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Regime shifts in interest rate volatility

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Abstract

I find evidence of regime shifts in interest rate volatility using short-rate data from the U.S., the U.K., Japan, and Canada. The regime shifts, if unaccounted for, could lead to spurious volatility persistence when the volatility processes are estimated with the stochastic volatility (SVOL) model. In contrast, the apparent persistence in volatility drops sharply in three out of the four countries when I estimate the volatility processes with the regime-switching stochastic volatility (RSSV) model. I also contribute to the literature by showing how to account for correlation in the regime-switching stochastic volatility model, which is important for modeling asymmetric volatility.

JEL: G10; G12
Keywords: Short-term interest rates; Stochastic volatility; Regime shifts; State-space model

1. Introduction

It has been a well-established empirical fact that the volatility of the U.S. short-term interest rate is itself volatile (e.g., Brenner et al., 1996). Moreover, Ball and Torous (1999) provide evidence that stochastic volatility (SVOL) is also a salient feature characterizing the short-rate dynamics of several other countries, such as the U.K., Japan, and so on.

A popular way to model the stochastic volatility in financial time series is the ARCH/GARCH models of Engle (1982) and Bollerslev (1986). However, a more attractive

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alternative might be the stochastic volatility (SVOL) model of Taylor (1986). As a competitor of the ARCH/GARCH type models, the SVOL model has a natural interpretation of being a discrete-time analog of continuous-time stochastic volatility diffusion models that are widely used in derivative pricing. In the SVOL approach, the underlying volatility is modeled as an unobserved state variable. Ball and Torous (1999) adopt this approach to model the volatility process of the short-term interest rates. A potential problem, however, is that the persistence in volatility may be overstated due to structural breaks in the volatility process. Lamoureux and Lastrapes (1990) first point out this misspecification problem in the context of the GARCH models. However, the same criticism may also be applied to the SVOL model.

In their review article, Chapman and Pearson (2001) point out the necessity to model structural breaks in interest rate volatility.

> However, inferences about the relation between the level and volatility of the short-rate are sensitive to the treatment of the years between 1979 and 1982, the so-called "Federal Reserve experiment." In particular, the data from this period suggest a very strong relation between volatility and the level of interest rates, while excluding this period or treating it as a distinct (lower probability) "regime" suggests a much weaker relation. Finally, modelling the volatility of interest rates requires more than a simple "level effect", i.e., there appears to be some sort of stochastic volatility. However, the additional volatility component can be described adequately (in a statistical sense) in a variety of competing ways.

Gray (1996) also has a similar viewpoint. He argues that structural breaks, such as the Federal Reserve experiment of 1979 to 1982, justify the use of a regime-switching model to model the short-rate volatility.

One way to account for structural breaks is to use the regime-switching model of Hamilton (1989). Several authors have attempted to model the conditional heteroskedasticity in financial assets in the regime-switching framework. Regime shifts in the ARCH model has been studied in Hamilton and Susmel (1994) as well as in Cai (1994). Cai applies the regime-switching ARCH model to the monthly returns of the 3-month U.S. Treasury bills, and identifies two periods of shifting regimes, associated with the oil crisis and the Federal Reserve monetary policy experiment from 1979 to 1982, respectively. Gray (1996) develops a generalized regime-switching model where regime shifts are incorporated into the GARCH model. He concludes that the regime-switching GARCH model outperforms its single-regime counterpart for the U.S. short rates.

Modeling regime shifts in the SVOL model has been considered in So et al. (1998) However, their approach is based on Bayesian methods and, therefore, computationally intensive. More importantly, compared with the method used in this paper, their approach is restrictive in the sense that their model does not allow for nonzero-correlation between the disturbance terms. Nor do they consider the case of time-varying transition probabilities. Kalimipalli and Susmel (2004) also propose using the Markov Chain Monte Carlo algorithm to model regime shifts in U.S. short-rate volatility. However, in their model, the volatility elasticity parameter γ is fixed at 0.5.

In this paper, we compare the SVOL approach with the regime-switching stochastic volatility (RSSV) approach in modeling short-rate volatility. With a similar estimation technique, Smith (2002) also estimates a Markov-switching stochastic volatility model for the 30-day U.S. Treasury bill, and compares its performance to the SVOL model and the Markov-switching model. However, Smith only uses monthly observations from 1964 to 1996, whereas we mainly focus weekly observations (monthly observations are also considered as a robustness check) and use a longer sample period. In addition, Smith only considers the U.S. case while we also take a look at the international evidence. The variety of regime-switching models estimated in this paper is also more extensive than Smith's paper.

Our contributions to the literature are as follows:

First, we find strong evidence that there exist regime shifts in short-rate volatility in four countries and especially in the U.S. and U.K. data. Using Vuong's nonnested likelihood ratio (LR) test, we find strong support the RSSV model in all four countries. Both in-sample and out-of-sample forecasting performance also indicates that RSSV model outperforms SVOL model both in U.S. and U.K., but not the other two countries. The regimes are very persistent and seems be associated with macroeconomic shocks in the U.S. and the U.K. cases.

Second, we extend the RSSV model by relaxing the independence assumption commonly used in the SVOL/RSSV models. We find evidence that modeling the correlation is important at least for the U.S. short-rate volatility.

Third, apart from a constant transition probability RSSV model, we also put forward a time-varying transition probability RSSV model where the transition probabilities are allowed to vary with other exogenous variables. We find evidence that a constant transition probability RSSV model is the more parsimonious specification.

