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Interpreting the predictive power of the consumption–wealth ratio

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Abstract

Lettau and Ludvigson [Lettau, M., Ludvigson, S.C., 2001a. Consumption, aggregate wealth and expected stock returns. Journal of Finance 56, 815–849] find that the estimated consumption–wealth ratio (*cay*) is a strong predictor of U.S. stock returns, while Brennan and Xia [Brennan, M.J., Xia, Y., 2005. *tay's* as good as *cay*. Finance Research Letters 2, 1–14] argue that the predictive power of *cay* arises from a look-ahead bias. In a unified framework, we examine how the presence of deterministic time trends affects the estimation and forecasting power of *cay*. We show that ignoring the presence of a deterministic time trend in estimating the cointegrating relationship among consumption, asset wealth, and labor income leads to a biased estimate of the consumption–wealth ratio. In the presence of a deterministic time trend, *cay* is a combination of the highly persistent bias component and the unbiased cointegrating residual, and most of the predictive power of *cay* is attributable to the bias component, casting doubt on the robustness of the forecasting power of *cay*.

JEL classification: E21; E44; G10
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1. Introduction

Building on the work of Campbell and Mankiw (1989) who derive the consumption–wealth ratio of a representative agent as a function of future returns on invested wealth and future

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consumption growth, Lettau and Ludvigson (2001a,b) estimate a proxy for the consumption–wealth ratio in the cointegrating regression among consumption, asset wealth and labor income. They find that the estimated consumption–wealth ratio (*cay*) is a strong predictor for aggregate stock returns and a powerful conditioning variable for explaining the cross section of stock returns.[2] They attribute the forecasting power of *cay* to the consumption smoothing behavior of forward-looking investors, arguing that *cay* is a good proxy for time-varying risk premia that summarizes investors' conditional expectations of future market returns.

One caveat of using the consumption–wealth ratio as a forecasting or conditioning variable is that it is not observable and therefore, must be estimated. In particular, when the consumption–wealth ratio is estimated using the full sample data, the estimated coefficients are not literally in the information set at time t. Brennan and Xia (2005) argue that the predictive power of *cay* arises, not necessarily because *cay* summarizes consumers' expectations of future returns, but mainly from a "look-ahead" bias introduced by estimating the cointegrating parameters in sample. Brennan and Xia replace consumption with an inanimate variable, a time trend, and regress the time trend on asset wealth and labor income. They find that the residual from this regression (*tay*) performs as well as, or better than, *cay*.

Brennan and Xia's findings suggest that the presence of deterministic time trends may be important in understanding the sources of the forecasting power of *cay*. This paper examines how the presence of deterministic trends affects the estimation of the consumption–wealth ratio and its forecasting power for stock returns. Aggregate variables such as consumption, asset wealth and labor income are usually assumed to have a deterministic linear time trend along the balanced growth path, which implies that the growth rates of these variables have nonzero drifts. While Lettau and Ludvigson implicitly impose the restriction of no time trend in the cointegrating regression, we allow for the possible presence of the time trend, and find that the cointegrating vector may not eliminate the deterministic trend. If the cointegrating vector eliminates the stochastic trend but not the deterministic trend, Lettau and Ludvigson's cointegrating regression is likely to be misspecified. This leads to biased cointegrating vector estimates, which translate into a biased estimate of the consumption–wealth ratio. This could be a serious problem since the deterministic trend asymptotically dominates the stochastic trend, implying that the stochastic trend plays no role in large samples. The bias in the estimated cointegrating error, in turn, affects the short-run dynamics and stock return forecasting. We show that the *cay* variable is a combination of a highly persistent bias component and an unbiased cointegrating residual. Furthermore, the bias component accounts for most of *cay's* strong predictive power for stock returns. This finding suggests that predictive regressions using *cay* may be spurious to the extent that the bias component is nonstationary, casting doubt on the robustness of the forecasting power of *cay*.[3]

Beyond the econometric justification, there exists a reasonable economic interpretation consistent with the presence of a deterministic trend in the equilibrium relationship among consumption, asset wealth, and labor income. Within the representative agent framework, the presence of a deterministic trend may imply the violation of the agent's budget constraint. The empirical analysis of the cointegrating relationship, however, relies on the consumption, asset wealth, and labor income data aggregated over heterogeneous consumers. Moreover, a degree of

[2] A number of predictors for aggregate stock returns have been suggested and tested in the literature. See, for example, Campbell and Shiller (1988), Fama and French (1988, 1989), Keim and Stambaugh (1986), Hodrick (1992), Lamont (1998), Bossaerts and Hillion (1999), and Ang and Bekaert (2004).

[3] Section 4.3 below explains in more detail how the bias component is related to the predictive power of *cay*.

heterogeneity among consumers that changes slowly over time is consistent with the presence of a deterministic time trend among the aggregate variables. Examples of such changes in consumer heterogeneity include changes in demography, income distribution, wealth distribution, and stock market participation. In particular, Carroll (2000) points out that the behavior of the median household may not resemble the behavior of a representative agent, as a typical household's wealth is much smaller than the wealth of a representative agent. Thus, differences in the consumption and savings behavior among heterogeneous consumers in an economy where the heterogeneity is slowly changing over time may generate a deterministic time trend in the aggregate data.

The paper is organized as follows. Section 2 reviews the theoretical background of *cay* and discusses how deterministic trends can be incorporated into the intertemporal budget constraint. Section 3 discusses the econometric issues in the estimation and test of cointegration with the deterministic trend. Section 4 presents the empirical results, and Section 5 concludes.

2. *cay*, *tay*, and deterministic trends

Consider the standard budget constraint of a representative agent in an economy where all wealth, including human capital, is tradable:

$$W_{t+1} = \left(1 + R_{w,t+1}\right)\left(W_t - C_t\right), \tag{1}$$

where C_t denotes consumption, W_t denotes aggregate wealth at the beginning of period t defined as the sum of asset wealth A_t and human capital H_t, and $R_{w,t+1}$ is the net return on aggregate wealth. Campbell and Mankiw (1989) use a log-linear approximation to the intertemporal budget constraint Eq. (1) and derive the log consumption–wealth ratio as a function of the expected future returns on aggregate wealth and future consumption growth:

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \rho_w^i \left(r_{w,t+i} - \Delta c_{t+i}\right), \tag{2}$$

where lower case letters denote log variables, Δ is the difference operator, and ρ_w is the steady-state value of $(W-C)/C$. Eq. (2) implies that the consumption–wealth ratio reveals the representative agent's conditional expectation of future returns on aggregate wealth and future consumption growth: a high consumption–wealth ratio today should be associated either with high future returns on aggregate wealth, or with low future consumption growth.

Since aggregate wealth is not observable, Lettau and Ludvigson (2001a) assume that the nonstationary component of log human capital, h_t, is well described by log labor income, y_t, implying $h_t = k + y_t + \eta_t$, where η_t is a mean-zero stationary random variable. By log-linear approximation, aggregate wealth W_t and its return $R_{w,t}$ can be written as $w_t \approx \omega a_t + (1-\omega)h_t$, and $r_{w,t} \approx \omega r_{a,t} + (1-\omega)r_{h,t}$, where a_t is log asset wealth, $r_{a,t}$ and $r_{h,t}$ are the returns on asset wealth and human capital, and ω is the share of asset wealth in aggregate wealth, A/W. With these assumptions, Lettau and Ludvigson express the consumption–wealth ratio in terms of the observable variables, c_t, a_t, and y_t:

$$c_t - \omega a_t - (1-\omega)y_t = E_t \sum_{i=1}^{\infty} \rho_w^i \left(\left(\omega r_{a,t+i} + (1-\omega)r_{h,t+i}\right) - \Delta c_{t+i}\right) - (1-\omega)\eta_t. \tag{3}$$

Since all the terms on the right-hand side of Eq. (3) are assumed to be stationary, the left-hand side should also be stationary, implying that consumption, asset wealth, and labor income are cointegrated if they are nonstationary random variables.

