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Diversification benefits of emerging markets subject to portfolio constraints

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Abstract

This paper examines the international diversification benefits subject to portfolio constraints—in particular, constraints on short selling. We show that the international diversification benefits remain substantial for U.S. equity investors when they are prohibited from short selling in emerging markets. This result is robust to investment restrictions on nonnative individuals. It is also unaffected by the fact that the U.S. equity index portfolio is not on the efficient frontier spanned by U.S. securities. The integration of world equity markets reduces, but does not eliminate, the diversification benefits of investing in emerging markets subject to short-sale constraints.

JEL classification: G11, Portfolio choice; G15, International financial markets; C11, Bayesian analysis; C15, Statistical simulation methods
Keywords: International diversification; Short-sale constraints; Home bias; Asset allocation; Bayesian inference

1. Introduction

An important issue in international economics concerns the size of benefits from diversifying over securities in foreign countries, especially securities in emerging markets. In theory, if foreign securities do not perfectly correlate with U.S. securities, domestic investors gain from international diversification. However, the magnitude of the diversification benefits in general depends on various portfolio constraints, such as investors' ability to take short positions. Given the existence of derivative securities on stock market indices in developed countries, it is often feasible for institutional investors to take short positions on

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developed markets. Investors nonetheless face short-sale constraints in many emerging markets. In this paper, we study the impact of short-sale constraints on the existence and magnitude of international diversification benefits to U.S. equity investors. The existence of substantial diversification benefits of investing in emerging markets subject to short-sale constraints will underscore the importance of international diversification.

Short-sale constraints have gained increasing attention in recent finance literature. Sharpe (1991) conjectures that departures from the CAPM might be small even in the extreme case where negative holdings are excluded. He postulates that institutional arrangements to improve investors' abilities to take negative positions facilitate the efficient allocation of risk in the economy. Hansen et al. (1994), He and Modest (1995), and Luttmer (1996) study how short-sale constraints and transactions costs affect consumption-based asset pricing models. For portfolio efficiency subject to short-sale constraints, Wang (1998) conducts Bayesian inference, Basak et al. (in press) develop an asymptotic test, and De Roon et al. (2001) carry out regression-based tests for mean–variance spanning.

Ignoring short-sale constraints, many studies have documented low correlation across international markets and substantial diversification benefits. The early literature of Grubel (1968), Levy and Sarnat (1970), and Lessard (1973) finds low correlation among equity returns in industrial countries and concludes that the gain from international diversification is substantial. Harvey (1995) shows that securities in emerging markets promise U.S. investors both high expected returns and risk, as well as low correlation with securities in developed markets. Bekaert and Urias (1996) reject the hypothesis that equity indices in industrial countries span the mean–variance frontier of all international equity indices and thus demonstrate the existence of diversification benefits in emerging markets. Using the international CAPM, De Santis and Gerard (1997) estimate that the expected gain from international diversification to a U.S. investor is on average 2.11% annually. Errunza et al. (1999) further show that the international diversification benefits can be obtained from investment in country funds and American Depository Receipts traded in U.S.

It remains unclear whether there exist substantial benefits from diversifying over emerging equity markets after imposing short-sale constraints. In the asset management industry, where short-sale constraints are routinely imposed in optimal portfolio choice problems, practitioners believe that the diversification benefits in emerging equity markets are substantial but, to our knowledge, there has not been any formal econometric inference. Glen and Jorion (1993) empirically show the existence of the benefits in currency hedging subject to short-sale constraints. Using mean–variance spanning tests, De Roon et al. (2001) argue that the evidence of diversification benefits in emerging markets disappears after imposing short-sale constraints. However, their statistical tests show strong evidence of the diversification benefits when investing in some individual Latin American or Asian countries, but no evidence of the benefits when investing optimally in the combination of these emerging markets. It seems odd to rule out diversification benefits in emerging markets when there are clear benefits derived from particular emerging markets. De Roon et al. (2001) explain that the counterintuitive results are driven by the loss of power in the asymptotic mean–variance spanning tests when more emerging markets are included.

The issue investigated in this paper is closely related to the home bias puzzle in finance. It is observed that U.S. investors tend to hold a substantially larger portion of their invested equities in domestic stocks than what is suggested by the diversified world market portfolio or an optimal portfolio that maximizes a commonly used utility function (e.g., French and Poterba, 1991)[1]. One explanation offered in the literature is that the diversification benefits do not exist in practice because of the difficulty of taking short positions in many non-U.S. equity markets. Thus, it is important to understand whether investing primarily at home is optimal when market frictions such as short-sale constraints are taken into account.

In this paper, we use Bayesian inference to examine the impact of short-sale constraints on the international diversification benefits to U.S. investors. Unlike the asymptotic mean–variance spanning tests, our Bayesian approach incorporates the uncertainty of finite samples into the posterior distributions of the diversification benefits. Two different measures of the diversification benefits are employed. The first one follows the work of Kandel et al. (1995) and Wang (1998) on portfolio efficiency. With equal variance, the expected return on the U.S. equity index portfolio is either smaller than or equal to the expected return on the internationally efficient portfolio. We use the difference between the expected returns on the two portfolios to measure the magnitude of the international diversification benefits. The advantage of this measure is that it tells us how far the U.S. equity index portfolio is away from the efficient frontier and has an intuitive interpretation as the gain in expected returns through diversification.

It is often argued that the main benefit of international diversification is the reduction in variance rather than the increase in returns. For example, a columnist in *The Wall Street Journal* wrote:[2] “The main reason to invest abroad isn’t to replicate the global market or to boost returns. Instead, what we’re trying to do by adding foreign stocks is to reduce volatility”. Elton and Gruber (1995; chapter 12) argue that, since there is no evidence to support an international CAPM, risk-averse investors with no ability to forecast expected returns might seek to minimize the variance of their portfolio. To capture this aspect of the diversification benefits, our second measure of the diversification benefits is the reduction in the standard deviation when investors switch from the U.S. equity index portfolio to the global minimum-variance portfolio. Given that expected returns are difficult to estimate, an advantage of this second measure is that it does not depend on expected returns. Hence, the estimated weights in the global minimum-variance portfolio may be more accurate and relatively stable over different sample periods. The global minimum-variance portfolio has been used by other researchers (e.g., Stambaugh, 1997). One way to think of it is that the global minimum-variance portfolio is efficient if expected returns are assumed to be equal across markets.

Using either of the two measures, we find that the diversification benefits of emerging equity markets remain substantial after imposing short-sale constraints in these markets. The result holds when we limit our analysis to investable stocks, i.e., stocks that are available to nonnative investors and meet minimum size and liquidity criteria. The result is also unaffected by the fact that the U.S. equity index portfolio is not on the efficient frontier spanned by U.S. securities. The integration of world equity markets reduces, but

[1] For a recent review of the home bias puzzle, we refer readers to Lewis (1999, 2000).

[2] Jonathan Clements: “Getting Going: International Investing Raises Questions on Allocation, Diversification, Hedging”, *The Wall Street Journal*, July 29, 1997.

does not eliminate, the diversification benefits of investing in emerging markets subject to short-sale constraints. The results produced by our Bayesian approach are not only robust but also sensible. Unlike the asymptotic mean–variance spanning test, our inference produces sensible results—adding more emerging markets always increases the diversification benefits. An important implication of our results is that the introduction of market frictions such as short-sale constraints does not resolve the home bias puzzle but instead reinforces it, at least for the emerging markets.

The rest of the paper proceeds as follows. In Section 2, we discuss measures of the international diversification benefits and the computation of the posterior distributions. In Section 3, we describe the data used in this study. In Section 4, we present our main empirical results. In Section 5, we examine how our result is affected by the integration of global equity markets over time and by the restrictions on nonnative investors. We also study if our result is due to the fact that the U.S. equity index portfolio is not on the efficient frontier of U.S. securities. In Section 6, we discuss some related issues along with our conclusion.

2. Measuring the diversification benefits

We measure the international diversification benefits in the mean–variance framework. Let $R=(r_1, \ldots, r_n)'$ be the vector of random returns of n assets. We assume that the vector of returns has a multivariate normal distribution, with mean $\mu=(\mu_1, \ldots, \mu_n)'$ and covariance matrix $\mathbf{\Omega}$. Let S be the set of all the real vectors $x=(x_1, \ldots, x_n)'$ such that $x_1+\ldots+x_n=1$. The vector of weights in a portfolio is a point in S. A set of constraints on portfolio weights is represented by a closed convex subset C of S. For example, the case in which portfolio holdings are unconstrained can be represented by $C=S$, and the case in which short sales are prohibited can be represented by $C=\{x\in S\colon x_i \geq 0,\ i=1, \ldots, n\}$.

