

The ex ante real rate and inflation premium under a habit consumption model[☆]

Leonardo Madureira

Department of Banking and Finance, Case Western Reserve University, United States

Accepted 18 September 2006

Available online 19 January 2007

Abstract

This paper analyzes and quantifies ex ante components of bond yields – real rate of returns and risk premiums – from observed prices of nominal and indexed bonds in the United Kingdom from 1983 to 2000. The estimation uses an asset pricing framework based on a habit consumption model together with a joint formulation of consumption growth and inflation. Nominal yields carry a time-varying inflation premium that is significant throughout the period, increasing in the bond's maturity and contributing up to 25 basis points to yearly nominal yields. The analysis allows the extraction of the ex ante real rate from indexed bonds by properly taking into account both the incomplete indexation on these instruments and the inflation premium embedded in the nominal bonds.

© 2006 Elsevier B.V. All rights reserved.

JEL classification: E21; E27; E31; E43; G12

Keywords: Indexed bonds; Real interest rates; Inflation premium; Ex ante risk premiums; Habit consumption model

1. Introduction

Measures of ex ante variables are critical factors in many types of financial problems. The ex ante real rate is a measure of expectations of future economic activity and can heavily influence the investment decisions of corporations and individuals. The ex ante inflation risk premium,

[☆] I am grateful to Wayne Ferson (the Editor) and the referee for insightful and constructive comments. I also thank Michael Brandt, Rüdiger Fahlenbrach, Martin Lettau, Jessica Wachter, Amir Yaron and seminar participants at the Wharton School, the 2003 Midwest Finance Association, the 2003 Western Finance Association and the 2004 Financial Management Association meetings for their helpful comments and suggestions, and Martin D. D. Evans and the Bank of England for providing the data on bond prices. The usual disclaimer applies.

E-mail address: leonardo.madureira@case.edu.

which assesses to what extent compensation is necessary for holding nominal instruments subject to the risk of inflation, is an additional cost of issuing these instruments, and an indirect measure of ex ante variation in inflation. The goal of this paper is to estimate these ex ante variables. I do this by comparing nominal and indexed bonds from the UK, and through the simulation of an intertemporal equilibrium model.

The conventional approach to estimating real rates and inflation premiums applies Fisher's (1930) idea that nominal yields are composed of the ex ante real yield plus the ex ante rate of inflation. However, the Fisher theorem does not hold under uncertainty, so nominal yields also include an inflation risk premium. In order to extract one of these unobservable components, one must use a proxy for the others.¹ The introduction of indexed bonds, whose nominal payments are adjusted for changes in purchasing power,² has provided a rich source of data with which to improve the methods for extracting ex ante variables.

Several papers use different methodologies to deal with this estimation.³ However, indexed bond yields are not exactly real yields, because of an indexation lag, which exposes the indexed bond yield to inflation fluctuations, so an inflation risk premium is still present. Part of the literature presents results that ignore this risk premium. For example, Woodward (1990) states that “no method was adopted to deal with inflation risk because none seemed superior to ignoring it altogether” (p. 377) while Barr and Campbell (1997) argue that it is difficult to take risk or liquidity premiums into account without specifying a complete equilibrium model.

Evans (1998) adopts an equilibrium model in order to derive relations between the nominal yield, the indexed yield, and the ex ante real yield. The real yield can be estimated from nominal and indexed yields once a measure for a specific risk premium component is obtained. Evans (1998) provides for this through an additional assumption regarding the joint evolution of inflation and nominal bond prices.⁴

This paper is an extension of Evans's (1998) work. I examine the ex ante real rates and inflation premium in the UK bond market⁵ using a two-step procedure. First, I estimate nominal and indexed yield curves. Second, I use an asset pricing model to relate these yields to the theoretical real yields and the risk premiums components. I extend Evans's work by using 60% more data, but more importantly in terms of the treatment of the equilibrium model. While Evans works with a non-parametric kernel, I allow for a parameterization of the intertemporal equilibrium model and, thus, of the kernel. The use on a parameterized kernel allows the extraction of the full time series of the examined variables, and might lead to better estimates of the components of nominal and indexed yields.

I look for a kernel based on a model that can explain the most relevant empirical features of the data. The standard consumption CAPM with power utility has poor performance in the US and

¹ For example, the ex ante real rate can be extracted by using survey measures of expected inflation (Cukierman, 1986) or by modeling expected inflation as an autoregressive process (Throop, 1998); expected inflation can be extracted by modeling the ex ante real rate as a constant (Fama, 1975) or as a random walk process (Fama and Gibbons, 1982; Jones et al., 1995); alternatively, a joint formulation of both ex ante variables can be used (Hamilton, 1985).

² Good reviews of how indexed bonds work and their relative merits and problems can be found in Campbell and Shiller (1996), Emmons (2000), and Wrase (1997).

³ The list is long, and includes Brown and Schaefer (1994), Deacon and Derry (1994), Kandel et al. (1996), Levin and Copeland (1992), and McCulloch and Kochin (1998).

⁴ Other approaches include Gong and Remolona (1996) and Gong et al. (1998), who use the CIR term structure model, and Evans (2003), who estimates a three-state Markov-switching model relating real and nominal term structures.

⁵ Constraints on data availability prohibit the application of the method to US data. The US started issuing indexed bonds only in 1997, while the UK had a well-established market since 1981.

UK with respect to empirical puzzles such as the equity premium and risk-free rate.⁶ Good alternatives in terms of solving these puzzles include models based on Epstein and Zin (1989) preferences and models based on habit formation (Abel, 1990; Campbell and Cochrane, 1999; Constantinides, 1990; Ferson and Constantinides, 1991). While any of these could be adopted for the purposes of this paper – and the literature is still very active in trying to determine the model that performs best – I adopt the habit model originated by Campbell and Cochrane (1999). Li (2001) and Liu (2003) present some evidence against the model from Campbell and Cochrane with respect to US data, but Hyde and Sherif (2005) show that the model performs better for UK bond data than the consumption CAPM, the Epstein-Zin and the model from Abel (1990).⁷

The estimation procedure in Evans (1998) is able to capture only a constant risk premium relating nominal, real and indexed yields. By using a parametric model, I can capture a time-varying one, which is more in line with observations of time variation in risk components. I am also able to extract other premiums, in particular the inflation risk premium, which has been deemed an important motivation to issue indexed securities. I also avoid a crucial derivation step in Evans (1998), which is the approximation of the asset pricing equation by means of log linearization.

Since the equilibrium model is established for a real economy and I examine bonds that are written in nominal terms, I need to complement the model with inflation. I assume a vector autoregression (VAR) process for consumption growth and inflation. Finally, by modelling the volatility of inflation as an asymmetric GARCH process, I incorporate the empirical patterns that the volatility of inflation is time-varying and positively correlated with the level of inflation.

The empirical results indicate that the habit model provides sensible measures of the components of the risk premium. The risk premiums justify their names in that they are highly dependent on the time-varying risk aversion of the agent (given by the ratio of the power exponent of the habit utility specification to the surplus consumption ratio variable).

The pure inflation premium in nominal bonds is the extra yield investors demand because of the uncertainty regarding future inflation — the component that the government can save by shifting investors from holding nominal bonds to holding real bonds. The estimation shows that this is an important component of nominal yields: it is significantly positive throughout the period, increasing in the bond maturity and contributing up to 25 basis points to yearly nominal yields.⁸

The adjustment factor linking nominal, indexed, and real yields – which allows extraction of the ex ante real rate from the indexed yields – is on average close to the constant adjustment factor obtained by the non-parametric specification of Evans (1998). Since the adjustment factor is small, the estimates of the ex ante real rates do not differ significantly on average from the results in Evans (1998). However, the adjustment factor is highly time-varying and increasing in the bond maturity, and the bias from assuming a fixed adjustment factor can reach up to 5 basis points when the surplus consumption ratio is very low.

⁶ The equity premium and the risk-free rate puzzles were first presented in Mehra and Prescott (1985) and Weil (1989), respectively. Discussions on how consumption models deal with these puzzles are found in Campbell (2000), Cochrane and Hansen (1992), Hansen and Jagannathan (1991), and Hansen and Singleton (1982) regarding the US market, and Allais, Cadiou, and Des (1991), Engsted (1998), and Hyde and Sherif (2005) regarding the UK market.

⁷ They do not examine internal habit models such as the ones from Constantinides (1990) and Ferson and Constantinides (1991).

⁸ The investigation assumes tax effects and liquidity premiums to be negligible when compared, for example, to the effects coming from the inflation and the real risk premium. Tax effects are alleviated when tax-free investors play a big role in the market, and possible liquidity premium effects are alleviated by the use of data from the most actively traded market for indexed bonds — in the second quarter of 2001, the indexed bonds reached 25.2% of the UK bond market, with a nominal value of £79 billion.

The organization of the paper is as follows. Section 2 presents the theoretical derivations of the risk premium components and of the intertemporal equilibrium model adopted in the study. Section 3 contains the estimation of the term structure for nominal and indexed bonds, the empirical examination of the equilibrium model, and the estimation of the ex ante variables. Section 4 concludes.

2. Theoretical derivation

2.1. Bond prices and risk premiums

Let $Q_t(h)$ denote the price at time t of a riskless bond delivering one nominal unit at $t+h$. For the theoretical real bond, let $Q_t^*(h)$ be the time t price of a riskless claim to one real unit to be delivered at $t+h$. To complete the notation, P_t is the price index at time t and M_{t+1} (M_{t+1}^*) is the nominal (real) kernel. Using these definitions with the basic equation of asset pricing theory leads to (see Appendix A for the complete derivation):

$$Q_t(h) = Q_t^*(h)E_t[P_t/P_{t+h}]Np_{t,h} \quad (1)$$

where

$$Np_{t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^h M_{t+i}^*, P_t/P_{t+h} \right]}{E_t \left[\prod_{i=1}^h M_{t+i}^* \right] E_t[P_t/P_{t+h}]} \quad (2)$$

Eq. (1) gives a flavor of the Fisher equation. The price of a nominal bond comes from the price of a real bond, after adjustment for expected inflation and for the inflation risk premium, represented by the term $Np_{t,h}$. To understand the role of $Np_{t,h}$ as the inflation risk premium, suppose for instance that the conditional covariance term in Eq. (2) is negative. In this case, negative shocks to P_t/P_{t+h} , i.e., increasing inflation and decreasing real returns for nominal bonds, are associated with positive shocks to $\prod_{i=1}^h M_{t+i}^*$, which means increase in future marginal utility, or, equivalently, a decrease in future consumption in excess of habit. In other words, nominal bonds deliver badly when needed most, so they should sell at a discount, which is provided by the negative sign of the conditional covariance term in Eq. (2).