This paper is organized as follows. The next section gives further background and sets up the models. Section 3 estimates the SVOL and RSSV models using the short-rate data of the U.S., the U.K., Canada, and Japan. A battery of robustness checks are also included. Section 4 gives some concluding remarks.

2. Model specifications

2.1. The stochastic volatility model for short-term interest rate

We start with the Chan et al. (1992) (CKLS) model of the short rate:

$$dr = (\alpha + \beta r) + \sigma r^{\gamma} \mathrm{d}W, \tag{1}$$

where r is the interest rate level, and W is a Brownian motion. Note that in the CKLS model, volatility is specified as a function of the interest rate level, capturing the so-called "level effect". Note that if we rewrite $\alpha+\beta r$ as $\beta(r-\theta)$ where $\theta \equiv -(\alpha/\beta)$, then β can be interpreted as the speed of mean reversion and θ the mean value of the interest rate. The parameter γ has the interpretation of being the volatility elasticity and is usually less than one in theoretical models. For example, γ=0.5 in the Cox et al. (1985) model and γ=0 in the Vasicek (1977) model. CKLS find that the unrestricted estimate of γ in their model is approximately 1.5, which contradicts the specifications of most theoretical models. In fact, CKLS is able to reject all models whose γ is less than 1. In addition to the fact that this is incompatible with most theoretical models, a γ greater than 1 has the undesirable implication that the short-term interest rate may become nonstationary at high interest rate levels.

As in Ball and Torous (1999), we augment the discrete-time CKLS model by incorporating the stochastic volatility into the short-rate process:

$$\Delta r_t = a + br_{t-1} + \sigma_{t-1} r_{t-1}^{\gamma} \varepsilon_t \tag{2}$$

$$h_t = \mu + \phi h_{t-1} + \sigma_\eta \eta_t, \tag{3}$$

where $\Delta r_t \equiv r_t - r_{t-1}$, $h_t \equiv \ln(\sigma_t^2)$, ε_t, and η_t are i.i.d. standard Gaussian innovations. We will relax the independence assumption later. Note that short-rate volatility is treated as a latent process in this SVOL model. One estimation strategy is based on the Kalman filter. By assuming normality, estimation of the SVOL model can be carried out by quasi-maximum likelihood (QML). The Kalman filter is used to obtain the prediction error decomposition of the Gaussian likelihood function, which is then numerically maximized. The QML approach to the estimation of the SVOL model has been discussed in Ruiz (1994) and extended to the multivariate case by Harvey et al. (1994).

This SVOL model can be viewed as a discrete-time approximation to a two-factor interest rate model, such as the Longstaff and Schwartz (1992) model, where the short rate and its volatility are specified as two factors.

To estimate the model given by Eqs. (2) and (3), we first estimate the drift parameters a and b by ordinary least squares (OLS). Let $y_t = \Delta r_t - (a + br_{t-1})$, and $x_t \equiv \ln(y_t^2)$. We obtain the following state-space model:

$$x_t = h_{t-1} + 2\gamma \ln(r_{t-1}) + \ln(\varepsilon_t^2) \tag{4}$$

$$h_t = \mu + \phi h_{t-1} + \sigma_\eta \eta_t. \tag{5}$$

Note that $\ln(\varepsilon_t^2)$ is distributed as a $\log-\chi^2$ variable with mean=-1.2704 and variance=$\pi/2$. Hence, to facilitate QLM estimation, we can rewrite Eq. (4) as follows:

$$x_t = h_{t-1} + 2\gamma \ln(r_{t-1}) - 1.2704 + \xi_t, \tag{6}$$

where $\xi_t = \ln(\varepsilon_t^2) + 1.2704$.

2.2. The regime-switching stochastic volatility model

Although the ARCH/GARCH and the SVOL models provide a nice way to account for volatility persistence that is commonly observed in financial data, there is a concern that the apparent volatility persistence may be overestimated because of the failure to account for structural shifts in volatility. Lamoureux and Lastrapes (1990) investigate this possibility in the case of the GARCH models subject to deterministic structural breaks. However, obviously, the same problem may also plague the SVOL model with random structural breaks.

In the absence of a perfect knowledge when such structural shifts might occur, the regime-switching model of Hamilton (1989) may be a useful tool to account for random structural shifts. Attempts to incorporate regime shifts into the ARCH/GARCH models have been made by Hamilton and Susmel (1994), Cai (1994), and Gray (1996), among others.

So et al. (1998) generalize the SVOL model by adding the regime-switching properties, which we refer to as the regime-switching stochastic volatility (RSSV) model. The switching dynamics is governed by a first-order Markov process. They estimate the RSSV model with a Bayesian approach (Markov-chain Monte Carlo), which is computationally intensive.

A more convenient way to estimate the RSSV model is to use Kim's filter. Kim (1994) extends Hamilton's regime-switching model to a general state-space form. Since the SVOL model is typically written in the linear state-space form, we can apply Kim's filter to the estimation of the RSSV model. For details of the algorithm, we refer to Kim (1994) and Kim and Nelson (1999). In the following discussion, we concentrate on a simple two-state RSSV model, where the regime variable is assumed to follow a two-state, first-order Markov process. We focus on three specifications of the RSSV model in this paper. The first model specification only allows the parameter μ in Eq. (5) to be regime-dependent. Hereafter we refer to this as the RSSV-1 model. Let s be an unobserved regime variable, where s=0 and s=1 denote two different volatility regimes. We rewrite Eq. (5) as follows:

$$h_t = \mu_s + \phi h_{t-1} + \sigma_\eta \eta_t \tag{7}$$

where μ_s is regime-dependent. The transition probability matrix X of the Markov process can be written as follows:

$$X = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix} \tag{8}$$

where $p=\Pr(s_t=0|s_{t-1}=0,I_{t-1})$, $q=\Pr(s_t=1|s_{t-1}=1,I_{t-1})$, and I_{t-1} is the information set.

