avoid an increase of volatility when the policy rate is modified. The aim is to communicate monetary policy measures clearly and not to give confusing signals to the market, especially when expectations regarding interest rates vary in the middle of a maintenance period. This aspect is studied by Gaspar et al. (2001) who, using a time series from January 1999 to March 2001, come to the conclusion that the market anticipates monetary policy changes, so that the announcement of a new rate does not cause important changes either in the level or volatility of overnight rates. However, Moschitz (2004a) using a wider series to February

Table 1
Descriptive statistics

Statistics	EONIA rate (r)	First difference of EONIA rate (Δr)
Mean	3.3740	−0.0006
Median	3.2900	0.0000
Max	5.7500	1.1600
Min	1.3400	−0.9800
Std	0.9021	0.1417
Skew	0.3199	0.8631
Kurt	2.1739	16.8901
JB	59.2737	10,628.38
(p -value)	0.0000	0.0000

Descriptive statistics of the daily rate and its first difference over the period January 1999 to December 2003. Mean denotes the sample arithmetic mean, Median is the sample median, Max and Min stand for maximum and minimum respectively, Std denotes standard deviation and Skew and Kurt stand for skewness and kurtosis respectively. All expressed in percentage terms. JB stands for the Jarque–Bera normality test statistic. Underneath this statistic is the p -value.

2004, found that banks react slowly to changes in the policy rate, with increased volatility around the day of the announcement of the monetary policy decision.

3. Data

This section analyses the EONIA interest rate sample of 1303 daily observations over the period from January 1999 – the starting date of Stage Three of the EMU – to December 2003. As Fig. 1 shows the EONIA rate has, in general, been close to the rate of the MRO, which shows the importance of these operations to the aims of steering interest rates. The fluctuations in the overnight rate, which can be seen in the graph, reflect liquidity conditions that are temporarily relaxed or restrictive on the money market. As we have already said, these fluctuations and the peaks are mainly related to the aforementioned averaging provisions for the reserve requirements, but also to the calendar effects and the fortnightly meetings of the Governing Council of the ECB.

Table 1 shows the descriptive statistics for the EONIA rate and its first difference. The maximum value (5.75%) was reached on 17 April 2001, and the minimum (1.34%) on 23 September 2003. The largest rise (1.16%) occurred on 24 May 2000 and the largest drop (0.98%) on 24 October 2001. We can see that both of them occurred on day 24 of the month, the first day

Table 2
Autocorrelation coefficients and unit root test

Autocorrelation coefficients	EONIA rate (r)	First difference of EONIA rate (Δr)	Policy spread ($r - r^*$)
ρ_1	0.987	−0.106	0.673
ρ_2	0.977	−0.152	0.422
ρ_3	0.970	−0.055	0.282
ρ_4	0.965	−0.054	0.186
ρ_5	0.961	−0.035	0.114
ρ_6	0.958	−0.034	0.068
ρ_7	0.956	0.004	0.042
ADF	−1.262	−20.102**	−15.897**

ADF denotes t -statistics of Augmented Dickey–Fuller tests with constant and lag length according to the Schwarz Information Criterion (SIC). The 5% (1%) critical value is −2.864 (−3.435); significance is denoted by * (**). All test results are robust if constant in ADF test is not included.

of each reserve maintenance period, when the EONIA recovers the value that it reached on the days before the end of the reserve maintenance period. Moreover, the Jarque–Bera statistic rejects normality for both the series: level (r_t) and first difference (Δr_t). Note that Δr_t shows a high level of asymmetry and leptokurtosis.

Table 2 provides the high persistence that the EONIA series exhibits by looking at the sample autocorrelation coefficients. Note the high values they show and how slowly they tend to decrease. This suggests evidence of the possible presence of a unit root, as ratified by the Augmented Dickey–Fuller (ADF) test. This test, however, rejects the presence of a unit root for the series of Δr_t . Note also that the correlation coefficients of Δr_t are small and negative. This behavior confirms the possibility of mean reversion.

Finally, as expected, Fig. 1 also shows that the close link between the EONIA and policy rate, denoted as r_t^* , is in line with the notion of a long-run relationship. We show that the policy spread, defined as $r_t - r_t^*$, is a stationary process since the ADF test in Table 2 clearly rejects the unit root. Note also how the correlations of the policy spread series are positive and tend to decrease very quickly. We implement, not reported here, an error-correction model for the first difference of EONIA rate with an asymmetric adjustment depending on the positive and negative policy spread variables. Our study also verifies mean-reversion with a higher value size for the negative spread. Nautz and Offermans (2006) also prove that r_t and r_t^* are cointegrated. They carry out a deeper study – by using non-linear error-correction equations – confirming a non-symmetric adjustment of EONIA to the policy spread such that the reversion to r_t^* is faster when this spread is negative. This fact strengthens the next section's way of modeling the conditional mean of Δr_t as a function of both a positive and negative spread, that is, a two mean reverting process. Ozgur (2006) also shows that the presence of a double, as opposed to a single, mean reversion for the 3-month T-bill rates is superior.¹¹

4. ARCH-Poisson–Gaussian model

A Poisson–Gaussian process with ARCH volatility is selected for modeling the dynamics of the change in the EONIA rate or Δr_t . This process appears to be suitable for the empirical characteristics of the series under study. As the model allow for jumps, we can perfectly capture the peaks observed in the overnight rate series and also the high level of leptokurtosis, which is displayed in Table 1, makes the use of these models very appropriate. Additionally, empirical evidence shows that jump models diminish the possible non-linearity of the conditional mean of the series. As regards the behavior of the interest rate volatility, it is widely accepted that these series have a high level of conditional heteroskedasticity in the variance. In this way, a GARCH family specification for volatility allows us to capture both the clustering effect and the high persistence intrinsic to Δr_t .

4.1. The model

Let us define the information set available at time t to be the history of daily interest rates, both the overnight and the MRO rates, and denoted as Φ_t . The model for the EONIA rate shows the following discrete version from its jump-diffusion continuous specification:

$$\Delta r_t = \mu_t + \sigma_t \Delta z_t + J_t \Delta n_t \quad (1)$$

¹¹ It consists of an augmentation of the nonlinear symmetric mean proposed by Ait-Sahalia (1996).

where μ_t is the conditional mean, which will be characterised later, when there is no jump; Δz_t is an independent standard normal variable; J_t is the jump size, which is also assumed normally and independently distributed with a time varying mean denoted as θ_t and variance ψ^2 ($\psi > 0$), which is assumed to be constant in order to simplify the model. Likewise, we assume that Δz_t and J_t are independent. Moreover Δn_t designates the Poisson process with λ_t as the time-varying intensity parameter for the number of jumps which occur in the interval from $t - 1$ to t , specifically $(t - 1, t]$. We approach the Poisson process with a Bernoulli distribution – see Das (2002) – conditioned to Φ_{t-1} with probability λ_t when there is a jump, that is $P(\Delta n_t = 1 | \Phi_{t-1}) = \lambda_t$, and a probability $1 - \lambda_t$ when there is no jump. Note that λ_t is interpreted as the ex-ante probability of a jump occurring. We specify the conditional variance σ_t^2 with an ARCH (1) feature¹² so as to limit the number of estimated parameters. Finally, it is important to mention that μ_t , σ_t , θ_t and λ_t are measurable with respect to the information set Φ_{t-1} .

This subsection is divided into three parts: the first (Subsection 4.1.1) shows both the likelihood function and some conditional moments – specifically the total mean and variance – for the process Δr_t . The second (Subsection 4.1.2) describes exactly what kind of process is adjusted for the no-jump component of the total conditional mean and finally, the third (Subsection 4.1.3) exhibits the starting time-varying pattern of λ_t as a step function.

4.1.1. Likelihood function and conditional moments

The hypothesis underlying Eq. (1) implies that the distribution of Δr_t , conditioned to the most recent information set and to j jumps, is normally distributed as

$$f(\Delta r_t | \Delta n_t = j, \Phi_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_t^2 + j\psi^2)}} \exp\left(-\frac{(\Delta r_t - \mu_t - j\theta_t)^2}{2(\sigma_t^2 + j\psi^2)}\right) \quad (2)$$

where, for our purposes, j takes either value 0 or 1. The conditional density function is obtained from

$$f(\Delta r_t | \Phi_{t-1}) = (1 - \lambda_t) \times f(\Delta r_t | \Delta n_t = 0, \Phi_{t-1}) + \lambda_t \times f(\Delta r_t | \Delta n_t = 1, \Phi_{t-1}) \quad (3)$$

and its log-likelihood, denoted as L , is calculated as $\sum_{t=1}^T \ln f(\Delta r_t | \Phi_{t-1})$ where T indicates the size of the sample. The conditional mean of Eq. (3) is

$$E_{t-1}(\Delta r_t) = \mu_t + \lambda_t \theta_t \quad (4)$$

where $E_{t-1}(\cdot)$ indicates the expectation conditioned to the information set Φ_{t-1} . The conditional variance is obtained as

$$V_{t-1}(\Delta r_t) = \sigma_t^2 + \sigma_{t,J}^2. \quad (5)$$

It can be seen that this conditioned variance contains two components. A first component defined as

$$\sigma_{t,J}^2 = \lambda_t [\psi^2 + (1 - \lambda_t) \theta_t^2] \quad (6)$$

is associated with the innovation of the jumps and secondly, σ_t^2 is an ARCH (1) structure:

$$\sigma_t^2 = \omega_0 + \omega_1 [\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2 \quad (7)$$

¹² The same volatility structure is imposed by Das (2002) for modeling the Fed Funds rate.

where it can be seen that $\omega_0, \omega_1 > 0$. Observe that Eq. (7) includes the effects of both innovations – normal innovations and innovations due to jumps – in the square of its past innovations.¹³

4.1.2. The no-jump component in the conditional mean

As we have already mentioned, because the central bank tries to keep the overnight rate close to the MRO rate, see Fig. 1, we have modeled the conditional mean μ_t as a process reverting to the MRO rate. This rate displays discontinuous behavior due to the discrete interventions carried out by the ECB. Accordingly, the rate of the MRO will represent the long-term average to which the EONIA rate should revert. One relevant characteristic of our model is the specification of two different speeds of reversion: one for high rates if the EONIA rate is above the MRO rate, and another for low rates when the daily rate is below the MRO rate. In short, the behavior of μ_t is guided by:

$$\mu_t = \alpha_1 (r_{t-1} - r_{t-1}^*)^+ + \alpha_2 (r_{t-1} - r_{t-1}^*)^- \quad (8)$$

where r_t is the overnight rate, r_t^* is the MRO or policy rate, both expressions $(r_{t-1} - r_{t-1}^*)^+$ and $(r_{t-1} - r_{t-1}^*)^-$ are equal to $\max(r_{t-1} - r_{t-1}^*, 0)$ and $\min(0, r_{t-1} - r_{t-1}^*)$ respectively. Finally, both α_1 and α_2 are the reverting coefficients, which must be negative to ensure that reversion to the policy rate exists. An asymmetric behavior will hold if it is verified that $\alpha_1 \neq \alpha_2$.