In order to define the price of an indexed bond, one has to take into consideration the indexation lag l . Holding the zero-coupon indexed bond entitles the bondholder to one nominal unit adjusted for inflation up to l months before maturity, which is given by P_{t+h-l}/P_t . Thus, the indexed bond price $Q_t^+(h)$ is the time t price of P_{t+h-l}/P_t nominal units to be delivered at $t+h$. An equation similar to Eq. (1), now referring to the indexed bond, can be derived:

$$Q_t^+(h) = Q_t^*(\tau)E_t[Q_{t+\tau}(l)]Ip_{t,h} \quad (3)$$

where $\tau=h-l$, and:

$$Ip_{t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^{\tau} M_{t+i}^*, Q_{t+\tau}(l) \right]}{E_t \left[\prod_{i=1}^{\tau} M_{t+i}^* \right] E_t[Q_{t+\tau}(l)]} \quad (4)$$

The term $Ip_{t,h}$ can be understood as the price adjustment bondholders demand for holding an asset whose real returns still fluctuate with inflation, i.e., the inflation risk premium for an indexed bond. An expression identifying the term premium relating nominal bonds of different maturities can also be derived:

$$Q_t(h) = Q_t(\tau) E_t[Q_{t+\tau}(l)] Tp_{t,h} \quad (5)$$

where the term premium is given by:

$$Tp_{t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^{\tau} M_{t+i}, Q_{t+\tau}(l) \right]}{E_t \left[\prod_{i=1}^{\tau} M_{t+i} \right] E_t[Q_{t+\tau}(l)]} \quad (6)$$

Finally, I combine Eqs. (3) and (5) to obtain:

$$Q_t^+(h) = Q_t^*(\tau) \frac{Q_t(h)}{Q_t(\tau)} \frac{Ip_{t,h}}{Tp_{t,h}} \quad (7)$$

In the equation above, $Q_t(\tau)$ and $Q_t(h)$ are readily observed, and $Q_t^+(\tau)$ can be estimated from the prices of coupon-indexed bonds, as it will be shown in Section 3.1. Therefore, in order to complete the estimate for the real bond price (and yield) it is necessary to get estimates of the premium components $Ip_{t,h}$ and $Tp_{t,h}$. In the next sections, I discuss how to estimate these risk premiums, as well as the inflation premium in Eq. (2).

2.2. The intertemporal equilibrium model

The kernel in the derivations of the asset pricing Eqs. (1)–(7) is the intertemporal marginal rate of substitution of the equilibrium model for the economy. I adopt an external, or “keep up with the Joneses” (Abel, 1990), model of habit formation devised by Campbell and Cochrane (1999) and Cochrane (2001) and its extensions from Brandt and Wang (2001) and Wachter (2006). According to the model from Campbell and Cochrane, identical agents maximize the utility function:

$$E_t \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\alpha} - 1}{1-\alpha} \quad (8)$$

where δ is the subjective discount factor, C_t is the real consumption at time t , and X_t stands for the level of habit. The relation between consumption and habit is captured by the surplus consumption ratio $S_t = (C_t - X_t)/C_t$. The value of this surplus is between 0 and 1: 0 for a very bad state when consumption equals the habit level, and 1 when consumption increases indefinitely with respect to the habit level.

In order to define how habit responds to consumption, an AR(1) stochastic process is postulated for $s_t = \log(S_t)$:

$$s_{t+1} = (1-\phi) \bar{s} + \phi s_t + \lambda(s_t) \epsilon_{t+1}^g \quad (9)$$

where ϵ_{t+1}^g represents shocks to the consumption growth, and $\lambda(s_t)$ acts as a “sensitivity function” controlling the sensitivity of the surplus consumption ratio to these shocks. As in Wachter (2006),

this function is chosen so that habit is predetermined at and near the steady state $s_t = \bar{s}$ and that the real interest rate can vary.⁹

In order to satisfy these two conditions, the sensitivity function takes the form:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1 & s_t \leq s_{\max} \\ 0 & s_t \geq s_{\max} \end{cases} \quad (10)$$

where the steady-state level is given by:¹⁰

$$\bar{S} = \sigma_g \sqrt{\frac{\alpha}{1 - \phi - b/\alpha}} \quad (11)$$

and $s_{\max}$ is the value of s_t such that $\lambda(s_{\max}) = 0$:

$$s_{\max} = \bar{s} + \frac{1}{2}(1 - \bar{S}^2) \quad (12)$$

To complete the model, I need to establish the dynamics of the consumption growth, which feeds Eq. (9) with its shocks. The original proposition from [Campbell and Cochrane \(1999\)](#) assumes consumption growth as an i.i.d. lognormal process. However, since I am investigating the term structure of nominal interest rates, it is important to allow the model to account for interdependence between consumption growth and inflation, which calls for a joint formulation of these two variables. [Boudoukh \(1993\)](#) extensively discusses the need for this joint formulation as well as the empirical evidence supporting this interdependence. Other references for the motivations on this interdependence include [Fama \(1981\)](#) and [Pennacchi \(1991\)](#); [Evans and Wachtel \(1992\)](#) also postulate the interdependence of consumption growth and inflation when examining nominal interest rates. I pursue this approach by assuming that consumption growth and inflation follow a stationary VAR(1) process:

$$\begin{bmatrix} g_t \\ \pi_t \end{bmatrix} = \Phi_0 + \Phi_1 \begin{bmatrix} g_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^g \\ \epsilon_t^\pi \end{bmatrix}, \quad (13)$$

where Φ_0 is a 2×1 matrix and Φ_1 is a 2×2 matrix, and the innovations ϵ_t^g and ϵ_t^π are jointly normally distributed with covariance Σ_t .

I model the volatility of the innovations in order to capture some stylized facts. First, the volatility of consumption seems to be homoskedastic, so I assume a constant σ^g . Second, the volatility of inflation seems to be time-varying and tends to be correlated with inflation levels, being high when inflation is high (e.g., [Engle, 1982](#); [Boudoukh, 1993](#); [Demetriades, 1989](#); [Grier and Perry, 1998](#)). Then, I assume, as in [Brandt and Wang \(2001\)](#), that the volatility of inflation σ_t^π follows an asymmetric GARCH process:

$$(\sigma_t^\pi)^2 = \begin{cases} \varphi_0 + \varphi_1(\sigma_t^\pi)^2 + \varphi_2(\epsilon_{t-1}^\pi)^2 & \text{if } \epsilon_{t-1}^\pi \geq 0 \\ \varphi_0 + \varphi_1(\sigma_t^\pi)^2 + (\varphi_2 + \varphi_3)(\epsilon_{t-1}^\pi)^2 & \text{otherwise} \end{cases} \quad (14)$$

⁹ The first condition is very important in that it provides that the habit level does not depend on consumption near or at the steady state, thus allowing for the interpretation that the habit level is exogenous ([Campbell and Cochrane, 1999](#)). The second condition is not adopted in [Campbell and Cochrane \(1999\)](#), whose formulation implies a constant real rate; however, it is important in this context because I am trying to specifically examine the term structure of interest rates.

¹⁰ As will be seen later, b is the constant of proportionality between the evolution of the risk-free rate and the model's state variable.

The coefficient φ_3 allows the volatility of inflation to respond differently to the inflation shock, according to the sign of φ_3 . The volatility of inflation will be positively correlated with the level of inflation as long as $\varphi_3 < 0$. To conclude the specification, I assume a constant correlation ρ between the innovations ϵ_t^g and ϵ_t^π . Thus, the time-varying conditional covariance of the innovations in Eq. (13) is given by:

$$\Sigma_t = \begin{bmatrix} (\sigma^g)^2 & \rho \sigma^g \sigma_t^\pi \\ \rho \sigma^g \sigma_t^\pi & (\sigma_t^\pi)^2 \end{bmatrix} \quad (15)$$

3. Empirical examination

The empirical examination is composed of four parts. First, I compute the term structures for nominal and indexed bonds. Second, the intertemporal equilibrium model is analyzed for the UK market. Then the yield curves and the parameters of the habit model are used to evaluate the risk premium components. Finally, real rates can be extracted from bond prices and the estimated risk premiums. The final subsection examines some robustness issues regarding the adopted model and the solution method.

3.1. Estimation of the nominal and indexed term structures

I adopt a pricing function specification from the methods used by the Bank of England to fit the nominal bond prices (Evans, 1998). The pricing function is:

$$\ln \hat{Q}(h) = \frac{-h}{100} \left(\beta_0 + \beta_1 \delta \left(\frac{h}{\mu_1} \right) + \beta_2 \left[\delta \left(\frac{h}{\mu_1} \right) - \exp \left(\frac{-h}{\mu_1} \right) \right] + \beta_3 \left[\delta \left(\frac{h}{\mu_2} \right) - \exp \left(\frac{-h}{\mu_2} \right) \right] \right) \quad (16)$$

with $\delta(a/b) = (b/a)(1 - \exp(-a/b))$.

The function is fitted to the nominal bond prices using the arbitrage condition that a coupon-paying bond can be seen as the sum of various pure-discount bonds, one for each coupon and one for the final redemption payment. Letting $Q_{i,t}^c(H)$ be the price of a bond i paying coupons C_i over the bond's life, and final redemption 1 at $t+H$, and $I_{i,t}(h)$ be an indicator function that is equal to 1 whenever $t+h$ is a period in which bond i pays coupons, the following relation holds:

$$Q_{i,t}^c(H) = \frac{C_i}{2} \sum_{h=1}^H I_{i,t}(h) Q_t(h) + Q_t(H) \quad (17)$$

A similar arbitrage condition relates coupon-paying indexed bonds to pure-discount nominal and indexed bonds. When the payment horizon for an indexed coupon is less than the indexation lag l , the coupon value is known at time t to have a nominal value of $\frac{1}{2} C_i \frac{P_{t+h-l}}{P_t}$ (where P_t is the base level of the price index, known when the bond is issued), so it can be related to a pure-discount nominal bond. Otherwise, the coupon value is to be related to a pure-discount indexed bond, since its nominal value will depend on the future evolution of purchasing power. Letting $Q_{i,t}^{c+}(H)$

represent the price of an indexed bond with an indexation lag of 8 months, the lag in the UK market,¹¹ the following condition must hold:¹²

$$Q_{i,t}^{c+}(H) = \frac{C_i}{2} \sum_{h=1}^8 I_{i,t}(h) Q_t(h) \frac{P_{t+h-8}}{P_i} + \frac{C_i P_t}{2 P_i} \sum_{h=9}^H I_{i,t}(h) Q_t^+(h) + Q_t^+(H) \frac{P_t}{P_i} \quad (18)$$

For the term structure of indexed bonds, Evans (1998) proposes a simpler pricing function than the one that the Bank of England originally used for the nominal bonds, because there are fewer indexed bonds issued:

$$\hat{Q}^+(h) = \hat{Q}(h) \hat{\lambda}(h-8) \quad (19)$$

where

$$\ln \hat{\lambda}(\tau) = \frac{-\tau}{100} \left[\beta_0^+ + \beta_1^+ \delta \left(\frac{\tau}{\mu^+} \right) \right]$$

The estimation method consists of two steps. For each t , I first fit Eq. (16) to nominal bond prices by minimizing the mean squared error between observed prices and the prices implied by Eq. (16). With the estimated parameter vector $(\beta_0, \beta_1, \beta_2, \beta_3, \mu_1, \mu_2)$ I can compute $\{Q_t(h)\}_{h=1}^8$, and then use these results as input for the second step, which consists of fitting Eq. (19) to observed indexed bond prices. The estimated parameters $(\beta_0^+, \beta_1^+, \mu^+)$ are then used to compute the indexed yield curve.¹³

This method is applied to data from the UK market. Data on monthly prices of nominal and indexed bonds were obtained from Evans (1998) (January 1983 until November 1995) and then complemented with data supplied by the Bank of England (December 1995 until June 2001). Prices of indexed bonds are linked to the Retail Price Index (RPI), whose time series was obtained from the StatBase®-Timezone service from National Statistics of UK. The results of applying the estimation method to the data are exemplified in Fig. 1, which plots some yield curves for zero-coupon nominal and indexed bonds over the sample period.¹⁴

¹¹ When a bond is traded between dates in which a coupon is paid, it is a market practice for the buyer to compensate the seller for the period in which the seller held the bond but for which she will receive no coupon payment. In order to calculate accrued interests on bonds, and given the fact that coupons on indexed bonds are paid every six months, the size of each dividend must be known up to six months before the payment is done; as this is linked to the price index, a lag of at least six months is needed. But the price index figures themselves are only available with a lag (the figure for a particular month typically being available only in the second week of the next month). A further month's lag is allowed because coupon payments on different bonds are processed at different times of the month.