Suppose x_{m} ($\in C$) is a given benchmark portfolio. Since our focus is on the international diversification benefits to U.S. investors, the benchmark in this paper is always the index portfolio of U.S. equity market. The expected return of the benchmark portfolio is $x_{\mathrm{m}}'\mu$, which will be smaller than the expected return of an efficient portfolio with the same variance, $x_{\mathrm{m}}'\mathbf{\Omega}x_{\mathrm{m}}$, unless portfolio x_{m} is mean–variance efficient. The benefits of diversifying from portfolio x_m to an efficient portfolio of all the n assets, controlling for variance, can be measured by

$$\delta(x_{\mathrm{m}}, C, \mu, \Omega) \equiv \max_x \{x'\mu - x_{\mathrm{m}}'\mu \mid x\in C,\ x'\Omega x \leq x_{\mathrm{m}}'\Omega x_{\mathrm{m}}\}. \tag{1}$$

The benchmark portfolio x_{m} is mean–variance efficient if and only if $\delta=0$.

The measure δ defined in Eq. (1) is the same measure of portfolio inefficiency used by Wang (1998), which is a straightforward extension of the measure Δ defined by Kandel et al. (1995). When portfolio weights are unconstrained, i.e., $C=S$, the measure δ defined here is equivalent to Kandel et al.'s Δ. The measure δ has a simple intuitive appeal: it is the gain in expected returns obtained by diversifying from the benchmark portfolio to an efficient portfolio with the same variance. In this paper, we report δ in annualized percentage points.

Since it is often argued that the main benefit of international diversification is the reduction of variance rather than the increase in returns, we also introduce a measure of the international diversification benefits in terms of variance reduction. The measure, which we call ϕ, is the distance in terms of the standard deviation of the benchmark portfolio from the efficient frontier:

$$\phi(x_{\mathrm{m}}, C, \mu, \Omega) \equiv \min_{x}\left\{1 - \sqrt{(x'\Omega x)/(x_{\mathrm{m}}'\Omega x_{\mathrm{m}})} \mid x \in C,\ x'\mu \geq x_{\mathrm{m}}'\mu\right\}. \tag{2}$$

In the presence of a risk-free asset, $1-\phi$ is the maximum correlation between a given benchmark portfolio and an efficient portfolio, as shown by Kandel and Stambaugh (1987) and Shanken (1987). When there are constraints on portfolio weights, $1-\phi$ is no longer the maximum correlation. Nevertheless, ϕ is the proportional reduction in volatility for an investor who switches to a diversified international portfolio from the U.S. equity index portfolio x_{m}. Clearly, ϕ is zero if and only if δ is zero. In this sense, it is an equivalent measure to δ. Moreover, the results using ϕ are qualitatively similar to those using δ. Therefore, in this paper, we do not report results for ϕ.

It is well known that expected returns are more difficult to estimate than variance. Elton and Gruber (1995; chapter 12) argue that risk-averse investors with no ability to forecast expected returns might seek to minimize the variance of their portfolio. Such investors do not think the return on any asset is higher than the returns on the others and consequently they assume that all the risky assets have the same expected returns. In this case, our measure of the diversification benefits, ϕ, is independent of expected returns and becomes the measure we introduce as follows,

$$\psi(x_{\mathrm{m}}, C, \Omega) \equiv \min_{x}\left\{1 - \sqrt{(x'\Omega x)/(x_{\mathrm{m}}'\Omega x_{\mathrm{m}})} \mid x \in C\right\}. \tag{3}$$

The measure ψ indicates the reduction in volatility when U.S. investors switch from the U.S. equity index portfolio to the global minimum-variance portfolio. The weights in the global minimum-variance portfolio solve the problem of $\min_x x'\boldsymbol{\Omega} x$ subject to $x \in C$.

The main purpose of this paper is to draw inferences from the observed financial data. We therefore assume that little is known, a priori, about the expectation and the covariance matrix of asset returns. Therefore, the prior probability density function is assumed to be

$$p(\mu, \Omega) = p(\mu) \cdot p(\Omega), \quad p(\mu) \propto \text{constant}, \quad p(\Omega) \propto |\Omega|^{-(n+1)/2}. \tag{4}$$

Suppose that there are T independent observations on the vector of returns. Denote the tth observation on the vector of returns by R_t and the matrix of all observed returns by $R = (R_1, \ldots, R_T)'$. Let $\hat{\mu}$ and $\hat{\boldsymbol{\Omega}}$ be the sample mean and covariance matrix of asset returns, respectively. It can be shown that the posterior probability density function is

$$p(\mu, \Omega \mid R) = p(\mu \mid \Omega, \hat{\mu}, T) \cdot p(\Omega \mid \hat{\Omega}, T). \tag{5}$$

The marginal posterior distribution $p(\mathbf{\Omega}|\hat{\mathbf{\Omega}}, T)$ is the inverted Wishart distribution with the scale matrix $T\hat{\mathbf{\Omega}}$ and degrees of freedom $T-1$. The conditional distribution $p(\mu|\mathbf{\Omega}, \hat{\mu}, T)$ is the multivariate normal distribution with mean μ and covariance matrix $(1/T)\mathbf{\Omega}$.[3]

Since the two measures of the diversification benefits, δ and ψ, are nonlinear functions of μ and $\mathbf{\Omega}$, we do not have analytical expressions for their posterior distributions. However, the Monte Carlo method can be used to estimate these posterior distributions. A random sample $\mathbf{\Omega}$ is drawn from the inverted Wishart distribution with parameter matrix $T\hat{\mathbf{\Omega}}$ and degrees of freedom $T-1$. Then, a random sample μ is drawn from the multivariate normal distribution with mean $\hat{\mu}$ and covariance matrix $(1/T)\mathbf{\Omega}$. The pair $(\mu, \mathbf{\Omega})$ is thus a random sample from the joint posterior distribution expressed in Eq. (5). Given μ and $\mathbf{\Omega}$, δ is obtained by solving the constrained optimization problem in Eq. (1), and ψ is obtained by variance minimization, which provides samples from the posterior distributions $p(\delta \mid R)$ and $p(\psi \mid R)$, respectively. By repeating this process, a large number (e.g., 10,000) of independent draws of δ and ψ are obtained from their respective posterior distributions. These samples are used to form the approximated posterior distributions of δ and ψ, as described by Geweke (1989).[4]

An issue with our two measures of diversification benefits, δ and ψ, is that the diffuse prior specified in Eq. (4) about the expectation and the covariance matrix of stock returns might correspond to strong prior beliefs about δ and ψ, which are nonlinear functions of the return distribution moments. This issue has been examined by both Kandel et al. (1995) and Wang (1998). To examine the implications of non-informative priors about μ and $\mathbf{\Omega}$ for the priors about δ, they compute the posterior distributions of the latter conditional on a small sample (i.e., a small T). Since a small sample provides little information, the posterior distributions corresponding to such a sample should reflect mostly prior information. Kandel et al. (1995) find that the prior distribution in Eq. (4) implies rather non-informative priors about δ, and Wang (1998) shows that this is true when portfolios are constrained. Our simulation (not reported) indicates that the implied prior about ψ is also diffuse when the number of return observations is small.

The combination of Bayesian inference and Monte Carlo simulation has several advantages. First, it is easy to implement. Once the samples of μ and $\mathbf{\Omega}$ are drawn, the calculation of the measures of the diversification benefits is straightforward. It is much easier to conduct Bayesian inference than the classical inference as carried out by De Roon et al. (2001) and Basak et al. (in press). Second, it allows us to examine δ and ψ subject to short-sale constraints. Although Basak et al. (in press) have developed the asymptotic distribution theory for the variance reduction by an efficient portfolio under short-sale constraints, the sampling distributions of δ and ψ based on the sample moments remain unknown. Finally, it allows us to draw exact inference on the magnitude of the benefits, besides testing the null hypothesis of no diversification benefits.

As a by-product of Monte Carlo simulation, we also obtain a set of independent draws from the posterior distribution of efficient portfolio weights. In classical inference,

[3] See, for example, Zellner (1971, pp. 224–227).