4.1.3. The jump component

As we have mentioned above, we know that there are certain days on which the EONIA rate has a greater jump probability: the last days of the maintenance period, end of month, quarter, half-year and year, and the days close to the periodical meetings of the Governing Council of the ECB. Accordingly, we make the jump arrival intensity λ_t of these days depending on dummy variables. In this way, we assume a piecewise dynamics for the specification of the jump intensity:

$$\lambda_t = \delta_0 D_t^0 + \delta_1 D_t^{\text{mp}} + \delta_2 D_t^{\text{m}} + \delta_3 D_t^{\text{meet}} + \delta_4 D_t^{\text{meet+mp}} + \delta_5 D_t^{\text{meet+m}} \quad (9)$$

where D_t is a certain dummy variable:

$D_t^{\text{mp}} = 1$	if the last four days of the maintenance period.
$D_t^{\text{m}} = 1$	if the last two days of each month ¹⁴ .
$D_t^{\text{meet}} = 1$	if day of meeting of the Governing Council of the ECB or next day.
$D_t^{\text{meet+mp}} = 1$	if either the day of the meeting of the Governing Council of the ECB or the following day coincide with the last four days of a maintenance period.
$D_t^{\text{meet+m}} = 1$	if either the day of the meeting of the Governing Council of the ECB or the following day coincide with the last two days of the month.
$D_t^0 = 1$	if any other day,

verifying that $D_t^{\text{mp}} \cap D_t^{\text{m}} \cap D_t^{\text{meet}} \cap D_t^{\text{meet+mp}} \cap D_t^{\text{meet+m}} \cap D_t^0 = \emptyset$.

¹³ Given Eqs. (6) and (7), the total conditional variance in Eq. (5) follows an ARCH (1) structure similar to Eq. (7) though exhibiting a time-varying constant equals to $w_0 + s_{t,j}^2$ which may depend on a set of dummy variables through the intensity parameter in case of being defined like Eq. (9) introduced later.

¹⁴ In order to reduce the number of dummy variables, we have grouped the end of month, quarter, half-year and year effects into a single variable labelled “end of month effect”.

Table 3

ARCH-Gaussian and ARCH-Poisson–Gaussian process with constant intensity parameter λ

Parameters	$\mu_t = \alpha(r_{t-1} - r_{t-1}^*)$		$\mu_t = \alpha_1(r_{t-1} - r_{t-1}^*)^+ + \alpha_2(r_{t-1} - r_{t-1}^*)^-$	
	Model 1 ARCH-Gaussian	Model 2 ARCH-Poisson–Gaussian	Model 3 ARCH-Gaussian	Model 4 ARCH-Poisson–Gaussian
α	−0.173 (−5.084)	−0.0317 (−4.542)	—	—
α_1	—	—	−0.1254 (−3.371)	−0.0315 (−4.384)
α_2	—	—	−1.155 (−21.184)	−0.4169 (−10.18)
ω_0	0.0047 (3.347)	0.00003 (6.418)	0.0032 (1.524)	0.00004 (6.108)
ω_1	2.2086 (2.623)	0.7708 (5.4333)	3.9393 (1.283)	0.7475 (6.1468)
γ	—	−0.3222 (−6.714)	—	−0.2896 (−5.675)
ψ	—	0.1916 (14.100)	—	0.1897 (13.413)
λ	—	0.2176 (11.483)	—	0.2191 (12.068)
Log-L [SIC]	1127.51 [1116.75]	2128.99 [2107.48]	1185.47 [1171.12]	2150.58 [2125.48]

Daily data of EONIA rate covering the period January 1999 to December 2003. This table contains the ARCH-Gaussian process (no jumps) and the ARCH-Poisson–Gaussian process under different features of μ_t . The ARCH-Poisson–Gaussian process with a constant intensity parameter λ is specified as follows:

$\Delta r_t = \mu_t + \sigma_t \Delta z_t + J_t \Delta n_t$; $\Delta z_t \sim N(0, 1)$; $J_t \sim N(\theta, \psi^2)$; Δz_t and J_t are independent;
 $\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$; $\sigma_t^2 = \omega_0 + \omega_1[\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2$ and μ_t is defined in the table.

Maximum likelihood estimates. Bollerslev and Wooldridge (1992) robust *t*-statistics are in parentheses. Log-L stands for the log-likelihood function. SIC is the Schwarz (1978) information criterion.

The aim of the dummy variables $D_t^{\text{meet} + \text{mp}}$ and $D_t^{\text{meet} + \text{m}}$ is to capture those days where some effects coincide. Note that it would be impossible to distinguish which part of the jump intensity is due to which effect without the introduction of these variables. More parsimonious specifications, leaving out these two effects, will be adopted in the following sections due to the introduction of more extended models.

The choice of the last four days of the maintenance period and the last two days of each month was made after repeating the analysis with different combinations and observing that the best fit, under the Schwarz (1978) Information Criterion (SIC), is obtained with the combination above. As observed by Pérez Quirós and Rodríguez Mendizábal (2006), there is no consensus on how many days are affected by the end of the maintenance period effect. In a simple graphic analysis, we observe that it is necessary to eliminate at least four days in order to remove peaks in the EONIA series. Since the time between the last MRO of the maintenance period and the end of that period is one week at most, then four days becomes a reasonable value.

With respect to the effect caused by the meetings of the Governing Council¹⁵ of the ECB, we looked at the jump intensity occurring on the day of the meeting and on the following day. The announcement of the result of the meeting, with the possible modification of the policy rate, is not known until 1:45 p.m. Indeed, the EONIA rate corresponding to the day of the meeting incorporates transactions made before the announcement and so it does not take into account the outcome of the meeting. All transactions carried out on the following day include the result

¹⁵ The Governing Council meets twice a month, on alternate Thursdays. At its first meeting, as a rule, the Governing Council assesses the economic situation and the stance of the monetary policy. Decisions on the key interest rates are normally taken during this meeting. At its second meeting, the Governing Council focuses on issues related to other tasks and responsibilities of the ECB and the Eurosystem. Obviously, if warranted by the circumstances, the Governing Council can still decide to change the key ECB interest rates at any time, regardless of previously scheduled meetings.

of the meeting. In case of approving a change of the policy rate, it comes into effect on the day after the meeting.

Finally, we will assume for the mean of the jump size θ_t two alternative specifications: constant average, i.e. $\theta_t = \theta$, or mean reversion to the MRO rate modeled as $\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$ verifying that $\gamma < 0$ to guarantee mean reversion to the policy rate.

4.2. Empirical results

The maximum-likelihood estimation permits formal tests of the relative fits of the alternative models. Nested specifications can be tested using the likelihood ratio test (LRT) for those models from this section (Tables 3 and 4). Another possible candidate statistic for model selection is the popular SIC statistic, choosing the model with a higher SIC value, although it is not a formal hypothesis test. Note that this criterion penalizes for the number of parameters. We will also use this statistic to decide between non-nested models.

Table 4
ARCH-Poisson–Gaussian process with λ_t as a piecewise function

Parameters	$\mu_t = \alpha(r_{t-1} - r_{t-1}^*)$		$\mu_t = \alpha_1(r_{t-1} - r_{t-1}^*)^+ + \alpha_2(r_{t-1} - r_{t-1}^*)^-$	
	Model i	Model ii	Model iii	Model iv
	$\theta_t = \theta$	$\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$	$\theta_t = \theta$	$\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$
α	−0.046 (−6.601)	−0.0344 (−6.365)	—	—
α_1	—	—	−0.0465 (−6.244)	−0.0388 (−7.832)
α_2	—	—	−1.0928 (−263.81)	−0.9943 (−61.837)
ω_0	0.00003 (5.123)	0.00003 (7.189)	0.0004 (10.127)	0.0004 (12.833)
ω_1	0.6240 (5.922)	0.6092 (5.542)	0.6651 (6.338)	0.6482 (6.053)
θ or γ	−0.0087 (−1.196)	−0.4152 (−8.179)	−0.009 (−1.170)	−0.2758 (−4.406)
ψ	0.2101 (12.939)	0.1903 (14.249)	0.2003 (12.294)	0.1930 (13.035)
δ_0	0.1209 (7.841)	0.1128 (6.917)	0.1025 (7.005)	0.1023 (7.209)
δ_1	0.5260 (10.201)	0.5271 (11.546)	0.5500 (10.714)	0.5347 (10.431)
δ_2	0.5257 (8.838)	0.5076 (9.830)	0.5233 (8.486)	0.5306 (8.460)
δ_3	0.1421 (3.582)	0.1309 (4.239)	0.1344 (3.493)	0.1476 (3.568)
δ_4	0.8307 (10.345)	0.8060 (10.055)	0.8763 (10.372)	0.8847 (10.998)
δ_5	0.3896 (2.741)	0.3569 (3.080)	0.4368 (2.843)	0.4373 (3.367)
Log-L [SIC]	2186.46 [2147.01]	2222.14 [2182.69]	2237.55 [2194.51]	2256.05 [2218.01]

Daily data of EONIA rate covering the period January 1999 to December 2003. This table contains the ARCH-Poisson–Gaussian process under different features of both μ_t and θ_t . Our more general ARCH-Poisson–Gaussian process is specified as follows:

$\Delta r_t = \mu_t + \sigma_t \Delta z_t + J_t \Delta n_t$; $\Delta z_t \sim N(0,1)$; $J_t \sim N(\theta_t, \psi^2)$; Δz_t and J_t are independent;
 $\sigma_t^2 = \omega_0 + \omega_1[\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2$ and both μ_t and θ_t are defined in the table.

