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Volatility estimation on the basis of price intensities

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Abstract

This paper investigates the use of price intensities, i.e. the time between price changes of a given size, to estimate volatilities based on high-frequency data. We interpret the conditional probability for the occurrence of a price event within a certain time horizon as a risk measure which allows us to obtain an estimator of the conditional volatility per time. To consider censoring effects caused by nontrading periods, we use a proportional hazard model. Seasonalities are taken into account by including regressors based on a flexible Fourier form capturing intraday and time-to-maturity seasonalities. Testing for serial correlation and controlling for unobservable heterogeneity permits us to check for misspecification on different aggregation levels. Empirical results are based on intraday transaction data of Bund future trading at the LIFFE, London. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The road to a feasible estimation of risk measures based on intraday data is strewed with considerable problems. Some of them originate in the very nature of

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the transaction process, like the irregular spacing between observations,[1] the discrete price changes between observations,[2] or the bounce effect[3] induced by trading costs in the form of a bid-ask spread. Others are attributed to the process of information diffusion and price discovery, like serial dependency found in many variables which are supposed to contain information about the state of the price process, e.g. the intensity of the price process,[4] i.e. the time between price changes, the variation of transaction costs or the volume traded in a certain time span. In spite of the important role these variables usually play in assessing market microstructure, they are generally considered as a nuisance in the context of risk analysis and the analysis of large-scale price changes.[5] The standard way to proceed would be to define a particular aggregation scheme which supposedly does away with these effects. Andersen and Bollerslev (1998b) show that the choice of an appropriate aggregation scheme is very crucial for these models. By choosing different aggregation levels, they obtain different estimates for the volatility per hour. They show that volatility forecasts can be improved significantly by determining an adequate aggregation level and by considering effects on various frequency levels. In this context, they illustrate that the volatility process is driven by the simultaneous interaction of different patterns at the intradaily and daily level. These results demonstrate that the choice of the aggregation scheme has a strong impact on the volatility estimation.

In order to circumvent these imponderabilities, we consider an avenue of research initiated by Cho and Frees (1988) and Engle and Russell (1998).[6] They quantify the instantaneous volatility of the price process by modelling price intensities, i.e. the time between price changes. Their main results concentrate on the serial dependency found in the process of price intensities and their relationship to hypotheses emanating from market microstructure literature. In this context, the volatility is based on the time, the price process spends in the neighborhood of a certain price level. The size of this neighborhood is the tuning parameter used to focus the empirical analysis. The strategy pursued here is to analyze the time it takes for the price process to generate a certain given price change. Hence, the volatility is determined by the conditional expected duration between two price events, i.e. the end of a price duration, given the explanatory

[1] See the seminal paper of Engle (1996) or Engle and Russell (1997) for a statistical analysis.

[2] See e.g. the early rounding models like Harris (1990), Gottlieb and Kalay (1985) or Ball (1988). For an analysis of the economic implications, see Harris (1994). Some more recent models take into account discreteness as well as the irregular spacing of observations, e.g. Hausman et al. (1992), Manrique and Shephard (1997), Engle and Russell (1998) or Rydberg and Shephard (1999).

[3] See the original work of Roll (1984) or also Hausman et al. (1992).

[4] See Easley et al. (1997) for a structural model of the price process.

[5] This is not meant to imply that the investigation of the relationship between trade frequency, traded volume and volatility as analyzed, e.g. by Clark (1973), Gallant et al. (1992) or Jones et al. (1994), is not worthwhile. Yet, for the assessment of risk in this context it is only of limited use.

[6] See also Giot (2000).

variables. This kind of volatility estimation differs from the classical strategy, where the conditional variances are modeled, either over a given interval as in the standard GARCH literature or over irregular time intervals, including more or less appropriate corrections for the intensity of the price process.[7]

Our approach circumvents the problems of correcting for the variability of trading intensity by building the volatility estimator directly on the durations between price changes. By using a semiparametric proportional hazard model, we estimate the conditional probability for a price event in the next time interval, given the time, the last price event dates back. These conditional probabilities can be interpreted as the risk for price changes in the next time interval. Based on these probabilities, we calculate a conditional volatility, which solves the problem of finding an appropriate aggregation level by defining explicitly the event of interest. Hence, the corresponding time intervals and thereby the frequency of observations are determined by the definition of the price event. This feature is the most important advantage of our approach. As it will be shown in the course of this paper, this redefinition of observations is a straightforward solution for most problems induced by the impact of market microstructure variables.

By analyzing price events at the LIFFE Bund future market, we have to take into account that this market is not open 24 h per day. Thus, spells can also end at the next trading day or can be terminated during the night. The fact that price durations which are terminated during the night are not observable leads to censoring mechanisms, i.e. the exact length of the spell cannot be measured. Since we know exactly when the spell started, but do not know when it finished, such an observation can only be registered by a lower and upper bound, where the upper bound is given by the time until the first price event at the next morning. In the following, we call these observations ‘pseudo-censored’. To account for such pseudo-censoring effects, we use a proportional hazard model for grouped durations based on the work of Han and Hausman (1990). The particular advantage of this approach is that pseudo-censored durations can be treated as grouped durations and thus, they enter the log likelihood by lower and upper bounds. To check for misspecifications, we employ a test on serial correlation based on the work of Gourieroux et al. (1985). A further advantage of this model is that it permits us to control for unobservable heterogeneity,[8] which is specified parametrically by a gamma distributed random variable entering the hazard function multiplicatively leading to a mixture model.

[7] Ghysels and Jasiak (1997) develop a class of ARCH models for transaction data which are unequally spaced in time. They combine a temporal aggregation procedure for GARCH models with ACD models for the duration between particular trades. In this bivariate ACD-GARCH model, they are able to test for Granger causal relationships between the volatility and inter-trade durations. Grammig and Wellner (1999) provide an extension of this approach.

[8] In duration literature, a wide range of investigations exists which analyze the impact of misspecified heterogeneity effects on the estimated baseline hazard rate, see e.g. Heckman and Singer (1984).

A further question which needs to be resolved is the assessment of seasonalities. Intraday seasonalities are a well-documented fact[9] and have to be taken into account when estimating intraday volatility. In the context of futures trading, a second source of seasonalities becomes apparent, namely the time to maturity. Both types of seasonalities are analyzed in this paper using a Fourier series expansion as suggested by Andersen and Bollerslev (1997).

The outline of our paper is as follows: Section 2 gives a brief overview of different ways to estimate volatility. Section 3 describes the econometric model based on a categorical proportional hazard framework. In Section 4, we describe the data and the inclusion of seasonalities. Empirical findings based on the Bund future trading at LIFFE, London, are given in Section 5, while Section 6 concludes.

2. Concepts of volatility estimation

In order to give a concise picture of the relationship between the new risk estimation concept proposed here and concepts proposed in previous work, it seems worthwhile to look at the standard definition of conditional volatility per time[10]

$$
\begin{aligned}
&\sigma_t^2 = \mathrm{E}\left[\frac{1}{\tau_t} (r_t - \bar{r}_t)^2 \middle| x_t \right], \qquad t = 1, \dots, T^{\circ}, \\
&\text{with } r_t := \frac{p_t - p_{t-1}}{p_{t-1}}, \\
&\bar{r}_t = E[r_t \mid x_t] = 0 \\
&\text{and } \tau_t := \vartheta_t - \vartheta_{t-1}.
\end{aligned} \tag{1}
$$

Here, ϑ_t denotes the calendar time, e.g. measured in seconds, since a reference date and t is just an index to order the events of interest not directly related to the calendar time. Therefore, in this standard definition of volatility, t orders the particular transactions and thus τ_t is the inter-trade duration. T° is the sample size of the observed transactions and p_t is the price at which transaction t was carried out. r_t denotes the corresponding return, while x_t contains a set of at least weakly exogenous regressors and may include lagged observations of the dependent variable.

[9] See e.g. Wood et al. (1985) and McInish and Wood (1992) for an analysis of seasonalities of trading at the NYSE, Dacorogna et al. (1993) and Guillaume et al. (1997) for the foreign exchange market or Andersen and Bollerslev (1997) for an analysis of intraday periodicity.

[10] For ease of exposition, no log returns are used. However, using log returns would not raise any substantial problems.

In order to apply standard GARCH-type procedures to the estimation problem in Eq. (1), τ_t is usually assumed to be constant, i.e. the event of interest is not the price at every transaction but the price observed at certain points of time. Hence, in this context, t orders the observations at certain calendar times ϑ_t, for which $\tau_t = \vartheta_t - \vartheta_{t-1} = \tau$ is constant. Intraday aggregates are typically used, e.g. on the basis of 5-min intervals (see Andersen and Bollerslev, 1998a; Andersen et al., 1999 for further references). Eq. (1) is quite unprecise with respect to the measure which is used within the integral. As it is clear from the preceding argument in the context of standard GARCH-type models, the price intensity is considered deterministic and constant overall t

$$\sigma^2_{t,(r)} = \mathop{\mathrm{E}}_{r}\left[r_t^2 \mid x_t \right] \frac{1}{\tau}. \tag{2}$$

In order to account for seasonalities some modifications are needed. This can be done by the inclusion of particular explanatory variables x_t as suggested by Andersen and Bollerslev (1997) (see also Section 4.2), by the introduction of time varying coefficients as in Bollerslev and Ghysels (1996) or by the use of concepts of time deformation in the spirit of Dacorogna et al. (1996). Such a procedure, however, raises the question of an optimal aggregation level as a major problem which has to be solved before estimators of this type can be used for risk assessment. Evidently, there are two main sources of errors introduced. First, it is an unresolved issue what the criteria of optimality for the determination of the aggregation level are. See the discussion of Andersen and Bollerslev (1998a) in the context of a continuous time DGP. Second, it is far from clear what kind of bias is introduced by a constant aggregation level particularly for assets having a ‘liquidity life cycle’ as it is the case for a future.

Some models specify indeed a bivariate process, which accounts for the stochastic nature of both the process of price changes and the process of price intensities. These specifications allow volatility estimates on the basis of the bivariate distribution. In this context, the volatility measure is based on the transaction process, therefore, here t orders the particular transactions. With the random variable $d_t := p_t - p_{t-1}$ we have

$$\sigma^2_{t,(d,\tau)} = \mathop{\mathrm{E}}_{d,\tau}\left[\frac{1}{\tau_t} \cdot d_t^2 \middle| x_t \right] \frac{1}{p_{t-1}^2}, \qquad t = 1, \ldots, T^{\mathrm{o}}. \tag{3}$$

The limitation of these models to the analysis of price changes d_t is standard and not substantial, considering, that these models are usually estimated on the basis of transaction data. Furthermore, it does not limit the applicability, given that this factor can be recovered ex post. Having already emphasized that these models are based on transaction data, the process of returns or price changes has a peculiar property which needs to be accounted for. Price changes take on only a few different values depending on the tick size and the liquidity of the traded asset. For an analysis of the effects of neglected discreteness see Harris (1990), Gottlieb and Kalay (1985), or Ball (1988). Most models concentrating on the bivariate process account explicitly for those market microstructure effects, like Russell and Engle (1998), Rydberg and Shephard (1999) or Gerhard and Pohlmeier (2000). Ghysels and Jasiak (1997) and Grammig and Wellner (1999) analyze a bivariate process including returns in a GARCH specification which accounts explicitly for the stochastic nature of the price intensity. Nevertheless, they refrain from taking market microstructure effects into account. However, the aforementioned procedures yield valuable information for the analysis of market microstructure, but they are of limited use for risk assessment since it is a tedious task to aggregate these models to obtain a valid risk estimator. Moreover, the results of these models mainly reflect the structure of the price process as measured from one transaction to the next, but yield only a limited amount of information for the overall risk of a series.