[4] The estimated posterior distribution appears to converge to the target distribution immediately, and produces virtually identical results when re-run with a different seed in the random number generator.

obtaining the sampling distribution of efficient portfolio weights based on the sample moments is a nontrivial task. It is only recently that Britten-Jones (1999) has established the sampling distribution theory for the case without portfolio constraints. The sampling distribution of efficient portfolio weights subject to short-sale constraints is still unknown. In Bayesian inference, we can easily use those independent samples of portfolio weights to estimate the posterior distribution of efficient portfolio weights.

3. Data

We use dollar-denominated monthly total returns on stock indices supplied by Morgan Stanley Capital International (MSCI) for the G7 group of developed countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States). We also use dollar-denominated monthly total returns on stock indices of eight emerging market countries: four Latin American markets (Argentina, Brazil, Chile, and Mexico) and four Asian markets (Hong Kong, South Korea, Singapore, and Thailand). Data for Hong Kong and Singapore are also obtained from MSCI. Data for other emerging markets are obtained from the International Finance (IFC). We focus on the period from January 1976 to December 1999 because the IFC indices of the above emerging markets start from January 1976. Table 1 lists the countries, their market capitalization, and market share as of December 1999. Table 2 reports the basic statistics. Similar statistics are provided by Harvey (1995), Bekaert and Harvey (1995), and Bekaert et al. (1998).

The means and standard deviations of returns in Table 2 do not provide a clear indication as to whether emerging markets offer diversification benefits to U.S. investors

Table 1
Countries and their market capitalization

Country	Symbol	Market cap	Weight
United States	USA	16,635.114	0.5497
Canada	CAN	800.914	0.0265
Japan	JAP	4546.937	0.1503
France	FRA	1475.457	0.0488
Germany	GER	1432.190	0.0473
Italy	ITA	728.723	0.0241
United Kingdom	UK	2933.280	0.0969
Argentina	ARG	83.887	0.0028
Brazil	BRA	227.962	0.0075
Chile	CHI	68.228	0.0023
Mexico	MEX	154.044	0.0051
Korea	KOR	308.534	0.0102
Thailand	THA	58.365	0.0019
Hong Kong	HK	609.090	0.0201
Singapore	SIN	198.407	0.0066

This table provides information on the countries whose equity indices are examined in the paper. The third column gives the stock market capitalization in billions of U.S. dollars of these countries as of 1999. The fourth column gives the weight of each country as a fraction of the total capitalization. The data on market capitalization are obtained from I/B/E/S.

Table 2
Basic statistics of the data

Country	Mean	Sdev	Country	Mean	Sdev
USA	16.34	14.64	ARG	53.11	89.50
CAN	13.27	19.24	BRA	25.25	56.21
JAP	15.76	23.25	CHI	29.55	36.50
FRA	17.39	22.88	MEX	24.96	42.68
GER	15.63	20.43	KOR	19.34	38.58
ITA	15.61	26.79	THA	16.24	35.04
UK	17.78	20.24	HK	23.08	33.15
			SIN	16.13	26.99

Correlations

	USA	CAN	JAP	FRA	GER	ITA	UK
CAN	0.71						
JAP	0.25	0.29					
FRA	0.44	0.43	0.42				
GER	0.37	0.33	0.33	0.61			
ITA	0.24	0.30	0.36	0.45	0.38		
UK	0.50	0.53	0.39	0.53	0.45	0.36	
	USA	CAN	JAP	FRA	GER	ITA	UK
ARG	0.05	0.14	0.00	0.06	0.02	0.09	− 0.03
BRA	0.13	0.09	0.10	0.09	0.12	0.14	0.13
CHI	0.09	0.16	0.08	0.08	0.10	0.09	0.07
MEX	0.32	0.27	0.14	0.17	0.15	0.12	0.23
KOR	0.19	0.19	0.33	0.08	0.08	0.14	0.17
THA	0.25	0.25	0.19	0.17	0.26	0.14	0.20
HK	0.37	0.41	0.27	0.28	0.33	0.25	0.42
SIN	0.48	0.47	0.32	0.26	0.28	0.20	0.42
	ARG	BRA	CHI	MEX	KOR	THA	HK
BRA	0.02						
CHI	0.13	0.10					
MEX	0.19	0.09	0.21				
KOR	− 0.05	0.05	0.10	0.13			
THA	0.06	0.07	0.19	0.26	0.32		
HK	0.04	0.17	0.19	0.25	0.16	0.39	
SIN	0.07	0.14	0.20	0.34	0.21	0.52	0.61

This table gives the sample means, standard deviations, and correlations of the dollar-denominated monthly returns on the equity indices. The mean is reported in percentage and annualized by multiplying 12. The standard deviation (Sdev) is also reported in percentage and annualized by multiplying $\sqrt{12}$. Dollar-denominated monthly returns on the equity indices are obtained from MSCI and IFC. The sample period covers January 1976 through December 1999.

beyond those offered by G7 countries. During the sample period, emerging markets generally have both higher means and higher standard deviations relative to G7 countries. The exceptions are Thailand and Singapore, whose mean returns are comparable to those of G7 countries but standard deviations are higher.

In contrast, the correlation matrix in Table 2 provides strong evidence of the benefits of diversifying over emerging markets, in addition to G7 countries. Most emerging market countries have low correlation, both among themselves as well as with G7 countries. The low correlation suggests that investors may benefit from long positions in emerging markets, although the relatively high standard deviation of emerging markets could mitigate the benefits of international diversification. In contrast to emerging markets, G7 countries have relatively high correlation among themselves. In particular, the correlation between U.S. and Canadian stock returns is over 0.7.

4. Main empirical results

We examine the international diversification benefits during the period from January 1976 to December 1999. The diversification benefit, δ, is measured as the increase in the expected annualized return when switching from the U.S. equity index portfolio to the efficient portfolio with the same variance. The diversification benefit, ψ, is measured as the decrease in the standard deviation, as a percentage of the standard deviation of the U.S. equity index portfolio, when switching from the U.S. equity index portfolio to the global minimum-variance portfolio. We consider the benefits of diversifying over several combinations of countries. For each combination of countries, we look at the benefits both with portfolio weights unconstrained and with portfolio weights constrained to be nonnegative.

4.1. Gain in expected returns

Let us first examine the measure δ. Table 3 provides the basic statistics of its posterior distribution. To be precise, these numbers are characteristics of the approximated posterior distribution of δ, based on Monte Carlo samples from its true distribution.[5] When portfolio weights are unconstrained, the benefits of diversifying over G7 and emerging markets are large, while the benefits of diversifying over only G7 countries are relatively small. As shown in the first panel of Table 3, the benefit of diversifying over G7 countries is at least 0.6% with a posterior probability of 0.99, while the benefit of diversifying over G7 and Latin American countries is at least 3.48% with a posterior probability of 0.99. The benefit of diversifying over G7 and Asian countries is at least 1.56% with a posterior probability of 0.99. The benefit of diversifying over all countries is at least 4.61% with a posterior probability of 0.99. The magnitude of these diversification benefits is probably big enough to justify the costs of investing in various emerging markets. On the other hand, the short positions required to achieve these diversification benefits may not be easy to implement.

As expected, the short-sale constraints shift the location of the posterior distribution of δ towards zero. By comparing the first and the second panels of Table 3, it can be seen that

[5] Since the estimated posterior distribution provides a good approximation as we pointed out in footnote 3, we simply refer to the approximated posterior distribution as the posterior distribution in all subsequent discussions.

Table 3
Posterior distributions of δ and ψ

Efficient frontier	δ					ψ				
	Mean	Sdev	1%	5%	Med	Mean	Sdev	1%	5%	Med
Unconstrained										
G	3.71	1.84	0.60	1.13	3.49	11.81	2.40	6.72	8.02	11.70
GL	8.62	2.59	3.48	4.67	8.45	15.36	2.66	9.55	11.10	15.29
GA	5.30	2.01	1.56	2.34	5.09	13.30	2.52	7.89	9.29	13.21
GLA	10.03	2.69	4.61	5.91	9.86	16.85	2.68	11.06	12.47	16.78
No short-sales on any markets										
G	2.26	1.84	0.00	0.00	1.95	11.15	2.38	6.18	7.45	11.04
GL	6.96	2.55	1.87	3.12	6.76	14.26	2.62	8.52	10.08	14.19
GA	3.40	1.97	0.00	0.56	3.16	11.88	2.46	6.67	8.00	11.78
GLA	7.53	2.60	2.28	3.54	7.39	14.72	2.62	9.05	10.54	14.68
No short-sales on emerging markets										
GL	8.47	2.59	3.32	4.56	8.30	15.03	2.64	9.25	10.83	14.96
GA	4.76	1.98	1.16	1.89	4.53	12.49	2.48	7.19	8.59	12.39
GLA	9.13	2.65	3.78	5.02	8.97	15.44	2.64	9.74	11.20	15.39

The posterior distributions of δ and ψ are calculated for various efficient frontiers. The first frontier is spanned by the equity indices of G7 countries (indicated by G), the second is spanned by the equity indices of G7 and four Latin American markets (GL), the third is by G7 and four Asian markets (GA), and the last is by all the 15 markets (GLA). In the calculation of posterior distributions of δ and ψ, portfolio weights are assumed to be either unconstrained or constrained to be nonnegative for all markets or only for emerging markets. The mean, standard deviation, first percentile, fifth percentile, and median of the posterior distributions are reported.

with all portfolio weights constrained to be nonnegative, the first percentile of the benefit of diversifying over G7 and Latin American countries drops from 3.48% to 1.87% in annualized returns, while the first percentile of the benefit of diversifying over G7 and Asian countries is zero. When constraints are imposed on all countries, the first percentile of the benefit of diversifying over all countries drops to 2.28% from 4.61%, while the posterior mean reduces to 7.53% from 10.03%. The short-sale constraints move the entire posterior distribution of δ towards zero by 1.5–2.5% in annualized returns.