¹² In the equation, P_t is released with a lag of two weeks, i.e., the investors do not necessarily know P_t when they apply the arbitrage condition. As Kandel et al. (1996) state, bond prices reflect expectations about yet unannounced past inflation.

¹³ Eq. (19) implies that the ratio of indexed to nominal bond prices is not time-varying but rather a deterministic function of maturity. However, this does not impose any restriction on the time-series of the risk premium components or the ex ante real rate. The pricing functions are fitted to observed bond prices at each time t — that is, different parameters are estimated for each point in time. Moreover, the covariance components defining the risk premiums do not depend directly on the ratio of indexed to nominal bond prices.

¹⁴ In carrying out the estimation I use the same simplifying assumptions used in Evans (1998). First, I assume there is no uncertainty about P_t at time t , which overcomes the potential problem in the pricing function (18) that investors in reality know about P_t with a lag. Second, tax effects are deemed unimportant by assuming a flat zero tax rate. Evans (1998) reports that the results are robust to these assumptions. Concerning the sample choice, for conventional bonds I opt not to consider double-dated and convertible bonds (since they have embedded options that affect the price) and I ignore the prices of bonds very close to either the issuance or the maturity of the bonds. (When estimating the yield curve, the Bank of England (Debt Management Office, 2000) excludes bonds trading when-issued as well as bonds with less than three months to maturity, given that in these situations a slight inaccuracy in the price can lead to a large error in the estimated yield.)

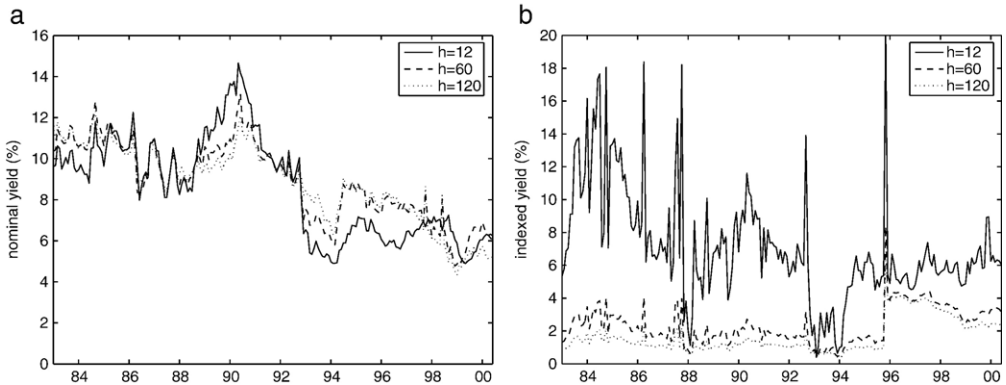


Fig. 1. Observed nominal and indexed yields. The first graph plots the time series of nominal yields estimated from conventional bonds, calculated as $y_t(h) = -(1200/h)\log(\hat{Q}_t(h))$, for h given in months. The second graph plots the time series of indexed yields estimated from conventional and indexed bonds, calculated as $y_t(h) = -(1200/h)\log(\hat{Q}_t^+(h))$, for h also given in months.

3.2. Evaluation of the consumption model

Data for UK consumption, inflation, and population were obtained from the StatBase®-Timezone service from National Statistics of UK, for the first quarter of 1955 through the first quarter of 2001. Throughout the paper I report results using seasonally adjusted consumption data. (I discuss the alternative of using the non-seasonally adjusted data in the Robustness section.) Quarterly series of seasonally adjusted real per capita consumption was computed by dividing the sum of real (constant prices) seasonally adjusted household nondurables and services consumption by population. Since I had access only to annual data on population, I assume a constant growth rate within the year in order to obtain the quarterly population series. Finally, the implicit price deflator was obtained by comparing the consumption series in current and constant prices. The evolution of consumption growth and inflation for the period in analysis is presented in Fig. 2.

The first step of the examination involves the estimation of the parameters defining the joint evolution of consumption growth and inflation. Table 1 presents estimates of the VAR (1)-TGARCH model. As expected, the innovations to inflation conform to the assumption of heteroskedasticity and correlation with the level of inflation. The coefficients for the TGARCH formulation are highly significant, and inflation volatility indeed seems to be correlated with the level of inflation; the response of inflation volatility to a positive shock ($\varphi_2 = .68$) is 17 times bigger than the response to a negative shock ($\varphi_2 + \varphi_3 = .04$) of the same magnitude.

The second step involves the examination of the intertemporal consumption model, i.e., of the remaining parameters $\Theta = [\alpha, \delta, \phi, \bar{s}]$ that govern preferences in Eq. (8) and the surplus consumption ratio evolution Eq. (9). Rather than trying to estimate the model from the data, I choose instead to calibrate these parameters around reasonable values and check how the resulting estimates are sensible to these choices.

Calibration is a common approach used throughout the literature of intertemporal equilibrium models. More importantly, though, calibration seems more appropriate to the objective of this research. Since I am assuming the validity of the equilibrium model, with calibration I am simply

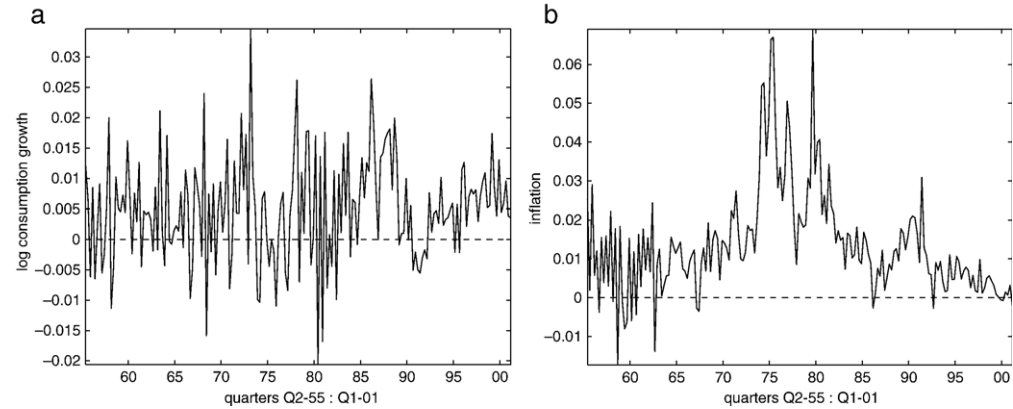


Fig. 2. Quarterly consumption growth and inflation in UK.

examining the sensitivity of the risk premium estimates to the “free” parameters, and this sensitivity would be carried out anyway even if I chose to estimate the model. Only in the scenario where my chosen parameters would be distant from the real ones would calibration be a problematic approach. Fortunately, there is not too much freedom of choice for these remaining parameters.

These parameters have been chosen to match market moments, even in studies where the model is estimated from the data (Campbell and Cochrane, 1999; Wachter, 2006). The parameter α is used so that the model produces the historical sharpe ratio. The subjective discount factor δ can be chosen to match the risk-free rate with the average return on nominal bonds. The persistence of the surplus consumption ratio ϕ has usually been taken as the first-order autocorrelation of the price-dividend ratio. Although the last parameter $\bar{s}$ can be treated as an isolated one, it is preferable to work with reasonable values established for it based on the other parameters.

Table 1
Dynamics of consumption growth and inflation

$$\begin{bmatrix} g_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0.0076 \\ (0.0010) \\ 0.0038 \\ (0.0013) \end{bmatrix} + \begin{bmatrix} -0.1314 & -0.1480 \\ (0.0755) & (0.0476) \\ 0.1117 & 0.6678 \\ (0.0939) & (0.0592) \end{bmatrix} \begin{bmatrix} g_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^g \\ \epsilon_t^\pi \end{bmatrix}$$

$SD[\epsilon_t^g]=0.0083$ $SD[\epsilon_t^\pi]=0.0104$ $Corr[\epsilon_t^g, \epsilon_t^\pi]=-0.2234$

$$\begin{matrix} (\sigma_t^\pi)^2 = 0.1210E-6 + & 0.6879(\epsilon_{t-1}^\pi)^2 - & 0.6406I(\epsilon_{t-1}^\pi < 0)(\epsilon_{t-1}^\pi)^2 + & 0.5441(\sigma_{t-1}^\pi)^2 \\ (0.0324E-6) & (0.1819) & (0.1683) & (0.0980) \end{matrix}$$

Results of the estimation of the VAR(1) model for the joint evolution of these variables (Eq. (13)) and TGARCH model for the volatility of inflation (Eq. (14)). g_t is consumption growth and π_t is inflation; ϵ_t^g and ϵ_t^π are respectively the innovations to consumption growth and inflation; σ_t^π is the volatility of inflation. In parentheses are asymptotic standard errors. The sample period is Q1:1955 through Q1:2001.

From expression (11) it can be observed that the only free parameter in $\bar{s}$ is b . This last parameter is recovered using the equation for the evolution of the real risk-free rate. From the kernel definition I derive:¹⁵

$$r_{t+1} = -\ln(\delta) + \alpha z_t - b(s_t - \bar{s}) - \frac{\alpha}{2} \left(1 - \phi - \frac{b}{\alpha} \right) \quad (20)$$

In this equation z_t is the predictable component of consumption growth. This expression also explains the role of the parameter b : It is the constant of proportionality linking the real risk-free rate to the state variable of the model. I take unconditional expectations on both sides and substitute for the first moments of the real risk-free rate and consumption growth in order to derive b :

$$b = 2 \left(E[r_t] + \ln(\delta) - \alpha E[z_t] + \frac{\alpha}{2} (1 - \phi) \right) \quad (21)$$

Thus, three out of the four remaining parameters $[\alpha, \delta, \phi, b]$ need to be defined. I opt to work on calibrated values for $\Theta_1 = [\alpha, \delta, \phi]$, and their initial values are presented in Table 2. I start from the basic values used by Campbell and Cochrane (1999) and Wachter (2006). A caveat, though, is that these papers look at the US market. I decided to apply them to the UK based on the similarity – in the context of this exercise – between the US and UK markets (see Dimson and Marsh, 2001). Given that I am adopting rather than estimating a model, there won't be a right value for the calibration — the issue is to verify to what extent my results are sensitive to the exact choice of the initial calibration values. I show later in the sensitivity analysis that the qualitative results derived here are robust to varying the calibrated parameters within ranges that reflect the magnitude of difference between the US and UK markets.