In the context of our model, we start with the assumption that a decision maker in need of a risk measure is indeed able to express the size of a significant price change. We call this size α. Using this α in the given context we recognize that the risk measure boils down to the question of how long it might take to realize this significant price change, i.e. the empirical analysis of first passage times. The bivariate distribution of r_t and τ_t is no longer of interest, since r_t is reduced to the ratio of a constant and a conditionally deterministic variable. Note that here t orders the events of interest which are determined by the price process, i.e. all observations for which $|p_t - p_{t-1}| \geq \alpha$. Then, $\tau_t = \vartheta_t - \vartheta_{t-1}$ are redefined as price durations. By this, the sample size is reduced from T^o to T, where T denotes the number of price events.[11] Hence, we can formulate a conditional volatility per time as[12]

$$\sigma^2_{t,(\tau)} = \mathop{\mathrm{E}}_{\tau}\left[\frac{1}{\tau_t}\middle| x_t\right]\left(\frac{\alpha}{p_{t-1}}\right)^2 \tag{4}$$

$$t,(\tau) = \sigma_t^{*\,2} \cdot \frac{1}{p^2_{t-1}}, \qquad t = 1,\ldots,T. \tag{5}$$

This procedure has the second significant benefit that for typical values of α, a data dependent aggregation scheme is obtained, which rids the researcher of many market microstructure effects that are a nuisance in the analysis of risk.

[11] A price event is defined as the end of price duration, i.e. when the price process leaves the interval $[p_{t-1} - \alpha;\ p_{t-1} + \alpha]$, where p_{t-1} denotes the price level of the previous price event. Note that this definition requires to initialize the time series of the price durations at a certain price level. See Section 4 for further details.

[12] For a similar specification of a marginal volatility, see Engle and Russell (1998).

σ_t^{*2} stands for the conditional volatility of price changes, from which the conditional volatility of returns can easily be recovered (see Eq. (5)). For ease of exposition and without implying a limitation of the suggested procedure, only the volatility of price changes is considered from here on.

3. The econometric approach

3.1. A proportional hazard model

Our strategy of volatility estimation is based on the time between price changes of a given size. If one investigates durations in financial markets, one has to account for the occurrence of event clustering. Short, respectively large durations are followed by durations of the same kind. This has already been found in different empirical investigations (see, e.g. Engle and Russell, 1997, 1998; Bollerslev and Domowitz, 1993; Andersen and Bollerslev, 1997). To take autocorrrelations between the durations into account, Engle (1996)[13] and Engle and Russell (1998) proposed the Autoregressive Conditional Duration (ACD) model which specifies the time flow directly via an autoregressive process. The ACD model is based on a dynamic parametrization of the conditional mean function and is the counterpart of the GARCH model for price processes.

The work of Engle and Russell was the starting point for a wide range of investigations in the class of ACD models. Ghysels and Jasiak (1997) combine the ACD and GARCH model by assuming a joint structure of the volatility of the price process and the intertrade durations. Bauwens and Giot (2000) propose a logarithmic ACD model which allows for the inclusion of covariates without imposing restrictions on the parameters to ensure the nonnegativity of the durations. A fractional ACD model to account for long run dependencies, potentially contained in the data, is specified by Jasiak (1999). Bauwens and Veredas (1999) introduce the Stochastic Conditional Duration (SCD) model by specifying a stochastic conditional mean function. Furthermore, more flexible distributions for the ACD error terms are proposed, like the Burr distribution (Grammig and Maurer, 2000), the Generalized Gamma distribution (Lunde, 2000) or the Generalized F distribution (Hautsch, 2001).

However, because of the existence of pseudo-censoring effects, these highly versatile ACD models cannot directly be applied for the given problem.[14] Note that news events may also occur overnight, leading to a price event whose exact

[13] This paper is now published as Engle (2000).

[14] Giot (2000) applies ACD models for volatility estimation but circumvents the problem of censoring by deleting overnight durations which is a justifiable way of proceeding in the context of small price changes.

timing cannot be observed. Hence, we may only know when the spell started, but we do not know exactly when it was finished, because the resulting price change can only be observed in the next morning. To account for these censoring structures, we employ a proportional hazard model for grouped durations. The specification is based on the work of Han and Hausman (1990) and Meyer (1990) and allows the simultaneous estimation of the coefficients of explanatory variables and of discrete points of the baseline survivor function. Hence, in this framework, pseudo-censored durations are treated as categorized durations which enter the log likelihood function by lower and upper bounds.

By examining the intraday and interday transaction price process, we define $t = 1, \ldots, T$ as the recorded ordering of transactions with price events and $\tau_t = \vartheta_t - \vartheta_{t-1}$ as the duration between these price events, which is assumed to be a random variable following a nonspecified distribution. The model is based on the proportional hazard specification of Cox (1972, 1975),

$$\lambda(\tau_t | x_t) = \lambda_0(\tau_t) \exp(-x_t' \beta), \tag{6}$$

where x_t is a vector of covariates, β is the corresponding vector of coefficients and $\lambda_0(\tau_t)$ is a nonspecified baseline hazard rate, corresponding to the hazard rate measured at $x_t = 0$.[15] Following Kiefer (1988) and Han and Hausman (1990), the proportional hazard model admits a convenient interpretation as a linear model

$$\tau_t^* = x_t' \beta + \varepsilon_t, \tag{7}$$

where

$$\tau_t^* \equiv \ln \Lambda_0(\tau_t) = \ln \int_0^{\tau_t} \lambda_0(s) \, \mathrm{d}s.$$

In this specification, the error term ε_t follows an i.i.d. type II extreme value distribution which does not depend on $\Lambda_0(\tau_t)$ and whose p.d.f. is given by[16]

$$f_\varepsilon(s) = \exp(s - \exp(s)).$$

Note that the approach corresponds to a linear regression model for the transformed duration τ_t^* with an extreme value distributed error term. The crucial point is that the form of the transformation depends on the (unknown) baseline hazard function and thus, is unobservable. Hence, in this context, τ_t^* can be interpreted as a latent variable.

In order to obtain a semiparametric estimation of λ_0 in the context of a categorical model, we partition the durations in K categories, where we define the durations $\tilde{\tau}_k$, $k = 1, \ldots, K-1$, as category bounds. These category bounds are

[15] For a broader exposition of hazard rate models, see e.g. Cox and Oakes (1984), Kalbfleisch and Prentice (1980) or Lancaster (1994).

[16] Note that the form of the distribution f_ε is not assumed directly but is driven by the underlying proportional hazard assumption.

associated with the corresponding thresholds of the latent variable based on the relationship

$$\delta_k \equiv \ln \Lambda_0(\tilde{\tau}_k). \tag{8}$$

Therefore, in this framework, the specification is interpreted as an ordered response model based on an extreme value distribution. The thresholds of the latent model are defined as the log integrated baseline hazard, calculated at the points of the corresponding category bounds. The direct relationship between the latent thresholds and the log integrated baseline hazard is one of the main advantages of this approach. The semiparametric baseline survivor function is directly obtained through the estimated thresholds. It can be calculated at the $K-1$ discrete points by

$$S_0(\tilde{\tau}_k) = \exp(-\exp(\delta_k)), \qquad k = 1, \dots, K-1. \tag{9}$$

3.2. Censoring effects and market closure

To account for price events which occur overnight, e.g. induced by information events on other markets, we extend the model to allow for pseudo-censoring effects. In this context, the length of a price duration cannot exactly be measured, because the timing of the end of the spell is not observable. We can only observe the time from the previous price change until the closure of the market, which is the minimum of the real price duration and is denoted by $\tau_{t\min}$. The price change is observable at the opening of the next trading day at the earliest, hence, the time from the last price event at the previous trading day until the next morning is the maximal length of the spell which is denoted by $\tau_{t,\max}$. The limits of the resulting censoring interval are associated with the corresponding categories $\tilde{\tau}_l, \tilde{\tau}_u \in \{\tilde{\tau}_1, \dots, \tilde{\tau}_{K-1}\}$, containing $\tau_{t,\min}$, $\tau_{t,\max}$. Hence, the pseudo-censoring of a duration implies that the latent variable lies between the thresholds $\delta_{l,t}$, $\delta_{u,t}$, where

$$\delta_{l,t} \equiv \ln \Lambda_0(\tilde{\tau}_l) \qquad \text{with } \tilde{\tau}_l = \tilde{\tau}_k \text{ and } \tau_{t,\min} \in (\tilde{\tau}_k, \tilde{\tau}_{k+1}],$$
$$\delta_{u,t} \equiv \ln \Lambda_0(\tilde{\tau}_u) \qquad \text{with } \tilde{\tau}_u = \tilde{\tau}_k \text{ and } \tau_{t,\max} \in (\tilde{\tau}_{k-1}, \tilde{\tau}_k].$$

Using the indicator variable

$$c_t = \begin{cases} 1 & \text{if } \tau_t \text{ is pseudo-censored} \\ 0 & \text{else,} \end{cases} \tag{10}$$

the log likelihhod function takes the form

$$\log \mathscr{L}(\beta, \delta) = \sum_{t=1}^{T} \sum_{k=1}^{K-1} (1-c_t) \mathbf{1}_{\{\tau_t \in (\tilde{\tau}_{k-1}; \tilde{\tau}_k]\}} \ln \int_{\delta_{k-1} - x_t' \beta}^{\delta_k - x_t' \beta} f_\varepsilon(s) \mathrm{d}s + \sum_{t=1}^{T} c_t \ln \int_{\delta_{l,t} - x_t' \beta}^{\delta_{u,t} - x_t' \beta} f_\varepsilon(s) \mathrm{d}s. \tag{11}$$

In this formula, the last term denotes the probability to observe a pseudo-censored price duration between the observable lower and upper duration bound, $\tau_{t,\min}$ and $\tau_{t,\max}$.

3.3. Accounting for unobserved heterogeneity

In the duration literature, a wide range of studies analyzes the impact of unobserved heterogeneity on the estimates of the hazard function and proposes different ways to account for such effects in duration models.[17] Along the lines of the work of Han and Hausman (1990), we specify heterogeneity effects by a random variable which enters the hazard function multiplicatively. An important advantage of this approach is that the survivor function of the compounded model can be calculated in closed form and does not require numerical integration. Including a gamma distributed random variable ω_t with $\mathrm{E}[\omega_t]=1$ and $\mathrm{Var}[\omega_t]=\theta^{-1}$, a mixed proportional hazard model is obtained. Under the assumption that ω_t is independent of x_t, the conditional hazard, given the covariates and unobservable heterogeneity effects, is

$$\lambda(\tau_t|x_t,\omega_t)=\lambda_0(\tau_t)\exp(-x_t'\beta)\cdot\omega_t, \qquad t=1,\ldots,T. \tag{12}$$

In this framework, the baseline survivor function is obtained by

$$S_0(\tilde{\tau}_k)=\frac{1}{\left[1+\exp(\delta_k-\ln(\theta))\right]^{\theta}}, \qquad k=1,\ldots,K-1. \tag{13}$$

It is easily shown that this model converges to the original model based on the extreme value distribution if $\theta^{-1}=\mathrm{Var}(\omega)\to 0$. Hence, if no unobservable effects exist, the compounded model corresponds to the standard proportional hazard model given in Eq. (6).