2.3. Modeling correlation in the regime-switching stochastic volatility model

The SVOL model chooses to model the log variance instead of volatility itself to ensure positive variances. However, by squaring the data, we may lose useful information unless the true correlation between ε_t and η_t is zero. Most SVOL models impose the zero-correlation assumption. Nevertheless, a priori, we have no reason to believe that this assumption is valid. As a matter of fact, empirical evidence from stock returns indicates that this zero-correlation assumption may be false because of the well-documented phenomenon of asymmetric volatility for stock returns. Namely, stock market volatilities increase (decrease) as stock prices drop (go up).[1]

In the case of interest rate data, it is unclear whether or not the correlation between ε_t and η_t is zero. For example, Ball and Torous (1999) assume zero-correlation when estimating the SVOL model. They argue that in their sample the correlations are low. However, their reported correlations (standard errors in parentheses) for the Euro-mark and

[1] See, for example, Black (1976).

the Euro-yen series are 0.163 (0.082) and 0.224 (0.091). It looks like at least for these two interest rate series, zero-correlation is not a very good assumption. Ball and Torous do not report the correlation for U.S. T-bill yields due to convergence problem.

Harvey and Shephard (1996) propose a method to handle the correlation between the two disturbances for the SVOL model. They show that the loss of information due to squaring may be recovered if we carry out inferences conditional on the signs of the observations.

Following Harvey and Shephard, we condition on the sign of the residuals to recover the information about correlation, and modify the SVOL model as follows:

$$x_t = h_{t-1} + 2\gamma\ln(r_{t-1}) - 1.2704 + \xi_t,$$

$$h_t = \mu + \phi h_{t-1} + g_t u^* + \sigma_\eta \eta_t^*, \tag{9}$$

$$\left(\begin{array}{c} \varepsilon_t \\ \eta_t{}^* \end{array}\right)\Bigg| g_t \rightarrow ID\left(\left(\begin{array}{c} 0 \\ 0 \end{array}\right), \left(\begin{array}{cc} \sigma_\xi^2 & \gamma^* g_t \\ \gamma^* g_t & \sigma_\eta^2 - u^{*2} \end{array}\right)\right), \tag{10}$$

where g_t is a variable that takes 1 (-1) if y_t is positive (negative). u^*=$E_+(\eta_t)$ and γ^*=cov$_+(\eta_t,\xi_t)$. E_+ and cov$_+$ denote the expectation and covariance conditional on ε_t being positive. Note that, in our model, σ_ξ^2 equals ($\pi^2/2$) and is not a parameter to be estimated.

When ε_t and η_t are bivariate normal with corr(ε_t,η_t)=ρ, Harvey and Shephard show that

$$u^* = 0.7979\rho\sigma_\eta, \tag{11}$$

$$\gamma^* = 1.1061\rho\sigma_\eta. \tag{12}$$

Because of the fact that Eqs. (6) and (9) still form a state-space model, QML estimation of the SVOL model with correlation can be carried out as usual, using the results in Eqs. (11) and (12).

Extending Harvey and Shephard's approach to the case of the RSSV model is fairly straightforward. We rewrite Eq. (9) as follows:

$$h_t = \mu_s + \phi h_{t-1} + g_t u^* + \sigma_\eta \eta_t^*. \tag{13}$$

Obviously, the above equation and Eq. (6) remain a state-space model. Therefore, we can still apply Kim's filter to estimate the RSSV model with correlation (hereafter RSSV-COL model). This is the second specification we consider in this paper.

2.4. Regime-switching ϕ

Next we consider the third specification of the RSSV model where both μ and ϕ are allowed to be regime dependent:

$$h_t = \mu_s + \phi_s h_{t-1} + \sigma_\eta \eta_t. \tag{14}$$

Replacing Eq. (7) with the above equation, we can proceed as usual. We dub this specification as the RSSV-2 model.

2.5. Time-varying RSSV model

In So et al. (1998), Smith (2002), and Kalimipalli and Susmel (2004), the transition probabilities are assumed to be constants, which is not very flexible. Here we show how to relax this assumption by allowing for time-varying transition probabilities.

Following Diebold et al. (1994), we specify the time-varying transition probabilities as follows:

$$p(s_t = j | s_{t-1} = j; I_{t-1}) = \frac{e^{a_j + b_j r_{t-1}}}{1 + e^{a_j + b_j r_{t-1}}}, \quad j = 0, 1. \tag{15}$$

Thus, the transition probabilities in this model specification are allowed to vary with the lagged interest rate levels. In fact, we can let the transition probabilities be a function of any other exogenous variables as well. Hence, this model specification is flexible and encompasses a constant transition probability model.

3. A comparison of the stochastic volatility and the regime-switching stochastic volatility models

In this section, we compare the SVOL model with its regime-switching counterpart the RSSV model using short-term risk-free interest rate data from four developed countries, the United States, Canada, Japan, and the United Kingdom.