3.3. Estimation of risk premium components and the ex ante real rate

Due to the limited availability of data, I am forced to work with a quarterly economy. I thus read expressions (1)–(7) as being developed in quarterly terms. However, the estimation cannot directly use some of the equations derived above, since the eight-month lag in the UK market is not a multiple of 3 months. Thus, for example, the quarterly economy cannot establish a stochastic evolution of the future bond price $Q_{t+\tau}$ ($h=8$ months); I follow the alternative to approximate it to the next possible quarterly representation, i.e., to take $Q_{t+\tau}$ ($h=9$ months) in place of $Q_{t+\tau}$ ($h=8$ months) in the risk premium equations.¹⁶ The inability of the eight-month lag to be captured by a quarterly economy carries over for the estimation of some risk premiums (the

¹⁵ From the definition $r_{t+1} = \ln \frac{1}{E_t[M_{t+1}^*]}$, develop the kernel expression $M_{t+1}^* = \delta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\alpha}$ and substitute expressions (9), (10), and (11).

¹⁶ Notice that these future bond prices are used in conditional expectations; in other words, this means in practice postulating that agents cannot at time t distinguish very well between the eight-months- and nine-months-to-maturity bond prices to be established at time $t+\tau$. This assertion does not seem to be very harmful, and certainly improves as the horizon τ increases.

Table 2
Preference parameters

Parameter	Value	Origin
α	2.0000	Calibrated
ϕ	0.9690	Calibrated
δ	0.9840	Calibrated
$E[r_t]$	0.0049	Observed
$E[z_t]$	0.0059	Observed
b	0.0194	Derived
$\bar{S}$	0.0813	Derived

This table shows the parameters to be used in the basic setup of the simulation of the model. α is the investor's risk-aversion coefficient, ϕ is the AR(1) component of the evolution of the surplus consumption ratio, δ is the subjective discount factor, r_t is the risk-free rate, z_t is the predictable component of the consumption growth process, b is the constant of proportionality linking the risk-free rate to the surplus consumption rate, and $\bar{S}$ is the steady-state level of the surplus consumption ratio.

estimation of $Np_{t,h}$ does not face this problem). I deal with it in a similar way, approximating the equations by assuming the lag $l=9$ months.¹⁷

The risk premium components are formed by combining the real kernel ($M_{t,t+j}^*$), the nominal kernel ($M_{t,t+j}$), and the purchasing price evolution (P_t/P_{t+j}). The setup embeds a direct assumption about the purchasing price evolution, and, by also including the evolution of consumption growth and of the surplus consumption ratio, it embeds knowledge about the real and the nominal pricing kernels' evolution, since these are essentially non-linear functions of these state variables. To see this, I derive expressions for the real kernel from t to $t+n$:

$$\begin{aligned} M_{t,t+n}^* &= \delta^n \left(\frac{S_{t+n} C_{t+n}}{S_t C_t} \right)^{-\alpha} \\ &= \delta^n \exp(-\alpha(s_{t+n} - s_t + g_{t+1} + \dots + g_{t+n})) \end{aligned} \quad (22)$$

and for the nominal kernel:

$$\begin{aligned} M_{t,t+n} &= \delta^n \left(\frac{S_{t+n} C_{t+n}}{S_t C_t} \right)^{-\alpha} \frac{P_t}{P_{t+n}} \\ &= \delta^n \exp(-\alpha(s_{t+n} - s_t + g_{t+1} + \dots + g_{t+n}) - (\pi_{t+1} + \dots + \pi_{t+n})) \end{aligned} \quad (23)$$

Estimation of the risk premium components is done through simulation, which works as follows. Each expression is approximated by an average over the subset of simulated paths for

¹⁷ Thus, in reality I estimate the risk premiums for a theoretical indexed bond that has a nine-month lag. At first glance, this could be problematic, since the procedure derives risk premium measures of an asset that is potentially riskier: an indexed bond with a nine-month lag provides less compensation for future inflation than one with an eight-month lag; therefore I would expect to get, for example, a higher inflation risk premium. To check the validity of my results, I later repeat the computations above assuming a six-month lag, to get a sense of what the actual premium would be for an asset with an in-between lag. I observe that for the six-month and nine-month lagged indexed bonds the risk premiums are similar in shape and very similar in levels. Therefore, I report results computed assuming the specification shown in the equations above, i.e., with a nine-month lag. Results for the six-month lag can be provided upon request.

which the state variables take on particular values. For example, the expression $E_t[M_{t,t+j}]$, the conditional expectation of the nominal kernel j quarters ahead, is obtained by:

$$E_t[M_{t,t+j}] = \frac{1}{S} \sum_{s=1}^S \hat{M}_{t,t+j}^s \quad (24)$$

where $\hat{M}_{t,t+j}^s$ denotes one simulated realization of the nominal kernel and S is the number of simulated paths.

The starting point of the simulation is to obtain the state variables as of time t , i.e., the observables g_t and π_t and the unobservable s_t . The log surplus consumption ratio s_t is determined through the transition Eq. (9) using the initial condition $s_0 = \bar{s}$ and the forecast errors from the joint evolution of consumption growth and inflation. The simulation effectively proceeds by generating a path $\{\hat{g}_{t+s}, \hat{\pi}_{t+s}, \hat{s}_{t+s}\}_{s=1}^j$ according to the surplus consumption ratio evolution (9) and to the adopted joint evolution of consumption growth and inflation, and by using pairs of independent draws from a standard normal distribution. Each path or simulated economy gives rise to the different values for the surplus consumption ratio, consumption growth, and inflation, because of the different sequences of innovations that feed into their stochastic formulations. In my example, this procedure allows the computation of a simulated nominal kernel:

$$\hat{M}_{t,t+j}^s = \delta^j \exp(-\alpha(\hat{s}_{t+n} - s_t + \hat{g}_{t+1} + \dots + \hat{g}_{t+n}) - (\hat{\pi}_{t+1} + \dots + \hat{\pi}_{t+n})) \quad (25)$$

and of any other variable that one might want to estimate, such as the real kernel, the purchasing price evolution, etc. This procedure is repeated S times, and the average of these realizations is taken as the conditional expectation. Finally, expressions involving conditional covariances are treated in a similar way, with the simulation being computed for the conditional expectations that compose the conditional covariance.

The next subsections present estimates of the various risk premiums analyzed in Section 2.1 and of the ex ante real rate. For the basic simulations I adopt the estimates of the VAR(1)-TGARCH model in Table 1, together with the preference parameters presented in Table 2. Variations on these parameters will be discussed below. The simulation size is $S=1,000,000$, and the period goes from the first quarter of 1983 to the second quarter of 2001.

3.3.1. Inflation risk premium

In order to analyze the inflation risk premium $Np_{t,h}$ represented in Eq. (2), I look at it as contributing to the nominal yield. For this I take logs of Eq. (1) and write:

$$-\log(Q_t(h)) = -\log(Q_t^*(h)) - \log(E_t[P_t/P_{t+h}]) - \log(Np_{t,h}) \quad (26)$$

In order to work with annualized yields, I multiply the equation above by $1200/h$ and use the definition of the yearly nominal yield on a bond h periods from expiring, $y_t(h) = -1200/h * \log(Q_t(h))$, to get:

$$y_t(h) = y_t^*(h) - \frac{1200}{h} \log(E_t[P_t/P_{t+h}]) - \frac{1200}{h} \log(Np_{t,h}) \quad (27)$$

The last term denotes the contribution of the nominal risk premium to the yield of the bond. Estimates of this contribution are obtained through the simulation method described in the previous section. Table 3 presents summary statistics of this contribution to the yields of bonds

Table 3
Contribution of inflation risk premium $-\frac{1200}{h}\log(Np_{t,h})$ to yearly nominal yields

Statistics for inflation premium $-\frac{1200}{h}\log(Np_{t,h})$					
	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
Mean	0.1297	0.1758	0.2099	0.2353	0.2558
SD	0.0802	0.0895	0.0868	0.0827	0.0791
Corr with:					
s_t	-0.6010	-0.6306	-0.7000	-0.7561	-0.8130
g_t	-0.5274	-0.5258	-0.5218	-0.5137	-0.5004
π_t	0.0302	0.0090	-0.0377	-0.0905	-0.1407
ϵ_t^g	-0.5532	-0.5535	-0.5551	-0.5497	-0.5367
ϵ_t^π	0.2220	0.2148	0.1731	0.1383	0.1239

Values for the contribution are reported in percentage terms, according to Eq. (27); s_t is the log surplus consumption ratio, g_t is consumption growth, and π_t is inflation; ϵ_t^g and ϵ_t^π are respectively the innovations to consumption growth and inflation. Results reported based on quarterly data from Q1:1983 to Q2:2001.

with maturities ranging from $h=12$ to 60 months, and the first graph in Fig. 3 depicts the time series of this contribution between the first quarter of 1983 and the second quarter of 2001.

On average, the inflation risk causes an increase of 13 to 25 basis points in the nominal yield. The premium increases with the bond maturity and is highly time-varying. It is highly negatively correlated with the surplus consumption ratio, indicating that in times of recession (low surplus consumption ratio) individuals are more risk-averse and so they demand a higher premium for holding nominal assets. Since the surplus consumption ratio evolves according to the consumption shocks, the premium is also negatively correlated with these shocks. The last line shows that inflation shocks greatly influence the inflation premium. Combining the negative correlation of the premium with consumption growth shocks and the positive correlation of the premium with inflation shocks explains the big spike in the beginning of the 1990s. In the quarter with the big spike, there was a combination of the biggest positive shock to inflation (+0.0215, when the average shock in the period was -0.002 and the next biggest shock was only +0.006) and the biggest negative shock to consumption (-0.0125, when the average shock was +0.0012); these were further amplified as the result of a sequence of negative consumption growth shocks that had already caused surplus consumption ratios to decrease toward a recession state.