3.4. Application to volatility estimation

The use of the described model for grouped durations becomes apparent if one reformulates σ_t^{*2} as in Eq. (5),

$$\sigma_t^{*2}=\alpha^2\sum_{k=1}^{K-1}\mathrm{Prob}\left[\tau_t\in(\tilde{\tau}_{k-1},\tilde{\tau}_k]|x_t\right]\cdot\mathrm{E}\left[\frac{1}{\tau_t}\middle|\tau_t\in(\tilde{\tau}_{k-1},\tilde{\tau}_k\right],x_t], \tag{14}$$

and notes that in the context of grouped durations, the approximation

$$\mathrm{E}\left[\frac{1}{\tau_t}\middle|\tau_t\in(\tilde{\tau}_{k-1},\tilde{\tau}_k\right],x_t]\approx\mathrm{E}\left[\frac{1}{\tau_t}\middle|\tau_t\in(\tilde{\tau}_{k-1},\tilde{\tau}_k\right]],$$

must be valid, so that there is an obvious sample estimator of the second factor in the weighted sum of Eq. (14). This does not exclude the conditioning information,

[17] See e.g. Cox (1972), Lancaster (1979), Heckman and Singer (1984), Kiefer (1988), Meyer (1990), Honoré (1990), Lancaster (1994), Bearse et al. (1994) and An (1998).

but merely expresses the fact that all information contained in the explanatory variables enters only the first factor $\text{Prob}[\tau_t \in [\tilde{\tau}_{k-1}, \ \tilde{\tau}_k) \,|\, x_t]$ for which an estimator is readily available, using

$$\Pr[\tilde{\tau}_{k-1} < \tau_t \leq \tilde{\tau}_k | x_t] = \int_{\delta_{k-1} - x_t'\beta}^{\delta_k - x_t'\beta} f_\varepsilon(s)\,\mathrm{d}s.$$

An important assumption in this ordered response approach is the conditional independence of the error terms. All serial dependence has to be captured by the x_t, leading to conditionally i.i.d. error terms. To check for this assumption, we use the approach of Gourieroux et al. (1985) to test for serial correlation in the latent errors. The test corresponds to a score test based on the concept of generalized residuals which is applied to the case of an extreme value distribution.[18]

4. Data

4.1. The generation of price events

Our sample contains intraday data of the LIFFE Bund future trading. Within the observation period, the Bund future was one of the most actively traded future contracts in Europe and was a notional 6% German government bond of DEM 250.000 face value which matured in 8.5–10.5 years at contract expiration. There were four contract maturities per year, March, June, September and December. Prices were denoted in basis points of the face value, thus one tick was equivalent to a contract value of DEM 25. This study uses data on 11 contracts and 816 trading days between 4/5/94 and 6/30/97 with a total of about $T^{\circ} \approx 2 \times 10^6$ transactions, which amounts to about 2500 transactions per day.

The dataset contains time stamped prices, bids, asks and volumes associated with the transactions. The price series consists of T° arrival times and T° prices $\{(\vartheta_1, p_1), \ldots, (\vartheta_{T^{\circ}}, p_{T^{\circ}})\}$. From these pairs price durations are obtained by thinning the process and selecting only some of the pairs. Thus, the price durations are obtained by the time between the points which are kept. Similar to Engle and Russell (1997), the retained series can be defined formally as follows:

(a) Retain the first point (ϑ_1, p_1) of the point process (point 1).
(b) Retain point (ϑ_t, p_t), $t > 1$, if $|p_t - p_r| \geq \alpha$, where r is the index of the most recent retained point (ϑ_r, p_r).

α is constant and can be chosen arbitrarily. In the empirical analysis we use values of $\alpha \in \{5, 10, 15, 20\}$, corresponding to 28,650, 7491, 3560, and 2166

[18] For more details, see e.g. Hautsch (1999).

observations (see Table 1 for details). As discussed in Section 3, price events can also be caused by news occurring overnight leading to pseudo-censored observations. Because the resulting price changes are observable only at the beginning of the next trading day, it is difficult to identify whether the price changes are driven by overnight news or by information which occurred recently. For this reason, we consider the first price event which occurs within the first 15 min of a trading day to be pseudo-censored, i.e. a price change which is driven by events occurring overnight. This assumption implies that price events occurring after the first 15 min of a trading day are driven by recent information. Thus, for these observations, the exact waiting time is measured as the time since the last observation of the previous trading day. Fig. 1 shows the arrival times of three transactions occurring on different trading days. The spell between A and B is assumed to be pseudo-censored and lasts at least 15 min and at most 15 h and 30 min, where the waiting time between A and C is assumed to be non-censored and, thus, is measured exactly as 16 h.

By using 5, 10, 15 and 20-tick durations we obtain 779, 600, 457 and 318 pseudo-censored durations.

As dependent variable we use the categorized price durations. We choose a categorization which ensures satisfactory frequencies in each category. Tables 2 and 3 in Appendix A show the categorization and the distribution of the categorized durations for the different sizes of price changes.

4.2. The inclusion of seasonalities

Intradaily seasonalities of the volatility are a well known problem in empirical research of market microstructure, see e.g. Wood et al. (1985) or Müller et al. (1997). Engle and Russell (1997) present strong empirical evidence for seasonalities in price durations. One solution is to generate seasonally adjusted series by partialling out the time-of-day effects. Engle and Russell (1997) regress the duration on a function of the time and obtain an estimator of the typical shape. By dividing the durations by their estimated seasonality factor, a seasonally adjusted series is obtained.

Here, a Fourier series approximation is employed, as proposed by Andersen and Bollerslev (1998b) based on the work of Gallant (1981). This framework

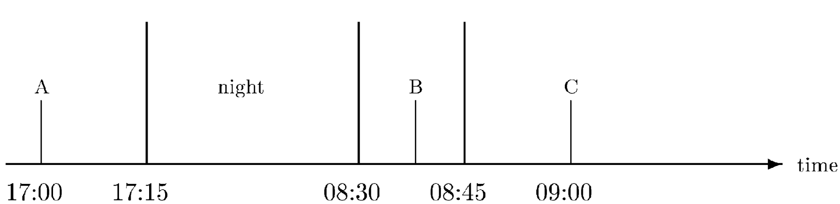

Fig. 1. Identification of pseudo-censored durations.

allows for a direct but parsimonious assessment of the volatility patterns based on different frequencies, like intraday seasonalities in contrast to time-to-maturity seasonalities. Additional regressors of the form t^*, $\cos(t^* \cdot 2\pi p)$, and $\sin(t^* \cdot 2\pi p)$ are included in the regression, where $t^* \in [0,1]$ denotes the standardized time of the day and p is the order of the term. The total effect of seasonalities using the coefficients μ_1, $\mu_{c,p}$, and $\mu_{s,p}$ and the order of approximation P is given by

$$s(\mu, t^*, P) = \mu_1 \cdot t^* + \sum_{p=1}^{P} \left(\mu_{c,p} \cos(t^* \cdot 2\pi p) + \mu_{s,p} \sin(t^* \cdot 2\pi p) \right). \tag{15}$$

By using terms up to order 6, it is possible to estimate volatility patterns on different frequency levels. The time-to-maturity regressors are based on a sample length of 150 days. To account for observations with values of more than 150 days to maturity, we include a dummy variable. For the analysis of intraday seasonalities

$$t_1^* = \frac{\text{seconds since 7:30}}{\text{seconds between 7:30 and 16:15}} \tag{16}$$

is used as a reference and for the analysis of time-to-maturity effects

$$t_2^* = \frac{\text{days to maturity}}{150}. \tag{17}$$

Note that explanatory variables are included with one lag, i.e. the seasonality variables are associated with the beginning of the particular spells.

5. Empirical findings

5.1. Estimation results

The investigation of volatility patterns based on price changes of different sizes gives an indication about the size of fluctuations of the process. Consider e.g. a price process which fluctuates frequently around a certain price level leading e.g. to a high number of five-tick price events but, in the extreme case, to no 20-tick price events. Such price movements are associated with a low 20-tick volatility combined with a high five-tick volatility. Consider now price movements causing the same frequency of five-tick price events but along with high amplitudes. In this case, the five-tick volatility is unchanged but the 20-tick volatility is higher. Hence, this example illustrates that the comparison of duration-driven volatilities based on price intervals of different sizes allows us to get deeper insight into the pattern of price movements.

Tables 4–7 give the results for the duration estimation using 5, 10, 15 and 20-tick price changes. For each tick size, we estimate three specifications: one model without any explanatory variables (panel A), one regression with only

Table 1
Number of (non-censored) and pseudo-censored observations per contract and overall

Contract	5 Ticks		10 Ticks		15 Ticks		20 Ticks	
	Obs.	Cens. Obs.	Obs.	Cens. Obs.	Obs.	Cens. Obs.	Obs.	Cens. Obs.
1	4001	77	1082	62	530	48	330	35
2	2332	70	580	49	277	38	170	21
3	2659	70	685	59	312	41	185	32
4	2693	67	735	57	338	49	196	36
5	1973	73	494	43	230	30	137	17
6	3104	73	818	59	371	46	221	33
7	2820	73	768	56	378	44	247	37
8	1760	69	433	50	216	40	129	23
9	1957	69	512	48	230	35	146	25
10	2773	68	731	57	366	42	205	27
11	2578	70	653	60	312	44	200	32
Total	28,650	779	7491	600	3560	457	2166	318

Based on BUND futures trading at LIFFE.

Table 2
Distribution of categorized price durations based on 5 and 10-tick price changes

Categories	5 Ticks				10 Ticks			
	Non-Censored			Cens.	Non-Censored			Cens.
	Obs	Mean	Std. Dv.	Obs	Obs	Mean	Std. Dv.	Obs
[0 min, 1 min]	2324	0.51	0.29	77	161	0.49	0.26	12
(1 min, 2 min]	2480	1.51	0.29	66	144	1.49	0.29	14
(2 min, 4 min]	4156	2.94	0.57	93	240	3.07	0.58	29
(4 min, 8 min]	5388	5.84	1.15	139	511	6.02	1.17	46
(8 min, 12 min]	3346	9.87	1.15	97	474	10.06	1.15	43
(12 min, 20 min]	3665	15.51	2.29	117	829	15.81	2.28	72
(20 min, 30 min]	2302	24.44	2.92	79	757	24.65	2.81	75
(30 min, 1 h]	2331	41.51	8.48	70	1350	43.03	8.69	149
(1 h, 2 h]	1019	81.55	16.41	21	974	84.46	16.81	79
(2 h, 3 h]	289	144.07	17.36	3	375	145.91	17.05	32
(3 h, 4 h]	99	205.54	15.68	2	214	207.81	17.09	5
(4 h, 6 h]	89	288.28	33.88	8	249	293.42	36.50	12
(6 h, 8 h]	19	388.32	24.98	5	89	407.12	33.21	21
(8 h, 24 h]	203	1097.81	150.73	2	311	1089.20	177.29	11
(24 h, 36 h]	40	1670.56	207.16	0	59	1621.92	168.52	0
(36 h, 48 h]	23	2668.96	213.18	0	36	2708.46	174.35	0
(48 h, ∞)	98	5238.70	3461.27	0	118	5050.50	2397.13	0
Total	27,871	47.87	395.07	779	6891	321.62	912.70	600

Based on BUND futures trading at LIFFE. Means and standard deviations (only for non-pseudo-censored durations) in minutes. Pseudo-censored observations are stated by their corresponding lower boundaries.