Lettau and Ludvigson estimate the cointegrating parameters of consumption, asset wealth and labor income using Stock and Watson's (1993) dynamic least squares (DLS) method as follows:

$$c_t = \alpha + \beta_1 a_t + \beta_2 y_t + \sum_{i=-k}^{k} b_{1,i} \Delta a_{t-i} + \sum_{i=-k}^{k} b_{2,i} \Delta y_{t-i} + \varepsilon_t, \tag{4}$$

and define the estimated consumption–wealth ratio as $cay_t \equiv c_t - \tilde{\beta}_1 a_t - \tilde{\beta}_2 y_t$, where $\tilde{\beta}_1$ and $\tilde{\beta}_2$ denote the estimates of β_1 and β_2 in Eq. (4). Lettau and Ludvigson find that *cay* is a strong predictor of excess returns on aggregate stock market indexes, exhibiting a higher explanatory power than conventional forecasting variables. They attribute the forecasting power of *cay* to the consumption smoothing behavior of forward-looking investors, arguing that *cay* summarizes the investors' expectations of future returns on the market portfolio.

One caveat of using *cay* as a forecasting variable is that it is not observable, but must be estimated. In particular, when *cay* is estimated using the full sample data, the estimated cointegrating parameters are not literally in the information set at time t. While Lettau and Ludvigson argue that since the estimated cointegrating vector is superconsistent, the estimated value can be treated as if it is the true parameter, Avramov (2002) shows that the results still suffer from "look-ahead" bias. He finds that the forecasting power of *cay* is substantially weaker and dominated by other conventional predictive variables (such as the term spread) if *cay* based on data available at the time of prediction is used in predictive regressions.

Brennan and Xia (2005) investigate the "look-ahead" bias introduced by estimating the cointegrating parameters in sample, from a different perspective. More specifically, Brennan and Xia replace consumption with an inanimate variable, a linear time trend t, and define *tay* as the residual from a simple OLS regression of t on a and y as follows:

$$tay = t - \tilde{\gamma}_0 - \tilde{\gamma}_1 a_t - \tilde{\gamma}_2 y_t, \tag{5}$$

where t is calendar time in months, and $\tilde{\gamma}_0$, $\tilde{\gamma}_1$, and $\tilde{\gamma}_2$ are the estimates from the OLS regression. They find that *tay* performs as well as *cay* in forecasting stock returns, although the linear time trend has nothing to do with the optimizing behavior of forward-looking investors. They interpret the result as indirect evidence that the forecasting power of *cay* does not necessarily imply that *cay* summarizes consumers' expectations of future returns. However, using *tay* as a forecasting variable has the following problem: as long as a_t and y_t are not cointegrated, *tay* is likely to be nonstationary, leading the predictive regression to be spurious.[4]

Despite the statistical concerns with *tay* as a forecasting variable, Brennan and Xia's finding raises an important issue regarding the presence of a deterministic time trend in the cointegrating relationship among consumption, asset wealth, and labor income. If consumption, asset wealth, and labor income are well described as I (1) processes with deterministic time trends, both the deterministic and stochastic components of cointegration should be taken into account in the statistical inference.

Lettau and Ludvigson implicitly assume the absence of a deterministic trend in the cointegration test and estimation, arguing that the presence of a deterministic trend implies an eventual violation of the budget constraint identity. This may well be a correct prior within the theoretical framework of a representative agent economy. The empirical analysis of the cointegrating relationship, however, relies on the consumption, asset wealth, and labor income data aggregated over heterogeneous consumers. In the presence of consumer heterogeneity that

[4] Lettau and Ludvigson (2005) also point out the problem of the persistence of *tay*.

slowly changes over time, the cointegrating relationship among the aggregate variables may well contain the deterministic time trend.

As a concrete example, let us illustrate how the steady growth in stock market participation in the U.S. economy in recent decades,[5] can induce a deterministic time trend in the cointegrating relationship among consumption, asset wealth, and labor income in aggregate data. Let λ_{1t} be the stockholders' share of aggregate consumption, λ_{2t} the stockholders' share of aggregate asset wealth, and λ_{3t} the stockholders' share of aggregate labor income at time t. Then, the stockholders' consumption, asset wealth, and labor income, denoted by C_t^s, A_t^s, and Y_t^s, are $C_t^s=\lambda_{1t}C_t$, $A_t^s=\lambda_{2t}A_t$, and $Y_t^s=\lambda_{3t}Y_t$, where $0<\lambda_{it}\leq 1$, $i=1, 2, 3$. Taking logs and assuming that $\ln \lambda_{it}$ can be described by a linear function of a time trend, $\ln \lambda_{it}=\pi_i^o+\pi_i^t$, we have:

$$c_t^s - \beta_1 a_t^s - \beta_2 y_t^s = (c_t - \beta_1 a_t - \beta_2 y_t) + (\pi_{10} - \beta_1 \pi_{20} - \beta_2 \pi_{30}) + (\pi_1 - \beta_1 \pi_2 - \beta_2 \pi_3)t. \tag{6}$$

The consumption–wealth ratio of the stockholders is expressed as the consumption–wealth ratio of the aggregate variables and a linear time trend. Although the linear time trend is merely a crude approximation of growing stock market participation, it may help to describe the equilibrium relationship among aggregate consumption, asset wealth, and labor income. As long as there exists an exogenous factor that is approximated by deterministic components, they should be taken into account in the empirical analysis. Moreover, there is a good reason why a time trend should be included in the cointegrating regression from a statistical point of view, which is examined in the following section.

3. Cointegration and deterministic trends

This section investigates how the presence of a deterministic trend in equilibrium can be tested and how its omission affects the estimation of the cointegrating vector. Let x_{1t} and x_{2t} be one-dimensional and m-dimensional mean-zero random variables. Suppose that x_{1t} and x_{2t} are individually integrated processes of order one with the first differenced series having nonzero drifts:

$$\Delta x_{1t} = \pi_1 + v_{1t}, \tag{7}$$

$$\Delta x_{2t} = \pi_2 + v_{2t}, \tag{8}$$

where v_{1t} and v_{2t} are mean-zero stationary processes. Under the assumption that the initial values of x_{1t} and x_{2t} are zeroes, recursive substitution shows that the data generating processes of x_{1t} and x_{2t} are given by:

$$x_{1t} = \pi_1 t + x_{1t}^o, \tag{9}$$

$$x_{2t} = \pi_2 t + x_{2t}^o, \tag{10}$$

where $x_{1t}^o = \sum_{s=1}^t v_{1s}, x_{2t}^o = \sum_{s=1}^t v_{2s}$, and t is a linear time trend. The time-series x_{1t} and x_{2t} are thus driven by both the deterministic linear trend t and the purely stochastic components x_{1t}^o

[5] Using data from the Surveys of Consumer Finances, Ameriks and Zeldes (2001) report that the percentage of U.S. households owning stocks has increased from 23.9% in 1962 to 33.4% in 1989 and 49.4% in 1998. The stockholding households include the households that hold stocks indirectly via mutual funds, trusts, DC pensions, and IRAs, as well as direct ownership of publicly traded stocks.

and x^o_{2t}. Suppose that the stochastic trends x^o_{1t} and x^o_{2t} are cointegrated with the cointegrating vector β,

$$x^o_{1t} = \beta' x^o_{2t} + u_t, \tag{11}$$

where u_t is a stationary process. It follows from the data generating processes that

$$x_{1t} = \pi t + \beta' x_{2t} + u_t, \tag{12}$$

with $\pi = \pi_1 - \pi_2'\beta$. Eq. (12) is the cointegrating regression specification under the presence of a linear time trend.[6]

It is well known that the limiting distribution of the OLS estimators in Eq. (12) is, in general, non-Gaussian and asymptotically biased. The asymptotic bias may not be a serious problem with respect to the parameter estimators since they are superconsistent. However, the non-standard limiting distribution leads to invalid hypothesis testing about the cointegrating vectors. First, an asymptotic bias arises if u_t is serially correlated. In this case, however, we can simply correct it by using the estimates of the long-run variance of u_t. The second source of the bias is the correlation between the innovations of the regressors and the regression errors. Several estimation procedures have been developed to correct for the cross-correlation. They include Stock and Watson's (1993) dynamic least squares (DLS), Phillips and Hansen's (1990) fully modified estimator (FME), and Park's (1992) canonical cointegrating regression (CCR).