Maximum likelihood estimates. Bollerslev and Wooldridge (1992) robust *t*-statistics are in parentheses. Log-L stands for the log-likelihood function. SIC is the Schwarz (1978) information criterion. The coefficients δ_1 , δ_2 , δ_3 , δ_4 , δ_1 , and δ_0 denote respectively the following probabilities of jump: (i) the last four days of the maintenance period, (ii) the last two days of the month, (iii) the day of the meeting of the ECB’s Governing Council and next day, (iv) the day of the meeting of the ECB’s Governing Council or the following day when they coincide with the last four days of a maintenance period, (v) the day of the meeting of the ECB’s Governing Council or the following day when they coincide with the last two days of the month and (vi) the remaining days. In short, the intensity of the jump is defined as follows:

$$\lambda_t = \delta_0 D_t^0 + \delta_1 D_t^{\text{mp}} + \delta_2 D_t^{\text{m}} + \delta_3 D_t^{\text{meet}} + \delta_4 D_t^{\text{meet+mp}} + \delta_5 D_t^{\text{meet+m}}$$

where D_t^0 , D_t^{mp} , D_t^{m} , D_t^{meet} , $D_t^{\text{meet+mp}}$, $D_t^{\text{meet+m}}$ are dummy variables.

Tables 3 and 4 show the results obtained by the Poisson–Gaussian model with ARCH variance – under the two alternative specifications of both μ_t and θ_t – with a constant intensity parameter λ and a time-varying intensity λ_t defined in Eq. (9) respectively. Note that model *ii* (model *iv*) nests model 2 (model 4) when all the coefficients in Eq. (9) are equal. We can appreciate some features. First, the piecewise specification fits better than a constant intensity parameter. Second, there is significant evidence of mean reversion, although the double one fits better.

Third, Table 4 also analyses the effect on changing the specification of θ_t . There is hardly any difference in the parameters when we compare either model *i* against model *ii* (single mean reversion) or model *iii* against model *iv* (double mean reversion). The SIC statistic reaches a slightly higher value for both situations when θ_t is not constant.¹⁶ It also holds that parameter γ is negative and significant. This result leads to a mean reversion to the MRO rate in terms of the mean of the jump size.

Fourth, modeling jumps in model 2 (model 4) clearly provides a significantly better fit than model 1 (model 3) with no jumps, as the SIC reports. The SIC of model 2 (model 4) is 2107.48 (2125.48) which is almost double the SIC of model 1 (model 3) of 1116.75 (1171.12).¹⁷ We conclude that the ARCH-Poisson–Gaussian model with double mean reversion and mean reversion structure for θ_t (model *iv*) outperforms the other models of Tables 3 and 4 according to SIC.

Fifth, if we concentrate on the parameters of the double mean reversion under any specification for either the intensity parameter or the mean of the jump size, we observe two main characteristics: first, the value of the parameters α_1 and α_2 are both negative and significant, confirming the reversion to the MRO interest rate and second, this reversion is asymmetric since α_1 and α_2 are different. Since $|\alpha_1|$ is lower than $|\alpha_2|$, it also holds that the speed of reversion for low rates (and lower than the policy rate) is greater than the speed of reversion for high rates (and higher than the policy rate). So, the EONIA rate reverts to the MRO rate more quickly from drops than from rises. This evidence is also supported by Nautz and Offermans (2006) as we mentioned before. An economic explanation can be found in Ayuso and Repullo (2003). They assume that the ECB minimises a loss function depending on the policy spread, which is asymmetric in the sense that the central bank is more concerned about the EONIA below the target than above it. For this reason, it supplies less liquidity than that required to keep the EONIA equal to the target rate. Nautz and Offermans (2006) show that the EONIA adjustment is significantly stronger for negative policy spreads under both auction formats, being more pronounced during the fixed rate tender period.

Last but not least, a comparison of the ARCH-Gaussian – models 1 and 3 in Table 3 – and any ARCH-Poisson–Gaussian model reveals a sharp drop in the conditional variance once jumps are implemented. Note that the ARCH (1) process becomes non-stationary¹⁸ since ω_1 is higher than 1 under models 1 and 3. This fact implies that jumps account for a substantial component of the volatility as Eq. (6) confirms.

Finally, jumps are more likely to occur on certain days than on others. Henceforth, our comments will be related to model *iv* in Table 4. As one might expect, the jump arrival intensity is very

¹⁶ Both models are non-nested due to the specification of q_t . Model *ii* (model *iv*) would subsume model *i* (model *iii*) if $\theta_t = \gamma + \gamma(r_t - 1 - r_t^*)$. We have not adopted this alternative in order to estimate a lower number of parameters since we will also use I_t in other more extended models.

¹⁷ Unfortunately, the LRT has a non-standard distribution since testing $\lambda = 0$ is on the boundary of the parameter space and the problem of non-identification of the parameters θ and ψ under the null. Hansen (1992, 1996) raises the point that there may be an issue with the LRT since we can obtain a flat likelihood surface. Hansen's test statistic is the supremum over a grid of standardized LRT which requires a lot of CPU hours. This is the reason we do not implement Hansen's method as well as the ample evidence by SIC suggesting processes with jumps.

¹⁸ This problem does not disappear with higher order GARCH terms. We have estimated a GARCH (1,1) but it is again non-stationary.

high at the end of the maintenance period, i.e. $\delta_1 = 0.535$. Likewise, the parameter which measures the jump intensity at the end of the month (δ_2) is nearly as high as δ_1 . The lowest jump intensity is on the day of the meeting and the following day, i.e. $\delta_3 = 0.148$. This result must be taken with caution because this parameter covers all the meetings, including those that implement a change in the policy rate and those that do not. The results of the parameters δ_4 and δ_5 , which indicate the jump intensity when the end of the maintenance period and the end of the month coincide with a meeting respectively, are not important results due to the low percentage that they represent of the whole. The coefficient δ_0 measures the impact by the occurrence of jumps due to the possible effects on the remaining days. It gives a value of 0.102, which means that the lowest probability of a jump occurs on those days in which none of the analysed effects takes place.

5. Autoregressive conditional jump intensity

In Section 4, we obtain a model for the EONIA rate that incorporates the end of the month effect, the end of the maintenance period effect, and the ECB's meetings in the intensity parameter. Note that the first two effects can be set as seasonal effects in the sense that they are exhibited each month. Here, we generalize the piecewise Eq. (9) for the dynamics of λ_t with a truly time-varying function. We implement the autoregressive conditional jump intensity (ARJI), see Chan and Maheu (2002) or Maheu and McCurdy (2004). The idea behind our first model, or Eq. (9), is to understand the in-sample dynamics of the jump intensity and therefore, which factors may affect its behavior. We are now interested in a more flexible and parsimonious specification of λ_t to explain not only the in-sample but also the out-of-sample behavior.

The meeting effect does not support this kind of seasonal evidence. The reason is that in our estimation period, the number of times the MRO rate changes is 15 over a total of 120 meetings for a period of 5 years (2 meetings per month). It suggests that although we know the date of the meeting each month, the frequency of a change in the interest rate is very low in order to consider the meeting dates important by fixing a dummy variable for these dates. Hence, we leave out D_t^{meet} while keeping both D_t^{mp} and D_t^{m} for the new dynamics of λ_t .¹⁹

Finally, we study the behavior of λ_t around the days of the ECB's meetings. Under the new modeling of λ_t , i.e. ARJI dynamics, we can obtain a daily time-varying ex-post probability of the occurrence of a jump. A higher value of this probability gives a clearer indication of a jump. This probability is useful to check the existence of jumps around those meetings where there is change in the policy rate.

5.1. The model

The dynamics for λ_t is given by

$$\lambda_t = \delta_0 + \delta_1 \lambda_{t-1} + \delta_2 \xi_{t-1} + \delta_3 D_t^{\text{mp}} + \delta_4 D_t^{\text{m}} \quad (10)$$

where D_t^{mp} (D_t^{m}) corresponds to a dummy variable for the last four days of the maintenance period (the last two days of each month). The standard ARJI model is nested in Eq. (10), for $\delta_3 =$

¹⁹ We also eliminate the dummy variables $D_t^{\text{meet} + \text{mp}}$ and $D_t^{\text{meet} + \text{m}}$ in order to get a more parsimonious structure. Now, they are grouped into the dummy for the maintenance period (end of month) those days where *meet* and *mp* (*meet* and *m*) coincide.

$\delta_4 = 0$. The coefficient δ_1 measures the level of persistence over the arrival of jump events (jump clustering). The term ξ_{t-1} is known as the jump intensity innovation and it is defined as

$$\xi_{t-1} \equiv E_{t-1}[\Delta n_{t-1}] - \lambda_{t-1} = P(\Delta n_{t-1} = 1 | \Phi_{t-1}) - \lambda_{t-1}$$

where $P(\Delta n_{t-1} = 1 | \Phi_{t-1})$ is called the filter and it is the ex-post inference on Δn_{t-1} given the information set Φ_{t-1} , while λ_{t-1} is by definition the conditional expectation of Δn_{t-1} given Φ_{t-2} . This implies that ξ_t is a martingale difference sequence with respect to Φ_{t-1} , that is, $E_{t-1}[\xi_t] = 0$. It also holds that $E[\xi_t] = 0$ and $\text{Cov}(\xi_t, \xi_{t-i}) = 0, \forall i$. The parameter δ_2 measures the effect of the most recent intensity residual or the change in the conditional forecast of Δn_{t-1} given the last available information set Φ_{t-1} which is larger than Φ_{t-2} . Eq. (10) will be a stationary process if $|\delta_1|$ is lower than one.