Table 3
Distribution of categorized price durations based on 15 and 20-tick price changes

Categories	15 Ticks				20 Ticks			
	Non-Censored			Cens.	Non-Censored			Cens.
	Obs	Mean	Std.Dv.	Obs	Obs	Mean	Std.Dv.	Obs
[0 min, 1 min]	44	0.58	0.20	6	11	0.49	0.20	3
(1 min, 2 min]	21	1.56	0.28	4	7	1.21	0.17	2
(2 min, 4 min]	35	3.08	0.57	4	11	2.88	0.72	2
(4 min, 8 min]	89	5.90	1.23	17	21	5.80	1.08	1
(8 min, 12 min]	117	10.08	1.18	15	27	10.30	0.96	7
(12 min, 20 min]	177	15.80	2.32	39	67	16.02	2.30	19
(20 min, 30 min]	207	25.15	2.94	46	65	25.47	3.04	21
(30 min, 1 h]	528	43.92	8.47	85	232	44.73	8.63	41
(1 h, 2 h]	489	43.92	16.64	85	243	87.33	17.46	55
(2 h, 3 h]	251	84.90	17.78	36	134	149.25	16.55	27
(3 h, 4 h]	146	146.65	17.23	15	85	209.46	17.03	13
(4 h, 6 h]	240	204.80	35.66	23	158	293.29	34.62	15
(6 h, 8 h]	138	294.75	34.35	30	130	406.71	37.31	30
(8 h, 24 h]	339	406.28	222.35	52	294	1145.20	218.72	79
(24 h, 36 h]	83	1110.03	148.70	0	116	1627.54	155.37	3
(36 h, 48 h]	40	2731.32	160.93	0	51	2694.04	170.45	0
(48 h, ∞)	163	4841.94	2931.29	0	196	4875.33	2809.73	0
Total	3103	539.95	1309.20	457	1848	968.74	1739.66	318

Based on BUND futures trading at LIFFE. Means and standard deviations (only for non-pseudo-censored duration) in minutes. Pseudo-censored observations are stated by their corresponding lower boundaries.

intraday seasonalities (panel B) and one specification including all seasonalities (panel C). As a means of comparison over the different specifications, we included the Bayes Information Criterion (BIC). A specification analysis based on a test on serial correlation and the assessment of unobserved heterogeneity complements the empirical results.

For all four different price durations, it is shown that the goodness of fit is significantly improved by the inclusion of time-to-maturity seasonalities. Looking at the BIC values, one can see that the explanatory power of the time-to-maturity seasonalities seems to increase for larger price changes while the explanatory power of included intraday patterns seems to decrease slightly. Hence, the larger the analyzed price changes, the higher the impact of the lower frequent time-to-maturity seasonality effects while the strength of intradaily patterns weakens.

Most of the intraday and time-to-maturity seasonalities' coefficients are significant on a 5% level, whereas the significance decreases slightly for higher price changes caused by smaller sample sizes.[19] This is true for all but the five-tick

[19] Note that with Eq. (6), we have $\partial \ln \lambda(\tau_t \mid x_t)/\partial x_t = -\beta$, i.e. β is interpreted as the constant proportional effect of the covariates x_t on the conditional failure probability. Note that a positive sign indicates higher durations and thus lower volatilities.

Table 4
Estimation of proportional hazard models for grouped durations

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Thresholds						
δ_1	−2.4670	0.0000	−2.4825	0.0000	−2.6196	0.0000
δ_2	−1.6909	0.0000	−1.7010	0.0000	−1.8324	0.0000
δ_3	−0.9729	0.0000	−0.9757	0.0000	−1.0983	0.0000
δ_4	−0.3494	0.0000	−0.3415	0.0000	−0.4518	0.0000
δ_5	−0.0180	0.0110	−0.0012	0.4856	−0.1024	0.0012
δ_6	0.3498	0.0000	0.3809	0.0000	0.2932	0.0000
δ_7	0.6133	0.0000	0.6589	0.0000	0.5840	0.0000
δ_8	0.9724	0.0000	1.0431	0.0000	0.9941	0.0000
δ_9	1.2299	0.0000	1.3170	0.0000	1.2945	0.0000
δ_{10}	1.3422	0.0000	1.4312	0.0000	1.4225	0.0000
δ_{11}	1.3906	0.0000	1.4778	0.0000	1.4752	0.0000
δ_{12}	1.4421	0.0000	1.5256	0.0000	1.5295	0.0000
δ_{13}	1.4546	0.0000	1.5368	0.0000	1.5423	0.0000
δ_{14}	1.6402	0.0000	1.7092	0.0000	1.7384	0.0000
δ_{15}	1.6948	0.0000	1.7614	0.0000	1.7978	0.0000
δ_{16}	1.7334	0.0000	1.7980	0.0000	1.8394	0.0000
Intraday seasonalities						
μ_1			0.2192	0.0004	0.1895	0.0022
$\mu_{s,1}$			0.2929	0.0000	0.2927	0.0000
$\mu_{s,2}$			−0.0108	0.2077	−0.0164	0.1135
$\mu_{s,3}$			−0.0379	0.0002	−0.0402	0.0001
$\mu_{s,4}$			0.0927	0.0000	0.0920	0.0000
$\mu_{s,5}$			−0.0226	0.0094	−0.0284	0.0017
$\mu_{s,6}$			−0.0600	0.0000	−0.0613	0.0000
$\mu_{c,1}$			−0.2216	0.0000	−0.2347	0.0000
$\mu_{c,2}$			0.0881	0.0000	0.0890	0.0000
$\mu_{c,3}$			−0.0333	0.0001	−0.0343	0.0001
$\mu_{c,4}$			0.0177	0.0232	0.0184	0.0212
$\mu_{c,5}$			0.0909	0.0000	0.0904	0.0000
$\mu_{c,6}$			−0.0419	0.0000	−0.0432	0.0000
Seasonalities over the future's maturity						
μ_1^*					−0.1600	0.0000
$\text{Dummy}_{>150}$					0.2315	0.0000
$\mu_{s,1}^*$					−0.0678	0.0000
$\mu_{s,2}^*$					0.2043	0.0000
$\mu_{s,3}^*$					−0.1048	0.0000
$\mu_{s,4}^*$					−0.0690	0.0000
$\mu_{s,5}^*$					−0.0785	0.0000
$\mu_{s,6}^*$					−0.0222	0.0096
$\mu_{c,1}^*$					0.0481	0.0000
$\mu_{c,2}^*$					0.0170	0.0335
$\mu_{c,3}^*$					−0.0008	0.4663
$\mu_{c,4}^*$					0.0083	0.1835

Table 4 (*continued*)

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Seasonalities over the future's maturity						
$\mu^*_{c,5}$					0.0304	0.0004
$\mu^*_{c,6}$					0.0328	0.0003
BIC						
	−63126		−62257		−61382	

Based on BUND future trading, LIFFE, using five-tick price changes; 28,650 observations, *p*-values based on asymptotic *t*-statistics.

price changes, which deviate in some respects substantially from the higher aggregates. This will subsequently be discussed in greater detail.

5.2. Seasonalities over the future's maturity

One set of explanatory variables in the model's specification captures the effects due to the decreasing time to maturity over the life of a BUND future. In this context, we employ the trigonometric expansion based on the future life cycle, as described in Section 4.2. A reasonable interpretation of the time-to-maturity seasonalities requires looking at all regressors within the Fourier series approximation.[20] Based on this flexible form, we calculate the price change volatility σ_t^{*2} based on all coefficients.

Fig. 2 shows the pattern of σ_t^{*2} depending on the time to maturity for 20 tick price changes, while the intraday seasonality coefficients are fixed on a value corresponding to 14:00. We find the highest volatility values within a time horizon of about 90 days to maturity. In the Bund future market, there are expiration days every 3 months. Therefore, these high trading activities seem to be caused by the roll-over from the previous contract to the front month contract. The strong increase of the volatility at the roll-over might be explained by a kind of price finding process concerning the front month contract leading to a higher frequency of 20-tick price changes. This process seems to be finished after 90 days to maturity leading to a strongly decreasing pattern. Note in this context, that it has been observed frequently, that the future's volatility is higher than the volatility of the underlying, see e.g. the analysis of MacKinlay and Ramaswamy (1988) for the S&P 500 stock index future. On the other hand, in the same paper by MacKinlay and Ramaswamy, the mean absolute mispricing of the future with respect to the

[20] The interpretation of single coefficients is not reasonable, because the seasonality patterns are based on the sum of all included terms. If only the significant coefficients were retained, some type of filtering would be applied, which is however hard to interpret as the expansion terms are not orthogonal.

Table 5
Estimation of proportional hazard models for grouped durations

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Thresholds						
δ_1	−3.8274	0.0000	−3.6079	0.0000	−4.7691	0.0000
δ_2	−3.1777	0.0000	−2.9559	0.0000	−4.1151	0.0000
δ_3	−2.5780	0.0000	−2.3545	0.0000	−3.5109	0.0000
δ_4	−1.8737	0.0000	−1.6481	0.0000	−2.7993	0.0000
δ_5	−1.4612	0.0000	−1.2341	0.0000	−2.3807	0.0000
δ_6	−0.9467	0.0000	−0.7176	0.0000	−1.8552	0.0000
δ_7	−0.5826	0.0004	−0.3508	0.0000	−1.4783	0.0000
δ_8	−0.0192	0.1111	0.2243	0.0000	−0.8768	0.0724
δ_9	0.3915	0.0000	0.6579	0.0000	−0.3983	0.0016
δ_{10}	0.5727	0.0000	0.8544	0.0000	−0.1671	0.1080
δ_{11}	0.6888	0.0000	0.9787	0.0000	−0.0153	0.4548
δ_{12}	0.8463	0.0000	1.1443	0.0000	0.1983	0.0712
δ_{13}	0.9147	0.0000	1.2157	0.0000	0.2974	0.0137
δ_{14}	1.2595	0.0000	1.5743	0.0000	0.8795	0.0000
δ_{15}	1.3493	0.0000	1.6628	0.0000	1.0370	0.0000
δ_{16}	1.4177	0.0000	1.7296	0.0000	1.1590	0.0000
Intraday seasonalities						
μ_1			0.5994	0.0000	0.6838	0.0000
$\mu_{s,1}$			0.4431	0.0000	0.4924	0.0000
$\mu_{s,2}$			−0.0406	0.0322	−0.0307	0.0936
$\mu_{s,3}$			−0.0120	0.2731	−0.0257	0.1030
$\mu_{s,4}$			0.0398	0.0184	0.0175	0.1824
$\mu_{s,5}$			−0.0332	0.0427	−0.0728	0.0001
$\mu_{s,6}$			−0.0576	0.0010	−0.0374	0.0231
$\mu_{c,1}$			−0.0074	0.3434	0.0676	0.0001
$\mu_{c,2}$			0.0891	0.0000	0.1394	0.0000
$\mu_{c,3}$			−0.0043	0.4028	0.00331	0.0294
$\mu_{c,4}$			0.0679	0.0001	0.0816	0.0000
$\mu_{c,5}$			0.0748	0.0000	0.0754	0.0000
$\mu_{c,6}$			−0.0214	0.1185	−0.0253	0.0808
Seasonalities over the future's maturity						
μ_1^*					−1.6062	0.0000
$\text{Dummy}_{>150}$					0.7014	0.0027
$\mu_{s,1}^*$					0.0463	0.2832
$\mu_{s,2}^*$					0.2011	0.0000
$\mu_{s,3}^*$					−0.0398	0.1247
$\mu_{s,4}^*$					0.0917	0.0008
$\mu_{s,5}^*$					0.0266	0.1392
$\mu_{s,6}^*$					−0.0170	0.2019
$\mu_{c,1}^*$					0.5790	0.0000
$\mu_{c,2}^*$					−0.0907	0.0002
$\mu_{c,3}^*$					−0.0290	0.1266
$\mu_{c,4}^*$					0.0061	0.4046

Table 5 (*continued*)

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Seasonalities over the future's maturity						
$\mu^*_{c,5}$					−0.0829	0.0003
$\mu^*_{c,6}$					0.0150	0.2445
BIC						
	−17488		−17394		−16709	

Based on BUND future trading at LIFFE, using 10-tick price changes; 7491 observations; *p*-values based on asymptotic *t*-statistics.

underlying is found to decrease over the future's life cycle, which is a quite intuitive result as the price of the future should converge to the spot price at maturity. Thus, it is not surprising that we find a decreasing volatility pattern after the roll-over. After about 80 days to maturity, we observe a stabilization of the volatility pattern at a relatively constant level where we cannot identify the beginning of the next roll-over period from this contract to the following front month contract. This might be explained by the fact that, theoretically, the stochastic properties of the future should converge to the underlying asset with decreasing time to maturity, i.e. the volatility pattern of the future should be mainly driven by the volatility of the underlying asset.