Consider the following two regression models:

$$x_{1t} = \hat{\pi}' t + \hat{\beta}' x_{2t} + \hat{u}_t, \tag{13}$$

$$x_{1t} = \tilde{\beta}' x_{2t} + \tilde{u}_t. \tag{14}$$

Eq. (13) is called the unrestricted model, while Eq. (14) is called the restricted model since $\pi = 0$ is implicitly imposed in the specification of Eq. (14). Suppose $\pi_1 = \pi_2'\beta$ so that $\pi = 0$. In this case, the cointegrating vector $\boldsymbol{\beta}$ eliminates not only the stochastic trends but also the deterministic trends among the variables. Following Ogaki and Park (1997) and Park (1992), one would say that if $\pi = 0$, x_{1t} and x_{2t} are "stochastically and deterministically" cointegrated, while if $\pi \neq 0$, x_{1t} and x_{2t} are "stochastically" cointegrated. Therefore, the restricted model (14) is valid only when the deterministic cointegration restriction, $\pi = 0$, is satisfied.

To test the deterministic cointegration restriction, $\pi = 0$, consider the null hypothesis of the form,

$$H_0 : R'\Pi = 0, \tag{15}$$

where $R = (1, 0_m')'$ and $\Pi = (\pi, \beta')'$. Park (1992) and Hansen (1992) show that the Wald test statistic for the hypothesis testing in the cointegration regression using the CCR or FME estimator yields a χ^2 asymptotic distribution.

For testing deterministic cointegration, the Wald test statistic converges in distribution to a χ^2 distribution with m degrees of freedom:

$$G(\hat{\pi}^*) = \frac{\hat{\pi}' \left(R'(x'x)^{-1} R \right)^{-1} \hat{\pi}}{\hat{\omega}_{11.2}} \xrightarrow{d} \chi^2_m, \tag{16}$$

where $\hat{\pi}$ is the CCR or FME estimates of π.[7]

[6] All the variables do not need to have deterministic trends. As long as one variable is governed by a time trend, Eq. (12) is the appropriate cointegrating regression specification.

[7] Define $p_t = (u_t, \Delta x_t')'$ where $x_t = (t, x_{2t})$ with the long-run covariance matrix $\boldsymbol{\Omega}$ partitioned conformably:

$$\boldsymbol{\Omega} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \Omega_{22} \end{bmatrix}.$$

Then, $\omega_{11.2}$ is the estimate of $\omega_{11.2} = \omega_{11} - \omega_{12}\, \boldsymbol{\Omega}_{22}^{-1}\, \omega_{21}$.

If the test statistic cannot reject the deterministic cointegration restriction ($\pi=0$), the restricted regression model (14) is strongly advocated due to an efficiency gain. However, if the deterministic cointegration restriction is rejected ($\pi\neq0$), the deterministic time trend should be included as a regressor.[8] Otherwise, the regression model is misspecified and the estimates of β are biased. Therefore, the appropriate strategy to estimate the cointegrating vector is to start with the unrestricted model (13), test for $\pi=0$, and then estimate the restricted model (14) if it is compatible with the data, that is, if $\pi=0$ is not rejected.

To understand the consequence of misspecification, suppose that the hypothesis $\pi=0$ is not true, but the time trend is not included in the cointegrating regression. Then the estimated coefficients primarily capture the relation among the individual deterministic time trends instead of the cointegration or common stochastic trend, at least asymptotically, because the time trend asymptotically dominates the stochastic trend.

Under the assumption of the data generating processes in Eq. (10), $\sum_{t=1}^{T} x_{2t} = \sum_{t=1}^{T}\left(\pi_2 t + x_{2t}^o\right)$. While the first term, $\sum_{t=1}^{T}\pi_2 t$, should be divided by T^2 to converge, the second term converges if it is divided by $T^{3/2}$ where T is the number of observations.[9] Thus, the deterministic time trend, $\pi_2 t$, asymptotically dominates the stochastic trend: $\frac{1}{T^2}\sum_{t=1}^{T} x_{2t} \xrightarrow{p} \frac{1}{2}\pi_2$ since $\sum_{t=1}^{T}\pi_2 t = O_p(T^2)$ and $\sum_{t=1}^{T} x_{2t}^o = O_p\left(T^{3/2}\right)$.

It follows that the variance of x_{2t} and the covariance between x_{1t} and x_{2t} should be divided by T^3 to converge.[10] Therefore, asymptotically,

$$\frac{1}{T^3}\sum_{t=1}^{T} x_{2t}x_{2t}' \xrightarrow{p} \frac{1}{3}\pi_2\pi_2' \tag{17}$$

$$\frac{1}{T^3}\sum_{t=1}^{T} x_{1t}x_{2t} \xrightarrow{p} \frac{1}{3}\pi_1\pi_2. \tag{18}$$

Thus, the coefficient in the restricted model converges to:[11]

$$\tilde{\beta} \xrightarrow{p} \pi_1\left(\pi_2\pi_2'\right)^{-1}\pi_2. \tag{19}$$

Without the time trend in the cointegration regression, the asymptotics of the cointegrating vector is dominated by the deterministic time trends. Accordingly, the estimates of the cointegrating vector would be biased.

To examine how the bias is introduced in the coefficient estimates and the cointegration error, we maintain the assumption that the variables are not deterministically cointegrated, i.e. $\pi\neq0$. On the other hand, Lettau and Ludvigson estimate the cointegrating vector among consumption, asset wealth, and labor income imposing the deterministic cointegration restriction ($\pi=0$) in the cointegrating regression without an explicit test.[12] The restricted regression specification can be written in matrix form as $Y=X\tilde{\beta}+\tilde{e}$, where X includes asset wealth and labor income and Y is

[8] Alternatively, individually detrended series can be used in the cointegration regression.

[9] Note that $T^{-(n+1)}\sum_{t=1}^{T} t^n \rightarrow \frac{1}{n+1}$ for $n=0, 1, \cdots$. See Hamilton (1994) for details.

[10] Note that $\sum_{t=1}^{T} x_{2t}x_{2t}' = \sum_{t=1}^{T}\left(\pi_2\pi_2' t^2 + 2t\pi_2 x_{2t}^{o'} + x_{2t}^o x_{2t}^{o'}\right) = O_p(T^3) + O_p\left(T^{5/2}\right) + O_p(T^2)$ and $\sum_{t=1}^{T} x_{1t}x_{2t} = \sum_{t=1}^{T}\left(\pi_1\pi_2 t^2 + \pi_1 t x_{2t}^o + \pi_2 t x_{1t}^o + x_{1t}^o x_{2t}^o\right). = O_p(T^3) + O_p\left(T^{5/2}\right) + O_p\left(T^{5/2}\right) + O_p(T^2)$.

[11] Since $\pi_2'\pi_2$ can be singular, $\left(\pi_2'\pi_2\right)^{-1}$ denotes a generalized inverse.

[12] On the other hand, Bansal et al. (2005) include a linear time trend in their cointegration specifications. To measure the exposure of a portfolio's dividend stream to consumption (consumption leverage), they examine the cointegrating relationship between portfolio dividends and aggregate consumption with the deterministic time trend.

consumption. We assume that each series is demeaned. If the true model is $Y=X\beta+t\pi+u$, then the coefficient estimator from the restricted model is:

$$\tilde{\beta}=\beta+(X'X)^{-1}X't\pi. \tag{20}$$

Therefore, $\tilde{\beta}$ is biased since $X't\neq 0$: $E(\tilde{\beta})=\beta+\boldsymbol{\Gamma}\pi$, where $\boldsymbol{\Gamma}=(X'X)^{-1}X't$. This is the "omitted variable problem" in the cointegration framework. In contrast, the estimator from the unrestricted model is unbiased: $E(\hat{\beta})=\beta$.