Some authors, such as Kandel et al. (1996) and Levin and Copeland (1992), have used a different definition for the inflation risk premium. The difference is that their counterpart to expression (1) uses expected inflation instead of the expected value of the inverse of the inflation (or depreciation of money), and this adds an additional term on the inflation premium because of Jensen’s inequality. To see how it works, I redevelop (1) by multiplying and dividing it by expected inflation, to get:

$$Q_t(h) = Q_t^*(h) \frac{1}{E_t[P_{t+h}/P_t]} Np_{t,h} E_t \left[\frac{P_t}{P_{t+h}} \right] E_t \left[\frac{P_{t+h}}{P_t} \right] \tag{28}$$

By applying logs and using the definition of yields:

$$y_t(h) = y_t^*(h) + \frac{1200}{h} \log \left(E_t \left[\frac{P_{t+h}}{P_t} \right] \right) - \frac{1200}{h} \log(Np_{t,h}) - \frac{1200}{h} \log \left(E_t \left[\frac{P_t}{P_{t+h}} \right] E_t \left[\frac{P_{t+h}}{P_t} \right] \right)$$

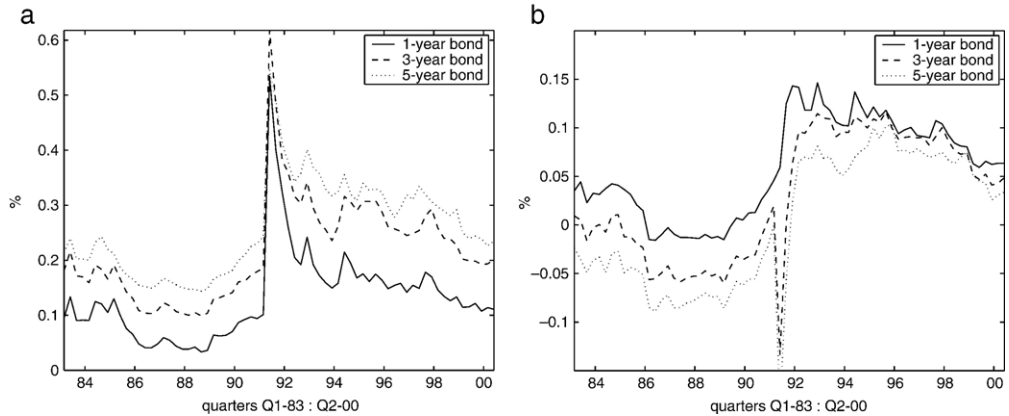


Fig. 3. Inflation risk premium. The first graph plots the time series of the contribution of inflation risk premium $-\frac{1200}{h} \log(Np_{t,h})$ to yearly nominal yields, according to Eq. (27). The second graph plots the contribution of the inflation risk premium $-\frac{1200}{h} \log(Np_{t,h})$ plus Jensen's component $-\frac{1200}{h} \log\left(E_t\left[\frac{P_t}{P_{t+h}}\right] E_t\left[\frac{P_{t+h}}{P_t}\right]\right)$ to yearly nominal yields, according to Eq. (29). Values for the contribution are reported in percentage terms.

Thus, there is an additional term once the relationship between nominal and real yields is established through expected inflation, and Jensen's inequality makes this term always negative.¹⁸ In the notation used by Kandel et al. (1996), the component involving $Np_{t,h}$ is called the *pure* inflation risk premium.¹⁹

In order to compare my results with these alternative definitions, in Table 4 I present the estimates of Jensen's component, together with the pure inflation premium and then the *total* inflation premium, and the second graph in Fig. 3 plots the pattern of the *total* inflation premium. Kandel et al. (1996) find evidence of the presence of this pure premium in Israel and find it to be positive, although insignificant. Campbell and Shiller (1996), using a completely different approach,²⁰ report estimates for the US varying from 20 to 100 basis points on a five-year nominal bond. Evans (1998) uses a regression approach – in which proxies of ex ante inflation are inserted together with measures of the ex ante rates – to find significant evidence of the total inflation premium in the UK market. Evidence for the UK is also documented by Chu et al. (1995). Levin and Copeland (1992) conclude that the total premium is significantly negative over the period 1982–1991, but they cannot isolate Jensen's component and, therefore, they do not measure the pure premium.

¹⁸ For any random variable x , Jensen's inequality establishes $E\left[\frac{1}{x}\right] - \frac{1}{E[x]} \geq 0 \Rightarrow E\left[\frac{1}{x}\right] \geq \frac{1}{E[x]} \Rightarrow E\left[\frac{1}{x}\right] E[x] \geq 1$. Applying the last inequality to the random variable $x = \frac{P_{t+h}}{P_t}$, I conclude that the log of the product of the expectations in the last term of Eq. (29) is positive, so the whole extra component is negative.

¹⁹ There is the question as to which Fisher expression is preferable, the one using the expected depreciation of money or the one using the expected inflation. Fama (1976) advocates that the appropriate version should use the expected depreciation of money (in which case the inflation risk premium is just the pure premium), because the real value of nominal assets is the relevant quantity for the pricing of nominal assets – and the depreciation of money is the direct indicator of the evolution of the value of nominal assets. However, I take an agnostic view here and present both definitions, in order to provide for a comparison with literature on the inflation risk premium that uses the two alternative formulations.

²⁰ They use a CAPM approach and estimate betas of nominal bonds, taking the inflation premium as proportional to the market equity premium, having these betas as the constant of proportionality.

Table 4
Contribution of inflation risk premium $-\frac{1200}{h}\log(Np_{t,h})$ plus Jensen's component $-\frac{1200}{h}\log\left(E_t\left[\frac{P_t}{P_{t+h}}\right]E_t\left[\frac{P_{t+h}}{P_t}\right]\right)$ to yearly nominal yields

Inflation premium statistics					
	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
<i>Panel A: pure premium</i>					
Mean	0.1297	0.1758	0.2099	0.2353	0.2558
SD	0.0802	0.0895	0.0868	0.0827	0.0791
<i>Panel B: Jensen's component</i>					
Mean	-0.0712	-0.1333	-0.1813	-0.2199	-0.2527
SD	0.0546	0.0754	0.0772	0.0734	0.0679
<i>Panel C: Pure premium+Jensen's component</i>					
Mean	0.0586	0.0425	0.0286	0.0154	0.0031
SD	0.0506	0.0582	0.0644	0.0670	0.0691

Values for the contribution are reported in percentage terms, according to Eq. (29). Results reported based on quarterly data from Q1:1983 to Q2:2001.

The results here are much more clear-cut regarding these components. The time series presented in the figures and tables makes it clear that the pure premium is positive, but it is smoothed out by Jensen's component as the horizon increases. The total premium is very small or even negative during the period analyzed by [Levin and Copeland \(1992\)](#), but positive from 1991 on. The negative spike on the figure (the figure was truncated in order to allow a better visualization of the general pattern of the total premium) is a direct effect of the biggest inflation shock mentioned earlier. According to the formulation of the volatility of inflation, the unusually big positive shock to inflation in that quarter led to a very high volatility, and given that Jensen's component is related to volatility of inflation,²¹ it caused Jensen's component to completely dominate the measure of the total inflation premium.

[Kandel et al. \(1996\)](#) argue that additional evidence for the existence of an inflation premium is the correlation between its empirical measure and the volatility of inflation. This is certainly true for Jensen's component. However, the pure inflation premium Np_t can still be seen as a measure of inflation uncertainty. The inflation risk premium is highly positively correlated with the volatility of inflation, with values ranging from 0.50 to 0.60 along the maturities analyzed.

3.3.2. Risk premiums $Ip_{t,h}$ and $Tp_{t,h}$ and the real rate

Since real bonds are not actually available, one might ask about a possible uncertainty premium when comparing the indexed and the theoretical real bonds — i.e., whether one could reduce costs by issuing real bonds instead of the bonds with incomplete indexation lag. For this, I can look at the premium for indexed bonds $Ip_{t,h}$ from Eqs. (3) and (4). Rewriting Eq. (3) in terms of yields allows me to recognize the term $-\frac{1200}{h}\log(Ip_{t,h})$ as the contribution to indexed yield

²¹ Jensen's component can be related to volatility such as in the simple setup where the random variable x is lognormal, $\log(x) \sim N(\mu_x, \sigma_x^2)$, which implies $E[x] = \exp(\mu_x + \sigma_x^2/2)$ and $E[x^{-1}] = \exp(-\mu_x + \sigma_x^2/2)$, so that $E[x]E[1/x] = \exp(\sigma_x^2)$.

Table 5

Contribution of risk premium in indexed bonds $-\frac{1200}{h}\log(Ip_{t,h})$ to yearly indexed yields

Statistics for premium $-\frac{1200}{h}\log(Ip_{t,h})$	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
Mean	0.1371	0.2949	0.3495	0.3691	0.3816
SD	0.1000	0.2133	0.2360	0.2325	0.2256

Values for the contribution are reported in percentage terms, according to Eq. (30). Results reported based on quarterly data from Q1:1983 to Q2:2001.

required by the investor because of uncertainty about future bond prices (and so about future inflation):

$$y_t^+(h) = \frac{\tau}{h}y_t^*(\tau) - \frac{1200}{h}\log(E_t[Q_{t+\tau}(I)]) - \frac{1200}{h}\log(Ip_{t,h}) \quad (30)$$

Table 5 presents the estimation of this premium. It is consistently positive (and increases in magnitude with the bond maturity), indicating that holders of indexed bond require a premium that ranges on average from 13 to 38 basis points for holding an asset that does not protect them completely from uncertainty when compared to the hypothetical real bond.²²

Notice, however, that while Eq. (30) can be used to analyze the relationship between indexed and real yields, it is not the best alternative for extracting the ex ante real rate. To back out the ex ante real rate, I refer to expression (7), whose only unobservable component, besides the real bond price, is the risk premium component expressed by the ratio $Ip_{t,h}/Tp_{t,h}$. Again, to provide an easy interpretation of this ratio as a contribution to yearly yields I take logs in that expression and derive:

$$y_t^+(h) = \frac{\tau}{h}y_t^*(\tau) + y_t(h) - \frac{\tau}{h}y_t(\tau) - \frac{1200}{h}\log\left(\frac{Ip_{t,h}}{Tp_{t,h}}\right) \quad (31)$$

That is, this risk premium can be seen as contributing to the yearly yields on indexed bonds through the expression $-\frac{1200}{h}\log(Ip_{t,h}/Tp_{t,h})$. Table 6 presents the results of estimating this expression. On average, the premium contributes up to 2 basis points to the yields, and the pattern of time variation is also very strong.

Evans (1998) has an expression similar to this one. However, because of the assumption of joint lognormality between the kernel and changes in the purchasing power, the log premium is further simplified as the covariance between the future bond price and the inflation during the indexation period. To measure this premium, Evans (1998) assumes a VAR model for the joint evolution of the bond price and the inflation, from which the conditional covariance can be estimated as a constant. His results point to the premium as contributing 1.5 basis points to annualized yields. Furthermore, this value is fairly constant across different maturities.

Thus, the risk premium derived from the habit consumption model is of the same order of magnitude as the risk premium from the basic VAR estimation from Evans (1998), but in the

²² Notice that this premium should not be compared directly with the nominal inflation premium, since the expressions denoting these premiums as contributions to yearly yields of respectively indexed and nominal bonds do not have comparable components.

Table 6

Contribution of $Ip_{t,h}/Tp_{t,h}$ risk premium $-\frac{1200}{h}\log(Ip_{t,h}/Tp_{t,h})$ to yearly indexed yields

Statistics for premium $-\frac{1200}{h}\log(Ip_{t,h}/Tp_{t,h})$					
	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
Mean	0.0046	0.0140	0.0165	0.0171	0.0194
SD	0.0039	0.0075	0.0058	0.0054	0.0064
Min	0.0025	0.0086	0.0091	0.0102	0.0045
Max	0.0316	0.0328	0.0424	0.0421	0.0512
Corr with:					
s_t	-0.1083	-0.2318	-0.5250	-0.7213	-0.5512
g_t	-0.3975	-0.4225	-0.5091	-0.4764	-0.2598
π_t	0.2134	0.1915	0.1134	-0.0415	-0.1301
ϵ_t^g	-0.3912	-0.4237	-0.5341	-0.4842	-0.3483
ϵ_t^π	0.4309	0.3331	0.3270	0.1093	0.0310

Values for the contribution are reported in percentage terms, according to Eq. (31). Results reported based on quarterly data from Q1:1983 to Q2:2001.

model developed here two additional features show up: The premium increases with the bond maturity, and the premium is highly time-varying.