Note that G7 countries do not offer clear diversification benefits before or after imposing short-sale constraints. When portfolios are unconstrained, as shown in the first panel of Table 3, the first and fifth percentiles of δ associated with G7 countries are 0.6 and 1.13 respectively, which are small relative to the percentiles of δ when emerging markets are added. With short-sale constraints imposed on G7 countries, as shown in the second panel of Table 3, the first and fifth percentiles of δ associated with G7 countries are both zero. That is, we cannot distinguish the diversification benefits of developed markets from zero with a posterior probability of 0.99 or 0.95. This implies that the observed international diversification benefits must come from emerging markets when short sales are not allowed in developed markets. These results highlight the importance of emerging markets for international diversification.

To gain a better understanding of the effect of short-sale constraints on the international diversification benefits, we report in Table 4 the posterior means and standard deviations

Table 4
Portfolio weights

Market	Efficient portfolio weights						GMV portfolio weights					
	UC		NS		EM		UC		NS		EM	
	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev
USA	0.70	0.17	0.36	0.13	0.65	0.17	0.63	0.07	0.54	0.05	0.60	0.07
CAN	−0.28	0.14	0.00	0.01	−0.31	0.14	−0.05	0.06	0.00	0.01	−0.07	0.06
JAP	0.10	0.09	0.06	0.07	0.08	0.09	0.13	0.04	0.11	0.04	0.12	0.04
FRA	−0.01	0.11	0.04	0.06	0.01	0.12	−0.06	0.05	0.00	0.01	−0.05	0.05
GER	0.08	0.12	0.07	0.09	0.07	0.12	0.15	0.05	0.13	0.04	0.15	0.05
ITA	0.02	0.08	0.03	0.05	0.02	0.08	0.06	0.03	0.04	0.03	0.06	0.03
UK	0.17	0.12	0.10	0.10	0.15	0.13	0.09	0.05	0.04	0.04	0.06	0.05
ARG	0.05	0.02	0.05	0.02	0.05	0.02	0.01	0.01	0.01	0.01	0.01	0.01
BRA	0.03	0.03	0.04	0.03	0.03	0.03	0.01	0.01	0.01	0.01	0.01	0.01
CHI	0.15	0.05	0.14	0.06	0.14	0.05	0.08	0.02	0.07	0.02	0.07	0.02
MEX	0.00	0.05	0.02	0.03	0.02	0.03	−0.02	0.02	0.00	0.00	0.00	0.00
KOR	0.05	0.05	0.05	0.05	0.05	0.04	0.02	0.02	0.02	0.02	0.02	0.02
THA	0.00	0.06	0.01	0.03	0.01	0.02	0.03	0.03	0.01	0.01	0.01	0.01
HK	0.05	0.07	0.03	0.04	0.03	0.04	−0.04	0.03	0.00	0.00	0.00	0.00
SIN	−0.11	0.10	0.00	0.02	0.00	0.02	−0.05	0.04	0.00	0.00	0.00	0.00

This table provides the posterior means and standard deviations of the weights in the efficient portfolio with the same variance as the U.S. index and in the global minimum-variance (GMV) portfolio of the 15 international markets considered in this paper. The portfolio weights are either unconstrained (indicated by UC), constrained to be nonnegative for all markets (NS), or only for emerging markets (EM).

of the weights in the efficient international portfolio that has the same variance as the U.S. equity index. Considering the mean portfolio weights in the unconstrained case, the only substantial short position in emerging markets is on Singapore at around 11%. In contrast, the short position on Canada is nearly three times as large, around 28%. When short-sale constraints are imposed, the weight on Canada is zero. It seems that the diversification benefits depend largely on taking short positions on developed countries like Canada. The standard deviations of these portfolio weights indicate that the posterior distribution of the weights is very noisy in the unconstrained case. This is consistent with the results reported by Britten-Jones (1999). For example, the optimal weight in U.S. is 0.70 and its two standard deviation range is 0.36–1.04.

The effect of short-sale constraints, for some countries, is to improve the precision of the estimated portfolio weights. The optimal weight in Italy, for example, increases from 0.02 in the unconstrained case to 0.03 when short-sale constraints are imposed on all countries, while its standard deviation decreases from 0.08 to 0.05. For other countries, the standard deviation remains high relative to the estimated portfolio weights, even after imposing constraints. More importantly, the optimal weight in U.S. is 0.36 when short-sale constraints are imposed on all countries and its two standard deviation range is 0.10–0.62.

Interestingly, the largest short position observed in our analysis is on Canada. The short position on Canada is statistically significant, as shown in Table 4, and it persists in all the subsequent analysis in this paper. Given the lower return and higher volatility of Canadian market compared to that of U.S., as well as the high correlation between these two markets, as previously reported in Table 2, the large short position on Canada is very reasonable.

If the diversification benefits mainly depend on shorting some developed countries, short-sale constraints on emerging markets should not have a big impact. This is confirmed by the posterior distributions of δ reported in the last panel of Table 3. Without short sales in emerging markets, the benefits of diversifying over G7 and Latin American countries and over G7 and Asian countries are almost as high as the benefits when portfolios are unconstrained. The benefit of diversifying over all countries is at least 3.78% in annualized returns with a posterior probability of 0.99. Although the benefit is about 0.8% lower than the benefit when portfolios are unconstrained, it is 1.5% higher than the benefit when all countries are constrained. Our results indicate that short-sale constraints on emerging markets have little impact on the diversification benefits to sophisticated institutional investors who can take short positions on developed countries.

In practice, foreign equity holdings among U.S. investors account for less than 10% of their total equity holdings. If U.S. investors cannot short sell any foreign stocks, Table 4 suggests that the optimal allocation to the non-U.S. developed countries is not significantly different from zero. For some emerging markets (e.g., Argentina and Chile), however, optimal weights are significantly different from zero. In contrast to the observed home bias of U.S. investors, the two standard deviation interval of the optimal weight on U.S. $(36 \pm 2(0.13))$ is below 62%. Thus, our results reinforce the home bias puzzle when emerging market countries and short-sale constraints are considered.

All the above results are based on the total equity returns, and the risk-free asset is not considered. We have also introduced the risk-free rate and implemented all our analysis with the excess returns. The risk-free rate is chosen to be the U.S. Treasury Bill returns. We find that the results based on the excess returns are very similar to the results based on the total returns. Thus, we elect not to report these results in the paper.

4.2. Reduction in standard deviations

We now examine ψ, our second measure of the international diversification benefits. According to the posterior distribution of ψ reported in Table 3, emerging markets continue to provide sizable diversification benefits to investors who are subject to short-sale constraints. For example, the first percentile of ψ for the global minimum-variance portfolio when diversifying over G7, Latin American, and Asian markets is over 11% in the unconstrained case and over 9% when all markets are subject to short-sale constraints.