It follows that the regression error in the restricted model, denoted as cay^R, is:[13]

$$cay^R=Y-X\tilde{\beta}=M_1Y=M_1t\pi+u, \tag{21}$$

where $M_1=I-X(X'X)^{-1}X'$. The first term of the above equation can be rewritten as:

$$M_1t\pi=t\pi-X(X'X)^{-1}X't\pi=(t-X\boldsymbol{\Gamma})\pi. \tag{22}$$

Since $\boldsymbol{\Gamma}$ is the slope coefficient vector in the regression of t on X, the term $(t-X\boldsymbol{\Gamma})$ is the residual from the regression of t on X. This is *tay* that Brennan and Xia construct. Note that if $\pi=0$, the first term becomes zero and thus cay^R is identical to the true cointegrating error. Thus, cay^R is the sum of two components: *tay* multiplied by π and the true cointegrating error from stochastic cointegration. Since $\boldsymbol{\Gamma}=(1/\pi)(\tilde{\beta}-\beta)$ from Eq. (20), Eq. (21) can be written as

$$\begin{aligned} cay_t^R &= c_t-\tilde{\beta}_1a_t-\tilde{\beta}_2y_t \\ &= \left[\pi t-\left(\tilde{\beta}_1-\hat{\beta}_1\right)a_t-\left(\tilde{\beta}_2-\hat{\beta}_2\right)y_t\right]+\left[c_t-\hat{\beta}_1a_t-\hat{\beta}_2y_t-\pi t\right] \\ &\equiv cay_t^B+cay_t^U, \end{aligned} \tag{23}$$

where $cay_t^B=\pi t-(\tilde{\beta}_1-\hat{\beta}_1)a_t-(\tilde{\beta}_2-\hat{\beta}_2)y_t$ and $cay_t^U=c_t-\hat{\beta}_1a_t-\hat{\beta}_2y_t-\pi t$. Since the deterministic trend is omitted in the restricted cointegrating regression, the coefficient estimates are biased. The bias in the coefficient estimates translates into the regression error so that the regression error, cay_t^R, is not equal to the true cointegration error, cay_t^U. The difference, cay_t^B, therefore, is expressed as a function of the bias in the coefficient estimates.

The above argument can be described in the framework of a simple two-step regression. Consider again the unrestricted model ($\pi\neq 0$):

$$x_{1t}=\hat{\pi}'t+\hat{\beta}'x_{2t}+\hat{u}_t. \tag{24}$$

Define $\mathbf{P}_{2x}\equiv x_{2t}(x_{2t}'x_{2t})^{-1}x_{2t}'$, the projection matrix on $R(x_{2t})$, the space spanned by x_{2t}. If x_{1t} is projected on $R(x_{2t})$, then the residual is Lettau and Ludvigson's cay^R, while if the time trend is projected on $R(x_{2t})$, then the residual is Brennan and Xia's *tay*. By the two-step regression, regressing cay^R on *tay* gives the same regression coefficient estimate of π and the residual is identical to the one from the unrestricted regression. That is,

$$x_{1t}^*=\hat{\pi}t^*+\hat{u}_t, \tag{25}$$

where $x_{1t}^*=(1-P_{2x})x_{1t}$ and $t^*=(1-P_{2x})t$. Note that $\hat{\pi}t^*=cay_t^B$ and $\hat{u}_t=cay_t^U$.

[13] The arguments in this section exactly hold when *cay* is estimated by OLS. Since Lettau and Ludvigson estimate *cay* by DLS, the argument in this section should be modified slightly. The main difference is that in the DLS framework, cay^B is not orthogonal to cay^U and cay^B is no longer a simple scaled version of *tay*, which is estimated by OLS. This difference is due to the fact that *cay* estimated by DLS is no longer a cointegrating regression error. The main point of our argument, however, remains intact as cay^R estimated by DLS still contains the bias component that is a function of the bias in the coefficient estimates.

If the unrestricted model is true, in the first step, cay^R and tay are obtained by the projections of consumption and the time trend on the space spanned by asset wealth and labor income. To estimate the true cointegrating error, cay^R should be projected on tay in the second step. Note that, from Eq. (25), cay_t^B (or tay_t) is orthogonal to the true cointegration error, cay_t^U. Eq. (24) and Eq. (25) are equivalent in the sense that they numerically give the same regression error.

4. Empirical results

4.1. Deterministic cointegration and stochastic cointegration

Table 1 reports summary statistics for the first differences in logs of consumption, asset wealth, and labor income.[14] Notably, all three variables have significantly positive sample means, which implies that the levels of the variables may have deterministic linear trends. Since the levels of the variables are believed to be integrated of order one, they are likely to be described by both stochastic and deterministic trends. As expected, asset wealth is the most volatile variable and consumption is a relatively smooth series.

Panel A of Table 2 reports the estimation results of the normalized cointegration vector in both the restricted and the unrestricted model.[15] The cointegration vector is estimated by a variety of cointegrating regression procedures, including OLS, DLS, CCR, and FME.[16] Despite its asymptotic bias, the OLS estimator is considered since it is superconsistent and the results in Section 3 exactly hold for the OLS procedure.

All the coefficient estimates are statistically significant. Moreover, the estimates of the cointegrating vector are quite similar across the estimation procedures.[17] The restricted and unrestricted models, however, deliver different coefficient estimates. In the case of the restricted model estimated by OLS, the coefficient estimate on asset wealth is 0.23 while the coefficient estimate on labor income is 0.67, which is consistent with Lettau and Ludvigson (2001a). However, in the unrestricted model, they are 0.09 and 0.55, while the coefficient on the linear time trend is 0.31 and highly significant.[18] The unrestricted model specifies that the equilibrium relationship is characterized by both deterministic and stochastic trends, whereas in the restricted model, only the common stochastic trend forms the equilibrium. As model misspecification

[14] We thank Martin Lettau and Sydney Ludvigson for making the data available on their Websites: http://pages.stern.nyu.edu/~mlettau/, and http://www.econ.nyu.edu/user/ludvigsons/. See Lettau and Ludvigson (2001a) for details on the data.

[15] If the unrestricted model is the correct specification so that the linear combination of consumption, asset wealth, and labor income with a time trend is stationary, the error process in the restricted model should be nonstationary. On the other hand, when the restricted model is true, but a linear time trend is included as a regressor, the regression tends to be spurious. Therefore, cointegration tests, in principle, can provide a guideline for the correct specification. However, it is well known that cointegration tests are often subject to size distortions. In many cases, cointegration tests cannot provide a conclusive answer for the correct model specification. Moreover, cointegration can only be tested under certain assumptions on the data generating processes of the time series. The limiting distribution of the error process depends on the particular assumptions maintained in the regression. Our specification test results are ambiguous as the test statistics in both specifications cannot rule out the possibility of the stationary error process.

[16] Lettau and Ludvigson employ the DLS procedure to estimate the consumption–wealth ratio, in which they include eight leads and lags of the changes in labor income and asset wealth.

[17] Johansen's (1988, 1991) maximum likelihood estimator (not reported) also produces similar results.

[18] In the cointegrating regression, the linear time trend is defined as t/T, $t=1, \cdots, T$ where T is the number of observations. Thus, $t/T \in (0, 1]$ regardless of the number of observations.

Table 1
Summary statistics

	Mean (%)	t-statistic	Standard deviation	Autocorrelation	Correlation matrix		
					Δc	Δa	Δy
Δconsumption (Δc)	0.497	15.061	0.005	0.340	1.000	0.252	0.521
Δasset wealth (Δa)	0.566	3.998	0.020	0.059	–	1.000	0.137
Δlabor income (Δy)	0.549	8.558	0.009	0.012	–	–	1.000

Δc, Δa, and Δy denote the first differences in logs of consumption, asset wealth, and labor income, respectively. The data are in quarterly frequency and the sample period is 1952:Q4–2002:Q4. See Lettau and Ludvigson (2001a) for details on the data.

produces biased estimates that cloud statistical inferences, the results suggest a need for testing whether the cointegrating vector eliminates the deterministic trend as well as the stochastic trend.