The results found for 20-tick price changes can be confirmed by analyzing the volatility patterns based on the price change durations for 15 and 10-tick price changes.[21] The results are mainly driven by the highly significant negative slope included in the time-to-maturity seasonals. This relates to the findings of Bessembinder and Seguin (1992) based on the S&P 500 stock index future that report evidence leading in the same direction. However, they report only an insignificant trend coefficient based on daily data. Note that the similarity of volatility patterns based on price changes of different sizes means that a higher frequency of small price changes is also combined with a higher frequency of larger price changes, i.e. fluctuations around a certain price level are combined with movements of the price level. Based on our findings, these relationships seem to hold for 10, 15 and 20-tick price changes.

The fact that five-tick price changes (Fig. 3) do not reflect this pattern and are characterized by a trend with a much lower but still significant coefficient can be well explained by the impact of transaction costs, which allow the future to fluctuate in a certain band without allowing for profitable trading strategies based on the forward-contract pricing relation. Thus, the roll-over seems not to alter the rate of five-tick price changes, compared to other periods in the future's life cycle.

[21] Because the volatility patterns are very similar, we do not present them in the paper.

Table 6
Estimation of proportional hazard models for grouped durations

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Thresholds						
δ_1	−4.4809	0.0000	−4.2363	0.0000	−5.1755	0.0000
δ_2	−4.0555	0.0000	−3.8097	0.0000	−4.7476	0.0000
δ_3	−3.5962	0.0000	−3.3491	0.0000	−4.2852	0.0000
δ_4	−2.9240	0.0000	−2.6749	0.0000	−3.6078	0.0000
δ_5	−2.4129	0.0000	−2.1623	0.0000	−3.0915	0.0000
δ_6	−1.9167	0.0000	−1.6657	0.0000	−2.5906	0.0000
δ_7	−1.5134	0.0000	−1.2621	0.0000	−2.1821	0.0000
δ_8	−0.8128	0.0000	−0.5565	0.0000	−1.4617	0.0000
δ_9	−0.3256	0.0000	−0.0537	0.1390	−0.9330	0.0000
δ_{10}	−0.0968	0.2481	0.1895	0.0001	−0.6687	0.0001
δ_{11}	0.0354	0.0618	0.3299	0.0000	−0.5130	0.0016
δ_{12}	0.2616	0.0000	0.5674	0.0000	−0.2431	0.0804
δ_{13}	0.4060	0.0000	0.7184	0.0000	−0.0648	0.3541
δ_{14}	0.9004	0.0000	1.2373	0.0000	0.6017	0.0003
δ_{15}	1.0360	0.0000	1.3748	0.0000	0.7958	0.0000
δ_{16}	1.1132	0.0000	1.4529	0.0000	0.0969	0.0000
Intraday seasonalities						
μ_1			0.5862	0.0000	0.5788	0.0000
$\mu_{s,1}$			0.3853	0.0000	0.3636	0.0000
$\mu_{s,2}$			−0.0700	0.0175	−0.0757	0.0116
$\mu_{s,3}$			−0.0464	0.0651	−0.0859	0.0020
$\mu_{s,4}$			0.0404	0.0906	0.0322	0.1405
$\mu_{s,5}$			−0.0584	0.0294	−0.0952	0.0009
$\mu_{s,6}$			−0.0483	0.0514	−0.0210	0.2353
$\mu_{c,1}$			0.0965	0.0005	0.2130	0.0000
$\mu_{c,2}$			0.1318	0.0000	0.1860	0.0000
$\mu_{c,3}$			−0.0228	0.2170	0.0141	0.3169
$\mu_{c,4}$			0.0165	0.2830	0.0221	0.2174
$\mu_{c,5}$			0.0741	0.0044	0.0616	0.0133
$\mu_{c,6}$			−0.0443	0.0557	−0.0447	0.0588
Seasonalities over the future's maturity						
μ_1^*					−1.2794	0.0000
$\text{Dummy}_{>150}$					0.6380	0.0173
$\mu_{s,1}^*$					0.0631	0.2751
$\mu_{s,2}^*$					0.2063	0.0003
$\mu_{s,3}^*$					−0.0490	0.1400
$\mu_{s,4}^*$					0.0988	0.0061
$\mu_{s,5}^*$					0.0379	0.1361
$\mu_{s,6}^*$					0.0147	0.3105
$\mu_{c,1}^*$					0.5430	0.0000
$\mu_{c,2}^*$					−0.0466	0.0803
$\mu_{c,3}^*$					−0.0291	0.0916
$\mu_{c,4}^*$					−0.0035	0.4576

Table 6 (*continued*)

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Seasonalities over the future's maturity						
$\mu^*_{c,5}$					−0.0625	0.0252
$\mu^*_{c,6}$					0.0353	0.1245
BIC						
	−8203		−8206		−7891	

Based on BUND future trading at LIFFE, using 15-tick price changes; 3560 observations; *p*-values based on asymptotic *t*-statistics.

A clear identification of seasonality effects over the future's maturity is not possible. In general, the volatility shape is less pronounced which indicates that time-to-maturity seasonalities seem to have a weaker influence on 5-tick price changes than on 20-tick price changes. This result confirms the findings above, that lower frequent seasonality effects have a stronger impact the larger the analyzed price changes are.

5.3. Intraday seasonalities

The seasonal patterns of the intraday price change volatility for a fixed time to maturity of 30 days based on 5 and 20-tick price durations are depicted in Figs. 4 and 5.[22] For the intraday shape of volatility, previous studies have pointed out that trading and especially the opening of related exchanges play a major role, see e.g. the analysis of Harvey and Huang (1991) on currency futures or the work of Gourieroux et al. (1999) on trading at the Paris Bourse. The main corresponding exchange apart from the DTB in Frankfurt, where a similar contract is traded, is probably the CBOT, which is the main exchange for futures on US government fixed income securities. They open at 7:20 CT which corresponds usually to 13:20 GMT. Trading at the NYSE on he other hand starts at 9:30 ET, corresponding to 14:30 GMT, usually.

For five-tick price changes, Fig. 5 indicates a volatility shape with a decreasing pattern after the opening, reaching a low around 12:30, which is quite intuitive given the well documented fact that intraday volatility drops after the opening phase (see e.g. Wood et al., 1985 and many others). The sharp increase in volatility around 13:20 fits in well with the opening of the CBOT. Volatility does not remain on a high level, but drops continually with an additional spike before 15:00 which may be associated with the opening of the NYSE. Yet, the volatility level remains for the entire afternoon on a higher level than in the morning. This is well in line with the above mentioned results of Harvey and Huang (1991) who

[22] Note that the time is given as GMT.

Table 7
Estimation of proportional hazard models for grouped durations

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Thresholds						
δ_1	−5.2785	0.0000	−5.0319	0.0000	−5.8568	0.0000
δ_2	−4.7841	0.0000	−4.5370	0.0000	−5.3512	0.0000
δ_3	−4.3038	0.0000	−4.0564	0.0000	−4.8797	0.0000
δ_4	−3.7532	0.0000	−3.5054	0.0000	−4.3274	0.0000
δ_5	−3.3131	0.0000	−3.0651	0.0000	−3.8857	0.0000
δ_6	−2.6638	0.0000	−2.4168	0.0000	−3.2352	0.0000
δ_7	−2.2682	0.0000	−2.0221	0.0000	−2.8385	0.0000
δ_8	−1.4343	0.0000	−1.1902	0.0000	−1.9980	0.0000
δ_9	−0.8912	0.0000	−0.6441	0.0000	−1.4390	0.0000
δ_{10}	−0.6493	0.0000	−0.3967	0.0000	−1.1824	0.0000
δ_{11}	−0.5074	0.0000	−0.2509	0.0002	−1.0297	0.0000
δ_{12}	−0.2574	0.0000	0.0063	0.4643	−0.7563	0.0001
δ_{13}	−0.0510	0.0453	0.2212	0.0008	−0.5230	0.0061
δ_{14}	0.5227	0.0000	0.8227	0.0000	0.1619	0.2189
δ_{15}	0.7383	0.0000	1.0444	0.0000	0.4379	0.0175
δ_{16}	0.8474	0.0000	1.1578	0.0000	0.5862	0.0025
Intraday seasonalities						
μ_1			0.5669	0.0000	0.4299	0.0005
$\mu_{s,1}$			0.2452	0.0000	0.1331	0.0084
$\mu_{s,2}$			−0.0853	0.0322	−0.0677	0.0688
$\mu_{s,3}$			−0.0455	0.1550	−0.1111	0.0059
$\mu_{s,4}$			−0.0027	0.4749	−0.0129	0.3811
$\mu_{s,5}$			−0.0255	0.2709	−0.0527	0.1067
$\mu_{s,6}$			−0.0736	0.0285	−0.0801	0.0194
$\mu_{c,1}$			0.0972	0.0079	0.1962	0.0000
$\mu_{c,2}$			0.0527	0.0847	0.1173	0.0013
$\mu_{c,3}$			−0.0699	0.0294	−0.0529	0.0806
$\mu_{c,4}$			0.0314	0.2007	0.0272	0.2374
$\mu_{c,5}$			−0.0049	0.4500	−0.0451	0.1202
$\mu_{c,6}$			−0.0122	0.3741	0.0072	0.4253
Seasonalities over the future's maturity						
μ_1^*					−1.0764	0.0023
$\text{Dummy}_{>150}$					0.6383	0.0372
$\mu_{s,1}^*$					0.0985	0.2151
$\mu_{s,2}^*$					0.2400	0.0003
$\mu_{s,3}^*$					−0.0555	0.1545
$\mu_{s,4}^*$					0.0713	0.0730
$\mu_{s,5}^*$					0.0201	0.3206
$\mu_{s,6}^*$					0.0334	0.1888
$\mu_{c,1}^*$					0.4458	0.0000
$\mu_{c,2}^*$					−0.0655	0.0627
$\mu_{c,3}^*$					−0.0862	0.0197
$\mu_{c,4}^*$					0.0004	0.4957

Table 7 (*continued*)

Variable	A		B		C	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Seasonalities over the future's maturity						
$\mu^{*}_{c,5}$					−0.0601	0.0704
$\mu^{*}_{c,6}$					0.0650	0.0483
BIC						
	−4850		−4910		−4764	

Based on BUND future trading at LIFFE, using 20-tick price changes; 2166 observations; *p*-values based on asymptotic *t*-statistics.

find a significantly increased volatility level when corresponding exchanges are opened. The spike in the beginning is well explained by a kind of price finding process at the opening, where the price information conveyed by US traders needs to be scrutinized.