For the measure ψ, the impact of short-sale constraints on all markets is moderate. Comparing the first and the second panels of Table 3, it can be seen that the posterior distribution of ψ shifts towards zero by about 0.5–2%. For example, the first percentile of the benefit of diversifying over G7 and Latin American countries drops from 9.55% to 8.52% in annualized returns, and the first percentile of the benefit of diversifying over G7 and Asian countries drops from 7.89% to 6.67%. The posterior mean of the benefit of diversifying over all countries reduces to 14.72% from 16.85% when constraints are imposed on all countries. By comparing the first and the third panels of Table 3, it can be seen that, when only portfolio weights in emerging markets are constrained to be nonnegative, the reduction in diversification benefits is smaller. The first percentile of the benefit of diversifying over G7 and Latin American countries drops from 9.55% to 9.25% in annualized returns, and the first percentile of the benefit of diversifying over G7 and Asian countries drops from 7.89% to 7.19%. The posterior mean of the benefit of diversifying over all countries reduces to 15.44% from 16.85% when constraints are imposed on emerging market countries.

The moderate impact of short-sale constraints on ψ can be explained by examining the posterior means and standard deviations of the portfolio weights in the global minimum-variance portfolio as reported in Table 4. The global minimum-variance portfolio requires relatively small short positions on a number of developed and emerging markets, rather than large short positions on a few developed countries. Therefore, for investors who only minimize variance, the impact of short-sale constraints on their international investments, and on G7 investments in particular, is relatively moderate. Table 4 also indicates that ceteris paribus, the portfolio weights in the global minimum-variance portfolio, are much more accurately estimated than the weights in the mean–variance efficient portfolio.

4.3. Individual emerging markets

It is important to determine whether the diversification benefits come from particular individual emerging markets or from the combination of all emerging markets. So far, our analysis has combined individual emerging equity markets into regional groups. Now, we compute the benefits of investing in each individual emerging market. In Table 5, we report the means, standard deviations, and percentiles of the posterior distributions of δ and ψ when the efficient portfolio is comprised of G7 and one of the eight emerging market countries.

When portfolios are unconstrained, individual emerging markets provide additional diversification benefits when added to G7 countries. The first percentile of δ for individual emerging markets ranges between 0.7% and 1.8% in annualized returns. When all portfolio weights are constrained to be nonnegative, individual emerging markets, with the exception of Argentina and Chile, offer no benefit when added to G7 countries. In contrast, when short-sale constraints are imposed only on emerging markets, individual emerging markets provide diversification benefits that are similar to the case when portfolio weights are unconstrained. This demonstrates that short-sale constraints on developed countries have a great impact on the diversification benefits of emerging markets.

We get qualitatively similar results from the estimates of ψ for individual emerging countries, as reported in Table 5. Short-sale constraints on all countries shift the distribution of ψ towards zero by 0.5–1%, whereas constraints on individual emerging markets leave the distribution virtually unchanged. These results confirm our earlier observation that the benefits of diversifying over emerging markets remain substantial with short-sale constraints on those markets.

Our results are in sharp contrast to the results reported by De Roon et al. (2001). When short-sale constraints are imposed on emerging markets but not on developed markets, they find that, while there are no benefits of diversifying over a group of Latin American countries, there are substantial benefits of diversifying into either Chile or Colombia. They also find similar results for Asian countries. Clearly, these findings contradict the basic notion of diversification benefits. Subject to short-sale constraints on

Table 5
Individual emerging markets

Frontier of G7 +	δ					ψ				
	Mean	Sdev	1%	5%	Med	Mean	Sdev	1%	5%	Med
Unconstrained										
ARG	5.77	2.19	1.58	2.51	5.55	12.44	2.48	7.11	8.46	12.36
BRA	4.24	1.92	0.83	1.50	4.02	12.13	2.46	6.88	8.22	12.06
CHI	6.43	2.41	1.80	2.85	6.20	14.39	2.60	8.79	10.27	14.30
MEX	4.19	1.89	0.82	1.49	3.96	11.95	2.45	6.67	8.06	11.88
KOR	4.17	1.91	0.79	1.44	3.94	12.23	2.47	7.02	8.32	12.14
THA	4.01	1.87	0.74	1.35	3.77	12.15	2.46	6.88	8.25	12.08
HK	4.13	1.86	0.81	1.47	3.89	12.17	2.47	6.82	8.25	12.09
SIN	3.95	1.84	0.72	1.33	3.72	12.08	2.46	6.80	8.17	11.99
No short-sales on any markets										
ARG	4.39	2.16	0.36	1.17	4.18	11.68	2.44	6.46	7.86	11.62
BRA	2.96	1.96	0.00	0.15	2.73	11.49	2.43	6.33	7.70	11.39
CHI	4.90	2.48	0.08	1.07	4.70	13.69	2.57	8.15	9.61	13.57
MEX	2.92	1.88	0.00	0.24	2.69	11.20	2.41	6.07	7.45	11.10
KOR	2.76	1.94	0.00	0.00	2.51	11.63	2.44	6.47	7.80	11.55
THA	2.51	1.88	0.00	0.00	2.26	11.51	2.43	6.33	7.71	11.41
HK	2.76	1.85	0.00	0.14	2.54	11.15	2.41	5.99	7.39	11.04
SIN	2.36	1.82	0.00	0.00	2.09	11.16	2.41	6.01	7.38	11.06
No short-sales on emerging markets										
ARG	5.77	2.19	1.58	2.51	5.55	12.43	2.48	7.11	8.46	12.36
BRA	4.22	1.93	0.80	1.46	4.00	12.12	2.46	6.88	8.21	12.05
CHI	6.43	2.41	1.80	2.85	6.20	14.39	2.60	8.79	10.27	14.30
MEX	4.17	1.89	0.79	1.46	3.94	11.86	2.45	6.58	7.97	11.79
KOR	4.14	1.92	0.76	1.40	3.91	12.22	2.47	7.02	8.31	12.13
THA	3.94	1.88	0.67	1.28	3.70	12.15	2.46	6.88	8.25	12.07
HK	4.10	1.86	0.77	1.44	3.87	11.81	2.45	6.56	7.91	11.73
SIN	3.77	1.83	0.60	1.18	3.54	11.82	2.45	6.51	7.96	11.73

The posterior distributions of δ and ψ are calculated for various efficient frontiers. Each frontier is spanned by the equity indices of G7 countries and one emerging market (indicated by the symbol for the country). In the calculation of posterior distributions of δ and ψ, portfolio weights are assumed to be either unconstrained or constrained to be nonnegative for all markets or only for emerging markets. The mean, standard deviation, first percentile, fifth percentile, and median of the posterior distributions are reported.

emerging markets, De Roon et al. test if emerging markets are mean–variance spanned by developed markets in the sense of Huberman and Kandel (1987). When only individual emerging markets are considered, their test performs well in finite samples as their simulations show. However, as explained by De Roon et al., when a group of emerging markets is considered, their test loses power in small samples, which causes them to fail rejecting the null hypothesis that the group of emerging markets is mean–variance spanned by developed markets.

Our Bayesian analysis does not have that kind of problem. By comparing the posterior distributions in Tables 3 and 5, it is easy to see that the diversification benefits of individual Latin American markets are always smaller than the diversification benefits of the combination of Latin American markets. The same also holds true for Asian countries. There are at least two reasons for our Bayesian analysis to provide more sensible results. First, unlike the spanning test that is based on asymptotic distributions in classical inference, Bayesian inference incorporates the uncertainty of finite samples into posterior distributions. Second, the short-sale constraints turn the spanning test into a nonlinear problem, and first-order approximation of a nonlinear function around the first moment is required in classic inference. In contrast, our Bayesian approximation uses the exact nonlinear function, δ or ψ, in the simulation of the posterior distribution and, by increasing the number of draws, can be as accurate as one wants (see Geweke, 1989).

5. Additional investigations

In this section, we examine whether the diversification benefits of emerging markets and the impact of short-sale constraints documented in the previous section are sensitive to the sample period, the composition of the emerging market indices, and the efficiency of the U.S. equity index. Specifically, in the first subsection, we investigate whether the benefits are different in the 1990s when global markets are known to have become more integrated. In the second subsection, we study how the benefits are affected by the use of investable indices that more accurately reflect the trading opportunities for nonnative investors. In the final subsection, we allow for the possibility that the U.S. equity index is not on the efficient frontier spanned by U.S. securities.

5.1. World market integration

It is well known that returns on emerging markets in the 1990s were quite different from those in the past due to increased integration in world markets. Therefore, it is reasonable to suspect that the international diversification benefits and the impact of short-sale constraints might have changed as emerging markets became more liberalized. It is also natural to question if our results are unduly influenced by the financial crises during 1997–1998. To examine these issues, we measure the diversification benefits separately for the period from January 1976 to December 1989 and the period from January 1990 to December 1999. According to Bekaert and Harvey (2000), since 1990, the emerging markets in our sample became integrated or liberalized, and correlation increases afterwards. As before, we look at the benefits with the portfolio weights both unconstrained and constrained to be nonnegative. Table 6 reports the basic statistics of the posterior distributions of δ and ψ for the two subperiods.