Real rates can be extracted from Eq. (7) using the estimates of nominal and indexed yields from Section 3.1 and estimates of the risk premium $Ip_{t,h}/Tp_{t,h}$ from the previous section. I can rearrange expression (31) into the formula used to extract real yields:

$$y_t^*(\tau) = \frac{h}{\tau}y_t^+(h) - \frac{h}{\tau}y_t(h) + y_t(\tau) + \frac{1200}{\tau}\log\left(\frac{Ip_{t,h}}{Tp_{t,h}}\right) \quad (32)$$

To exemplify this, Fig. 4 presents time series of ex ante real rates estimated for some maturities. Notice that since the estimates of the premium attached to Eq. (32) are quite similar in terms of magnitude to what Evans (1998) estimated, the results reported in his paper concerning real term behavior – that is, the rejection of the Fisher hypothesis and of the expectations hypothesis – will also obtain here.

Table 6 also presents the minimum and maximum values for the estimated premium $-\frac{1200}{h}\log(Ip_{t,h}/Tp_{t,h})$ for each maturity. These, together with the first moments of the estimated premium, give a sense of the possible problems of computing the ex ante real rate without taking into account the adjustment due to the indexation lag. The adjustment factor in Eq. (32) is obtained by multiplying the $Ip_{t,h}/Tp_{t,h}$ premium by $-h/\tau$. Thus, real rates obtained without adjusting for the indexation lag are biased upwards and the bias – for bond maturities from 24 to 60 months – can reach up to 5 basis points. Moreover, the correlations in Table 6 indicate that the maximum biases are reached when the surplus consumption ratio is very low.²³

3.4. Robustness

In this section we discuss some robustness issues. Of particular concern is the sensitivity of the results to the preference parameters that were used in the intertemporal equilibrium model, given

²³ Corroborating this number as a good estimate of the maximum bias is the fact that the low values of surplus consumption reached during the period 1983–2000 are among the lowest from the period 1955–2000 (results not shown).

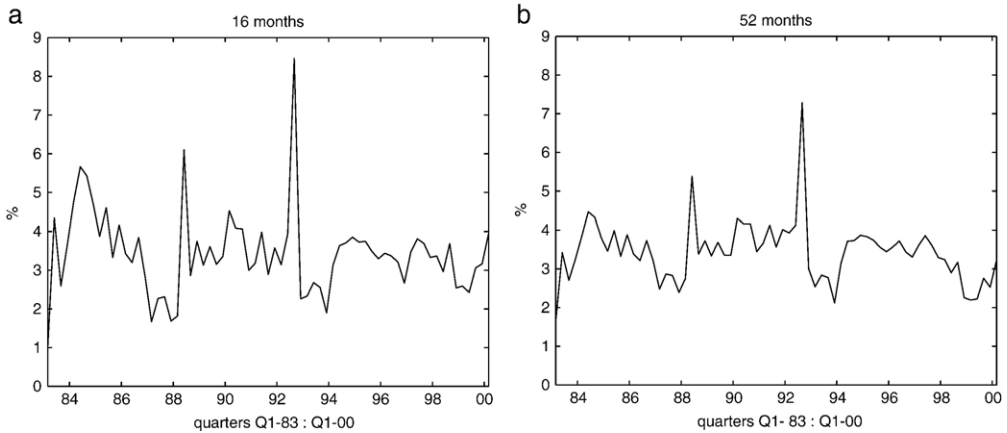


Fig. 4. Ex ante real rates in the UK.

that they were initially calibrated using US data. Appendix B details that the results show here are robust to the choice of these preference parameters. Appendix B also reports that the results are robust to the simulation method. Next I turn my attention to the equilibrium model itself, discussing whether it is a suitable choice for the UK bond price data.

All the results examined so far are conditional on the validity of the consumption model formulation described in Section 2.2. Campbell et al. (1997) point out that “economists have not yet produced a generally accepted model of the stochastic discount factor” (p. 334), and that the research on intertemporal equilibrium models is very active. While it is generally argued that the habit consumption model is a good alternative in order to overcome many deficiencies of simpler models, in this section I discuss in more detail some important features, and weaknesses, of the adopted model.

I want to examine the extent to which the model accrues to the reality. The measures of risk premiums derived above cannot be used in this assessment, since they are unobservables, so that I have nothing to compare in order to check whether the model is working well. For this, I can use nominal bond yields, which are readily observed and can be generated from within the model by the expression $Q_t(h) = E_t[M_{t,t+h}]$. As before, these fitted yields are generated by simulation, and the first graph in Fig. 5 plots the resulting estimates for different maturities, together with time series of the surplus consumption ratio variable, S_t .

Recall that S_t is the main state variable of the habit consumption model. The surplus consumption ratio can be seen as a measure of the wellness of the agent. Low surplus means consumption approaches the habit level, and thus the variable is referred to as a measure of the recession state. Also, it directly determines the relative risk aversion of the agent, by the expression α/S_t . Thus, the model generates very high values of relative risk aversion. These are necessary to overcome the equity premium puzzle. The approach differs from setting a high risk-aversion coefficient in the traditional power utility consumption model because, first, the time series of risk aversion is time-varying, and, second, having high values does not lead to unrealistically low interest rates. There is also a clear pattern of highly negative correlations between the surplus consumption ratio and the fitted yields (going from 0.94 for 12-month bonds to 0.99 in the case of 60-month bonds), which asserts the importance of this

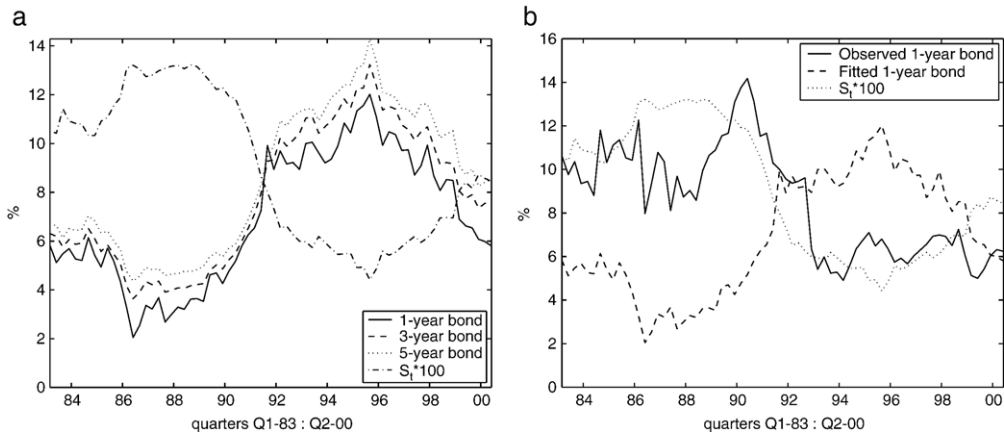


Fig. 5. Yields generated by the model. The first graph plots yields on zero-coupon nominal bonds generated by the model and the surplus consumption ratio S_t . The second graph compares observed yields and yields generated from the model, together with the surplus consumption ratio.

state variable – as compared to either consumption growth or inflation – in generating bond yields.

However, when I plot in the second graph of Fig. 5 the surplus consumption ratio generated by the model against observed yields, the pattern of correlation between yields and surplus consumption ratio is not as clear. Each yield follows the other's pattern most of the time, although not in levels. However, in the beginning of the 1990s, the surplus consumption ratio is actually positively correlated with the observed yields, and the yields move in completely opposite directions, with observed yields going down and generated yields going up. The yields generated by the model in this period go in the wrong direction from where they should go.

Campbell and Cochrane (1999) had already hinted at this problem for the US market, commenting on how “growth in consumption of nondurables and services was surprisingly slow in the early 1990s, bringing consumption near our implied habit level” (p. 236), so that their model actually predicted a pattern not observed in the data. My results confirm that the same pattern exists in the UK market. As is evident from Fig. 2, there is a clustering of low consumption growth in the beginning of the 1990s, resulting in the same problem the model presented for the US market.²⁴

²⁴ One significant event for the UK market in the early 1990s was the UK's brief participation in the European ERM (Exchange Rate Mechanism). The ERM was a fixed-exchange rate regime established by the European Community to keep the participants' exchange rates within specific bands in relation to each other; the UK joined the ERM in October 1990 and left in September 1992. Leaving aside the discussion of whether participation in the ERM caused it, the fact is that the UK's economy was badly hit during this subperiod, suffering its worst recession in 60 years. This certainly explains the clustering of low consumption growth mentioned above. Interestingly, the spikes on the estimated premiums all occur during this subperiod, and the mismatch between observed and fitted yields occurs exactly along this subperiod; for both the periods pre-ERM and post-ERM, the correlation between observed and fitted yields goes from 0.4 to 0.7, depending on the bond's maturity. These events might suggest a change in regime, e.g., it is possible that agents' perception of the real rate, inflation, and other economic quantities had changed during this period. Also, similar to the results presented by Lettau et al. (2003) for the US, there is evidence of a decrease of volatility of consumption in the 1990s for the UK, which could proxy for a decrease in macroeconomic risk.

There are caveats regarding the econometric use of the consumption data. Measurement errors and methodological imperfections can impair the use of the consumption data — e.g., [Breedon et al. \(1989\)](#) and [Wilcox \(1992\)](#). There is also a concern about using seasonally adjusted consumption in the estimation of consumption-based asset pricing models ([Ferson and Harvey, 1992](#); [Wilcox, 1992](#)). Seasonal adjustment is obtained through data smoothing — both backward and forward — that might not be mirrored in asset returns. To address this issue, I repeat the empirical examination using the original, non-seasonally adjusted, consumption data. The nature of the consumption growth and inflation processes is altered with non-seasonally adjusted data: The standard deviation of the consumption growth series is one order of magnitude higher than for seasonally adjusted data, while the correlation between consumption growth and inflation is reduced from -0.2234 to -0.0407 . More importantly, the estimation of the premium components using non-seasonally adjusted consumption data leads to results that are substantially different from the ones reported here. For example, the nominal inflation premium becomes significantly negative across all maturities and positively correlated with the surplus consumption ratio — which would imply that investors demand a lower inflation premium in periods of low surplus consumption and high levels of risk aversion. Reconciling these results with the basic intuition of the surplus consumption ratio as a measure of recession is an interesting issue for future research to explore.

Finally, I checked the sensitivity of the time series of surplus consumption ratio to both the formulation of the consumption growth process — which feeds s_t with its shocks — and the calibration parameters. For the former, I repeated the estimation of the implied S_t from different specifications of the consumption growth process — based on i.i.d., VAR(4), or VARMA(1,1) processes. The innovations are very similar and cannot account for a different pattern of S_t . I also repeated the whole process by varying the parameters ϕ and $\bar{s}$ of the transition Eq. (9). Again, the general pattern remains.

4. Conclusion

Measures of ex ante variables such as real rates of return are important in many financial problems, but unfortunately they are not observable. This paper uses an intertemporal consumption-based model with habit preferences to extract quarterly measures of ex ante risk premiums in UK bonds. In an extension to [Evans \(1998\)](#), this parametric approach allows for the extraction of the full time series of the premiums, and is flexible enough to account for time variation in the premium components. The extracted premiums vary over time with the surplus consumption ratio, the model's state variable that identifies recessions. The inflation risk premium relating nominal and real bonds is shown to be positive and economically significant, contributing up to 25 basis points to the nominal yields. A positive premium still persists when comparing indexed and hypothetical real bonds, because of the indexation lag in UK indexed bonds.