Analyzing the 20-tick volatility indicates a less pronounced intradaily pattern. This result confirms the findings based on the time-to-maturity volatility shapes, i.e. intraday volatility seems to be influenced mainly by more frequent but smaller price changes, while time-to-maturity volatility patterns are dominated by lower frequent but larger price changes. The results for 20-tick price changes (Fig. 4) are somewhat similar with respect to the peak around 13:20. Clearly, the CBOT opening influence can still be identified, yet the volatility levels drops faster than for the five-tick price changes, due to the fact that additional US traders do not generate price events above 20 ticks, after the initial information dissemination. The various spikes in the morning are somewhat harder to understand, yet, casual evidence suggests that these relate to meetings of German central bank's governing body (Zentralbankratsitzungen) and interest rate changes announced afterwards. A closer analysis of these effects is, however, clearly beyond the scope of this paper.

5.4. Serial correlation and heterogeneity

To check for misspecification due to serial dependence in the error terms, we use a test on serial correlation derived from Gourieroux et al. (1985).[23] Tables 8–10 show the test statistics based on the regressions for 5, 10, 15 and 20-tick durations for all three models A, B, and C. Comparing the test statistics of the different specifications, it is apparent that the intraday and time-to-maturity seasonalities capture serial correlation structures. Thus, serial dependency can

[23] Here, some attention has to be given to the fact that the distribution is not from the linear exponential family. See Hautsch (1999) for an extended discussion of the test.

Table 8
Test of regressions in Tables 4, 5, 6 and 7, columns (A), on misspecification due to serial correlation based on generalized residuals, lags 1–15

	5 Ticks	10 Ticks	15 Ticks	20 Ticks
Lag 1	3791.0519	138.2295	26.1135	2.0112
Lag 2	3040.2352	99.0246	59.5995	0.0297
Lag 3	2546.1531	225.1310	49.7752	0.0120
Lag 4	2069.2367	184.0805	2.7822	0.0283
Lag 5	1877.4584	76.8120	16.4400	0.1522
Lag 6	1639.0206	93.4919	6.2720	0.0065
Lag 7	1482.1860	59.6506	1.3449	0.4143
Lag 8	1442.4890	77.0252	2.1646	0.2685
Lag 9	1232.5707	30.3741	0.6818	1.0024
Lag 10	1228.1525	22.6624	29.7690	8.7705
Lag 11	1113.4457	20.2531	1.2279	0.0126
Lag 12	997.8533	41.3155	18.2551	0.0436
Lag 13	1118.5379	31.3082	10.7979	0.4059
Lag 14	1054.7821	34.8707	0.0570	0.1062
Lag 15	1020.5851	37.9009	8.1900	0.2438

H_0: No serial correlation at lag i. Critical values at 1%, 5% and 10% level: 6.635, 3.841 and 2.706.

partly be attributed to seasonalities on different frequency levels. This confirms the result of Engle and Russell (1997, 1998) who also show that the serial correlation in price durations can be reduced by seasonal adjustments.

Furthermore, we find that the test statistics are lower if larger price changes are considered, i.e. the larger the investigated price durations, the weaker serial dependency in the error terms. Based on five-tick price changes, we find extremely high test statistics for all three specifications indicating the existence of serial correlations. While for 10-tick price changes (specification C) only a slight serial dependency in the errors is found, these effects diminish for 15 and 20-ticks. These results support the assumption of conditional independence of the error terms, given the covariates, as assumed in Section 3.

To check for unobservable heterogeneity effects, we also examine a gamma compounded hazard model[24] (see Table 11). The variance of the heterogeneity variable increases the larger the price changes. For 20 ticks, we find for the heterogeneity variance a value of 0.7233, i.e. this specification almost corresponds to an ordered logit approach.[25] The increasing heterogeneity effects may be caused by the stronger impact of unobservable news effects. The relative importance of

[24] The volatility patterns computed based on these regressions are very similar to the figures obtained in the standard case. For this reason, we do not show these figures in the paper.
[25] Note that the BurrII (1) distribution is identical to the logistic distribution.

Table 9
Test of regressions in Tables 4, 5, 6 and 7, columns (B), on misspecification due to serial correlation based on generalized residuals, lags 1–15

	5 Ticks	10 Ticks	15 Ticks	20 Ticks
Lag 1	2778.4539	31.8606	9.6541	1.9492
Lag 2	2253.1421	37.2283	47.0633	0.0008
Lag 3	1952.0922	127.5896	35.8254	0.0868
Lag 4	1659.3659	121.3210	0.8140	0.0416
Lag 5	1588.9543	53.4275	1.5870	0.2243
Lag 6	1456.4895	68.2622	1.4101	0.0590
Lag 7	1360.6450	24.8154	0.1531	1.0953
Lag 8	1411.9811	47.7679	0.1553	0.0251
Lag 9	1214.0267	10.8690	0.2576	0.9905
Lag 10	1257.6358	7.9792	7.9216	6.1718
Lag 11	1155.1365	8.2022	0.9604	0.1325
Lag 12	1035.3788	10.4162	2.6921	0.0007
Lag 13	1134.8300	9.3055	2.5769	0.3632
Lag 14	1085.1326	7.2006	0.4668	0.0007
Lag 15	1087.4755	19.6313	0.3666	0.8676

H_0: No serial correlation at lag i. Critical values at 1%, 5% and 10% level: 6.635, 3.841 and 2.706.

macro news, which is not captured by explanatory variables, increases with the size of the analyzed price changes and decreases with the sample size. This feature coincides with the result of Hautsch (1999), who obtained only very small

Table 10
Test of regressions in Tables 4, 5, 6 and 7, columns (C), on misspecification due to serial correlation based on generalized residuals, lags 1–15

	5 Ticks	10 Ticks	15 Ticks	20 Ticks
Lag 1	2266.7398	2.0146	0.4018	0.1680
Lag 2	1657.6089	3.1089	5.3400	0.4804
Lag 3	1475.0306	9.3399	11.3430	0.3076
Lag 4	1088.9026	21.7761	0.1554	0.3422
Lag 5	1123.3051	4.6887	0.2617	0.0559
Lag 6	958.04800	10.2703	1.6982	0.2731
Lag 7	919.46822	0.2062	0.2520	0.1246
Lag 8	906.83266	2.5044	0.2601	0.6475
Lag 9	750.75089	0.0051	0.4018	1.0296
Lag 10	801.14390	0.1704	2.6998	5.7707
Lag 11	729.86078	0.2846	0.0298	0.1706
Lag 12	647.31812	1.5852	1.3803	0.6927
Lag 13	654.15629	1.2160	2.1455	0.0001
Lag 14	683.06744	0.0429	1.5153	0.6727
Lag 15	594.86937	1.8349	0.4841	0.0902

H_0: No serial correlation at lag i. Critical values at 1%, 5% and 10% level: 6.635, 3.841 and 2.706.

Table 11
Estimation of gamma compounded proportional hazard models for grouped durations

Variable	5 Ticks		10 Ticks		15 Ticks		20 Ticks	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Thresholds								
δ_1	−2.8493	0.0000	−5.1680	0.0000	−5.5049	0.0000	−6.0458	0.0000
δ_2	−2.0191	0.0000	−4.5042	0.0000	−5.0697	0.0000	−5.5451	0.0000
δ_3	−1.2116	0.0000	−3.8865	0.0000	−4.5974	0.0000	−5.0574	0.0000
δ_4	−0.4487	0.0000	−3.1473	0.0000	−3.8995	0.0000	−4.4953	0.0000
δ_5	−0.0038	0.4662	−2.7028	0.0000	−3.3600	0.0000	−4.0432	0.0000
δ_6	0.5404	0.0000	−2.1276	0.0000	−2.8272	0.0000	−3.3734	0.0000
δ_7	0.9759	0.0000	−1.6968	0.0000	−2.3800	0.0000	−2.9582	0.0000
δ_8	1.6534	0.0000	−0.9555	0.0000	−1.5379	0.0000	−2.0466	0.0000
δ_9	2.2153	0.0000	−0.3081	0.0768	−0.8474	0.0034	−1.3912	0.0001
δ_{10}	2.4827	0.0000	0.0216	0.4608	−0.4783	0.0664	−1.0677	0.0024
δ_{11}	2.6010	0.0000	0.2454	0.1341	−0.2531	0.2159	−0.8688	0.0117
δ_{12}	2.7294	0.0000	0.5729	0.0059	0.1571	0.3172	−0.4984	0.1028
δ_{13}	2.7607	0.0000	0.7286	0.0008	0.4452	0.0935	−0.1636	0.3435
δ_{14}	3.2650	0.0000	1.6577	0.0000	1.6471	0.0000	0.9500	0.0221
δ_{15}	3.4303	0.0000	1.9293	0.0000	2.0314	0.0000	1.4510	0.0023
δ_{16}	3.5510	0.0000	2.1399	0.0000	2.2617	0.0000	1.7335	0.0006
Intraday seasonalities								
μ_1	0.0014	0.4938	0.9573	0.0000	1.1864	0.0000	0.8655	0.0001
$\mu_{s,1}$	0.3640	0.0000	0.7276	0.0000	0.7588	0.0000	0.3817	0.0003
$\mu_{s,2}$	−0.0730	0.0001	−0.0009	0.4898	−0.0115	0.4144	−0.1190	0.0380
$\mu_{s,3}$	−0.0951	0.0000	−0.0006	0.4911	−0.0995	0.0137	−0.1838	0.0024
$\mu_{s,4}$	0.1500	0.0000	0.0755	0.0052	0.0650	0.0695	0.0030	0.4799
$\mu_{s,5}$	−0.0458	0.0009	−0.0925	0.0002	−0.1652	0.0001	−0.1170	0.0346
$\mu_{s,6}$	−0.1022	0.0000	−0.0411	0.0513	−0.0786	0.0335	−0.1039	0.0357
$\mu_{c,1}$	−0.3725	0.0000	0.0094	0.03800	0.2544	0.0000	0.3018	0.0000
$\mu_{c,2}$	0.1311	0.0000	0.1876	0.0000	0.3490	0.0000	0.2580	0.0001
$\mu_{c,3}$	−0.0752	0.0000	0.0387	0.0652	0.0784	0.0277	−0.0795	0.0688
$\mu_{c,4}$	0.0106	0.2214	0.1392	0.0000	0.1077	0.0075	0.0520	0.1813
$\mu_{c,5}$	0.1432	0.0000	0.1498	0.0000	0.1403	0.0005	0.0135	0.4054
$\mu_{c,6}$	−0.0976	0.0000	−0.0225	0.1804	−0.0842	0.0170	−0.0096	0.4307
Seasonalities over the future's maturity								
μ_1^*	−0.2655	0.0000	−2.1196	0.0000	−1.8308	0.0005	−1.3739	0.0181
$\text{Dummy}_{>150}$	0.2744	0.0000	1.1195	0.0019	1.1802	0.0124	1.1166	0.0412
$\mu_{s,1}^*$	−0.1157	0.0000	0.0972	0.2152	0.1223	0.2529	0.1961	0.1867
$\mu_{s,2}^*$	0.2828	0.0000	0.3230	0.0000	0.3795	0.0001	0.4406	0.0006
$\mu_{s,3}^*$	−0.1609	0.0000	−0.0197	0.3464	−0.0165	0.4132	−0.0069	0.4705
$\mu_{s,4}^*$	−0.1005	0.0000	0.1566	0.0001	0.1739	0.0037	0.1484	0.0364
$\mu_{s,5}^*$	−0.1415	0.0000	0.0553	0.0543	0.0486	0.1793	0.0337	0.3054
$\mu_{s,6}^*$	−0.0315	0.0088	−0.0225	0.2049	−0.0035	0.4681	0.0263	0.3187
$\mu_{c,1}^*$	0.0772	0.0000	0.8124	0.0000	0.8512	0.0000	0.6818	0.0000
$\mu_{c,2}^*$	−0.0013	0.4618	−0.1277	0.0005	−0.0730	0.0867	−0.1043	0.0573
$\mu_{c,3}^*$	−0.0075	0.2814	−0.0431	0.1209	−0.0720	0.0876	−0.1595	0.0114
$\mu_{c,4}^*$	−0.0111	0.1989	−0.0007	0.4922	−0.0116	0.4114	−0.0186	0.3875