The diversification benefits remain evident in the post-liberalization period, both with and without constraints. However, the magnitude of δ and ψ is smaller and the impact of short-sale constraints on emerging markets is larger, compared to the pre-liberalization period. For the period of 1976–1989, the first percentile of the benefit δ of diversifying over all countries is 9.78% when there are short-sale constraints only on emerging markets, compared to 10.54% when there are no constraints. Other statistics of the posterior distribution of δ also show little change when short-sale constraints are imposed only on emerging markets. For the period of 1990–1999, the

Table 6
Posterior distributions of δ and ψ for the subperiods

Efficient frontier	δ					ψ				
	Mean	Sdev	1%	5%	Med	Mean	Sdev	1%	5%	Med
For the period of 1976–1989										
Unconstrained										
G	9.72	3.93	2.19	3.83	9.39	17.69	3.60	9.94	11.96	17.60
GL	16.78	4.79	7.12	9.57	16.45	23.43	3.79	15.09	17.19	23.39
GA	16.01	4.89	5.99	8.51	15.67	27.35	3.85	18.47	21.08	27.35
GLA	22.06	5.38	10.54	13.69	21.83	32.20	3.83	23.32	25.84	32.26
No short-sales on any markets										
G	8.16	4.06	0.13	1.87	7.89	15.99	3.56	8.42	10.36	15.86
GL	14.90	4.85	4.96	7.49	14.61	21.92	3.81	13.49	15.75	21.90
GA	13.18	4.74	3.00	5.69	12.96	25.22	3.89	16.25	18.91	25.23
GLA	18.61	5.18	7.35	10.28	18.46	29.62	3.90	20.59	23.14	29.64
No short-sales on emerging markets										
GL	16.66	4.82	6.85	9.39	16.33	23.31	3.80	15.03	17.07	23.26
GA	15.30	4.88	5.41	7.85	14.97	26.57	3.86	17.71	20.27	26.59
GLA	20.97	5.34	9.78	12.64	20.74	30.77	3.86	21.96	24.37	30.79
For the period of 1990–1999										
Unconstrained										
G	4.05	2.23	0.52	1.09	3.68	9.55	3.31	3.04	4.63	9.25
GL	6.68	2.83	1.78	2.71	6.32	14.50	3.86	6.54	8.53	14.29
GA	7.98	3.18	2.29	3.48	7.63	13.11	3.72	5.46	7.40	12.92
GLA	10.92	3.78	4.11	5.52	10.47	18.62	4.18	9.60	12.03	18.49
No short-sales on any markets										
G	0.73	1.30	0.00	0.00	0.00	8.97	3.33	2.50	4.02	8.70
GL	2.46	2.01	0.00	0.05	2.01	10.10	3.53	3.08	4.84	9.85
GA	1.33	1.55	0.00	0.00	0.81	9.05	3.40	2.49	3.94	8.76
GLA	2.74	2.15	0.00	0.14	2.28	10.27	3.60	3.13	4.72	10.01
No short-sales on emerging markets										
GL	5.59	2.65	1.13	1.92	5.21	10.71	3.51	3.79	5.45	10.47
GA	4.59	2.37	0.78	1.41	4.23	9.66	3.39	3.10	4.55	9.41
GLA	5.98	2.81	1.24	2.15	5.59	10.93	3.58	3.72	5.45	10.69

Using the data during the two subperiods, the posterior distributions of δ and ψ are calculated for various efficient frontiers. The first frontier is spanned by the equity indices of G7 countries (indicated by G), the second is spanned by the equity indices of G7 and four Latin American markets (GL), the third is by G7 and four Asian markets (GA), and the last is by all the 15 markets (GLA). In the calculation of posterior distributions of δ and ψ, portfolio weights are assumed to be either unconstrained or constrained to be nonnegative for all markets or only for emerging markets. The mean, standard deviation, first percentile, fifth percentile, and median of the posterior distributions are reported.

first percentile of the benefit δ of diversifying over all countries is 4.11% when there are no constraints and falls to 1.24% when short-sale constraints are imposed only on emerging markets. A similar comparison shows that the median benefit drops from 10.47% to 5.59%. The changes are large relative to the magnitude of the diversification benefits. As also shown in Table 6, similar conclusions emerge when we use the measure ψ. After imposing short-sale constraints on emerging markets, the distribution of ψ is virtually unchanged in the pre-liberalization period, while the changes are large in the later period.

To understand the different impact of short-sale constraints for the two subperiods, we report in Table 7 the posterior means and standard deviations of the weights of the efficient portfolio used to calculate δ in Table 6. For the pre-liberalization period of 1976–1989, the only substantial short positions in the efficient portfolio are on Singapore

Table 7
Portfolio weights for the subperiods

Market	Efficient portfolio weights						GMV portfolio weights					
	UC		NS		EM		UC		NS		EM	
	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev	Mean	Sdev
For the period of 1976–1989												
USA	0.30	0.23	0.04	0.08	0.22	0.23	0.49	0.08	0.59	0.05	0.45	0.07
CAN	−0.16	0.20	0.01	0.03	−0.17	0.20	−0.02	0.06	0.00	0.02	−0.03	0.07
JAP	0.30	0.15	0.21	0.15	0.29	0.15	0.17	0.05	0.00	0.05	0.16	0.05
FRA	0.02	0.14	0.03	0.06	0.04	0.15	−0.05	0.05	0.04	0.01	−0.04	0.05
GER	−0.07	0.15	0.02	0.05	−0.07	0.15	0.09	0.05	0.04	0.04	0.09	0.05
ITA	−0.03	0.11	0.02	0.05	−0.03	0.11	0.05	0.04	0.03	0.03	0.05	0.04
UK	0.08	0.16	0.04	0.07	0.05	0.16	−0.01	0.05	0.12	0.01	−0.04	0.05
ARG	0.06	0.02	0.06	0.03	0.06	0.02	0.02	0.01	0.02	0.01	0.02	0.01
BRA	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.02	0.02	0.02	0.04	0.02
CHI	0.11	0.06	0.11	0.07	0.10	0.06	0.05	0.02	0.09	0.02	0.05	0.02
MEX	−0.01	0.06	0.03	0.04	0.02	0.03	−0.02	0.02	0.01	0.00	0.00	0.00
KOR	0.16	0.08	0.16	0.10	0.16	0.08	0.08	0.03	0.01	0.03	0.08	0.03
THA	0.27	0.11	0.19	0.12	0.23	0.11	0.20	0.04	0.00	0.03	0.18	0.03
HK	0.04	0.09	0.03	0.05	0.04	0.05	−0.01	0.03	0.02	0.01	0.00	0.01
SIN	−0.11	0.13	0.01	0.04	0.01	0.03	−0.05	0.04	0.00	0.00	0.00	0.00
For the period of 1990–1999												
USA	0.93	0.21	0.59	0.22	0.83	0.20	0.58	0.12	0.50	0.11	0.54	0.13
CAN	−0.26	0.19	0.00	0.02	−0.26	0.18	0.08	0.11	0.04	0.06	0.00	0.11
JAP	−0.03	0.10	0.00	0.02	−0.06	0.09	0.07	0.05	0.04	0.04	0.03	0.05
FRA	0.03	0.20	0.04	0.09	0.03	0.19	0.07	0.10	0.04	0.06	0.03	0.11
GER	0.05	0.18	0.04	0.09	0.01	0.16	0.01	0.10	0.03	0.05	0.01	0.10
ITA	0.04	0.09	0.03	0.06	0.05	0.09	0.05	0.05	0.05	0.04	0.05	0.05
UK	0.15	0.19	0.12	0.16	0.25	0.19	0.23	0.11	0.23	0.10	0.26	0.11
ARG	−0.01	0.05	0.02	0.03	0.01	0.02	−0.03	0.03	0.00	0.00	0.00	0.00
BRA	−0.01	0.04	0.02	0.02	0.01	0.02	−0.04	0.02	0.00	0.00	0.00	0.00
CHI	0.15	0.09	0.09	0.09	0.09	0.07	0.14	0.05	0.06	0.04	0.06	0.04
MEX	−0.01	0.08	0.01	0.03	0.01	0.03	−0.01	0.04	0.00	0.00	0.00	0.01
KOR	0.03	0.06	0.01	0.02	0.01	0.02	−0.01	0.03	0.00	0.01	0.00	0.01
THA	−0.13	0.07	0.00	0.00	0.00	0.00	−0.04	0.04	0.00	0.00	0.00	0.00
HK	0.18	0.11	0.02	0.04	0.03	0.04	−0.01	0.06	0.00	0.00	0.00	0.00
SIN	−0.12	0.14	0.00	0.01	0.00	0.01	−0.08	0.07	0.00	0.00	0.00	0.00

For the two subperiods, this table provides the posterior means and standard deviations of the weights in the efficient portfolio with the same variance as the U.S. index and in the GMV portfolio of the 15 international markets considered in this paper. The portfolio weights are either unconstrained (indicated by UC), constrained to be nonnegative for all markets (NS), or only for emerging markets (EM).

for emerging markets and on Canada for developed countries. In contrast, for the post-liberalization period of 1990–1999, there are more substantial short positions on emerging markets in the efficient portfolio and the magnitude of the short position on Canada almost doubles. For example, the short position on Thailand is 0.11 and is statistically significant. These short positions on emerging markets reflect the relatively poor performance of emerging markets in the latter half of the 1990s, compared to developed markets, and explain the larger impact of short-sale constraints on emerging markets during the same period.