Let $E_t[\lambda_{t+k}]$ denote the forecasts of the future jump intensities conditioned to nowadays which is denoted as period t . Since $E_t[\lambda_{t+1}] = \lambda_{t+1}$, for the case of $k \geq 2$ we have:

$$E_t[\lambda_{t+k}] = \delta_0 \left(\frac{1 - \delta_1^{k-1}}{1 - \delta_1} \right) + \delta_1^{k-1} \lambda_{t+1} + \delta_3 \sum_{j=2}^k \delta_1^{k-j} D_{t+j}^{\text{mp}} + \delta_4 \sum_{j=2}^k \delta_1^{k-j} D_{t+j}^{\text{m}}.$$

Finally, once we have observed r_t and using the Bayes rule, we can infer the ex-post or filtered probability of the occurrence of one jump at time t defined as

$$P(\Delta n_t = 1 | \Phi_t) = \frac{f(\Delta r_t | \Delta n_t = 1, \Phi_{t-1}) P(\Delta n_t = 1 | \Phi_{t-1})}{f(\Delta r_t | \Phi_{t-1})}$$

where $P(\Delta n_t = 1 | \Phi_{t-1}) = \lambda_t$ and so, $P(\Delta n_t = 0 | \Phi_t) = 1 - P(\Delta n_t = 1 | \Phi_t)$.

5.2. Empirical results

Table 5 reports the parameter estimates for the ARCH-Poisson–Gaussian model, where μ_t is a double mean-reverting process and θ_t shows mean reversion, under the dynamics of λ_t in Eq. (10), or model B, and some alternative versions of (10) given by: $\delta_1 = \delta_2 = 0$ or model A, the standard ARJI process or model C, and finally, an extension of model C by incorporating time-varying instead of constant variance for the jump size or model D.

The structure of λ_t in model A is a piecewise function similar but easier than λ_t of Eq. (9), see model iv in Table 4. The results show evidence of time-variation in the arrival of jump events since all the parameters of λ_t in model B are significantly different from zero. We observe that the same conclusions for the analysis of the double mean reversion in Table 4 are also maintained here. The parameter γ , which defines the speed of reversion of θ_t , is also negative. Note that the interpretation of the jump intensity for either the maintenance period (mp) or the end of the month (m) from both models A and B are now different with respect to Eq. (9). These effects, measured respectively by the parameters δ_3 and δ_4 , show the increment that λ_t suffers when any of these effects occurs, that is, $\lambda_t = \delta_0 + \delta_3$ for the maintenance period and $\lambda_t = \delta_0 + \delta_4$ for the end of the month. We can conclude that model B beats model A according to both SIC and LRT²⁰ values.

²⁰ Since there is again a problem of no identification under the null, we do not apply Hansen's methodology here as we did in the last section. The high magnitude of 78.6 corresponding to LRT suggests that model B provides a significant improvement over model A.

Table 5
ARCH-Poisson–Gaussian process with an autoregressive specification for λ_t

Parameter	Model A	Model B	Model C	Model D
α_1	−0.0379 (−7.486)	−0.0383 (−7.330)	−0.0270 (−6.272)	−0.0273 (−4.654)
α_2	−1.0574 (−87.213)	−0.9095 (−44.226)	−0.9134 (−46.390)	−0.9121 (−62.036)
ω_0	0.00004 (6.685)	0.00003 (7.763)	0.00003 (5.057)	0.00004 (0.044)
ω_1	0.7313 (6.035)	0.7011 (7.166)	0.5899 (12.052)	0.4020 (8.962)
γ	−0.3218 (−2.812)	−0.4878 (−13.057)	−0.4499 (−14.428)	−0.4897 (−17.602)
ψ (or ψ_0)	0.1944 (12.478)	0.1920 (11.621)	0.1853 (13.054)	0.1537 (8.392)
ψ_1	—	—	—	6.4190 (2.282)
δ_0	0.1038 (7.180)	0.0317 (4.249)	0.1375 (8.352)	0.1273 (8.584)
δ_1	—	0.4031 (7.373)	0.1021 (1.935)	0.1484 (2.260)
δ_2	—	0.6140 (7.090)	0.9456 (13.664)	0.9194 (17.876)
δ_3	0.4965 (10.080)	0.2806 (8.818)	—	—
δ_4	0.3951 (6.774)	0.3064 (8.745)	—	—
Log-L [SIC]	2248.07 [2215.79]	2287.39 [2247.94]	2178.75 [2146.48]	2190.20 [2154.34]

Daily data of EONIA rate covering the period January 1999 to December 2003. We estimate an ARCH-Poisson–Gaussian process with the following specification:

$\Delta r_t = \mu_t + \sigma_t \Delta z_t + J_t \Delta n_t$; $\Delta z_t \sim N(0,1)$; $J_t \sim N(\theta_t, \psi^2)$ where Δz_t and J_t are independent;
 $\mu_t = \alpha_1(r_{t-1} - r_{t-1}^*) + \alpha_2(r_{t-1} - r_{t-1}^*)^2$; $\sigma_t^2 = \omega_0 + \omega_1[\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2$, $\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$ and λ_t is defined as: $\lambda_t = \delta_0 + \delta_3 D_t^{\text{mp}} + \delta_4 D_t^{\text{m}}$ (model A), $\lambda_t = \delta_0 + \delta_1 \lambda_{t-1} + \delta_2 \xi_{t-1} + \delta_3 D_t^{\text{mp}} + \delta_4 D_t^{\text{m}}$ (model B) and $\lambda_t = \delta_0 + \delta_1 \lambda_{t-1} + \delta_2 \xi_{t-1}$ (models C and D). Remark on model D that $J_t \sim N(\theta_t, \psi^2)$ where $\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$ but now the variance is time-varying: $\psi_t^2 = \psi_0^2 + \psi_1 \sigma_t^2$ such that σ_t^2 is the ARCH (1) specification defined above.

Maximum likelihood estimates. Bollerslev and Wooldridge (1992) robust *t*-statistics are in parentheses. Log-L stands for the log-likelihood function. SIC is the Schwarz (1978) information criterion.

Note also that model B, by introducing the seasonal effects through the two dummy variables, outperforms the standard ARJI in model C. Finally, model D is the same as model C except that it displays variance for the jump size as a function of the ARCH (1) structure, i.e. $\psi_t^2 = \psi_0^2 + \psi_1 \sigma_t^2$. Note that there is no a significant improvement according to SIC between both models. Note also that the impact on the variance ψ_t^2 of the ARCH component measured through ψ_1 is high and significant. A high value of ψ_1 is also found in the study on stock returns carried out by Chan and Maheu (2002).²¹

If we analyse the contribution of each component to the total variance in Eq. (5), we verify that for model B the mean (median) of the series $\sigma_{t,J}^2 / V_{t-1} (\Delta r_t)$ is 0.817 (0.976) which is higher than 0.161 (0.033) for the mean (median) of $\sigma_t^2 / V_{t-1} (\Delta r_t)$.

We also study how the intervention made by the ECB's meetings can be a source of jumps. The empirical evidence of jumps can be supported by a high value of the ex-post probability. Table 6 reports these probabilities for a window of ± 4 around the days where an intervention of the policy rate occurs. We study a total of 15 interventions – the policy interest rate drops (rises) eight (seven) times – given our sample period. Those situations in which prior to the meeting date the EONIA rate leads to a high ex-post probability, might be interpreted as the market anticipating a change in the policy rate in the next meeting. Meanwhile, high ex-post probabilities after the meeting date could be interpreted as market reactions due to either an incorrect forecast of the interest rate level before the meeting, leading to a correction once the new MRO rate is known or those cases where

²¹ They also show that a standard ARJI model (or model C) having two or more jumps could be seen as one jump but using a larger jump size variance depending on the GARCH structure (or model D). Remember that our specification for the Poisson process in Eq. (1) only admits a maximum of one jump.

Table 6

Ex-post probabilities for a window of ± 4 days around the days of change in the MRO rate