Table 11 (*continued*)

Variable	5 Ticks		10 Ticks		15 Ticks		20 Ticks	
	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value	Coeff.	*p*-value
Seasonalities over the future's maturity								
$\mu^*_{c,5}$	0.0480	0.0001	−0.1388	0.0001	−0.1123	0.0152	−0.1181	0.0422
$\mu^*_{c,6}$	0.0482	0.0002	0.0355	0.1160	0.0213	0.3218	0.1196	0.0220
Heterogeneity variance								
$V[\omega_t]$	0.5280	0.0000	0.4369	0.0000	0.6628	0.0000	0.7233	0.0000
BIC								
	−61219		−16664		−7844		−4749	

Based on BUND future trading at LIFFE using 5, 10, 15 and 20 tick price changes; 28,650, 7491, 3560 and 2166 observations; *p*-values based on asymptotic *t*-statistics.

heterogeneity effects by analyzing intertrade durations. Preliminary studies on the basis of individual contracts support the hypothesis that unobservable heterogeneity is caused by news effects. This is surely an issue for further research but a thorough analysis is not an issue in this study.

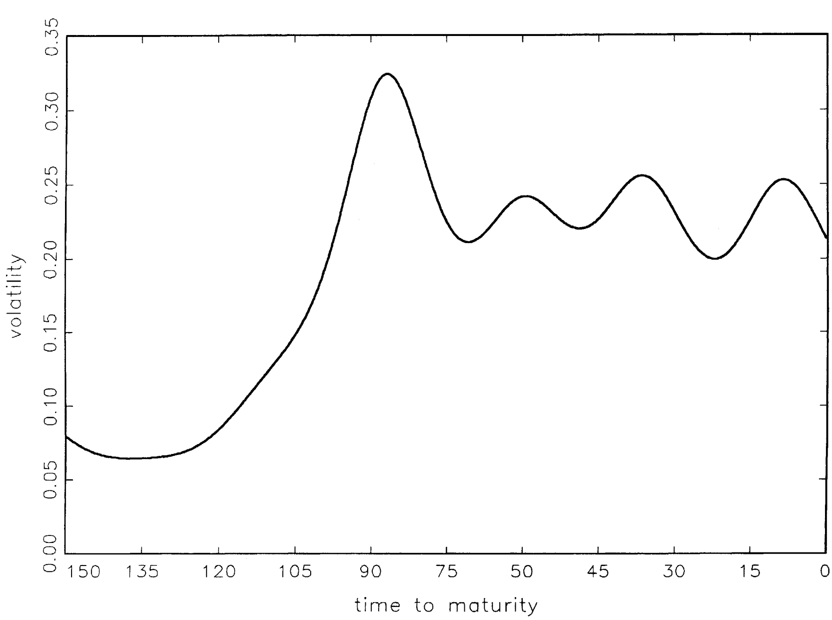

Fig. 2. Twenty-tick price change volatility vs. time to maturity, 14:00, Bund future trading, LIFFE, London.

6. Conclusions

In this study, we estimate volatilities based on transaction data by using price intensities, i.e. the time between price changes of a given size. In this context, we build a volatility estimator on the time between price events. These price events are defined as the end of a price duration, i.e. the time it takes to complete a price change of a certain tick size which is exogenously given. Using these price durations in order to estimate volatilities solves the problem of finding an appropriate aggregation level by defining explicitly the event of interest. Hence, the frequency of observations is determined automatically by the definition of the price event.

The fact that a market is not open 24 h a day leads to censoring mechanisms, since price events occurring overnight are not observable. We only know when the spells started, but do not know exactly when they were finished, because the resulting price change can only be observed at the beginning of the following trading day. Hence, for these durations, we can only register lower and upper bounds, and thus we call these durations ‘pseudo-censored’. To account for such effects, we use a proportional hazard model for grouped durations based on the

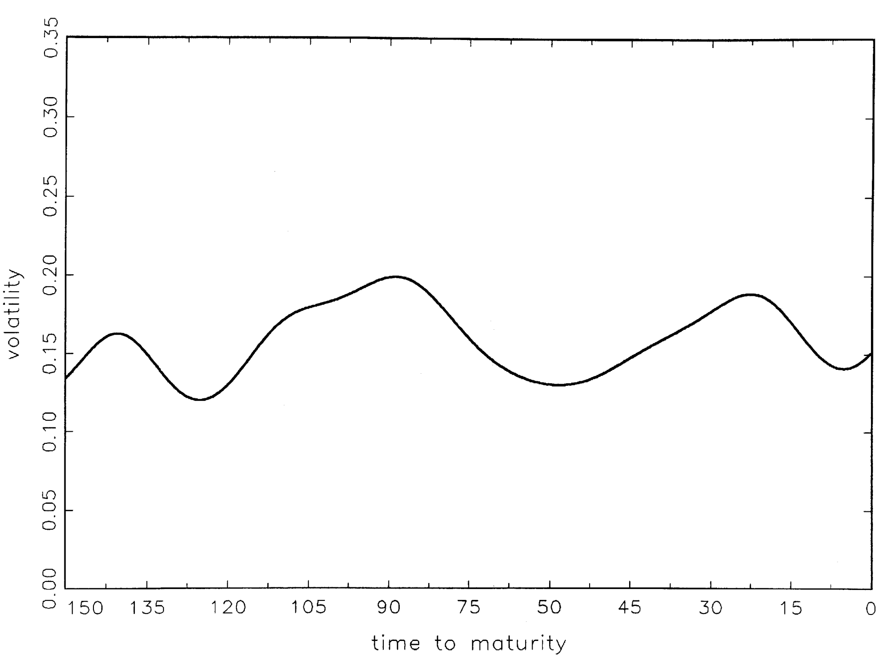

Fig. 3. Five-tick price change volatility vs. time to maturity, 14:00, Bund future trading, LIFFE, London.

work of Han and Hausman (1990). In this framework, a pseudo-censored observation enters the log likelihood function by its lower and upper bound.

We use a dataset containing LIFFE Bund future transactions on 11 contracts. To take intraday seasonalities and time-to-maturity seasonalities into account, we include a Fourier series approximation proposed by Andersen and Bollerslev (1998b) which allows for an assessment of volatility patterns based on the different frequencies.

It is shown that the goodness of fit is significantly improved by the inclusion of time-to-maturity seasonalities, where the explanatory power of these variables seem to increase the larger the analyzed price durations are. Hence, the larger the underlying price changes, the higher the impact of lower frequent time-to-maturity effects, while the impact of intradaily seasonalities weakens.

Comparing volatility shapes based on price changes of different sizes allows us to gain deeper insight into the size and the frequency of fluctuations of the price process. Based on the 20-tick volatility, we find a significant peak in the volatility around 90 days to maturity, caused by the roll-over from the previous contract to the front month contract. This peak is followed by a decreasing pattern which might be explained by the convergence of the future's volatility to the volatility of

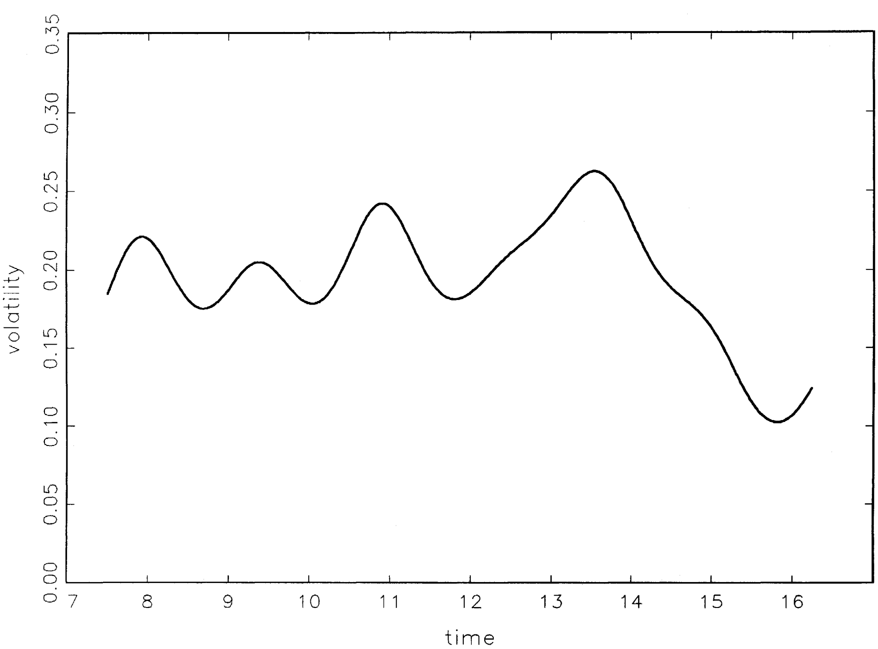

Fig. 4. Twenty-tick price change intraday volatility vs. time, 30 days to maturity, Bund future trading, LIFFE, London.

the underlying with decreasing time to maturity. This finding is confirmed by the volatility patterns based on 10 and 15 ticks. The five-tick volatility pattern does not reflect this shape and indicates a relatively constant frequency of five-tick price changes over the future's life cycle.

The shape of intraday price change volatilities seems to be dominated mainly by higher frequent but smaller price changes leading to a more pronounced intraday volatility pattern based on five-tick price changes. We find a decreasing pattern after the opening with a low around 12:30 and volatility peaks at 13:20 and before 15:00, which may be associated with the opening of the CBOT and the NYSE. The 20-tick volatility pattern is less pronounced but somewhat similar with respect to the 13:20 peak. Furthermore, the opening of the NYSE cannot be identified based on 20-tick price changes, thus, the entry of additional US traders in the market seems not to generate sufficiently large price changes.