3.1. Data

The U.S. interest rate data includes 2492 weekly observations of the 3-month Treasury bill rates obtained from the Federal Reserve site, ranging from January 1954 to October 2001. While the majority of studies on interest rate models focus on U.S. data, it is interesting to see what kind of evidence might emerge from the international data. Hence, we also examine the interest rate data for three other industrialized countries: Canada, Japan, and the United Kingdom. The data for these countries are obtained from Datastream. The interest rate series include: Canada Treasury bill 1 month (CN13883) from January 1980 to December 2000, a total of 1095 observations; Japan Interbank 1 month offered rate (JPIBK1M) from December 1985 to December 2000, 782 observations, and UK interbank 1 month middle rate (LDNIB1M) from January 1975 to December 2000, 1356 observations.

These short-rate data exhibit very distinctive patterns. Both very high interest rate levels and extremely low interest rate levels are observed. The highest interest rate is 21.55% from the Canadian interest rate series while the lowest is 0.086% from Japan. A common feature is that the interest rate movements typically exhibit very volatile behavior. For example, the U.S. interest rate shows some dramatic swings during the Federal Reserve experiment period of 1979 to 1982, which coincides with high interest rate levels. On the other hand, in cases of Canada and Japan, it seems that excessive volatility can also occur at median to low interest rate levels.

3.2. Model estimation

We first estimate the drift parameters with ordinary least squares (OLS), $\Delta r_t = a + br_{t-1} + \varepsilon_t$. Note that OLS gives consistent parameter estimates even in the presence of stochastic volatility. Estimates for a and b are reported in panel A of Table 1. We notice that the estimates for b are negative in all four countries, which is consistent with the interpretation that b is the mean reversion parameter for linear drift models.

We then subtract the estimated drift terms to obtain the residuals. Residuals from the OLS model seem to suggest that constant volatility is not a good assumption. We formally test for the ARCH/GARCH effects in the residuals using Engle's LM test with up to five lags. We find the null of no ARCH effects is strongly rejected by the data in all four countries.

Panel B of Table 1 reports the parameter estimates for the SVOL model consisting of Eqs. (5) and (6). We find that the γ estimates are less than one for all four countries. In the case of the U.S. short rates, it is about 0.71, significantly less than the CKLS estimate of 1.5. In fact, the highest γ estimate is 0.92 in the U.K. case. Hence, the estimates obtained here imply stationary interest rate processes. Moreover, the standard errors of these estimates are relatively small. Hence, it looks like the CKLS puzzle of unreasonable high γ may be due to their failure to model the stochastic volatility of short rates.

We also notice that the estimates for ϕ are very high for all countries. The highest number is 0.9905 for Japan and the lowest is 0.9580 for the U.K. data with the estimates for the U.S. and Canada in the middle. In addition, the standard errors for these ϕ estimates are relatively small. Such high estimates of ϕ imply very persistent volatility processes. As

Table 1
Parameter estimates of the SVOL model

Parameter	U.S.	Canada	Japan	UK
Panel A: OLS drift parameter estimates				
a	0.0174	0.0179	0.0007	0.0558
	(0.0094)	(0.0201)	(0.0085)	(0.0324)
b	−0.0031	−0.0030	−0.0027	−0.0060
	(0.0015)	(0.0022)	(0.0021)	(0.0031)
Panel B: SVOL model parameter estimates				
ϕ	0.9662	0.9803	0.9905	0.9580
	(0.0386)	(0.0201)	(0.0065)	(0.0709)
μ	−0.2272	−0.1165	−0.0649	−0.3237
	(0.2597)	(0.1267)	(0.0445)	(0.5474)
σ_η	0.3458	0.2231	0.2787	0.3916
	(0.1949)	(0.1219)	(0.0961)	(0.3247)
γ	0.7093	0.5486	0.7542	0.9211
	(0.0483)	(0.0780)	(0.0417)	(0.1000)
Log-likelihood	−5909.61	−2550.52	−1968.12	−3221.09

This table reports the parameter estimates of the stochastic volatility (SVOL) model for short-term interest rates from four countries. Panel A reports the drift parameter estimate for the following model with OLS: $\Delta r_t = a + br_{t-1} + \varepsilon_t$ Panel B reports the parameter estimates for the SVOL model consisting of Eqs. (5) and (6). Standard errors are reported in parentheses.

indicated by Lamoureux and Lastrapes (1990), the pricing of contingent claims, such as options, relies on perceptions of how permanent volatility shocks are. A transitory volatility shock will have a smaller impact on the price of an option with a relatively long maturity, and vice versa. Hence, correct specification of the persistence in volatility process is an important issue at least from the perspective of pricing derivative securities.

Lamoureux and Lastrapes (1990) argue that the apparent highly persistent volatility found with the GARCH models could be misleading if we do not account for possible structural breaks in the volatility process. Obviously, the same criticism is also applicable to the SVOL model used here. Therefore, we proceed by estimating the RSSV model to account for possible regime shifts in short-rate volatility.