Panel B of Table 2 provides the formal test results for deterministic cointegration. The test statistic follows the asymptotic χ^2 distribution in the framework of CCR (Park, 1992) or FME (Hansen, 1992). The test statistics decisively reject the deterministic cointegration restriction. This implies that, even though the cointegrating vector eliminates the stochastic trend among consumption, asset wealth, and labor income, it does not eliminate the deterministic trend among the variables. Therefore, the appropriate specification in the cointegrating regression is to include the linear time trend as a regressor.

If the time trend is not included in the regression, the effects of the time trend are likely to be translated into the coefficients on asset wealth and labor income. It implies that the estimated coefficients in the restricted model are a mixture of the true cointegrating vector and the coefficient on the deterministic time trend. In this sense, the cointegrating vector in the restricted

Table 2
Cointegrating regression

Panel A: cointegration vector estimates

	Restricted				Unrestricted			
	OLS	DLS	CCR	FME	OLS	DLS	CCR	FME
Asset wealth	0.227	0.275	0.251	0.251	0.086	0.129	0.093	0.093
s.e.	(0.021)	(0.010)	(0.031)	(0.031)	(0.022)	(0.021)	(0.032)	(0.032)
Labor income	0.665	0.616	0.643	0.643	0.545	0.541	0.559	0.559
s.e.	(0.023)	(0.010)	(0.033)	(0.033)	(0.020)	(0.013)	(0.030)	(0.030)
Time trend					0.308	0.264	0.284	0.284
s.e.					(0.039)	(0.035)	(0.057)	(0.057)

Panel B: deterministic cointegration tests

	CCR	FME
Wald	24.636	24.540
p-value	(0.000)	(0.000)

The restricted model imposes a zero coefficient restriction on the linear time trend in the cointegrating regression in which the dependent variable is consumption. Panel A reports the cointegrating vector estimates with the associated Newey–West adjusted standard errors in parentheses. OLS denotes ordinary least squares, DLS denotes Stock and Watson's (1993) dynamic least squares, CCR denotes Park's (1992) canonical cointegrating regression, and FME denotes Phillips and Hansen's (1990) fully modified estimator. Eight quarters of leads and lags are included parametrically for DLS and used for the estimation of the long-run covariance matrix nonparametrically in CCR and FME. Panel B reports the deterministic cointegration test results with p-values in parentheses. The null hypothesis is that consumption, asset wealth, and labor income are deterministically cointegrated. The Wald test statistics are calculated by CCR and FME. The sample period is 1952:Q4–2002:Q4.

model is biased as the deterministic trend asymptotically dominates the stochastic trend shared by consumption, asset wealth, and labor income. Such bias in the cointegrating vector, in turn, introduces distortion in the cointegrating error.

4.2. Short-run dynamics and error correction representation

The bias in the cointegrating vector affects the estimation of short-run dynamics of consumption, asset wealth, and labor income. If the cointegrating vector $B'=(1, -\beta_1, -\beta_2)$ characterizes the long-run equilibrium relationship among consumption, asset wealth, and labor income, then there exists an error correction representation of Engle and Granger (1987):

$$\Delta Z_t = \mathbf{A}B'Z_{t-1} + \sum_{k=1}^{p} \Phi_k \Delta Z_{t-k} + \epsilon_t, \tag{26}$$

where $Z_t=(c_t, a_t, y_t)'$. The system describes that a deviation from long-run equilibrium, $B'Z_{t-1}$, is adjusted in the direction of the space spanned by the column vector of the error correction coefficient matrix, $\mathbf{A}$. Therefore, if the error correction mechanism works, Z_t will eventually return to the long-run equilibrium path that is defined by $B'Z=0$ due to the presence of cointegration and the error correction mechanism in the system. This implies that at least one of the variables contributes to the adjustment process and thus the corresponding element in the error correction coefficient matrix $\mathbf{A}$ is expected to have statistical significance. Given the long-run relationship represented by the cointegration and the short-run dynamics implied by the error correction representation, one may view the error correction mechanism as a prediction by the cointegrating error in the dynamic system.

Table 3 presents the estimation results for the error correction coefficients. The lag length in the error correction model is set equal to one or two, and the cointegrating vector is estimated by OLS. If the cointegrating vector from the restricted model is imposed, the error correction term is significantly different from zero only in the asset wealth equation as documented by Lettau and Ludvigson. The error correction term is not significant for consumption growth and labor income growth. From this empirical finding, Lettau and Ludvigson conclude that deviations from the common stochastic trend among consumption, asset wealth, and labor income are primarily described as temporary components in asset wealth rather than those in consumption and labor income. In other words, asset wealth adjusts to restore the equilibrium relationship.

Table 3
Estimation of error correction coefficients

	Lag=1		Lag=2	
	Restricted	Unrestricted	Restricted	Unrestricted
Δconsumption (Δc)	−0.036	−0.100	−0.030	−0.109
	(0.027)	(0.043)	(0.029)	(0.045)
Δasset wealth (Δa)	0.399	0.255	0.393	0.148
	(0.126)	(0.204)	(0.130)	(0.212)
Δlabor income (Δy)	0.022	0.107	0.025	0.094
	(0.056)	(0.088)	(0.058)	(0.091)

The estimated error correction coefficients on Δc, Δa, and Δy are reported with the associated standard errors in parentheses. Δc, Δa, and Δy denote the first differences in logs of consumption, asset wealth, and labor income, respectively. The cointegrating vector is estimated by OLS. The restricted model imposes a zero coefficient restriction on the linear time trend in the cointegrating regression. Lag denotes the number of lags for the variables (Δc, Δa, and Δy) in the dynamic system, Eq. (26) in the paper. The sample period is 1952:Q4–2002:Q4.

Table 4
Summary statistics of the estimated consumption–wealth ratio

	Variance	Autocorrelation	Correlation matrix		
			cay^R	cay^B	cay^U
cay^R	1.381	0.832	1.000	0.779	0.627
cay^B	0.838	0.940		1.000	0.000
cay^U	0.543	0.784			1.000
σ	1.542				

The cointegrating vector is estimated by OLS. cay^R and cay^U are estimated from the restricted and the unrestricted models, respectively. The restricted model imposes a zero coefficient restriction on the linear time trend in the cointegrating regression in which the dependent variable is consumption. cay^B is the difference between cay^R and cay^U. σ is the ratio of the variance of cay^B to the variance of cay^U. The sample period is 1952:Q4–2002:Q4.

In contrast, if the unbiased cointegrating vector from the unrestricted model is used, the error correction term is significant for consumption growth. The error correction term is not significant in the asset wealth and labor income equations. When the deterministic trend is taken into account, the short-run dynamics embodied in the error correction coefficients show that consumption, not asset wealth, primarily adjusts to restore the equilibrium.

4.3. Impact of the bias in forecasting stock returns

We now evaluate the forecasting power of the proxies for the consumption–wealth ratio estimated in both the restricted and the unrestricted model. Table 4 reports summary statistics. cay^R refers to the consumption–wealth ratio estimated in the restricted model as in Lettau and Ludvigson. cay^U is the unbiased estimate of the consumption–wealth ratio from the unrestricted model, in which the deterministic time trend is included as a regressor. cay^U is, therefore, the true cointegration error, reflecting deviations from the common stochastic trend. cay^B represents the bias of cay^R defined as the difference between cay^R and cay^U. If the cointegrating vector is estimated by OLS, cay^B and cay^U are orthogonal and cay^B is identical to Brennan and Xia's *tay* multiplied by the coefficient on the linear time trend in the cointegrating regression.

cay^R has a larger sample variance and autocorrelation than cay^U. The variance of cay^B and possibly the variance of cay^R, in fact, cannot be defined if they are nonstationary. Thus, the variance should be interpreted as a sample variation in the given sample period rather than the variance per se. The autocorrelation of cay^B is over 0.94, while cay^U is much less persistent.