By checking for misspecifications due to serial correlation and unobserved heterogeneity, we find evidence for the fact that intraday and time-to-maturity seasonalities play an important role in capturing serial dependency and that serial correlation tends to diminish the larger the analyzed price changes are. Furthermore, we find empirical evidence for the fact that the heterogeneity effects seem to

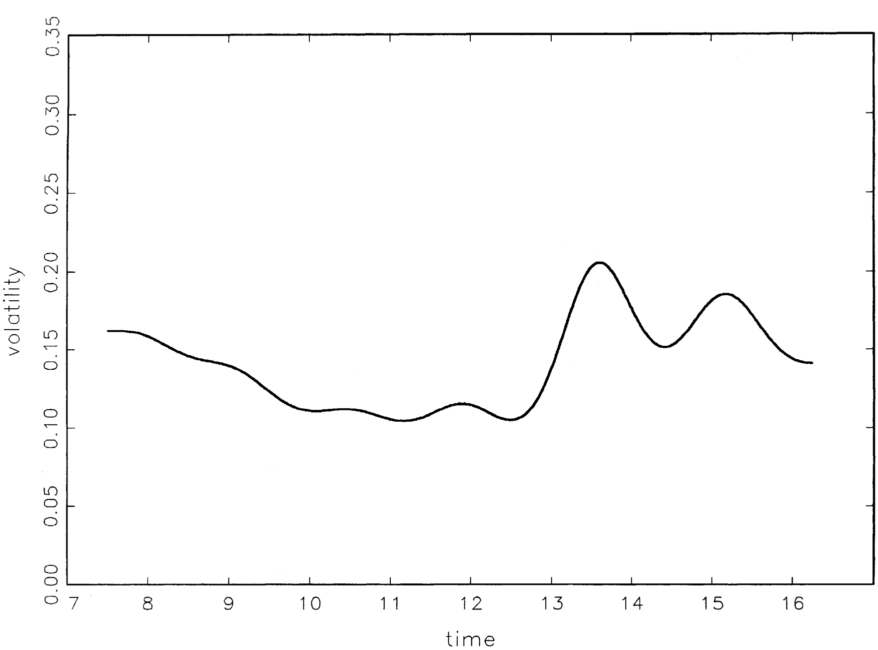

Fig. 5. Five-tick price change intraday volatility vs. time, 30 days to maturity, Bund future trading, LIFFE, London.

increase the larger the analyzed price changes are. A fact that might be explained by stronger impacts of unobservable news effects due to smaller sample sizes.

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References

An, M.Y., 1998. Consistent estimation of regression coefficients in mixed proportional hazard models. Discussion Paper June, presented at the ‘Conference on panel Data and Structural Labor Market Models’, Sandbjerg, Danemark.

Andersen, T.G., Bollerslev, T., 1997. Intraday periodicity and volatility presistence in financial markets. Journal of Empirical Finance 4, 115–158.

Andersen, T.G., Bollerslev, T., 1998a. Answering the skeptics: yes, standard volatility models do provide accurate forecasts. International Economic Reviews 39 (6023), 885–905.

Andersen, T.G., Bollerslev, T., 1998b. Deutsche mark-dollar volatility: intraday activity patterns, macroeconomics announcements, and longer run dependencies. Journal of Finance 53, 219–265.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 1999. The distribution of the exchange rate volatility, Discussion Paper 6961, NBER Working Paper Series.

Ball, C., 1988. Estimation bias induced by discrete security prices. Journal of Finance 43, 841–865.

Bauwens, L., Giot, P., 2000. The logarithmic ACD model: an application to the bid/ask quote process of two NYSE stocks. Annales d’Economie et de Statistique 60, 117–149.

Bauwens, L., Veredas, D., 1999. The stochastic conditional duration model: a latent factor model for the analysis of financial durations, Discussion Paper 9958, CORE.

Bearse, P., Canals, J., Rilstone, P., 1994. Semiparametric maximum likelihood estimation of duration models with unobserved heterogeneity, Discussion paper, Department of Economics, York University.

Bessembinder, H., Seguin, P.J., 1992. Futures-trading activity and stock price volatility. Journal of Finance 47, 2015–2034.

Bollerslev, T., Domowitz, I., 1993. Trading patterns and prices in the interbank foreign exchange market. Journal of Finance 48, 1421–1443.

Bollerslev, T., Ghysels, E., 1996. Periodic autoregressive conditional heteroscedasticity. Journal of Business and Economic Statistics 14, 139–151.

Cho, D.C., Frees, E.W., 1988. Estimating the volatility of discrete stock prices. Journal of Finance 43, 451–466.

Clark, P.K., 1973. A subordinated stochastic process model with finite variance for speculative prices. Econometrica 41, 135–155.

Cox, D.R., 1972. Regression models and life-tables. Journal of the Royal Statistical Society, Series B 34, 187–200.

Cox, D.R., 1975. Partial Likehood. Biometrika 62, 269.

Cox, D.R., Oakes, D., 1984. Analysis of Survival Data. Chapman & Hall, London.

Dacorogna, M.M., Müller, U.A., Nagler, R.J., Olsen, R.B., Pictet, O.V., 1993. A geographical model for the daily and weekly seasonal volatility in the foreign exchange market. Journal of International Money and Finance 12, 413–438.

Dacorogna, M.M., Gauvreau, C.L., Müller, U.A., Olsen, R.B., Pictet, O.V., 1996. Changing time scale for short-term forecasting in financial markets. Journal of Forecasting 15, 203–227.

Easley, D., Kiefer, N.M., O'Hara, M., 1997. The information content of the trading process. Journal of Empirical Finance 4, 159–186.

Engle, R.F., 1996. The econometrics of ultra-high frequency data, Discussion paper, UCSD.

Engle, R.F., 2000. The econometrics of ultra-high frequency data. Econometrica 68 (1), 1–22.

Engle, R.F., Russell, J.R., 1997. Forecasting the frequency of changes in quoted foreign exchange prices with the autoregressive conditional duration model. Journal of Empirical Finance 4, 187–212.

Engle, R.F., Russell, J.R., 1998. Autoregressive conditional duration: a new model for irregularly spaced transaction data. Econometrica 66, 1127–1162.

Gallant, R.A., 1981. On the bias in flexible functional forms and an essential unbiased form: the fourier flexible form. Journal of Econometrics 15, 211–245.

Gallant, A.R., Rossi, P.E., Tauchen, G., 1992. Stock prizes and volume. Review of Financial Studies 5, 199–242.

Gerhard, F., Pohlmeier, W., 2000. On the simultaneity of absolute price changes and transaction duration, Discussion paper, University of Konstanz.

Ghysels, E., Jasiak, J., 1997. GARCH for irregularly spaced financial data: the ACD-GARCH model, Discussion Paper 97s-06, CIRANO, Montreal.

Giot, P., 2000. Time transformations, intraday data and volatility models. Journal of Computational, 4.

Gottlieb, G., Kalay, A., 1985. Implications of the discreteness of observed stock prices. Journal of Finance 40, 135–154.

Gourieroux, C., Monfort, A., Trognon, A., 1985. A general approach to serial correlation. Econometric Theory 1, 315–340.

Gourieroux, C., Jasiak, J., LeFol, G., 1999. Intra-day market activity. Journal of Financial Markets 2, 193–226.

Grammig, J., Maurer, K.-O., 2000. Non-monotonic hazard functions and the autoregressive conditional duration model. Econometrics Journal 3, 16–38.

Grammig, J., Wellner, M., 1999. Modeling the interdependence of volatility and inter-transaction duration processes, Discussion Paper, Humboldt-University, Berlin, 21.

Guillaume, D.M., Dacorogna, M.M., Dave, R.R., Müller, U.A., Olsen, R.B., Pictet, O.V., 1997. From the bird's eye to the microscope: a survey of new stylized facts of the intra-daily foreign exchange markets. Finance and Stochastics 1, 95–129.

Han, A., Hausman, J.A., 1990. Flexible parametricestimation of duration and competing risk models. Journal of Applied Econometrics 5, 1–28.

Harris, L., 1990. Estimation of stock variances and serial covariances from discrete observations. Journal of Financial and Quantitative Analysis 25, 291–306.

Harris, L.E., 1994. Minimum price variations, discrete bid-ask spreads, and quotation sizes. Review of Financial Studies 7, 149–178.

Harvey, C.R., Huang, R.D., 1991. Volatility in the foreign currency futures market. Review of Financial Studies 4, 543–569.

Hausman, J.A., Lo, A.W., MacKinlay, A.C., 1992. An ordered probit analysis of transaction stock prices. Journal of Financial Economics 31, 319–379.

Hautsch, N., 1999. Analyzing the time between trades with a gamma compounded hazard model. an application to LIFFE bund future transactions, Discussion Paper 99/03, CoFE, Konstanz.

Hautsch, N., 2001. The generalized F ACD model, Mimeo, University of Konstanz.

Heckman, J., Singer, B., 1984. Econometric duration analyses. Journal of Econometrics 24, 63–132.
Honoré, B.E., 1990. Simple estimation of a duration model with unobserved heterogeneity. Econometrica 58, 453–473.
Jasiak, J., 1999. Persistence in intratrade durations, Discussion Paper, York University, Montreal.
Jones, C.M., Kaul, G., Lipson, M.L., 1994. Transactions, volume, and volatility. Review of Financial Studies 7, 631–651.
Kalbfleisch, J.D., Prentice, R.L., 1980. The Statistical Analysis of Failure Time Data. Wiley.
Kiefer, N.M., 1988. Economic duration data and hazard functions. Journal of Economic Literature 26, 646–679.
Lancaster, T., 1979. Econometric methods for the duration of unemployment. Econometrica 47, 939–956.
Lancaster, T., 1994. The Econometric Analysis of Transition Data. Cambridge Univ. Press, New York.
Lunde, A., 2000. A generalized gamma autoregressive conditional duration model, Discussion paper, Aarlborg University.
MacKinlay, A.C., Ramaswamy, K., 1988. Index-futures arbitrage and the behavior of stock index futures prices. Review of Financial Studies 1, 137–158.
Manrique, A., Shephard, N., 1997. Likehood analysis of a discrete bid/ask price model for a common stock quoted on the NYSE, Discussion paper, Nuffield College, Oxford University.
McInish, T., Wood, R., 1992. An analysis of intraday patterns in bid/ask spreads for NYSE stocks. Journal of Finance 47, 753–764.
Meyer, B., 1990. Unemployment insurance and unemployment spells. Econometrica 58, 757–782.
Müller, U.A., Dacorogna, M.M., Dave, R.R., Olsen, R.B., Pictet, O.V., Weizäcker, J.E.V., 1997. Volatilities of different time resolutions-analyzing the dynamics of market components. Journal of Empirical Finance 4, 213–239.
Roll, R., 1984. A simple implicit measure of the effective bid-ask spread in an efficient market. Journal of Finance 39, 1127–1139.
Russell, J.R., Engle, R.F., 1998. Econometric analysis of discrete-valued, irregularly-spaced financial transactions data using a new autoregressive conditional multinomial model. Discussion paper, presented at Second International Conference on High Frequency Data in Finance, Zurich, Switzerland.
Rydberg, T.H., Shephard, N., 1999. Dynamic of trade-by-trade price movements: decomposition and models, Discussion paper, Nuffield College, Oxford, presented at Symposium on Microstructure and High Frequency Data, Paris, France.
Wood, R.A., McInish, T.H., Ord, J.K., 1985. An investigation of transactions data for NYSE stocks. Journal of Finance 40, 723–741.