The risk premium measures can be combined with observed nominal and indexed yields in order to extract measures of the ex ante real rate of return. The adjustment factor linking nominal, indexed, and real yields has the same order of magnitude as in the non-parametric specification of [Evans \(1998\)](#), but existing methods that do not properly incorporate this adjustment factor lead to estimates with a bias of up to 5 basis points.²⁵

²⁵ The procedure to estimate the ex ante real rates proposed by the Bank of England in [Anderson and Sleath \(2001\)](#) assumes this adjustment factor to be 0.

Future research can extend this work in many ways. Different applications can make use of the time series of the extracted ex ante data, for example for understanding how these unobservables correlate with other financial market variables, for tests of rational expectations, or for analysis of macroeconomic implications. Results will always be conditional, though, on the validity of the adopted equilibrium model — in this case, an extension of the habit consumption model from [Campbell and Cochrane \(1999\)](#). The simulation technique employed here can also be adapted to other classes of intertemporal consumption models.

Appendix A. Derivation of risk premium equations

Some aspects of this section draw on [Evans \(1998\)](#), [Balsam et al. \(1998\)](#), and [Campbell and Cochrane \(1999\)](#), and some of the notation is adopted from these papers.

I start from the basic equation of asset pricing theory:

$$1 = E_t[M_{t+1}R_{t+1}] \quad (33)$$

where R_{t+1} stands for the gross return from holding an asset from t to $t+1$ and M_{t+1} is the stochastic kernel. This equation is the first-order condition for optimal consumption and portfolio formation of an economy with a representative agent. By convention, Eq. (33) is expressed in nominal terms: R_{t+1} is computed in nominal terms and the corresponding M_{t+1} is the *nominal* kernel. The asset pricing equation is then used to price bonds.

Let $Q_t(h)$ denote the price at time t of a riskless bond delivering 1 nominal unit at $t+h$. At time $t+1$, this asset is equivalent to the nominal bond with $h-1$ periods to maturity, so that they have the same price. Therefore, the gross return from holding the nominal bond from t to $t+1$ is $Q_{t+1}(h-1)/Q_t(h)$. Substituting in Eq. (33) I have:

$$Q_t(h) = E_t[M_{t+1}Q_{t+1}(h-1)] \quad (34)$$

For the theoretical real bond, let $Q_t^*(h)$ be the time t price of a riskless claim to 1 real unit to be delivered at $t+h$; 1 real unit corresponds, in nominal terms, to a claim to P_{t+h}/P_t , where P_t is the price index at time t . Since at $t+1$ the price of a claim to P_{t+h}/P_{t+1} is $Q_{t+1}^*(h-1)$, the value of a claim to P_{t+h}/P_t is $Q_{t+1}^*(h-1)(P_{t+h}/P_t)$. Thus, using the asset pricing equation:

$$Q_t^*(h) = E_t[M_{t+1}Q_{t+1}^*(h-1)P_{t+1}/P_t] \quad (35)$$

For the zero-coupon indexed bond, I have a similar equation to the one derived above. Taking into consideration the indexation lag l , holding the zero-coupon indexed bond entitles the bondholder to 1 nominal unit adjusted for the inflation up to l months before maturity, which is given by P_{t+h-l}/P_t . Thus, let $Q_t^+(h)$ be the time t price of P_{t+h-l}/P_t nominal units to be delivered at $t+h$. Since at $t+1$ the price of a claim to P_{t+h-l}/P_{t+1} is $Q_{t+1}^+(h-l)$, the value of a claim to P_{t+h-l}/P_t is just $Q_{t+1}^+(h-l)(P_{t+1}/P_t)$, which gives:

$$Q_t^+(h) = E_t[M_{t+1}Q_{t+1}^+(h-l)P_{t+1}/P_t] \quad (36)$$

for $l < h$; for $l \geq h$ the indexed bond gives the same payoff as the pure nominal bond, whence $Q_t^+(l) = Q_t(h)$ when $l \geq h$.

Iterating these equations forward, using the law of iterated expectations and observing that $Q_t(0)=1$ and $Q_{t+h-l}^+(l)=Q_{t+h-l}(l)$ lead to:

$$Q_t(h) = E_t \left[\prod_{i=1}^h M_{t+i} \right] \quad (37)$$

$$Q_t^*(h) = E_t \left[\prod_{i=1}^h M_{t+i}^* \right] \quad (38)$$

$$Q_t^+(h) = E_t \left[\prod_{i=1}^{h-l} M_{t+i}^* Q_{t+h-l}(l) \right] \quad (39)$$

where $M_{t+i}^* = M_{t+i}(P_{t+i}/P_{t+i-1})$.

Substituting M_{t+i}^* in Eq. (33) I end up with $1 = E_t[M_{t+1}^* R_{t+1}^*]$, which is the asset pricing equation expressed in real terms, and thus I call M_{t+i}^* the *real* kernel. Under an intertemporal consumption model with a representative agent, the real kernel corresponds to the model's intertemporal marginal rate of substitution (IMRS), the rate at which the agent is ready to trade consumption at time t for consumption at time $t+1$. Then, the nominal kernel M_t is obtained from the real kernel M_t^* , after correcting for the change in purchasing power along the period of evaluation of the IMRS.

Applying the definition of the conditional covariance to Eq. (37) leads to:

$$\begin{aligned} Q_t(h) &= E_t \left[\prod_{i=1}^h M_{t+i}^* \right] E_t[P_t/P_{t+h}] Np_{t,h} \\ &= Q_t^*(h) E_t[P_t/P_{t+h}] Np_{t,h} \end{aligned} \quad (40)$$

where

$$Np_{t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^h M_{t+i}^*, P_t/P_{t+h} \right]}{E_t \left[\prod_{i=1}^h M_{t+i}^* \right] E_t[P_t/P_{t+h}]} \quad (41)$$

Working in a similar way on Eq. (39), I derive the following:

$$\begin{aligned} Q_t^+(h) &= E_t \left[\prod_{i=1}^{\tau} M_{t+i}^* \right] E_t[Q_{t+\tau}(l)] Ip_{t,h} \\ &= Q_t^*(\tau) E_t[Q_{t+\tau}(l)] Ip_{t,h} \end{aligned} \quad (42)$$

where $\tau = h - l$, and:

$$Ip_{t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^{\tau} M_{t+i}^*, Q_{t+\tau}(l) \right]}{E_t \left[\prod_{i=1}^{\tau} M_{t+i}^* \right] E_t[Q_{t+\tau}(l)]} \quad (43)$$

Another expression can be obtained if I include in the framework an expression for the term premium. Starting from expression (37), I derive:

$$\begin{aligned} Q_t(h) &= E_t \left[\prod_{i=1}^h M_{t+i} \right] = E_t \left[\prod_{i=1}^{\tau} M_{t+i} \prod_{j=\tau+1}^h M_{t+j} \right] = E_t \left[\prod_{i=1}^{\tau} M_{t+i} \right] E_t \left[\prod_{j=\tau+1}^h M_{t+j} \right] T_{p,t,h} \\ &= E_t \left[\prod_{i=1}^{\tau} M_{t+i} \right] E_t [Q_{t+\tau}(l)] T_{p,t,h} = Q_t(\tau) E_t [Q_{t+\tau}(l)] T_{p,t,h} \end{aligned} \quad (44)$$

where I use the law of iterated expectations in the penultimate equality, and $\tau = h - l$. The risk premium component for this equation is identified as:

$$T_{p,t,h} = 1 + \frac{\text{Cov}_t \left[\prod_{i=1}^{\tau} M_{t+i}, Q_{t+\tau}(l) \right]}{E_t \left[\prod_{i=1}^{\tau} M_{t+i} \right] E_t [Q_{t+\tau}(l)]} \quad (45)$$

My final expression simply combines Eqs. (42) and (44):

$$Q_t^+(h) = Q_t^*(\tau) \frac{Q_t(h) I_{p,t,h}}{Q_t(\tau) T_{p,t,h}} \quad (46)$$

Appendix B. Robustness issues

B.1. Sensitivity to Preference Parameters

From all preference parameters used in the equilibrium model, Table 2 isolates α , ϕ , and δ as the ones that were calibrated. These are the best candidates to look at in terms of sensitivity analysis.

The first parameter of interest is α , the one that directly governs the investor's risk aversion. Since the time series of relative risk aversion is given by α/S_t , I expect a particular pattern of the sensitivity of the risk premiums to the calibration of the parameter α . If agents are more risk-averse (increase in α), they will demand a higher premium for holding instruments that are riskier than an alternative. Notice that the parameter α also affects the value of the surplus consumption ratio S_t according to Eqs. (9)–(12), but the first effect dominates.

Table 7 presents the estimation of various risk premium components for increasing values of α . Recall that the pure inflation premium measures the extra yield that agents require in order to hold an asset (nominal bond) that is riskier than the alternative (the hypothetical real bond). Increases in risk aversion should lead to increases in the premium, which is exactly what the results show. Results (not reported) also indicate that the premium $I_{p,t,h}$ increases with α . The table also shows that the adjustment factor $-\frac{1200}{h} \log(I_{p,t,h}/T_{p,t,h})$ linking nominal, indexed, and real yields is very stable across different values of α .²⁶

²⁶ Campbell (1998) discusses how the habit model's outputs are somewhat insensitive to the choice of the preference parameter. In particular, the equity risk premium rises only with the square root of the preference parameter.

Table 7

Sensitivity of the risk premiums to the parameter α

Mean value of $-\frac{1200}{h} \log(Np_{t,h})$					
α	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
1.5	0.1282	0.1734	0.2116	0.2375	0.2522
2.0	0.1297	0.1758	0.2099	0.2353	0.2558
3.0	0.1463	0.2035	0.2483	0.2782	0.2981

Mean value of $-\frac{1200}{h} \log(Ip_{t,h}/Tp_{t,h})$					
α	Bond maturity (months)				
	$h=12$	$h=24$	$h=36$	$h=48$	$h=60$
1.5	0.0046	0.0145	0.0152	0.0191	0.0222
2.0	0.0046	0.0140	0.0165	0.0171	0.0194
3.0	0.0034	0.0111	0.0132	0.0170	0.0192

Average values are reported in percentage terms. All other parameters used in the estimation procedure come from Table 2. Results reported based on quarterly data from Q1:1983 to Q2:2001.

Similar conclusions apply to the remaining parameters ϕ and δ . Their influence on the relative risk aversion is determined by the time series of the surplus consumption ratio. The parameters ϕ and δ affect the surplus consumption ratio through Eqs. (9)–(12). To examine this sensitivity, I repeat the estimation procedure by using values of each parameter around the ones used in Table 2. The results indicate a small sensitivity of the premiums to ϕ in the same direction as the sensitivity to α , e.g., a slight increase in the nominal premium as ϕ increases; for the parameter δ , the sensitivity goes in the other direction. More importantly, though, is the fact that $-\frac{1200}{h} \log(Ip_{t,h}/Tp_{t,h})$ barely varies with the changes in the parameters. These results suggest that my calibration choices for the parameters in Table 2 are reasonable.²⁷

B.2. Sensitivity to simulation method

There might be a concern that computing expectations by simulation can be problematic in the setup of this paper. In the Campbell and Cochrane (1999) model, as well as extensions such as the one adopted here, probabilities of rare events are very important, which implies that the simulation methods converge very slowly.