Table 2 reports the parameter estimates of the RSSV-1 model (Eqs. (6) and (7)) using the interest rate data from the four countries under consideration. We find strong evidence of regime shifts in short-rate volatility. We notice that the estimated transition probabilities are very close to 1. In fact, for all four countries, their transition probabilities p and q are above 0.99, which implies very distinct volatility regimes. In addition, the standard errors for these transition probability estimates are very small, suggesting the transition probabilities are precisely estimated. High transition probabilities imply the regimes are indeed persistent. In fact, we can calculate the expected duration of these different regimes using the transition probability estimates. The expected duration of a particular regime is given by $1/(1-pr)$ where pr is the estimated transition probability for that regime. We find the expected duration in our data last for years instead of months, which implies that these regimes might be related to some fundamental state variables with long-lasting effects rather than short-term transient market movements. Formal hypothesis testing regarding the number of regime states requires the use of numerical methods that are very costly to compute. See Hansen (1992) and the related erratum (1996). However, we note that the

Table 2
Parameter estimates of the RSSV-1 model

Parameter	U.S.	Canada	Japan	UK
p	0.9985	0.9951	0.9975	0.9947
	(0.0012)	(0.0051)	(0.0029)	(0.0034)
q	0.9975	0.9960	0.9957	0.9916
	(0.0020)	(0.0036)	(0.0039)	(0.0060)
ϕ	0.9592	0.7585	0.6071	0.6205
	(0.0185)	(0.0850)	(0.0606)	(0.1260)
μ_0	−0.2156	−1.4719	−2.3735	−2.2464
	(0.1069)	(0.5578)	(0.3767)	(0.8053)
μ_1	−0.2902	−1.7468	−3.0907	−2.9305
	(0.1395)	(0.6718)	(0.4953)	(1.0365)
σ_η	0.2758	0.6462	1.4675	0.8235
	(0.0631)	(0.1425)	(0.1266)	(0.1568)
γ	0.4851	0.7533	0.8447	0.6682
	(0.1449)	(0.2176)	(0.0684)	(0.1539)
Log-likelihood	−5682.71	−2516.74	−1907.53	−3153.45

This table reports the parameter estimates of the regime switching stochastic volatility model (RSSV-1) consisting of Eqs. (6) and (7) using short-term interest rates from U.S., UK, Japan, and Canada. Standard errors are reported in parentheses.

pseudo-likelihood ratio test statistics are all extremely significant, with the highest p-value being 8.8×10^{-14} in the case of Canada and lowest p-value 3.3×10^{-97} in the case of the U.S. data. Alternatively, Smith (2002) proposes the use of Vuong's (1989) likelihood ratio (LR) test for nonnested competing models. Vuong's test statistic is as follows:

$$n^{-1/2}LR_n/\hat{\omega}_n \rightarrow N(0,1),$$

where $\mathrm{LR}_n = L_n^{\mathrm{RSSV}} - L_n^{\mathrm{SVOL}}$, L is the log-likelihood value, n is the number of observations, and $\hat{\omega}_n^2$ is the variance of the LR statistic. Vuong's LR statistic for the RSSV-1 model over the SVOL model is as follows (p-values in parentheses): for U.S., it is 9.9070 (0.0000); for Canada, it is 3.7909 (0.0000); Japan 4.7885 (0.0000); U.K. 5.3340 (0.0000). Hence, Vuong's LR test unanimously chooses the RSSV-1 model over the SVOL model for all four countries.

Another interesting phenomenon is related to the parameter estimates for ϕ. If we compare the ϕ estimates obtained from the SVOL model versus those from the RSSV model, we find the parameter estimates drop dramatically for three out of the four countries. For the Canada short-rate volatility, ϕ decreases from 0.98 to 0.76, for Japan from 0.99 to 0.61, and for the United Kingdom from 0.96 to 0.62. These represent decreases of 22% to 38%. In addition, the standard errors are relatively small compared with the magnitude of the decreases. The same parameter estimates decrease slightly in the U.S. case and are not statistically different from each other. This seems to confirm the our concern that the SVOL model could lead to overstated volatility persistence due to its failure to account for regime shifts in the volatility.

The γ estimates are broadly similar for the two models under consideration. We note that all the estimates of γ are less than one. In other words, they all imply stationary interest rate processes. This reaffirms that the CKLS puzzle is possibly due to model misspecification.

We plot the high-volatility regime probabilities along with the filtered conditional volatility for the four countries in Fig. 1. First, we notice that the probability plots are consistent with the conditional volatility plots. We find periods of high conditional volatility matching with high regime probabilities. Second, the evidence from the regime probability plot for the U.S. data seems to suggest a correlation between high volatility regimes and macroeconomic shocks. For example, the high volatility regime from the 70s to the mid-80s seems related to the Federal Reserve monetary policy experiment, the oil crisis, as well as the NBER-dated recessions.

To further investigate the relation between the regimes and macroeconomic variables, we calculate the average annualized CPI inflation rates and GDP growth rates within each regime. Interestingly, we find out that for the U.S. and the U.K., the inflation rates during the high volatility regime are approximately twice as high as those in the low volatility regime. The numbers are as follows. In the U.S. case, the average inflation rates are 0.0482 (high volatility regime) and 0.0267 (low volatility regime). In the U.K. case, the average inflation rates are 0.0830 (high volatility regime) and 0.0418 (low volatility regime). All numbers are highly significant. In addition, the t tests for mean equality unanimously reject the null that the inflation rates are the same across the two regimes at the 1% level. We also take a look at the nominal GDP growth rates. In the U.S. case, the average GDP growth rates are 0.0743 (high volatility regime) and 0.0604 (low volatility regime). In the