Down				Up			
Date	Ex-post	Date	Ex-post	Date	Ex-post	Date	Ex-post
April 2, 1999	0.172	Nov. 2, 2001	0.050	Oct. 29, 1999	1.000*	Jun. 2, 2000	0.015
April 5, 1999	0.029	Nov. 5, 2001	0.063	Nov. 1, 1999	0.251	Jun. 5, 2000	0.012
April 6, 1999	0.014	Nov. 6, 2001	1.000	Nov. 2, 1999	0.386	Jun. 6, 2000	0.009
April 7, 1999	0.999	Nov. 7, 2001	0.339	Nov. 3, 1999	0.053	Jun. 7, 2000	1.000
April 8, 1999	0.338	Nov. 8, 2001	0.828	Nov. 4, 1999	0.040	Jun. 8, 2000	0.847
April 9, 1999	1.000	Nov. 9, 2001	0.266	Nov. 5, 1999	0.037	Jun. 9, 2000	0.152
April 12, 1999	0.369	Nov. 12, 2001	0.032	Nov. 8, 1999	0.045	Jun. 12, 2000	0.132
April 13, 1999	0.057	Nov. 13, 2001	0.897	Nov. 9, 1999	0.032	Jun. 13, 2000	0.029
April 14, 1999	1.000	Nov. 14, 2001	0.995	Nov. 10, 1999	0.031	Jun. 14, 2000	1.000
May 4, 2001	0.010	Nov. 29, 2002	1.000*	Jan. 28, 2000	1.000*	Aug. 25, 2000	0.122
May 7, 2001	0.003	Dec. 2, 2002	0.186	Jan. 31, 2000	0.612*	Aug. 28, 2000	0.049
May 8, 2001	0.003	Dec. 3, 2002	0.932	Feb. 1, 2000	0.154	Aug. 29, 2000	0.012
May 9, 2001	0.005	Dec. 4, 2002	1.000	Feb. 2, 2000	0.634	Aug. 30, 2000	0.056
May 10, 2001	1.000	Dec. 5, 2002	0.311	Feb. 3, 2000	0.054	Aug. 31, 2000	0.153
May 11, 2001	0.999	Dec. 6, 2002	0.130	Feb. 4, 2000	0.267	Sept. 1, 2000	0.229
May 14, 2001	0.259	Dec. 9, 2002	0.096	Feb. 7, 2000	0.039	Sept. 4, 2000	0.012
May 15, 2001	0.010	Dec. 10, 2002	0.003	Feb. 8, 2000	0.004	Sept. 5, 2000	0.036
May 16, 2001	0.097	Dec. 11, 2002	0.004	Feb. 9, 2000	0.008	Sept. 6, 2000	0.011
Aug. 24, 2001	0.046	Feb. 28, 2003	0.999*	Mar. 10, 2000	0.134	Sept. 29, 2000	1.000*
Aug. 27, 2001	0.087	Mar. 3, 2003	0.769	Mar. 13, 2000	0.048	Oct. 2, 2000	0.406
Aug. 28, 2001	0.015	Mar. 4, 2003	0.587	Mar. 14, 2000	0.018	Oct. 3, 2000	0.090
Aug. 29, 2001	1.000	Mar. 5, 2003	0.212	Mar. 15, 2000	0.012	Oct. 4, 2000	0.012
Aug. 30, 2001	0.903*	Mar. 6, 2003	0.099	Mar. 16, 2000	0.003	Oct. 5, 2000	0.952
Aug. 31, 2001	0.576*	Mar. 7, 2003	0.157	Mar. 17, 2000	0.004	Oct. 6, 2000	0.796
Sept. 3, 2001	0.235	Mar. 10, 2003	0.018	Mar. 20, 2000	0.081	Oct. 9, 2000	0.183
Sept. 4, 2001	0.043	Mar. 11, 2003	1.000	Mar. 21, 2000	0.091	Oct. 10, 2000	0.015
Sept. 5, 2001	0.008	Mar. 12, 2003	0.602	Mar. 22, 2000	1.000**	Oct. 11, 2000	0.005
Sept. 11, 2001	0.009	May. 30, 2003	0.546*	April 21, 2000	0.605**		
Sept. 12, 2001	1.000	Jun. 2, 2003	0.097	April 24, 2000	0.215		
Sept. 13, 2001	0.601	Jun. 3, 2003	0.063	April 25, 2000	0.537		
Sept. 14, 2001	0.140	Jun. 4, 2003	1.000	April 26, 2000	0.125		
Sept. 17, 2001	0.037	Jun. 5, 2003	0.528	April 27, 2000	0.263		
Sept. 18, 2001	1.000**	Jun. 6, 2003	0.178	April 28, 2000	1.000*		
Sept. 19, 2001	0.683**	Jun. 9, 2003	0.121	May 1, 2000	0.263		
Sept. 20, 2001	0.947**	Jun. 10, 2003	0.005	May 2, 2000	0.048		
Sept. 21, 2001	0.470**	Jun. 11, 2003	0.038	May 3, 2000	0.035		

This table contains the ex-post probability of the occurrence of one jump at time t , i.e. $P(\Delta n_t = 1 | \Phi_t)$ which can be seen in Section 5.1, for a window of ± 4 days around the days of change in the MRO rate. The meeting days are recorded here in bold. Down (Up) contains those dates where the MRO rate has moved downwards (upwards) by intervention. Those probabilities higher than 0.5 around any intervention date (± 4 days) which do not match with either the maintenance or end of the month period have also been recorded in bold. The symbol (*) denotes a date corresponding to any of the last two days of the month while (**) shows any of the last four days from the maintenance period.

there is no anticipation of a possible change of the MRO rate before the meeting. An example of the latter case might be the reason for an ascending ex-post probability in the following meetings: October 5, 2000 (upward movement) and May 10, 2001 (downward movement). We can also appreciate that this probability is not always upwards on the same day as the meeting. There is also evidence of a cluster of jumps around each meeting, except March 16, 2000 (upward movement). Table 6 also exhibits, for the selected window around the meeting date, that other

types of events may have occurred in some cases: the end of the month and the last days of the maintenance period denoted as the symbols “*” and “**” respectively. It happens that the ex-post probabilities tend to be very high in these situations. Note that we could separate the meeting effect against any other effect in every case with the exception of those days around the meetings held on August 30, 2001 and September 17, 2001. We can also appreciate that there is a high ex-post probability on the last day of many windows. A possible reason is because this day corresponds to the first auction after the last meeting date where there is an intervention on the policy rate (e.g. April 14, 1999 where the last meeting is on April 8, 1999). Finally, remember the attack on Twin Towers on September 11, 2001. Note how the ex-post probabilities rise on the following days.

6. A regime-switching process with jumps

So far, a single-regime model is set for the mean reversion of EONIA to the policy rate. This leads to the assumption that the speed of either the single or double reversion is the same throughout the sample. A more general model such as a regime-switching (RS) process has the advantage, for instance, of being more flexible by allowing a different speed of reversion at different times throughout the sample. The RS process allows us to capture several patterns which could be hidden under a single regime and hence, a better understanding of the EONIA dynamics. Empirical evidence, reported among others by Gray (1996), Dahlquist and Gray (2000), Ang and Bekaert (2002a,b)), provides two states characterised by a low-volatility/random-walk (or near random-walk) regime where the short rate spends the majority of the time and a high-volatility/mean-reverting regime.

A possible answer to this behavior could be found in Mankiw and Miron (1986). They argue that the smoothing actions of the US Fed make the short rate behave like a random walk corresponding to ‘normal’ periods, that is why volatility is low under this regime.

Nevertheless, when there are extraordinary shocks, interest rates shift to another state or second regime where its volatility is higher and they show a strong mean reversion.

We develop an extension of our jump process to accommodate regime changes in the dynamics of EONIA. This approach can be found in Das (2002) though our model is more general. It accommodates either a single or double mean reversion, jump structure and ARCH (1) within each regime. Gray (1996) implements a generalized RS model with a GARCH (1,1) structure augmented with the level of the interest rate as opposed to a low-order ARCH process as in either Cai (1994) or Hamilton and Susmel (1994). Our model will be in line with the latter two studies in the sense of implementing an ARCH (1) structure and keeping the parameter of the ARCH effect the same in both states.²²

6.1. The model

Since we postulate a two-regime process, let S_t denote the unobserved state or latent variable of the system where $S_t = 0$ or 1. S_t is assumed to follow a first-order Markov process with the transition probabilities:

$$\begin{aligned} P[S_t = 0 | S_{t-1} = 0] &= p_{00}; & P[S_t = 1 | S_{t-1} = 0] &= 1 - p_{00} \\ P[S_t = 1 | S_{t-1} = 1] &= p_{11}; & P[S_t = 0 | S_{t-1} = 1] &= 1 - p_{11}. \end{aligned} \quad (11)$$

²² Another way of capturing the dynamics of the volatility would be a stochastic volatility model whose mean is subject to changes in regime, see Kalimipalli and Susmel (2004).

where $p_{00} = \exp(q_0) / (1 + \exp(q_0))$ and $p_{11} = \exp(q_1) / (1 + \exp(q_1))$ are defined as logistic functions. Note that these transition probabilities are constant at first. We will also implement the RS model with time-varying transition probabilities. This augmented model will be shown in this section later.

For our ARCH-Poisson–Gaussian process with shifting regime, we will impose several restrictions on the number of parameters and also easier specifications for the behavior of λ_t in order to make the estimation of the new model feasible as we will explain later. We will implement for μ_t either a simple or double mean reversion process:

$$\mu_{S_t,t} = \alpha^{S_t}(r_{t-1} - r_{t-1}^*); \quad \mu_{S_t,t} = \alpha_1^{S_t}(r_{t-1} - r_{t-1}^*)^+ + \alpha_2^{S_t}(r_{t-1} - r_{t-1}^*)^-;$$

$$\alpha^{S_t} = \alpha^0(1 - S_t) + \alpha^1 S_t; \quad \alpha_k^{S_t} = \alpha_k^0(1 - S_t) + \alpha_k^1 S_t, k = 1, 2.$$

For the ARCH (1) process defined in Eq. (7), we will only assume a shifting regime specification for the parameter ω_0 :

$$\sigma_{S_t,t}^2 = \omega_0^{S_t} + \omega_1[Ar_{t-1} - E_{t-2}(Ar_{t-1})]^2 \quad \text{where} \quad \omega_0^{S_t} = \omega_0^0(1 - S_t) + \omega_0^1 S_t.$$

A constant variance model will be defined as the above process subject to the restriction of $\omega_1 = 0$. With respect to the jump component, we will make the following assumptions on both λ_t and the volatility of the jump size:

$$\lambda_{S_t} = \lambda_0(1 - S_t) + \lambda_1 S_t; \quad \psi_{S_t} = \psi_0(1 - S_t) + \psi_1 S_t.$$

Note that both λ_{S_t} and ψ_{S_t} are step functions where each admits only two possible values depending on the state. Meanwhile, the mean of the jump size is presumed to be the same value across the different states to simplify. Here, we will adopt the specification of $\theta_t = \gamma(r_{t-1} - r_{t-1}^*)$. The Appendix shows the construction of both the likelihood function and smoothed probabilities.

We will also implement an augmentation of the above RS but a time-varying structure for the transition probabilities in Eq. (11) is allowed. We will assume two alternative specifications:

$$p_{ii,t} = \frac{\exp(\beta_0^i + \beta_1^i |r_{t-1} - r_{t-1}^*|)}{1 + \exp(\beta_0^i + \beta_1^i |r_{t-1} - r_{t-1}^*|)} \quad (12)$$

and

$$p_{ii,t} = \frac{\exp(\beta_0^i + \beta_1^i |r_{t-1} - r_{t-1}^*| + \beta_2^i D_t^{\text{mp}} + \beta_3^i D_t^{\text{m}})}{1 + \exp(\beta_0^i + \beta_1^i |r_{t-1} - r_{t-1}^*| + \beta_2^i D_t^{\text{mp}} + \beta_3^i D_t^{\text{m}})} \quad (13)$$

where $p_{ii,t} \equiv P[S_t = i | S_t = i]$ such that $i = 0, 1$.²³ Eq. (12) depends on the size of the spread at time $t - 1$. This model nests the constant transition probability, or Eq. (11), when $\beta_1^0 = \beta_1^1 = 0$. Meanwhile, Eq. (13) – which contains Eq. (12) under the restriction $\beta_2^i = \beta_3^i = 0$ – incorporates the well-known seasonal pattern captured by both the maintenance period and the end of the month effect. Eq. (13) nests the constant transition probability under $\beta_1^i = \beta_2^i = \beta_3^i = 0$.