Table 7 also reports the posterior means and standard deviations of the weights in the global minimum-variance portfolio used to calculate ψ in Table 6. As in the full sample case, the portfolio weights in the globally minimum-variance portfolio are more accurately estimated. For the pre-liberalization period of 1976–1989, the global minimum-variance portfolio requires taking small short positions on both developed and emerging markets. For the post-liberalization period of 1990–1999, the global minimum-variance portfolio requires taking short positions mainly on emerging markets. This explains why short-sale constraints on emerging markets have little impact on the diversification benefit ψ during the early period and a greater impact in the later period.

5.2. Investable indices of emerging markets

The index data in our analysis thus far have included stocks that may not be available to nonnative investors for legal or practical reasons. Legal restrictions include, among others, limits on foreign holding of general classes of shares or exclusion of foreign investment in particular sectors. Even if a stock is available for investment, nonnative investors may find it difficult to trade because the stock is too small and illiquid. To examine the effect of these restrictions on the diversification benefits, we use returns on the IFC investable indices for emerging markets (except for Hong Kong and Singapore, which do not have investable indices). The IFC investable indices are calculated in a similar fashion as the total return indices used in our earlier analysis, but cover a subset of index constituents that is available to nonnative investors and meets minimum size and liquidity requirements.

A problem in using the IFC investable indices is their relatively short history. The data are not available before January 1989 and, in the case of Korea, the earliest observation starts in January 1992. The common approach is to truncate the data to the most recent starting date. The disadvantage with this approach is that substantial information is lost and the resulting sample is very small. To utilize all available information, we use Gibbs sampling (Gelfand and Smith, 1990) in combination with data augmentation (Tanner and Wong, 1987) to estimate the diversification benefits.[6]

The observations of returns are still assumed to have an identical and independent normal distribution with mean μ and covariance matrix $\mathbf{\Omega}$. At each time t, returns on some

[6] Alternatively, for the case where the missing observations only occur at the beginning of the sample period, Stambaugh (1997) has developed an analytical solution. We have tried both Gibbs sampling and Stambaugh's approach and the results are the same.

assets are observed while others may be unobservable. Let R_{ot} be the vector of observed returns at time t and the set of all observed returns is $R_o=\{R_{ot}\}_{t=1,\ldots,T}$. Let U be the set of dates on which at least one asset return is not observed. Let R_{ut} be the vector of unobserved returns at time $t \in U$ and the set of all unobserved returns is $R_u=\{R_{ut}\}_{t\in U}$. The set of both observed and unobserved returns is $R=R_o \cup R_u$. Let μ_{ot} be the mean of R_{ot} and $\boldsymbol{\Omega}_{out}$ the covariance matrix between R_{ot} and R_{ut}, etc.[7]

In order to obtain the posterior distribution $p(\delta|R_o)$, we need to generate samples of $(\mu, \boldsymbol{\Omega})$ from the posterior distribution

$$p(\mu, \Omega \mid R_o) = \int p(\mu, \Omega, R_u \mid R_o) \mathrm{d}R_u. \tag{6}$$

The details of the Gibbs procedure are as follows. We first set a starting value for $(\mu, \boldsymbol{\Omega})$, say $(\mu^0, \boldsymbol{\Omega}^0)$.[8] For given $(\mu^i, \boldsymbol{\Omega}^i)$ where $i=0, 1, 2, \ldots$, we can generate a sample of unobserved returns R^i_{ut} from the conditional distribution $p(R_{ut}|\mu^i, \boldsymbol{\Omega}^i, R_o)$. This conditional distribution is normal with mean $\mu^i_{ut}+\boldsymbol{\Omega}^{i\prime}_{out}(\boldsymbol{\Omega}^i_{oot})^{-1}(R_{ot}-\mu^i_{ot})$ and covariance matrix $\boldsymbol{\Omega}^i_{uut}-\boldsymbol{\Omega}^{i\prime}_{out}(\boldsymbol{\Omega}^i_{oot})^{-1}\boldsymbol{\Omega}^i_{out}$. It is therefore easy to generate samples of unobserved data, R^i_u, conditional on $(\mu^i, \boldsymbol{\Omega}^i)$ and the observed data R_o. We then generate samples of $(\mu^{i+1}, \boldsymbol{\Omega}^{i+1})$ from $p(\mu, \boldsymbol{\Omega}|R_o, R^i_u)$, which is the posterior distribution assuming R^i_u is observed. This posterior distribution is exactly the same as in the case where all returns are observed. Conditional on $(\mu^{i+1}, \boldsymbol{\Omega}^{i+1})$, we generate new samples of unobserved data and iterate the same process. Under some general conditions, the distribution of $(\mu^i, \boldsymbol{\Omega}^i)$ converges to $p(\mu, \boldsymbol{\Omega}|R_o)$ when i is large (Tierney, 1994). We discard the samples of $(\mu^i, \boldsymbol{\Omega}^i)$ generated from the first 1000 iterations and retain the subsequent 10,000 samples for computing the posterior distribution of δ and ψ.

Table 8 reports the means, standard deviations, and percentiles of the posterior distributions of δ and ψ using the investable index returns. All the earlier results hold qualitatively. However, the impact of short-sale constraints is greater for investable indices than for total return indices. Since most of the data on investable indices are from the 1990s, this is consistent with our earlier result that the impact of short-sale constraints on emerging markets is greater during the post-liberalization period. For example, when short-sale constraints are imposed only on emerging markets, as reported in the last panel of Table 8, the first percentile of δ for all countries falls from 4.60% to 2.49% for investable indices, while (referring back to Table 3) the reduction is from 4.61% to 3.78% for total return indices. We find similar results for ψ. For example, when short-sale constraints are imposed only on emerging markets, as reported in the last panel of Table 8, the first percentile of ψ for all countries reduces from 11.45% to 8.14% for investable

[7] The mean μ_{ot} and covariance matrix Ω_{out} depend on t because we allow the set of observed and unobserved assets to change over time. For example, the return on an asset might be unobservable at one time but become observable at a later time.

[8] The starting value for μ and $\boldsymbol{\Omega}$ does not affect the results in Gibbs sampling. One can calculate the sample mean and covariance matrix by truncating the data to the same length for all assets. The sample mean and covariance matrix can then be used as the starting value.

Table 8
Investable indices

Efficient frontier	δ					ψ				
	Mean	Sdev	1%	5%	Med	Mean	Sdev	1%	5%	Med
Unconstrained										
G	3.72	1.83	0.59	1.14	3.50	11.82	2.40	6.66	8.01	11.72
GL	8.18	2.96	2.65	3.85	7.90	15.92	3.01	9.40	11.14	15.80
GA	6.27	2.37	1.85	2.86	6.03	13.81	2.59	8.20	9.68	13.72
GLA	11.28	3.46	4.60	6.13	10.94	18.72	3.29	11.45	13.45	18.64
No short-sales on any markets										
G	2.26	1.83	0.00	0.00	1.95	11.16	2.38	6.11	7.43	11.06
GL	5.98	2.70	0.96	2.04	5.70	12.61	2.65	6.94	8.48	12.48
GA	2.91	1.85	0.00	0.23	2.70	11.25	2.41	6.13	7.47	11.15
GLA	6.34	2.71	1.25	2.37	6.09	13.38	2.78	7.42	8.92	13.28
No short-sales on emerging markets										
GL	7.76	2.93	2.28	3.51	7.47	13.58	2.72	7.76	9.25	13.48
GA	4.27	1.87	0.86	1.55	4.07	11.90	2.44	6.67	8.06	11.82
GLA	8.02	2.90	2.49	3.72	7.77	14.30	2.84	8.14	9.72	14.21

The posterior distributions of δ and ψ are calculated for various efficient frontiers. The first frontier is spanned by the equity indices of G7 countries (indicated by G), the second is spanned by the equity indices of G7 countries and the investable indices of four Latin American markets (GL), the third is by the equity indices of G7 countries and the investable indices of four Asian markets (GA), and the last is by the equity indices of G7 countries and the investable indices of all the eight emerging markets (GLA). In the calculation of posterior distributions of δ and ψ, portfolio weights are assumed to be either unconstrained or constrained to be nonnegative for all markets or only for emerging markets. The mean, standard deviation, first percentile, fifth percentile, and median of the posterior distributions are reported.

indices, whereas (referring back again to Table 3) the reduction is from 11.06% to 9.74% for total return indices.