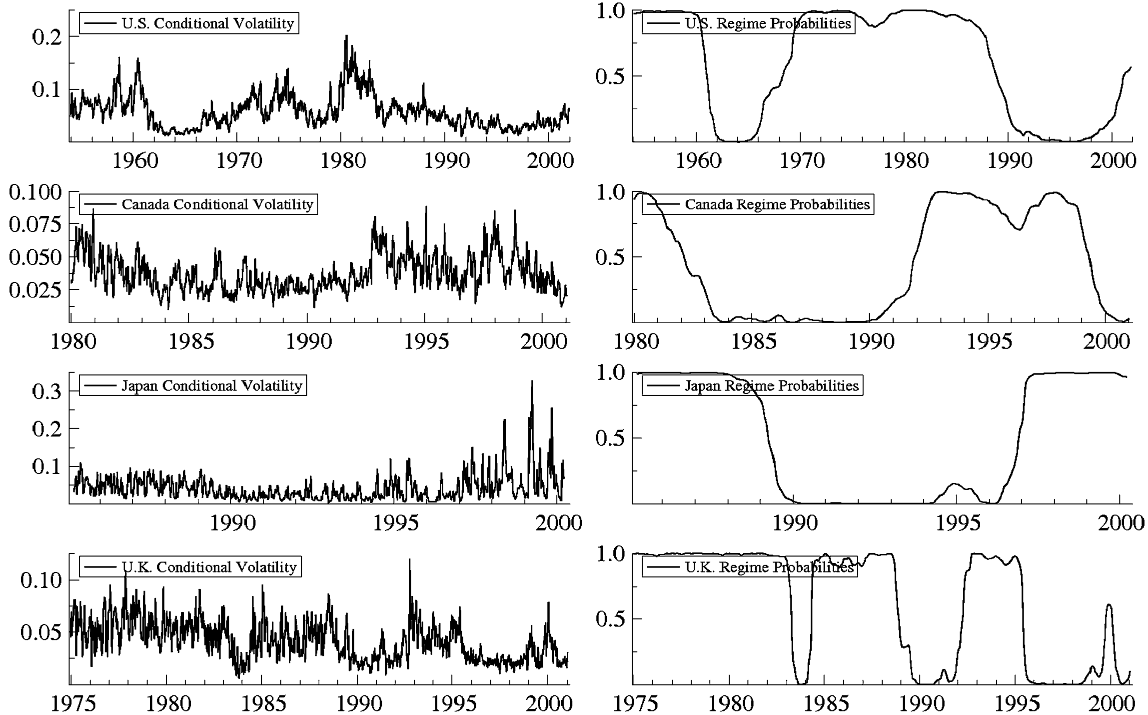

Fig. 1. Plot of regime probabilities and conditional volatility.

U.K. case, the average GDP growth rates are 0.0232 (high volatility regime) and 0.0128 (low volatility regime). Hence, it looks like the high (low) volatility regime is associated with higher (lower) inflation and lower (higher) real GDP growth. In the cases of Japan and Canada, the inflation rates and GDP growth rates across the regimes are statistically indistinguishable, possibly due to their relatively short sample periods.

The results for the RSSV-COL model are reported in Table 3. We notice that the parameter estimates look very similar to those reported in Table 2. If we take a look at the estimated correlation coefficients, only the correlation for the U.S. short-rate data is statistically different from zero. For the other three countries, zero-correlation does not seem to be a bad assumption. In fact, the regime probability plots for this model also look similar to those plots shown in Fig. 1. Overall, in our sample, we find no evidence of correlation in the data except for the U.S. short rates. However, in other cases, such as stock returns, modeling correlation might be important, and the modeling approach taken in this article could prove to be useful under those circumstances.

The third model specification is the RSSV-2 model. The results are reported in Table 4. There seems to be little evidence that this model specification outperforms the more parsimonious RSSV-1 model that we estimate earlier. We find the parameter estimates obtained from this specification are very similar to those reported in Tables 2 and 3. In terms of log-likelihood values, they are also very close. In addition, for all four countries under consideration, the two ϕ estimates are not statistically different from each other. The regime probabilities are almost identical to the other specifications. To formally compare the RSSV-1 specification with the RSSV-COL and RSSV-2 models, we use the LR test. The LR test statistics are insignificant for all four countries in the case of RSSV-1 model

Table 3
Parameter estimates of the RSSV-COL model

Parameter	U.S.	Canada	Japan	UK
p	0.9985	0.9953	0.9974	0.9947
	(0.0012)	(0.0048)	(0.0031)	(0.0035)
q	0.9977	0.9960	0.9958	0.9917
	(0.0019)	(0.0035)	(0.0038)	(0.0061)
ϕ	0.9524	0.7595	0.6426	0.6333
	(0.0158)	(0.0847)	(0.0630)	(0.1254)
μ_0	−0.2725	−1.4584	−2.1519	−2.1388
	(0.0945)	(0.5536)	(0.3929)	(0.8164)
μ_1	−0.36145	−1.7304	−2.8024	−2.7975
	(0.1222)	(0.6642)	(0.5124)	(1.0467)
σ_η	0.2944	0.6466	1.4290	0.8079
	(0.0541)	(0.1425)	(0.1302)	(0.1588)
γ	0.6287	0.7432	0.8397	0.6508
	(0.1312)	(0.2118)	(0.0721)	(0.1608)
ρ	−0.1683	0.0172	0.1171	0.0452
	(0.0750)	(0.0916)	(0.0776)	(0.0791)
Log-likelihood	−5680.28	−2516.72	−1906.36	−3153.28

This table reports the parameter estimates of the regime switching stochastic volatility model with correlation (RSSV-COL) consisting of Eqs. (6) and (13) using short-term interest rates from U.S., UK, Japan, and Canada. Standard errors are reported in parentheses.