6.2. Empirical results

Table 7 exhibits the estimates of the RS model with constant transition probabilities under alternative specifications of both the conditional mean $\mu_{S_t,t}$ and conditional variance $\sigma_{S_t,t}^2$. A total

²³ Note that $p_{ii,t} \equiv P[S_t = i | S_t = i]$ is an abbreviated form since it is also conditioned to other information sets.

Table 7
Poisson–Gaussian model with a regime-switching process (constant transition probabilities)

State	Parameters	Single mean reversion			Double mean reversion	
		$\mu_{S,t}=\alpha^S_t(r_{t-1}-r_{t-1}^*)$			$\mu_{S,t}=\alpha^S_1(r_{t-1}-r_{t-1}^*)^++\alpha^S_2(r_{t-1}-r_{t-1}^*)^-$	
		$\alpha^S_t=\alpha^0(1-S_t)+\alpha^1S_t$			$\alpha^S_k=\alpha^0_k(1-S)+\alpha^1_kS_t$ where $k=1,2$	
		Gaussian	Poisson–Gaussian			
		(I) ARCH (1)	(II) Constant variance	(III) ARCH (1)	(IV) Constant variance	(V) ARCH (1)
0	α^0	−0.0421 (−3.622)	−0.0182 (−0.993)	−0.0273 (−5.325)	–	–
1	α^1	−0.5452 (−10.343)	0.0313 (0.817)	0.0875 (2.286)	–	–
0	a^0_1	–	–	–	−0.0206 (−1.542)	−0.0274 (−4.634)
1	a^1_1	–	–	–	0.0234 (0.968)	0.0578 (0.988)
0	a^0_2	–	–	–	−1.0909 (−123.61)	−1.0855 (−43.054)
1	a^1_2	–	–	–	0.1347 (1.732)	0.1709 (2.168)
0	a^0_0	0.00004 (5.379)	0.00003 (7.267)	0.00003 (6.786)	0.00004 (6.669)	0.00003 (7.042)
1	ω^1_0	0.0365 (6.946)	0.0032 (3.760)	0.0033 (5.120)	0.0028 (4.599)	0.0030 (4.136)
0, 1	ω_1	0.4391 (3.304)	–	0.0056 (1.380)	–	0.0046 (0.799)
0, 1	γ	–	−1.2783 (−3.598)	−1.8410 (−7.547)	−1.2612 (−3.246)	−1.6295 (−3.269)
0	ψ_0	–	0.0058 (1.951)	0.0068 (2.325)	0.0064 (1.135)	0.0072 (2.714)
1	ψ_1	–	0.2643 (17.918)	0.2519 (15.092)	0.2646 (17.238)	0.2521 (14.020)
0	λ_0	–	0.1393 (3.431)	0.1127 (5.310)	0.1370 (4.026)	0.1292 (4.430)
1	λ_1	–	0.4079 (10.877)	0.4146 (10.480)	0.3817 (9.316)	0.3739 (8.283)
0	P_{00}	0.8339 (41.623)	0.8045 (43.378)	0.8016 (47.075)	0.8124 (47.531)	0.8095 (48.605)
1	P_{11}	0.5901 (12.661)	0.7850 (29.590)	0.7703 (32.667)	0.8001 (34.149)	0.7732 (22.023)
Log-L [SIC]		2153.59 [2128.49]	2255.01 [2215.56]	2266.70 [2223.65]	2273.10 [2226.48]	2299.43 [2249.22]

Daily data of EONIA rate covering the period January 1999 to December 2003. This table contains Poisson–Gaussian models with a regime-switching process and constant transition probabilities under different specifications for the conditional mean (single or double mean reversion) and conditional variance (constant or ARCH (1) variance). The system switches between two regimes, $S_t=0,1$, where each state follows a Poisson–Gaussian process except model I (Gaussian process in each state). In each regime, the parameters for the conditional mean and variance with no jumps, denoted as $\mu_{S_t,t}$ and $\sigma^2_{S_t,t}$ respectively; the jump intensity λ_{S_t} and the jump volatility ψ_{S_t} change. The Markov chain is defined by the switching probabilities in Eq. (11). The ARCH (1) specification is $\sigma^2_{S_t,t} = \omega^S_0 + \omega_1[\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2$ where $\omega^S_0 = \omega^0_0(1 - S_t) + \omega^1_0 S_t$. The constant variance is defined as $\sigma^2_{S_t,t} = \omega^S_0$, i.e. the above process subject to $\omega_1=0$. Maximum likelihood estimates. Bollerslev and Wooldridge (1992) robust *t*-statistics are in parentheses. Log-L stands for the log-likelihood function. SIC is the Schwarz (1978) information criterion.

of five RS models are estimated: model I is a Gaussian process with ARCH (1) structure and single mean reversion in each state. The remaining models in Table 7, and those in Table 8, will always be characterised by a Poisson–Gaussian process under alternative specifications. Model II shows single mean reversion and constant variance, model III presents single mean reversion but an ARCH (1) structure, model IV shows a double mean reversion with constant variance and finally, model V shows a double mean reversion with ARCH (1). Models III and V nest models II and IV when there is no ARCH effect respectively. Note also that model IV nests model II when there is no double mean reversion. The models in Table 8, denoted as VI to IX, are characterised as follows: models VI and VII are similar to models II and III respectively, but now with a time-varying transition probability $p_{ii,t}$ defined in Eq. (12). Meanwhile, models VIII and IX are the same as models II and III respectively but $p_{ii,t}$ is driven by Eq. (13). To ensure that we identify the global maximum, we re-estimate each model with different sets of starting values.

Looking at Tables 7 and 8, we can appreciate some features: first, modeling the EONIA rate under a regime-switching ARCH process but without the introduction of the jump component in

Table 8

Poisson–Gaussian models with a regime-switching process (time-varying transition probabilities)

State	Parameters	$p_{ii,t} = \varphi(\beta_0^i + \beta_1^i r_{t-1} - r_{t-1}^*)$		$p_{ii,t} = \varphi(\beta_0^i + \beta_1^i r_{t-1} - r_{t-1}^* + \beta_2^i D_t^{\text{mp}} + \beta_3^i D_t^{\text{m}})$	
		(VI) Constant variance	(VII) ARCH (1)	(VIII) Constant variance	(IX) ARCH (1)
0	α^0	−0.0236 (−3.036)	−0.0263 (−4.981)	−0.0311 (−4.406)	−0.0311 (−5.450)
1	α^1	0.0215 (0.538)	0.0479 (1.399)	0.0376 (1.108)	0.0491 (1.371)
0	ω_0^0	0.00004 (7.232)	0.00003 (7.116)	0.00004 (6.495)	0.00003 (5.173)
1	ω_0^1	0.0033 (3.110)	0.0038 (5.668)	0.0044 (4.312)	0.0043 (4.734)
0, 1	ω_1	–	0.0047 (1.345)	–	0.0074 (0.710)
0, 1	γ	−1.2727 (−2.655)	−1.7986 (−5.486)	−1.1190 (−3.322)	−1.5556 (−5.120)
0	ψ_0	0.0061 (2.006)	0.0067 (2.061)	0.0241 (4.266)	0.0212 (5.332)
1	ψ_1	0.2667 (18.770)	0.2601 (15.787)	0.2654 (17.838)	0.2607 (16.575)
0	λ_0	0.1298 (4.141)	0.1135 (6.637)	0.1892 (5.020)	0.1762 (6.137)
1	λ_1	0.4015 (9.699)	0.3918 (9.871)	0.4635 (9.096)	0.4403 (8.776)
0	β_0^0	1.4681 (6.505)	1.3792 (8.253)	3.0148 (7.968)	2.8766 (7.588)
1	β_0^1	0.0758 (0.322)	0.1176 (0.495)	−1.5327 (−2.984)	−1.3691 (−2.700)
0	β_1^0	−1.6935 (−0.471)	−0.0874 (−0.040)	−2.7050 (−0.556)	−3.1237 (−0.730)
1	β_1^1	9.9756 (3.943)	8.5354 (4.415)	16.0894 (3.734)	13.526 (2.783)
0	β_2^0	–	–	−2.8226 (−8.046)	−2.6925 (−8.001)
1	β_2^1	–	–	16.7859 (0.917)	19.142 (0.3556)
0	β_3^0	–	–	−2.8747 (−7.788)	−2.7674 (−8.406)
1	β_3^1	–	–	12.5492 (0.427)	11.525 (2.559)
	Log-L [SIC]	2286.80 [2240.18]	2293.21 [2243.00]	2384.62 [2323.66]	2393.14 [2328.59]

Daily data of EONIA rate covering the period January 1999 to December 2003. This table contains Poisson–Gaussian models with a regime-switching process and time-varying transition probabilities. The system switches between two regimes, $S_t = 0, 1$, where each state follows a Poisson–Gaussian process. In each regime, the parameters for the conditional mean and variance with no jumps, denoted as $\mu_{S_t,t}$ and $\sigma_{S_t,t}^2$, respectively; the jump intensity λ_{S_t} and the jump volatility ψ_{S_t} , change. The Markov chain is defined by the switching probabilities $p_{ii,t} \equiv P[S_t = i | S_{t-1} = i]$ where $i = 0, 1$ and $p_{ii,t} = \varphi(x_t)$ where $\varphi(x_t) = \exp(x_t) / (1 + \exp(x_t))$. The specification of $\mu_{S_t,t}$ and $\sigma_{S_t,t}^2$ are:

$\mu_{S_t,t} = \alpha^i (r_{t-1} - r_{t-1}^*)$ where $\alpha^i = \alpha^0 (1 - S_t) + \alpha^1 S_t$
 $\sigma_{S_t,t}^2 = \omega_0^i + \omega_1 [\Delta r_{t-1} - E_{t-2}(\Delta r_{t-1})]^2$ where $\omega_0^i = \omega_0^0 (1 - S_t) + \omega_0^1 S_t$; ($\omega = 0$: constant variance).