5.3. Inefficiency of the U.S. equity market

Since there is considerable evidence that the U.S. equity index portfolio is mean–variance inefficient, the benefits from international diversification found so far may be further reduced if a U.S. investor is able to improve on the efficiency of the U.S. equity index portfolio. We address this issue by including the size and book-to-market portfolios constructed by Fama and French (1993). We start by considering the diversification benefits of moving from the benchmark U.S. market index to the efficient frontier generated by the Fama-French portfolios. Then, similar to our earlier analysis, we successively add market indices of G7, Latin American, and Asian countries to the Fama-French portfolios. Table 9 reports the results.

Without imposing any short-sale constraints, the U.S. equity index is clearly inefficient relative to the frontier spanned by the Fama-French portfolios. The first percentile of δ corresponding to the Fama-French portfolios is 5.96% in annualized return and both the posterior mean and the median are above 12%. It is natural to question whether there are

Table 9
Fama-French portfolios

Efficient frontier	δ					ψ				
	Mean	Sdev	1%	5%	Med	Mean	Sdev	1%	5%	Med
Unconstrained										
F	12.70	3.17	5.96	7.77	12.54	16.33	2.70	10.40	12.00	16.26
FG	15.62	3.45	8.23	10.15	15.51	22.59	2.85	16.15	17.95	22.53
FGL	19.49	3.76	11.62	13.58	19.34	25.74	2.92	18.99	20.93	25.76
FGA	17.48	3.58	9.95	11.93	17.33	24.48	2.90	17.81	19.76	24.46
FGLA	21.37	3.87	13.32	15.29	21.20	27.55	2.97	20.69	22.68	27.52
No short-sales on any markets										
F	2.83	1.46	0.15	0.66	2.70	5.54	1.81	1.94	2.80	5.42
FG	4.71	1.97	0.67	1.65	4.63	15.03	2.63	9.26	10.83	15.00
FGL	8.45	2.66	3.00	4.29	8.33	17.67	2.76	11.44	13.13	17.64
FGA	5.36	2.09	1.08	2.10	5.26	15.87	2.68	9.86	11.49	15.82
FGLA	8.93	2.71	3.30	4.74	8.76	18.26	2.83	11.89	13.58	18.19
No short-sales on emerging markets										
FGL	19.24	3.76	11.42	13.33	19.10	25.42	2.92	18.68	20.62	25.44
FGA	16.53	3.52	9.16	11.04	16.39	23.53	2.89	16.97	18.82	23.51
FGLA	20.03	3.80	12.18	13.98	19.86	26.03	2.99	19.21	21.15	26.01

The posterior distributions of δ and ψ are calculated for various efficient frontiers. The first frontier is spanned by the Fama-French six portfolios (indicated by F), the second is spanned by the Fama-French portfolios and the equity indices of G7 countries (FG), the third is spanned by the Fama-French portfolios, G7 and four Latin American markets (FGL), the fourth is by the Fama-French portfolios, G7 and four Asian markets (FGA), and the last is by the Fama-French portfolios and all the 15 international markets (FGLA). In the calculation of posterior distributions of δ and ψ, portfolio weights are assumed to be either unconstrained or constrained to be nonnegative for all markets or only for emerging markets. The mean, standard deviation, first percentile, fifth percentile, and median of the posterior distributions are reported.

further diversification benefits if we start with an efficient U.S. portfolio rather than the U.S. index portfolio. The answer is yes. The first percentile of δ corresponding to the combination of the Fama-French portfolios and all the international markets is 13.32% in annualized return.

The impact of short-sale constraints reported in Table 9 is similar to what we find in Table 3. With or without short-sale constraints, adding emerging markets substantially increases the diversification benefit δ. Imposing short-sale constraints on emerging markets, but not on G7 markets, has a minimal effect on the posterior distribution of δ. The results for ψ are also similar to what we find in Table 3. Imposing short-sale constraints on all markets has a moderate to large effect on the diversification benefit ψ, while imposing short-sale constraints only on emerging markets has a minimal effect on the benefit.

6. Conclusion

We use Bayesian inference to examine the impact of short-sale constraints on the magnitude of the international diversification benefits to U.S. investors during the period of 1976–1999. Two different measures of the diversification benefits are employed. The first one captures the increase in the expected return when switching from the U.S. equity index portfolio to the efficient international portfolio with equal variance. The second one captures the reduction in the standard deviation when investors switch from the U.S. equity index portfolio to the global minimum-variance portfolio. We show that the international diversification benefits remain substantial for U.S. investors after imposing short-sale constraints on emerging markets but not after imposing short-sale constraints on G7 countries. This result is robust to investment restrictions on nonnative investors. It is also unaffected by the fact that the U.S. equity index portfolio is not on the efficient frontier spanned by U.S. securities. The integration of world equity markets reduces, but does not eliminate, the diversification benefits of investing in emerging markets subject to short-sale constraints. Our results reinforce the home bias puzzle with respect to investments in emerging markets.

Our analysis of the international diversification benefits has at least three advantages. First, it is easy to implement. Once the samples of μ and $\mathbf{\Omega}$ are drawn, the calculation of the measures of the diversification benefits is straightforward. Second, most studies focus on rejection of the null hypothesis of no diversification benefits and gauge the magnitude of the benefits by examining the strength of the rejection. In contrast, the combination of Bayesian inference and Monte Carlo simulation allows us to draw exact inference on the magnitude of the diversification benefits. Finally, our approach can be used to examine a variety of market frictions. In Section 2, we assume only that the portfolio constraints form a closed convex set. Since this assumption is rather general, it allows for other realistic constraints such as margin and collateral requirements, as well as limitations on portfolio proportions for fund managers. It also allows us to study transaction costs. For example, it is well known that proportional transaction costs can be represented by short-sale constraints. It is interesting to know if these diversification benefits can be wiped out by transaction costs.[9]

There are two caveats to our analysis in this paper. First, our analysis of δ assumes that the prior belief is non-informative. Therefore, the impact of short-sale constraints on δ examined in this paper is for investors whose prior belief is non-informative. Those with a non-informative prior belief use only the sample information from historical returns to form their posterior belief. However, if one believes that the observed historical returns are too high or too low and thus will converge to some equilibria, his/her belief in equilibrium models should be incorporated into the prior distribution. Black and Litterman (1992) and Pastor and Stambaugh (2000) develop frameworks for introducing equilibrium models into prior beliefs. The diversification benefits perceived by investors with informative priors can be very different from the benefits perceived by those with non-informative priors. This issue is examined by Wang (2002).

Second, the asset allocation problem we considered is static rather than conditional or dynamic. Hodrick (1981) and Harvey (1991) document the time-varying risk and expected returns for international equities. Bekaert and Hodrick (1992), Ferson and Harvey (1993), and Harvey (1995) find that international stock returns are predictable. Hodrick et al.

[9] Bekaert et al. (1997) provide in-depth discussions on transaction costs in emerging markets.

(1999) examine the hedging demands when international asset returns are predictable. Ang and Bekaert (in press) study how the hedging demands affect the international diversification benefits in dynamic portfolio choices with regime-switching models. The point we want to make in this paper is that, with short-sale constraints, diversification benefits exist even in the static framework. However, our analysis does not necessarily predict future diversification benefits of emerging markets. To predict future diversification benefits, it is necessary to extend our study to examine the impact of short-sale constraints on the benefits of a conditionally or dynamically diversified portfolio. Out-of-sample evaluations of such conditional studies will also be required.

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