Fig. 1 illustrates the time-series of cay^R, cay^B, and cay^U estimated by OLS, and suggests that cay^B may not be stationary. Stochastic shocks to cay^B do not die out within several years. On the other hand, cay^U does not exhibit a stochastic trend, suggesting that cay^U is a stationary series. The time-series behavior of cay^R seems to be in between that of cay^U and cay^B.

We assess the predictive ability by regressing excess stock returns or real stock returns on the one-period lagged consumption–wealth ratio as follows:[19]

$$r_{t+1} = a + bcay_t + u_{t+1}, \tag{27}$$

[19] The predictive regression can be viewed as an error correction model where all the coefficients on the lagged variables are restricted to be zeros, as in $\Delta Z_t = \mathbf{A}B'Z_{t-1} + \epsilon_t$. Thus, even though $\mathbf{A}B'$ is identified, $\mathbf{A}$ and B' are not individually identified. For any nonsingular matrix $\mathbf{Q}$, one can write $\mathbf{A}B' = (\mathbf{AQ})(\mathbf{Q}^{-1}B') = \mathbf{A}^*B^{*\prime}$ where $\mathbf{A}^* = \mathbf{AQ}$ and $B^{*\prime} = \mathbf{Q}^{-1}B'$. Thus, in general, one should be careful in interpreting the absolute size of the slope coefficient in the predictive regression unless the model is further identified in a structural way. For the purpose of forecasting, however, this identification is not necessary.

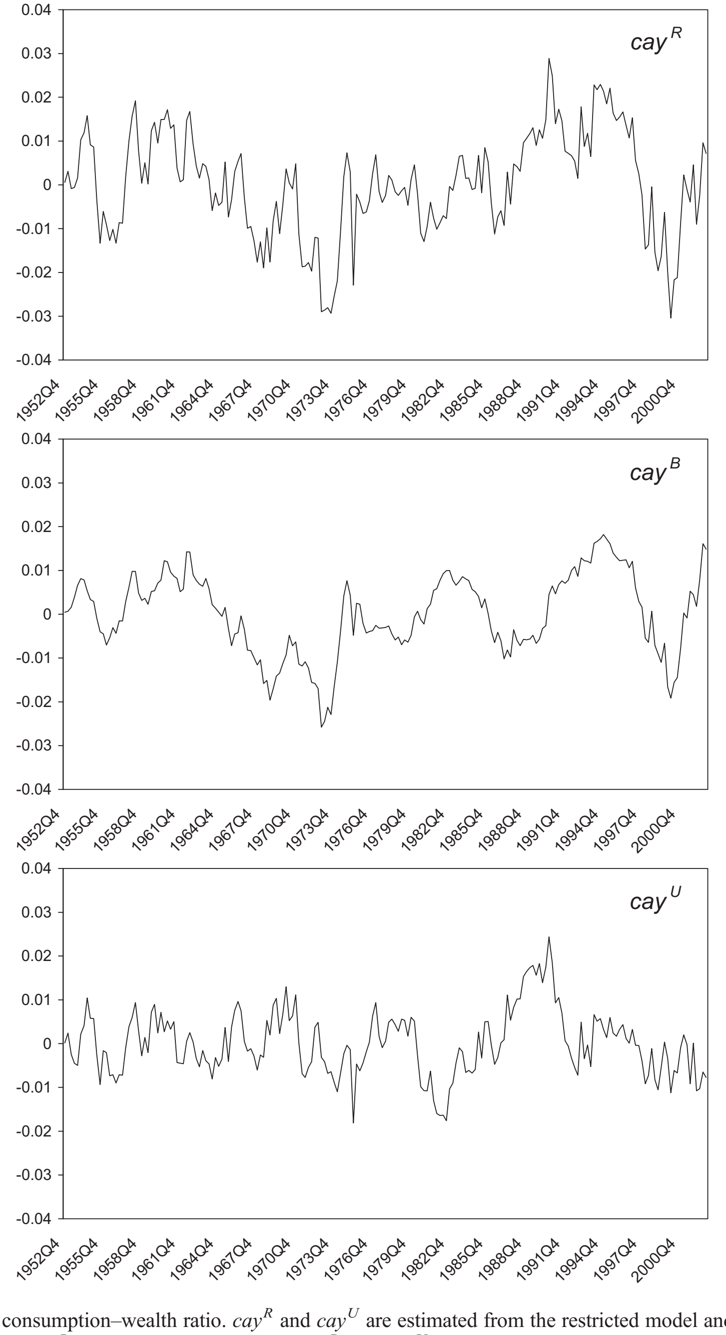

Fig. 1. Estimated consumption–wealth ratio. cay^R and cay^U are estimated from the restricted model and the unrestricted model, respectively. cay^B is the difference between cay^R and cay^U. The sample period is 1952:Q4–2002:Q4.

where r_{t+1} denotes the excess stock return or real stock return, and the period is quarterly. The excess stock returns are the returns on the value-weighted CRSP index in excess of the one-month Treasury bill rate. The real returns are the returns on the CRSP index minus one-period CPI inflation rates.

Table 5
One-period ahead forecasting: stock returns

	cay^R		cay^B		cay^U	
	Coefficient	R^2	Coefficient	R^2	Coefficient	R^2
Forecasting excess returns						
OLS	2.155	9.171	2.666	8.417	1.419	1.560
	(0.480)		(0.622)		(0.797)	
DLS	2.029	9.260	2.941	8.267	2.022	3.543
	(0.449)		(0.693)		(0.746)	
CCR	2.191	9.866	3.159	9.011	2.029	3.349
	(0.468)		(0.710)		(0.771)	
Forecasting real returns						
OLS	2.598	11.676	3.466	12.461	1.326	1.192
	(0.505)		(0.650)		(0.854)	
DLS	2.133	8.963	3.404	9.706	1.780	2.406
	(0.481)		(0.734)		(0.802)	
CCR	2.505	11.293	3.719	10.937	2.200	3.450
	(0.496)		(0.750)		(0.823)	

The excess return is the return on the value-weighted CRSP index in excess of the Treasury bill rate. The real return is the return on the CRSP index minus the one-period CPI inflation rate. The period is in quarterly frequency. cay^R and cay^U are estimated from the restricted model and the unrestricted model, respectively. cay^B is the difference between cay^R and cay^U. OLS denotes ordinary least squares, DLS denotes Stock and Watson's (1993) dynamic least squares, and CCR denotes Park's (1992) canonical cointegrating regression. Eight quarters of leads and lags are included parametrically for DLS and used for the estimation of the long-run covariance matrix nonparametrically in CCR. The numbers in parentheses are Newey–West adjusted standard errors. The sample period is 1952:Q4–2002:Q4.

Table 5 reports the slope coefficient estimates with the Newey and West (1987) adjusted standard errors and the associated R^2 statistics. cay^R explains more than 9% of the next period's variation of excess returns. The excess returns are also well predicted by the bias component cay^B: the R^2 is 8.4% for OLS and 9.0% for CCR. However, the unbiased cointegration error term, cay^U, explains only 1.6% (OLS) and 3.5% (DLS) of the subsequent variation of the excess returns. In particular, the slope coefficients on cay^U are not statistically significant in the case of OLS. The results are qualitatively similar for the real stock return forecasting.

Recall that cay^R is the sum of the bias component cay^B and the unbiased cointegrating residual cay^U. The high R^2 for cay^B and the much lower R^2 for cay^U in the predictive regression imply that the forecasting power of cay^R is mainly due to the bias component cay^B. However, the forecasting power of cay^B is subject to statistical and economic concerns. From Eq. (23), we can see that cay^B is the difference between the linear time trend (πt) and the linear combination of asset wealth and labor income, $cay_t^B = \pi t - (\tilde{\beta}_1 - \hat{\beta}_1)a_t - (\tilde{\beta}_2 - \hat{\beta}_2)y_t$. Given the sizes of the two bias terms, $(\tilde{\beta}_1 - \hat{\beta}_1)$ and $(\tilde{\beta}_2 - \hat{\beta}_2)$, and the fact that asset wealth is more volatile than labor income, cay^B is likely to largely reflect the fluctuation of asset wealth. Since asset wealth is nonstationary, this suggests that cay^B measures the trend deviations of asset wealth, which reflects the stochastic trend scaled by the bias of the parameters.[20] In this case, cay^B would contain the sum of the past shocks to asset wealth, implying that the variance would diverge asymptotically. In the predictive regression, therefore, the estimators are likely to be biased.