Maximum likelihood estimates. Bollerslev and Wooldridge (1992) robust t -statistics are in parentheses. Log-L stands for the log-likelihood function. SIC is the Schwarz (1978) information criterion.

each state (model I) leads to the lowest values of both the likelihood function and SIC. For instance, model III shows a SIC value of 2223.65 against 2128.49 for model I. This large difference of more than one hundred is due to the lack of the jump component in model I.

Second, there is only ARCH (1) effect in model I. The magnitude of ω_1 decreases sharply going from model I to another model. For example, $\omega_1 = 0.0056$ in model III against $\omega_1 = 0.4391$ in model I. Hence, once we plug the jump component into each state in model I, thus obtaining model III, the ARCH contribution in the variance of each state becomes less important.

Third, both variance components, i.e. $\omega_0^{S_t}$ from $\sigma_{S_t,t}^2$ (no jump) and $\psi_{S_t}^2$ (jump) and the intensity parameter λ_{S_t} are higher in state 1. This suggests that state 0 is a period of lower volatility than state 1. The percentage of days, though not reported, containing only those days for both the end of the maintenance period (mp) and the end of month (m) tend to be more concentrated in regime 1 across the different models. This empirical fact might help us to understand why regime 1 exhibits higher volatility than regime 0. It is why the probability of occurring a jump is also higher in regime 1 since, as we mentioned in the previous sections, λ_t is higher for either mp or m than any other day.

Fourth, model I shows a significant mean reversion in both states. This result is contrary to the evidence observed in the rest of the models characterised by a unique regime of mean reversion.

We will concentrate on the behavior of all models except model I since its SIC value is worse than the others because it does not model the jump effect in each state. The misspecification of jumps leads model I to exhibit mean reversion in the two states but the introduction of jumps gives a better performance, according to the SIC value, and shows a pattern of mean reversion that is completely different in each state and that is more in accordance with the empirical evidence shown by Gray (1996), among others. Specifically, the sign of the mean reversion (both single and double) is always negative in state 0 while it is positive in state 1 (except for model I). This evidence suggests that the EONIA rate is mean reverting to the MRO rate in state 0 characterised by a period of low volatility while the mean reversion does not hold in state 1. The mean reversion tends to be more significant under the ARCH (1) structure. Indeed, the mean reversion is always significant, except for models II and IV. It is also verified that the double mean reversion beats the single one. For example, the LRT value of $\chi^2_2 = 36.2$ ($H_0 = \text{II}; H_1 = \text{IV}$) where the 1% critical level for χ^2_2 is 9.21.

Last but not least, both higher SIC and significant LRT values conclude that models under time-varying transition probabilities improve the performance.²⁴ This evidence is also obtained by using the popular RCM statistic proposed by Ang and Bekaert (2002a) to measure the quality of the regime classification.²⁵

Finally, we must remark that our evidence on both low volatility and mean reversion in state 0 and the opposite in state 1 is different with respect to the other works cited previously. It is worth emphasizing that all those papers employ interest rate series having either a weekly or monthly frequency while we use daily data. The selection of a higher frequency might be a possible reason why this study brings out a different result. A possible answer to our results can be found in the analysis of the sign of the parameters which define $p_{ii,t}$ in the Eqs. (12) and (13). Taking model VI or VII and considering an increase in the size of the spread $|r_{t-1} - r_{t-1}^*|$: since β_1^0 is negative (state 0), the probability of staying in this regime decreases as the size of the spread increases. Conversely, since β_1^1 is positive (state 1) the probability of staying in this regime increases as the size of the spread increases. Therefore, when the size of the spread raises the probability of staying or switching to regime 1 increases. In short, high interest rate levels take place in regime 1 in addition to both a higher probability of occurring jumps and the failure to hold the mean reversion in this state. Now, if we consider model VIII or IX, the same conclusion holds on the sensitivity of $p_{ii,t}$ to changes in $|r_{t-1} - r_{t-1}^*|$, i.e. $\beta_1^0 < 0$ and $\beta_1^1 > 0$. The sensitivity of the end of the maintenance period or the end of month in regime 0 is measured through the parameters β_2^0 and β_3^0 respectively and they are negative while they show the opposite sign in regime 1. This evidence reinforces our suspicion or hypothesis that if tomorrow $D_t^{\text{mp}} = 1$ or $D_t^{\text{m}} = 1$ then the probability of staying or switching to regime 1 increases.

7. Forecasting performance

Since forecasting volatility is one of the main practical uses, it is important to provide evidence of the forecasting performance. We estimate the parameters of a particular model over the in-sample period which are held fixed to compute a time series of conditional variances over the out-of-sample period. We repeat this process several times for alternative in-sample periods. Let $\varepsilon_t = \Delta r_t - E_{t-1}[\Delta r_t]$ denote the innovation to the interest rate process. We compare the conditional

²⁴ We do not estimate RS under double mean reversion with time varying transition probabilities. The reason is based on the large number of parameters.

²⁵ LRT and RCM statistics are not exhibited but they are available upon request.

variance $E_{t-1}[\varepsilon_t^2]$ with the actual squared innovation ε_t^2 . The difference between ε_t^2 and $E_{t-1}[\varepsilon_t^2]$ is the error denoted as e_t . We will use the following common statistics for our analysis: the root mean squared error (RMSE), the mean absolute error (MAE) and the R^2 measure from the regression with dependent and independent variables ε_t^2 and $E_{t-1}[\varepsilon_t^2]$ respectively and imposing an intercept of zero and a slope of one. Hence, R^2 becomes a direct measure of the goodness of fit of the variance forecast. Note that negative values of R^2 are possible. For a time series $\{e_t\}$, these statistics are defined as

$$\text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^T e_t^2}; \quad \text{MAE} = \frac{1}{T} \sum_{t=1}^T |e_t| \quad \text{and} \quad R^2 = 1 - \frac{\sum_{t=1}^T e_t^2}{\sum_{t=1}^T \varepsilon_t^4}.$$

Another statistic is the SIC measure that will be analysed later. Table 9 exhibits the out-of-sample test for different periods. We compare the performance of some models of both one regime and two regimes. From the first group, we select both the single and the double mean reverting ARCH-Poisson–Gaussian processes where λ_t is constant, which represent respectively models 2 and 4 in Table 3, and the double mean-reverting ARCH-Poisson–Gaussian but with an autoregressive time-varying λ_t defined in Eq. (10), i.e. model B in Table 5. In the second group, we select the ARCH-Gaussian with shifting regime specification, or model I in Table 7, and all constant variance²⁶ Poisson–Gaussian with shifting regime specifications: models II and IV in Table 7 (constant transition probabilities) and models VI and VIII in Table 8 (time-varying transition probabilities). We attain the following conclusions from Table 9:

First, the best forecast according to either the lowest value of MAE or RMSE is classified depending on the mean reversion. For the single mean reversion, we select model 2 under MAE except for the first two periods where model I scores the highest performance. Nevertheless, we choose model I under RMSE except for the last two periods where model 2 is the best. For the double mean reversion, we select model 4 under MAE but model IV under RMSE except for the last two periods where model 4 is the winner again. If we compare models 4 and B, it holds that under RMSE model B beats model 4 except for the last two periods.

Second, if we compare a double mean reversion against a single one, under one regime model 2 is always better under MAE. Meanwhile, under RMSE model B wins except for the last two periods where model 2 wins. For two regimes, model I is the winner under MAE but there is a trade-off between models I and IV depending on the selected period under RMSE. Again, this comparison is under constant transition probabilities.

Third, comparing one regime against two regimes it happens that model I beats under MAE until May 6, 2004 and afterward model 2 wins. Models 4, I and IV win depending on the period under RMSE though the two regime models are better in most of the periods.

Fourth, contrary to the in-sample analysis, though not reported here,²⁷ there is no improvement going from constant to time-varying transition probability models. A reason might be that the more variables we include in the model, the lower RMSE will be even if they cannot add extra out-of-sample forecasting power; although they will improve the model's fit on historical data. This effect addresses the possibility of overfitting (Diebold, 2004) the in-sample data. We need to

²⁶ Remember that the ARCH (1) specifications in Tables 7 and 8 were not significant, that is why we only concentrate on constant variance models apart from model I.

²⁷ The in-sample analysis is available upon request. The in-sample results show that model VIII attains the lowest RMSE for every period. Under MAE, model VIII wins most of periods.