To address this issue, I follow the procedure suggested in Judd (1999) in order to analyze the quality of the estimation method. I repeat the simulation method described in Section 3 m times, each time with a different set of shocks for the consumption growth and inflation. For each variable of interest, I compute the mean of its m estimates as well as the standard deviation of its m estimates. The ratio of the standard deviation to the mean represents, in percentage terms, the accuracy of the estimated quantity. Repeating this procedure for different sizes S of simulation

²⁷ Results not reported here. I experiment with values of ϕ ranging from 0.959 to 0.979, and values of δ ranging from 0.969 to 0.994 (the range around ϕ matches the difference between the serial correlation of log price dividend ratio – from which the parameter is taken – for the US and the UK data; see Campbell and Cochrane, 1995, Table 11). The variation of the risk premium components is of the same order of magnitude as the variation seen in the results reported in Table 8.

Table 8
Analysis of simulation convergence

Ratio of standard deviation to the mean of $m=100$ estimates			
	Estimated variable, for $h=36$ and $t=1983$ Q4		
	$-\frac{1200}{h} \log(Np_{t,h})$	$-\frac{1200}{h} \log(Ip_{t,h})$	$-\frac{1200}{h} \log(Ip_{t,h}/Tp_{t,h})$
$S=50,000$	0.0490	0.165	0.281
$S=150,000$	0.0292	0.103	0.167
$S=250,000$	0.0221	0.088	0.133
$S=500,000$	0.0180	0.049	0.074
$S=1,000,000$	0.0139	0.037	0.060

The table shows the ratio of standard deviation to the mean of $m=100$ simulated values of each variable, for different sizes of the simulation procedure, for the fourth quarter of 1983.

allows for an assessment of the tradeoff between the size and the convergence of the simulation method.

Table 8 shows the results of the analysis for $m=100$. To save space, I tabulate results of the estimated premiums only for $h=36$ and only for the fourth quarter of 1983. At first, the table shows that the standard error of the estimated variables decreases with the size of the simulation procedure, with gains that are particularly important for the more complex measures, such as the $Ip_{t,h}/Tp_{t,h}$ risk premium.

Estimation of the contribution of nominal premium $Np_{t,h}$ does not seem to present a challenge in terms of convergence; the standard error of the estimated values is small, around 4% even for a smaller simulation size of $S=50,000$. The fact that the simulation procedure presents a slow rate of convergence is highlighted by the analysis of the other premiums: for a simulation size of $S=50,000$, the estimated value for the $Ip_{t,h}/Tp_{t,h}$ risk premium still has a remarkable standard error of about 28% of the estimated mean. However, increasing the size of the simulation procedure leads to standard errors that are more reasonable. In particular, for a simulation size of $S=1,000,000$, which is the simulation size adopted throughout the paper, the standard error of the $Ip_{t,h}$ risk premium goes down to 3%, and for the $Ip_{t,h}/Tp_{t,h}$ risk premium it goes down to 6%.

References

- Abel, A., 1990. Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80, 38–42.
- Allais, O., Cadiou, L., Des, S., 1991. Consumption habit and equity premium in the G7 countries. *Journal of Political Economy* 99, 263–286.
- Anderson, N., Sleath, J., 2001. New estimates of the UK real and nominal yield curves. Working Paper. Bank of England.
- Balsam, A., Kandel, S., Levy, O., 1998. Ex ante real rates and inflation risk premiums: a consumption based approach. Working Paper. Tel-Aviv University.
- Barr, D., Campbell, J., 1997. Inflation, real interest rates, and the bond market: a study of UK nominal and index-linked government bond prices. *Journal of Monetary Economics* 39, 361–383.
- Boudoukh, J., 1993. An equilibrium model of nominal bond prices with inflation-output correlation and stochastic volatility. *Journal of Money, Credit and Banking* 25, 636–665.
- Brandt, M., Wang, K., 2001. Time-varying risk aversion and unexpected inflation. Working Paper. University of Pennsylvania.
- Breeden, D., Gibbons, M., Litzenberger, R., 1989. Empirical tests of the consumption-oriented CCAPM. *Journal of Finance* 44, 231–262.
- Brown, R., Schaefer, S., 1994. The term structure of real interest rates and the Cox, Ingersoll, and Ross Model. *Journal of Financial Economics* 3, 3–42.

- Campbell, J., 1998. Asset Prices, Consumption and the Business Cycle. NBER Working Paper, vol. 6485.
- Campbell, J., 2000. Asset pricing at the millenium. *Journal of Finance* 55, 1515–1567.
- Campbell, J., Cochrane, J., 1995. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. NBER Working Paper, vol. 4995.
- Campbell, J., Cochrane, J., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.
- Campbell, J., Shiller, R., 1996. A scorecard for government indexed debt. NBER Macroeconomics Annual Conference.
- Campbell, J., Lo, A., MacKinlay, A., 1997. *The Econometrics of Financial Markets*. Princeton University Press, Princeton.
- Chu, Q., Lee, C., Pittman, D., 1995. On the inflation risk premium. *Journal of Business Finance & Accounting* 22, 881–892.
- Cochrane, J., 2001. *Asset Pricing*. Princeton University Press, Princeton.
- Cochrane, J., Hansen, L., 1992. Asset pricing explorations for macroeconomics. NBER Macroeconomics Annual. MIT Press, Cambridge.
- Constantinides, G., 1990. Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519–543.
- Cukierman, A., 1986. Measuring inflation expectations: a review essay. *Journal of Monetary Economics* 17, 315–324.
- Deacon, M., Derry, A., 1994. Estimating Market Interest Rate and Inflation Expectations from the Prices of UK Government Bonds. Working Paper, vol. 23. Bank of England.
- Debt Management Office, 2000. The DMO yield curve model. Technical Paper. Bank of England.
- Demetriades, P., 1989. The relationship between the level and variability of inflation: theory and evidence. *Journal of Applied Econometrics* 4, 239–250.
- Dimson, E., Marsh, P., 2001. UK financial market returns, 1955–2000. *Journal of Business* 74, 1–31.
- Emmons, W., 2000. The information content of treasury-indexed securities. Working Paper. Federal Reserve Bank of St. Louis.
- Engle, R., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* 50, 987–1008.
- Engsted, T., 1998. Evaluating the consumption-capital asset pricing model using Hansen–Jagannathan bounds: evidence from the UK. *International Journal of Finance and Economics* 3, 291–302.
- Epstein, L., Zin, S., 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework. *Econometrica* 57, 937–969.
- Evans, M., 1998. Real rates, expected inflation and inflation risk premia. *Journal of Finance* 53, 187–218.
- Evans, M., 2003. Real risk, inflation risk and the term structure. *The Economic Journal* 113, 345–389.
- Evans, M., Wachtel, P., 1992. Interpreting the movement in short-term interest rates. *Journal of Business* 65, 395–429.
- Fama, E., 1975. Short-term interest rates as predictors of inflation. *American Economic Review* 65, 269–282.
- Fama, E., 1976. Inflation uncertainty and expected returns on treasury bills. *Journal of Political Economy* 84, 427–448.
- Fama, E., 1981. Stock returns, real activity, inflation and money. *American Economic Review* 71, 545–565.
- Fama, E., Gibbons, M., 1982. Inflation, real returns and capital investment. *Journal of Monetary Economics* 9, 297–323.
- Ferson, W., Constantinides, G., 1991. Habit persistence and durability in aggregate consumption: empirical test. *Journal of Financial Economics* 29, 199–240.
- Ferson, W., Harvey, C., 1992. Seasonality and consumption-based asset pricing. *Journal of Finance* 47, 511–552.
- Fisher, I., 1930. *The Theory of Interest*. MacMillan, New York.
- Gong, F., Remolona, E., 1996. Inflation Risk in the US Yield Curve: The Usefulness of Indexed Bonds. Federal Reserve Bank of New York Research Paper, vol. 9637.
- Gong, F., Remolona, E., Wickens, M., 1998. What was the Market's View of UK Monetary Policy? Estimating Inflation Risk and Expected Inflation with Indexed Bonds. Federal Reserve Bank of New York Staff Reports, vol. 57.
- Grier, K., Perry, M., 1998. On inflation and inflation uncertainty in the G7 countries. *Journal of International Money and Finance* 17, 671–689.
- Hamilton, J., 1985. Uncovering financial market expectations of inflation. *Journal of Political Economy* 93, 1224–1241.
- Hansen, L., Jagannathan, R., 1991. Implications of security market data for models of dynamic economies. *Journal of Political Economy* 99, 225–262.
- Hansen, L., Singleton, K., 1982. Stochastic consumption, risk aversion and the temporal behavior of asset returns. *Journal of Political Economy* 91, 249–265.
- Hyde, S., Sherif, M., 2005. Consumption asset pricing models: evidence from the UK. *The Manchester School* 73, 343–363.
- Jones, C., Kahl, D., Stevens, J., 1995. Treasury bill rates as proxies for expected inflation. *Quarterly Journal of Business and Economics* 34, 80–89.

- Judd, K., 1999. *Numerical Methods in Economics*. The MIT Press, Cambridge.
- Kandel, S., Ofer, A., Sarig, O., 1996. Real interest rates and inflation: an ex-ante empirical analysis. *Journal of Finance* 51, 205–225.
- Lettau, M., Ludvigson, S., Wachter, J., 2003. The declining equity premium: what role does macroeconomic risk play? Working Paper. New York University.
- Levin, E., Copeland, L., 1992. Reading the message from UK indexed bond market: real interest rates, expected inflation and the risk premium. *The Manchester School of Economic and Social Studies* 61, 13–34.
- Li, Y., 2001. Expected returns and habit persistence. *Review of Financial Studies* 3, 861–899.
- Liu, L., 2003. It takes a model to beat a model: volatility bounds. Working Paper. Boston College.
- McCulloch, J., Kochin, L., 1998. The inflation premium implicit in the US real and nominal term structure of interest rates. Working Paper. Ohio State University.
- Mehra, R., Prescott, E., 1985. The equity premium: a puzzle. *Journal of Monetary Economics* 15, 145–161.
- Pennacchi, G., 1991. Identifying the dynamics of real interest rates and inflation: evidence using survey data. *The Review of Financial Studies* 4, 53–86.
- Throop, A., 1998. An evaluation of alternative measures of expected inflation. Federal Reserve Bank of San Francisco Economic Review 27–43.
- Wachter, J., 2006. A consumption-based model of the term structure of interest rates. *Journal of Financial Economics* 79, 365–399.
- Weil, P., 1989. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics* 24, 401–421.
- Wilcox, D., 1992. The construction of US consumption data: some facts and their implications for empirical work. *American Economic Review* 82, 922–941.
- Woodward, G., 1990. The real thing: a dynamic profile of the term structure of real interest rates and inflation expectation in the United Kingdom, 1982–89. *Journal of Business* 63, 373–398.
- Wrase, J., 1997. Inflation-indexed bonds: how do they work? Working Paper. Federal Reserve Bank of Philadelphia.