Table 6 shows that the unbiased cointegrating residual cay^U forecasts consumption growth, but its forecasting power for asset wealth growth is rather weak. The bias component, cay^B,

[20] Brennan and Xia (2005) make similar arguments.

Table 6
One-period ahead forecasting: consumption, asset wealth, and labor income

		cay^R		cay^B		cay^U	
		Coefficient	R^2	Coefficient	R^2	Coefficient	R^2
Δconsumption	OLS	−0.075	3.569	−0.052	1.014	−0.113	3.140
		(0.028)		(0.036)		(0.044)	
	DLS	−0.077	4.282	−0.074	1.681	−0.118	3.887
		(0.026)		(0.040)		(0.042)	
	CCR	−0.073	3.533	−0.064	1.202	−0.114	3.358
		(0.027)		(0.041)		(0.043)	
Δasset wealth	OLS	0.385	5.104	0.449	4.163	0.295	1.177
		(0.117)		(0.152)		(0.191)	
	DLS	0.372	5.427	0.513	4.385	0.400	2.413
		(0.110)		(0.169)		(0.180)	
	CCR	0.392	5.510	0.541	4.606	0.390	2.160
		(0.115)		(0.174)		(0.186)	
Δlabor income	OLS	0.007	0.008	−0.080	0.636	0.140	1.285
		(0.055)		(0.070)		(0.087)	
	DLS	−0.025	0.121	−0.132	1.416	0.080	0.476
		(0.051)		(0.078)		(0.082)	
	CCR	−0.001	0.000	−0.116	1.039	0.126	1.087
		(0.054)		(0.080)		(0.085)	

cay^R and cay^U are estimated from the restricted model and the unrestricted model, respectively. cay^B is the difference between cay^R and cay^U. OLS denotes ordinary least squares, DLS denotes Stock and Watson's (1993) dynamic least squares, and CCR denotes Park's (1992) canonical cointegrating regression. The numbers in parentheses are Newey–West adjusted standard errors. The sample period is 1952:Q4–2002:Q4.

however, does not forecast consumption growth or labor income growth. This result is not surprising since cay^B largely reflects the stochastic trend in asset wealth.

How then are the coefficient estimates on cay^R, cay^B, and cay^U related to one another in the predictive regressions? Since cay^R contains the bias term cay^B, its forecasting power is also biased. Note that $cay^R = cay^B + cay^U$, and cay^B is orthogonal to cay^U in the case of OLS. Let b^R, b^B, and b^U be the slope coefficients in the regression of returns, r_{t+1}, on cay^R, cay^B, and cay^U, respectively. Then, since $\mathrm{cov}(cay_t^B, cay_t^U)=0$, the slope coefficient on cay^R can be decomposed as[21]

$$
\begin{aligned}
b^R &= \frac{\mathrm{cov}\left(r_{t+1}, cay_t^R\right)}{\mathrm{var}\left(cay_t^R\right)} \\
&= \frac{\mathrm{cov}\left(r_{t+1}, cay_t^B\right) + \mathrm{cov}\left(r_{t+1}, cay_t^U\right)}{\mathrm{var}\left(cay_t^B\right) + \mathrm{var}\left(cay_t^U\right)} \\
&= \frac{\sigma}{1+\sigma} b^B + \frac{1}{1+\sigma} b^U, \qquad (28)
\end{aligned}
$$

where $\sigma = \frac{\mathrm{var}\left(cay_t^B\right)}{\mathrm{var}\left(cay_t^U\right)}$. The slope coefficient on cay^R turns out to be a weighted average of the var(cay_t^U coefficient on cay^B and the coefficient on cay^U. Given the parameter values of b^B and b^U, if $\sigma > 1$ (i.e., $\mathrm{var}(cay^B) > \mathrm{var}(cay^U)$), then b^R is likely to be closer to b^B.[22] Table 4 shows that

[21] Note again that var(cay^B) is undefined if it is nonstationary. var(cay^B) should be interpreted as a sample variation rather than the variance per se in the following arguments.

[22] One should be cautious in interpreting the argument since the coefficient b is likely to change as s changes.

Table 7
Forecasting long horizon excess returns

Horizon		2	3	4	8	12	16	20
cay^R	Coeff.	3.993	5.387	6.839	11.255	12.928	13.092	14.112
	s.e.	(0.974)	(1.477)	(1.997)	(3.982)	(5.561)	(6.475)	(7.022)
	R^2	14.819	18.604	23.297	36.726	39.203	36.278	31.250
cay^B	Coeff.	5.158	7.466	9.756	16.364	19.124	20.863	23.992
	s.e.	(1.221)	(1.883)	(2.587)	(5.399)	(7.586)	(9.270)	(10.859)
	R^2	14.640	21.084	28.044	45.851	51.623	56.060	56.119
cay^U	Coeff.	2.397	2.532	2.776	4.165	3.528	0.957	−1.903
	s.e.	(1.682)	(2.413)	(3.064)	(5.205)	(6.483)	(7.289)	(7.674)
	R^2	2.092	1.594	1.476	1.921	1.175	0.080	0.237

The cointegrating vector is estimated by OLS. cay^R and cay^U are estimated from the restricted model and the unrestricted model, respectively. cay^B is the difference between cay^R and cay^U. The numbers in parentheses are the Hodrick (1992) standard errors. The sample period is 1952:Q4–2002:Q4.

σ is larger than one in the sample period. When the consumption–wealth ratio is estimated by OLS, the weight on b^B, $\sigma/(1+\sigma)$, is 0.622. Therefore, b^R, the coefficient on cay^R in the predictive regression, is likely to be largely influenced by b^B, the coefficient on the bias component cay^B. Moreover, as the number of observations increases, var(cay^B) is expected to diverge since cay^B is nonstationary. Therefore, σ converges to infinity and thus $\sigma/(1+\sigma)\rightarrow 1$, leading $b^R\rightarrow b^B$ asymptotically.

In the predictive regression for stock returns, b^R is largely determined by b^B for two reasons. First, b^B is greater than b^U and, second, a larger weight is given to b^B.[23] Thus, b^R is biased toward b^B. Furthermore, the bias arises even when cay^B is not correlated with the predicted variables. Table 6 shows that the covariance between the consumption growth and cay^B is essentially zero, i.e., $b^B\approx 0$. If b^B is zero, then b^R is biased toward zero. That is,

$$b^R = \frac{b^U}{1+\sigma}. \tag{29}$$

Eq. (29) may explain why cay^R does not forecast consumption growth or labor income growth. This can be viewed as an "errors-in-variable" problem. In particular, since cay^B is not stationary, σ converges to infinity, and therefore b^R converges to zero. Note also that the R^2 statistic can be written as:

$$R^2 = \frac{(b^R)^2 \mathrm{var}\left(cay_t^R\right)}{\mathrm{var}(r_{t+1})} = \frac{\mathrm{var}\left(cay_t^B\right)+\mathrm{var}\left(cay_t^U\right)}{\mathrm{var}(r_{t+1})}\left(\frac{b^B\sigma+b^U}{1+\sigma}\right)^2. \tag{30}$$

Thus, the same argument can be applied to the R^2 statistic in the predictive regression.