Table 4
Parameter estimates of the RSSV-2 model

Parameter	U.S.	Canada	Japan	UK
p	0.9985	0.9953	0.9976	0.9940
	(0.0012)	(0.0048)	(0.0028)	(0.0037)
q	0.9975	0.9960	0.9959	0.9897
	(0.0021)	(0.0035)	(0.0036)	(0.0076)
ϕ_0	0.9581	0.74302	0.5797	0.6253
	(0.0204)	(0.0969)	(0.0815)	(0.1143)
ϕ_1	0.9601	0.7752	0.6313	0.5092
	(0.0195)	(0.0928)	(0.0719)	(0.2764)
μ_0	−0.2106	−1.3866	−2.5369	−2.2157
	(0.1119)	(0.5867)	(0.4970)	(0.7204)
μ_1	−0.2980	−1.882	−2.8945	−3.8143
	(0.1534)	(0.7952)	(0.5805)	(2.1577)
σ_η	0.2752	0.64559	1.4680	0.8426
	(0.0633)	(0.1424)	(0.1266)	(0.1491)
γ	0.4833	0.77493	0.8412	0.6646
	(0.1475)	(0.2419)	(0.0661)	(0.1577)
Log-likelihood	−5682.70	−2516.67	−1907.38	−3153.31

This table reports the parameter estimates of the regime switching stochastic volatility model (RSSV-2) consisting of Eqs. (6) and (14) using short-term interest rates from U.S., UK, Japan, and Canada. Standard errors are reported in parentheses.

vs. RSSV-2 model. The p-values are 0.89 (U.S.), 0.71 (Canada), 0.58 (Japan), and 0.60 (U.K.). In the case of RSSV-1 model vs. RSSV-COL model, the p-values are as follows: 0.03 (U.S.), 0.84 (Canada), 0.13 (Japan), and 0.56 (U.K.). This suggests that the RSSV-1 model is a parsimonious specification.

Finally, we also estimate the time-varying transition probability version of the RSSV-1 model using Eq. (15). It turns out that the results are very similar to the constant transition probability model. For example, in the case of the U.S. short-rate data, the parameter estimates for a_0, a_1, b_0, and b_1 are 6.09, 5.74, 0.07 and 0.05 respectively. For a_j values close to 6, the implied transition probability is about 0.9975, which is very close to what we get from the constant transition probability model. In fact, a likelihood ratio test can not reject the null of a constant transition probability model and the p-value is as high as 0.96. In addition, the regime probabilities are almost identical to those in the constant transition probability model. Hence, modeling time-varying probability does not seem to have an additional advantage at least for our sample.

3.3. Robustness tests

We check the robustness of our results by looking at different drift specifications and monthly data frequency.

3.3.1. Nonlinear drift

Aït-Sahalia (1996) proposes the following drift function for short-rate dynamics: $\mu(r,\theta)=\alpha_0+\alpha_1 r+\alpha_2 r^2+\alpha_3/r$. He argues that the nonlinearity of the drift effectively makes the interest rate process stationary. Conley et al. (1997) also consider the same parameterization of the drift as in Aït-Sahalia (1996). Nonlinearities in the drift are shown to be important for very high-variance elasticities (greater than 4) but not for low ones. Similar results are also reported in Stanton (1997) and Ahn and Gao (1999).

The apparent success of nonlinear drift models, however, is inconclusive. Pritsker (1998) questions the specification test developed in Aït-Sahalia (1996). The argument is that interest rates are known to be highly correlated whereas the nonparametric technique used in Aït-Sahalia's paper is very sensitive to the dependence in the data. Chapman and Pearson (2000) study the finite-sample properties of the nonparametric estimators used in Aït-Sahalia (1996) and Stanton (1997) by applying them to simulated sample paths of a square-root diffusion. Although the drift is linear, the nonparametric estimators suggest nonlinearities of the type and magnitude reported in Aït-Sahalia (1996) and Stanton (1997). Chapman and Pearson conclude that nonlinearity of the short-rate drift is not a robust stylized fact.

To check whether our results are sensitive to the specification of the drift function. We reestimate the RSSV model using the nonlinear drift function. The estimation technique remains unchanged. The only difference is that we run the following regression to get the OLS residuals:

$$\Delta r_t = a + br_{t-1} + cr_{t-1}^2 + dr_{t-1}^{-1} + \varepsilon_t.$$

The results for the RSSV-2 model are reported in Table 5. Despite some slight differences, the overall results continue to support the existence of two volatility regimes and they look similar to the linear drift specification especially in the U.S. case.

3.3.2. ARIMA(1,1,0) specification

Using Box-Jenkins methods, Kalimipalli and Susmel (2004) find that an ARIMA(1,1,0) model provides a satisfactory fit for the U.S. short rates that are highly autocorrelated. As an additional robustness check, we reestimate our RSSV model with this ARIMA(1,1,0) specification for the conditional mean. Namely, we obtain the OLS residuals after running the following regression: $\Delta r_t = a + b\Delta r_{t-1} + \varepsilon_t$. Once again, we find the results are very similar to those reported in the previous tables.[2] The transition probability estimates are still very close to one, indicating two sharply defined volatility regimes. Hence, we conclude our results are robust to difference drift function specifications.