Table 9
Out-of-sample performance

Period (sample size, T)	Statistic	One regime			Two regimes				
		Table 3		Table 5	Table 7			Table 8	
		Model 2 (single)	Model 4 (double)	Model B (double)	Model I (single)	Model II (single)	Model IV (double)	Model VI (single)	Model VIII (single)
Dec 31, 2003– Mar 30, 2005 (326)	MAE	0.0130	0.0131	0.0148	0.0126	0.0149	0.0140	0.0147	0.0130
	RMSE	0.0398	0.0398	0.0392	0.0284	0.0301	0.0291	0.0318	0.0312
	SIC	0.0018	0.0018	0.0019	0.0009	0.0011	0.0011	0.0013	0.0013
	R^2	0.0591	0.0675	0.2081	0.3361	0.1933	0.2384	0.1036	0.1622
Mar 4, 2004– Mar 30, 2005 (280)	MAE	0.0135	0.0135	0.0153	0.0128	0.0151	0.0142	0.0151	0.0135
	RMSE	0.0427	0.0426	0.0417	0.0303	0.0316	0.0307	0.0340	0.0329
	SIC	0.0021	0.0021	0.0022	0.0011	0.0012	0.0012	0.0015	0.0015
	R^2	0.0651	0.0733	0.2233	0.3543	0.2293	0.2672	0.1166	0.1892
May 6, 2004– Mar 30, 2005 (235)	MAE	0.0112	0.0113	0.0134	0.0118	0.0140	0.0132	0.0140	0.0126
	RMSE	0.0360	0.0357	0.0350	0.0258	0.0267	0.0256	0.0284	0.0277
	SIC	0.0015	0.0015	0.0016	0.0008	0.0009	0.0009	0.0011	0.0011
	R^2	0.0190	0.0321	0.1866	0.3149	0.1865	0.2397	0.0840	0.1489
Jul 1, 2004– Mar 30, 2005 (195)	MAE	0.0111	0.0113	0.0139	0.0118	0.0144	0.0135	0.0144	0.0131
	RMSE	0.0386	0.0385	0.0379	0.0273	0.0285	0.0273	0.0300	0.0295
	SIC	0.0018	0.0018	0.0019	0.0009	0.0011	0.0011	0.0013	0.0014
	R^2	0.0188	0.0401	0.1929	0.3274	0.1943	0.2529	0.1153	0.1648
Sep 2, 2004– Mar 30, 2005 (150)	MAE	0.0102	0.0104	0.0133	0.0112	0.0139	0.0131	0.0139	0.0124
	RMSE	0.0392	0.0389	0.0389	0.0263	0.0269	0.0260	0.0287	0.0281
	SIC	0.0019	0.0019	0.0022	0.0009	0.0010	0.0010	0.0013	0.0014
	R^2	–0.0222	0.0066	0.1532	0.3736	0.2394	0.2847	0.1594	0.1985
Nov 4, 2004– Mar 30, 2005 (105)	MAE	0.0043	0.0045	0.0083	0.0079	0.0116	0.0110	0.0112	0.0105
	RMSE	0.0060	0.0057	0.0118	0.0096	0.0141	0.0136	0.0136	0.0157
	SIC	4.7e–05	4.4e–05	0.0002	0.0001	0.0003	0.0003	0.0003	0.0005
	R^2	–0.7723	–0.6926	–4.7860	–3.0637	–7.8151	–7.3720	–7.1741	–9.7939
Jan 6, 2005– Mar 30, 2005 (60)	MAE	0.0038	0.0040	0.0090	0.0072	0.0102	0.0094	0.0096	0.0101
	RMSE	0.0047	0.0046	0.0127	0.0087	0.0123	0.0113	0.0111	0.0149
	SIC	3.3e–05	3.4e–05	0.0003	0.0001	0.0003	0.0003	0.0003	0.0007
	R^2	–1.3531	–2.0029	–23.945	–7.4391	–17.959	–17.957	–13.918	–30.128

Daily data of EONIA rate covering the period January 1999 to March 2005, a total of 1628 observations. This table contains mean absolute errors (MAE), root mean squared errors (RMSE) where the error e_t is the difference between the actual daily variance (ε_t^2 where $\varepsilon_t = \Delta r_t - E_{t-1}[\Delta r_t]$) and the conditional expected variance $E_{t-1}[\varepsilon_t^2]$, the SIC statistic is equal to $\text{RMSE}^2 \times T^{k/T}$ where T is the sample size and k is the number of parameters and the R^2 measure – with constant and slope equal zero and one respectively – defined as $R^2 = 1 - \sum_{t=1}^T e_t^2 / \sum_{t=1}^T \varepsilon_t^4$. Finally, “single” and “double” denote single and double mean reversion respectively.

penalize for degrees of freedom used – the number of parameters estimated – by implementing the SIC statistic defined as $\text{RMSE}^2 \times T^{k/T}$ where T is the sample size and k is the number of parameters. The out-of-sample results show that model I wins for the first five periods according to SIC in Table 9. Meanwhile, model 4 and model 2 win respectively in the periods November 4, 2004 to March 30, 2005 and January 6, 2005 to March 30, 2005. Hence, we show that SIC-selected models are more parsimonious.

Fifth, the R^2 measure is positive for all the periods except the last two. For the case of $R^2 > 0$, model I scores the highest value. R^2 is also negative for the period going from September 2, 2004 to March 30, 2005 but only for model 2. The evidence of $R^2 < 0$ in the last two periods suggests

that all models tend to forecast systematically daily variances higher than the actual variances. Note that this overprediction is lower for models 2 and 4.

Finally, we may conclude that a parsimonious two-regime process such as model I, an ARCH (1) process which does not include either jumps or double mean reversion, performs best in most periods in Table 9.

8. Conclusions

This paper analyses the daily time series of the Euro overnight interest rate (EONIA). We conclude that jumps are an essential component of modeling EONIA. The different models we implement contribute to a much better in-sample fit once jumps are considered under either one or two regimes. Three effects such as the end of maintenance period effect, the calendar effect and the meeting effect show evidence of the existence of jumps. These effects have been considered for modeling jumps and specifically, the jump intensity or the ex-ante probability of a jump occurring. We find that both the end of maintenance period effect and the calendar effect cause greater jumps than the effect of the meetings of the Governing Council of the ECB. The lowest jump intensity corresponding to the days on which none of the effects occur leads to the conclusion that the main causes of the jumps are covered by our model. Also, the persistence of the ARCH effects tend to decrease with the introduction of jumps. Finally, the ARCH structure is not significant under two regimes with jumps.

The ECB's aim of keeping the overnight rate close to the policy rate seems to sustain given the high level of reversion that empirical evidence suggests. We have analysed whether the speed of reversion is different when EONIA is higher than the MRO rate to when it is lower. It suggests that reversion is much greater in the latter case. In short, a two mean reversion specification has been implemented giving better results than a one mean reversion process.

The first two models implemented here are based on extensions of the well-known Poisson–Gaussian-ARCH structure. The difference between the two candidates can be found in the dynamics of the jump intensity λ_t . Our first approach consists of a step function where λ_t captures the three effects mentioned before. The second model exhibits a really time-varying pattern for λ_t , in concrete, the autoregressive conditional jump intensity (ARJI) by Chan and Maheu (2002), which also includes both the end of the maintenance period and the calendar effects since these effects behave like seasonal components. We conclude that both models lead to the same results, although the latter gives better performance. Under the ARJI dynamics, we obtain daily time-varying ex-post probabilities which can be used to understand better the reaction of EONIA around the ECB's meetings where an intervention over the policy rate happens.

Last but not least, we develop a two-regime-switching model where each state is driven by an ARCH-Poisson–Gaussian under different specifications for the transition probabilities. The empirical results indicate that there is one regime characterised by high volatility and no reversion to the policy rate and the other by low volatility and mean reversion. This evidence is in accordance with other studies such as Gray (1996) among others in the sense that the mean reversion does not always hold, that is why two regimes might result to be more appropriate than a single regime to model interest rates. Moreover, two-regime models outperform single-regime ones in our volatility forecasting analysis most of the time. In short, an ARCH (1) two-regime process without either jumps or double mean reversion structure performs best in most periods in our out-of-sample analysis. According to our more generalized structure for the transition probability, it is shown that the probability of staying or switching to the regime of high volatility/no mean reversion is higher when either the maintenance period or the end of the month

effect happens. This regime is also characterised by a higher probability of jumps occurring and higher levels of interest rates.

Finally, since we have adopted an exogenous behavior for the policy rate r_t^* , a fruitful avenue for future research would be modeling the dynamics of r_t^* as a pure jump process in the spirit of Piazzessi (2005).²⁸ It holds that the timing of jumps is deterministic and coincides with the exact dates of the ECB's meetings. The intensity parameter of jumps might depend on several state variables such as macroeconomic factors like both the output gap and inflation as in Dewachter et al. (2006) and so forth. This could improve the forecast of both the level and volatility of EONIA.

Appendix

The density of Δr_t in each state is:

$$f(\Delta r_t | S_t, \Phi_{t-1}) = (1 - \lambda_{S_t}) \times f(\Delta r_t | S_t, \Delta n_t = 0, \Phi_{t-1}) + \lambda_{S_t} \times f(\Delta r_t | S_t, \Delta n_t = 1, \Phi_{t-1})$$

where $\Delta r_t | S_t, \Delta n_t = j, \Phi_{t-1}$ follows a normal distribution with mean $\mu_{S_t, j} + j\theta_t$ and variance $\sigma_{S_t, j}^2 + j\psi_{S_t}^2$. So, the conditional density of Δr_t to construct the log-likelihood is:

$$f(\Delta r_t | \Phi_{t-1}) = p_{0,t} \times f(\Delta r_t | S_t = 0, \Phi_{t-1}) + p_{1,t} \times f(\Delta r_t | S_t = 1, \Phi_{t-1})$$

where $p_{S_t, t} \equiv P[S_{t-1} = i | \Phi_{t-1}]$ for $i=0,1$ is defined as

$$p_{0,t} = p_{00} \times P[S_{t-1} = 0 | \Phi_{t-1}] + (1 - p_{11}) \times P[S_{t-1} = 1 | \Phi_{t-1}]; p_{1,t} = 1 - p_{0,t}$$

such that p_{00} and p_{11} are the transition probabilities in Eq. (11) that can also be time-varying and both $P[S_{t-1}=0|\Phi_{t-1}]$ and $P[S_{t-1}=1|\Phi_{t-1}]$ denote the ex-post or filtered probabilities from the previous period. The filtered probabilities follow Bayes' Theorem:

$$P[S_t = 0 | \Phi_t] = \frac{P[S_t = 0 | \Phi_{t-1}] \times f(\Delta r_t | S_t = 0, \Phi_{t-1})}{f(\Delta r_t | \Phi_{t-1})}; \quad P[S_t = 1 | \Phi_{t-1}] = 1 - P[S_t = 0 | \Phi_t].$$

The smoothed probabilities $P[S_t | \Phi_T]$, where $t=1, 2, \dots, T$, are also of interest in determining if and when regime switches occur. These are obtained by the smoothing algorithm in Kim and Nelson (1999):

$$P[S_t = i | \Phi_T] = \sum_{j=0}^1 P[S_t = i, S_{t+1} = j | \Phi_T]$$

where

$$P[S_t = i, S_{t+1} = j | \Phi_T] = \frac{P[S_{t+1} = j | \Phi_T] \times P[S_t = i | \Phi_t] \times P[S_{t+1} = j | S_t = i]}{P[S_{t+1} = j | \Phi_t]}.$$

²⁸ The continuous time dynamics of r_t^* is defined as a pure Poisson process: $dr_t^* = 0.0025 (dN_t^U - dN_t^D)$ where the steps are multiples of 25 basis points (bp), or 0.0025. Both N_t^U and N_t^D are Poisson processes with stochastic intensity parameters λ_t^U and λ_t^D respectively. The conditional probability of a target increase by 25 bp during $[t, t + dt]$ is then $\lambda_t^U dt \times (1 - \lambda_t^D dt)$.

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