Finally, Table 7 presents the estimation results for forecasting the long horizon excess return with the Hodrick (1992) standard error. Under the assumption of a stationary predictor, Hodrick (1992) develops an alternative way of conducting inference for long horizon forecasting with better small sample properties. The fractions of the long horizon return variation predicted by cay^R and cay^B are remarkable. The R^2's are over 31% and 56% for a five-year forecasting horizon. On the other hand, the unbiased cointegration error, cay^U, does not forecast the long

[23] Note that $b^R > b^U$ if and only if $b^B > b^R$.

horizon returns, which is consistent with the results in Table 5. Table 4 indicates that cay^B is more persistent than cay^R, and cay^R is more persistent than cay^U. Since the long horizon predictability can be driven partly by the persistence of predictors, it is not surprising that cay^B and cay^R produce a higher R^2 than cay^U. In addition to the persistence of predictors, the use of overlapping returns in forecasting long horizon returns may also cause serious statistical inference problems, as shown by Kirby (1997). Kirby provides evidence that serial correlation induced by the use of overlapping returns generally leads to a substantial increase in the standard error of the sample R^2, making it difficult to draw precise inferences on the predictability of long horizon returns.

4.4. Subsample results

Hahn and Lee (2001) report that the cointegrating relation among consumption, asset wealth and labor income is unstable under the restricted model specification. They argue that the forecasting power of *cay* is driven by observations in the early sample period. Following Hahn and Lee, we estimate the cointegration vector and test the deterministic cointegration restriction for the two subsample periods with an equal number of observations. The first half corresponds to the period 1952:Q4–1977:Q4 and the second half corresponds to the period 1978:Q1–2002:Q4. We also consider the sample period 1978:Q1–1995:Q4 because Lettau and Ludvigson (2004) document that post-1995 data are responsible for potential parameter instability. Lettau and Ludvigson (2005) argue that using small samples to estimate the cointegrating parameters limits the ability to find support for the model even if it were true. However, as in any time-series analysis, long time-series is more likely to be subject to structural changes or slowly changing parameters. Noting these pros and cons, we conduct an empirical analysis for the subsamples.

Panels A and B of Table 8 provides the cointegrating vector estimates and the deterministic cointegration test results for the two subsample periods. For the 1952:Q4–1977:Q4 sample period, the cointegrating vector estimates are largely consistent with those of the full sample period and the deterministic cointegration restriction is rejected. However, the estimation results are somewhat different for the 1978:Q1–2002:Q4 and the 1978:Q1–1995:Q4 sample periods: the coefficient estimate on labor income is larger while the coefficient estimates on asset wealth and the time trend are smaller relative to those from the full sample. In particular, the coefficient estimate on time trend is not statistically significant and the deterministic cointegration restriction cannot be rejected for the 1978:Q1–1995:Q4 sample period. This suggests that the presence of the cointegrating parameter instability is not simply due to the post-1995 data.

Panel C documents the one-quarter ahead forecasting regression results. In the 1952:Q4–1977:Q4 sample period, cay^R explains 9% of the subsequent fluctuations of the excess return on the CRSP index. However, the deterministic cointegration test result suggests that cay^R is biased. The unbiased cointegration error, cay^U, forecasts only 2% of the excess return variation, and the estimated coefficient in the forecasting regression is statistically insignificant. This result implies that the forecasting power of the consumption–wealth ratio is likely to be biased upward in the early sample period.

Since the deterministic cointegration is not rejected for the second half of the sample period, the restricted model is preferred in estimating the cointegration error. In the 1978:Q1–2002:Q4 sample period, the forecasting power of cay^R is much weaker than in the first half of the full sample, but it is still substantial with an R^2 of 4.9%. For this sample period, although the deterministic cointegration restriction cannot be rejected, the coefficient

Table 8
Subsample results

Panel A cointegrating vector estimates

	1952:Q4–1977:Q4		1978:Q1–2002:Q4		1978:Q1–1995:Q4	
	Restricted	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted
Asset wealth	0.173	0.094	0.100	0.078	0.167	0.162
s.e.	(0.047)	(0.027)	(0.038)	(0.037)	(0.046)	(0.047)
Labor income	0.674	0.515	0.894	0.695	0.847	0.765
s.e.	(0.036)	(0.029)	(0.071)	(0.112)	(0.075)	(0.104)
Time trend		0.164		0.098		0.026
s.e.		(0.022)		(0.045)		(0.023)

Panel B deterministic cointegration tests

	1952:Q4–1977:Q4		1978:Q1–2002:Q4		1978:Q1–1995:Q4	
Wald (p-value)	36.238	(0.000)	0.097	(0.756)	0.314	(0.575)

Panel C stock return forecasting

	1952:Q4–1977:Q4		1978:Q1–2002:Q4		1978:Q1–1995:Q4	
	Coefficient	R^2	Coefficient	R^2	Coefficient	R^2
cay^R	2.756	8.901	2.031	4.894	1.665	2.078
s.e.	(0.900)		(0.900)		(1.357)	
cay^B	3.096	7.059	3.725	3.390	2.392	0.229
s.e.	(1.144)		(1.998)		(5.928)	
cay^U	2.110	1.994	1.573	2.360	1.627	1.875
s.e.	(1.509)		(1.017)		(1.397)	

The restricted model imposes a zero coefficient restriction on the linear time trend in the cointegrating regression. Panel A reports the cointegrating vector estimates with the associated Newey–West adjusted standard errors in parentheses. The dependent variable is consumption. The cointegrating vector is estimated by OLS. Panel B reports the deterministic cointegration test results with the associated p-values in parentheses. The null hypothesis is that consumption, asset wealth, and labor income are deterministically cointegrated. The Wald test statistics are calculated by CCR. Panel C reports the estimation results for forecasting one-quarter ahead excess stock returns, where the excess return is the return on the value-weighted CRSP index in excess of the Treasury bill rate. cay^R and cay^U are estimated from the restricted model and the unrestricted model, respectively. cay^B is the difference between cay^R and cay^U.

on the time trend is significantly different from zero. Thus, it is unclear whether cay^R is biased or not.[24]

The overall results in Table 8 are consistent with our hypothesis that deterministic trend is important in understanding the fore casting power of consumption–wealth ratio. When the coefficient estimate on time trend is economically and statistically important in the long-run relationship among consumption, asset wealth, and labor income, as in the 1952:Q4–1977:Q4 sample period, the estimated consumption–wealth ratio from the restricted model is highly correlated with subsequent changes in excess returns. However, in this case, the forecasting power stems from the bias component cay^B. If the time trend is insignificant the long-run relationship, as in the 1978:Q1–1995:Q4 sample period, cay^R exhibits much weaker forecasting power.

[24] This ambiguity is attributable to the different estimation procedures in the cointegrating regression. We estimate the cointegrating vector by OLS while the deterministic cointegration test is based on CCR. The coefficient estimate on the time trend turns out to be statistically insignificant when CCR is employed. We report the OLS estimation results because CCR does not guarantee that cay^R is the sum of cay^B and cay^U and that cay^B is orthogonal to cay^U.

5. Conclusion

The presence of deterministic trends in the aggregate data on consumption, asset wealth, and labor income may bias the estimation of the consumption–wealth ratio. In contrast to Lettau and Ludvigson (2001a), who assume that consumption, asset wealth, and labor income are deterministically as well as stochastically cointegrated, we test for the deterministic cointegration restriction and find that the presence of a deterministic trend cannot be rejected for the sample period 1952 to 2002. This finding suggests that the cointegrating regression specification without a linear time trend is misspecified, and we show that the consumption–wealth ratio is a combination of the bias component due to misspecification and the unbiased cointegrating residual. Most of the consumption–wealth ratio's forecasting power for stock returns can be attributed to the highly persistent bias component, while the unbiased cointegrating residual shows substantially weaker forecasting power at both short and long horizons.

The theoretical and empirical evidence in this paper, however, does not fully resolve potential problems in time-series analysis of the consumption–wealth ratio. First, if the data generating processes of consumption, asset wealth, and labor income are described only by the stochastic components without any deterministic components, then Lettau and Ludvigson's specification is valid. Therefore, the results depend on the assumptions about the data generating processes. Second, deterministic cointegration tests are likely to be subject to small sample bias. In fact, all the statistical theories on cointegration are based on asymptotic distribution theory and, thus, the test procedures, including the cointegration test itself, have power and size problems.

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