3.3.3. Monthly data

Our results are derived using the weekly short-term interest rate data. As Brenner et al. (1996) point out, the weekly data have the advantage that a discrete time approximation to a continuous time model holds better with higher frequency data. However, the monthly data have the advantage that 30-day bills are closer to the true short rate that the SVOL/RSSV models are designed to analyze. In addition, a number of studies, such as Smith (2002), use monthly U.S. short-rate data. To verify that our results are not sensitive to the change in data frequency, we reestimate our RSSV model using the U.S. 1-month risk-free rates (January 1954 to October 2001) obtained from CRSP. The parameter estimates for the RSSV-2 model are as follows (standard errors in parentheses): p=0.98776 (0.02533),

[2] To conserve space, we have omitted the results. All unreported results are available upon request.

Table 5
Parameter estimates of the RSSV Model: nonlinear drift specification

Parameter	U.S.	Canada	Japan	UK
p	0.9971	0.9911	0.9974	0.9941
	(0.0035)	(0.0062)	(0.0027)	(0.0051)
q	0.9937	0.9949	0.9830	0.9845
	(0.0056)	(0.0033)	(0.0171)	(0.0111)
ϕ_0	0.9922	0.27674	0.68292	0.83044
	(0.0048)	(0.1851)	(0.0737)	(0.0805)
ϕ_1	0.92008	0.3512	−0.06565	0.34737
	(0.0580)	(0.1308)	(0.3419)	(0.4209)
μ_0	−0.0410	−4.8115	−2.1973	−1.2708
	(0.0272)	(1.2690)	(0.5001)	(0.6077)
μ_1	−0.4016	−5.436	−2.6767	−6.0287
	(0.2941)	(1.1985)	(0.9285)	(3.8650)
σ_η	0.2094	1.324	1.0818	0.5733
	(0.0348)	(0.1249)	(0.1534)	(0.1432)
γ	0.2232	0.99802	0.8120	0.9827
	(0.1225)	(0.1356)	(0.0797)	(0.1677)
Log-likelihood	−5658.82	−2618.42	−1856.62	−3109.44

This table reports the parameter estimates of the regime switching stochastic volatility model (RSSV-2) consisting of Eqs. (6) and (14) under the nonlinear drift specification $\mu=\alpha_0+\alpha_1 r+\alpha_2 r^2+\alpha_3/r$. Standard errors are reported in parentheses.

q=0.97999 (0.02718), ϕ_0=0.93042 (0.06230), ϕ_1=0.72850 (0.45166), μ_0=−0.24331 (0.20143), μ_1=−1.2185 (2.05819), σ_η=0.39794 (0.15504), and γ=0.68079 (0.21339). These results are comparable to those obtained using the weekly data.

3.3.4. Model comparison

To compare the various model specifications, we first consider the two most commonly used information criteria, AIC and SBC: $AIC=-2(l/T)+2(k/T)$ and $SBC=-2(l/T)+k\log(T)/T$. Interestingly, we find both criteria favor the RSSV models. Among the RSSV models, the more parsimonious RSSV-1 specification is picked as the best model by both AIC and SBC in three out of four countries. The only exception is Japan, where RSSV-2 is chosen as the best model.

We also compare the in-sample one-step-ahead forecasting performance of various models for conditional volatility. Three metrics are used here[3]: mean-squared error (MSE), mean absolute error (MAE), and R^2. RSSV models seem to dominate the SVOL model in U.S. and U.K., whereas the contrary is true for Canada and Japan. RSSV-2 seems to work best for U.S. data, while RSSV-COL model has the best in-sample forecasting performance for U.K. data.

We also consider out-of-sample forecasting performance using the three metrics. To achieve this goal, we use 2/3 of our sample to reestimate the various models and use the remaining 1/3 observations as our holdout sample to test forecasting performance. The results are consistent with the in-sample results.

[3] Note that heteroskedasticity adjusted MSE and MAE statistics can also be used.

4. Concluding remarks

In this paper, we investigate the possibility of regime shifts in short-rate volatility.

First, we find strong evidence in support of the existence of regime shifts in short-rate volatility in four countries, particularly in the U.S. and the U.K., based on both Vuong's nonnested LR test, standard model selection criteria, and both in-sample and out-of-sample forecasting performance metrics.

Second, we find parameter estimates from the SVOL model imply highly persistent short-rate volatility for all the four countries under consideration. In contrast, we estimate the RSSV model using the same data set and find that the previously found persistence in volatility falls dramatically for the U.K., Canada, and Japan data. In the case of the U.S. short-rate volatility, the volatility is still highly persistent. The evidence presented here highlights the importance of accounting for possible structural breaks in the volatility process.

Third, we propose several extensions of the RSSV model. In particular, we show how to account for correlation in the RSSV model and how to estimate a time-varying transition probability RSSV model. In our data, we find some evidence of negative correlation for the U.S. short-rate volatility, but essentially zero-correlation for other countries. The empirical evidence also favors the more parsimonious constant transition probability model.

The volatility regimes identified in this paper are very distinct and seem to last for a fairly long period of time. In the U.S. and the U.K. samples, we find that the high (low) volatility regime is associated with higher (lower) inflation rate and lower (higher) real GDP growth rate, which appears consistent with the notion that the regimes are related to macroeconomic shocks.

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