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Coskewness and cokurtosis in futures markets

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Abstract

The contribution of the third and fourth moments in explaining the return-generating process in futures markets remains unresolved. This study attempts to resolve this issue by using a four-moment model and by sampling 28 futures contracts and nine market proxies. Such sampling provides wide representation of futures markets and lends a high degree of robustness to the results. Our results show that the second, third and fourth moments are all important in explaining futures returns. Evidence from regression tests show increases in explanatory power as the third and fourth moments are included. The results are robust to the market proxy used. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The nature of the return-generating process in futures markets remains an unresolved issue. Although the distribution of futures returns is well known, empirical tests that examine risk premia in futures markets yield conflicting results. Many studies in the past fail to detect evidence of risk premia in futures markets. For instance, studies that employ a traditional Capital Asset Pricing

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Model (CAPM) or an Arbitrage Pricing Framework (APT), often fail to detect significant risk premia in futures prices (see, e.g., Dusak, 1973; Baxter et al., 1985; Ehrardt et al., 1987), and those that use a pricing framework that departs from traditional models provide mixed results. For example, Raynauld and Tessier (1984) and Chang (1985) detect significant risk premia while Junkus (1991) does not. To shed new evidence on the return-generating process in futures markets, we examine 28 futures contracts and use nine different proxies for the market. The wide sampling of futures contracts ensures comprehensive representation of all futures markets, while the use of different market proxies ensures the results are robust.

Early tests of the return-generating process were mainly confined to equity markets. Such tests followed the development of the widely accepted two-moment CAPM, developed by Sharpe (1964), Lintner (1965), Mossin (1966), and later Black (1972). Tests of the CAPM are conducted by Friend and Blume (1970), Black et al. (1972), and Basu (1977), among others. These tests provide much insight into the functioning of financial markets. However, violations of the linear pricing kernel proposed by the CAPM found by some (see, e.g., Chen et al., 1986; Fama and French, 1992, 1993), prompted the search for alternative pricing models. Suggestions included modifications to the CAPM, such as the addition of higher moments (e.g., Arditti and Levy, 1975).

This study provides a four-moment extension to the two-moment CAPM, much in the spirit of Kraus and Litzenberger's (1976) three-moment extension to the standard CAPM. Fang and Lai (1997) and Dittmar (1999) present four-moment extensions in other contexts. The basic inferences of our model are consistent with these models.

2. Asset pricing, coskewness, and cokurtosis

The literature has noted the importance of higher moments for some time. For instance, Levy (1969) suggests that expected utility depends on all of the distribution's moments and that higher moments cannot be neglected. He notes that higher moments should be added even if they add little information about the shape of the distribution. Studies in equity markets address the importance of higher moments. Badrinath and Chatterjee (1988) using the g and h and $g \times h$ distributions proposed by Tukey (1977), as they provide a more general framework, find complicated forms of skewness and elongation (kurtosis) in the Center for Research in Security Prices (CRSP) value-weighted and equally-weighted indexes. They conclude that the estimation of skewness and kurtosis must be carried out in concert.

More recent studies in equity markets have refocused attention on higher moments. Harvey and Siddique (2000) make a strong case for the inclusion of skewness in an asset-pricing framework. They note that the standard CAPM does not adequately price equity returns, especially the smallest capitalized deciles, and suggest the inclusion of a third moment. The smallest deciles also display the most skewed returns. Their findings suggest that systematic skewness is economically important.

Dittmar (1999) makes a strong case for the inclusion of the fourth moment. He notes that a necessary condition for standard risk aversion is decreasing absolute prudence and theoretically links such preferences to an aversion for kurtosis (see Eqs. (1) and (2) in Dittmar, 1999). This, then, provides a rationale for the inclusion of kurtosis. Dittmar (1999) finds evidence that a four-moment CAPM is able to price the cross-section of returns much better than the traditional CAPM, and that it outperforms multifactor alternatives. More evidence supporting a four-moment model for pricing equity returns is provided by Fang and Lai (1997). Their findings show that expected excess returns are related to systematic variance, systematic skewness, and systematic kurtosis.

Clearly, strong arguments can be made for the inclusion of skewness and kurtosis in modeling equity returns. Such arguments have important implications for traders in futures markets, as these markets are characterized by evidence of higher moments. For instance, Stevenson and Bear (1970) examine corn and soybeans and conclude that their distributions are leptokurtic. Dusak (1973) finds evidence of substantial nonlinearities in futures distributions, and Hudson et al. (1987) find significant levels of leptokurtosis in their sample of futures. Given these findings and prudence considerations, higher moments are likely to be important in determining risk premia in futures markets. Thus, it is surprising that the literature examining the pricing of higher moments in futures markets is particularly sparse. While few studies examine the pricing of the third moment in futures markets, the study of the fourth moment has been neglected. This study attempts to fill this gap.

In general, documenting the importance of higher moments in futures markets could not only aid market participants to devise better trading strategies but could also provide useful information on market efficiency. However, issues relating to the importance of higher moments in futures markets are arguably more important beyond these usual motives. Evidence from this study is expected to add insight into the nature of the return-generating process in these markets. If the results show that the third and fourth moments are priced in futures markets, then asset-pricing models employed by investors will need to account for this.

Our priors on the pricing of higher moments in the return-generating process follow established theory. Scott and Horvath (1980), for instance, show that investors with a positive preference for positive skewness exhibit a negative preference for the fourth moment, if investors display consistent risk aversion and strict consistency of moment preference. Theoretical considerations lead us to hypothesize that investors will exhibit a preference for positive coskewness and a dislike for cokurtosis.

The results from our study show that both coskewness and cokurtosis are priced in futures markets. To illustrate, when we employ the S&P 500 index futures as the market proxy, the average R^2 values increase from 0.07 for the two-moment regression to 0.13 for the three-moment regression and to 0.22 for the four-moment regression. Almost identical results are obtained with the other market proxies. Much like the results of Dittmar (1999) in the equity market, we find that the four-moment CAPM improves upon the performance of the traditional CAPM.

The rest of this paper is organized as follows. In Section 3, a theoretical framework that models the return-generating process with a third and fourth moment is presented. The empirical model is then filtered out. In Section 4, important econometric issues are discussed, and in Section 5, the data and methodology are described. The empirical results are presented in Section 6, and a summary and conclusions offered in Section 7.

3. Theoretical framework: four-moment extension of the CAPM

Like Kraus and Litzenberger (1976) and Lim (1989), we examine the effects of unconditional skewness, a measure of asymmetry in the distribution. A distribution characterized by a longer tail in the positive direction displays positive skewness. As Harvey and Siddique (2000) point out, a rational investor will prefer positive skewness to negative skewness, ceteris paribus.

We also introduce unconditional kurtosis, a measure of flatness or peakedness in a distribution, into our model. This is motivated by the documentation of leptokurtosis in futures distributions in earlier studies and by the more recent findings of the importance of kurtosis by Dittmar (1999) in equity markets. Long-tailed distributions, termed leptokurtotic distributions, suggest that the outcomes are likely to be at the extremes. Since we are interested in the contribution of an asset to a portfolio's skewness (coskewness) and to a portfolio's kurtosis (cokurtosis), our unconditional measures of skewness and kurtosis are designed to measure an asset's coskewness and cokurtosis.

We denote the first four moments of investor's end-of-period wealth as follows

$$\overline{W} = \sum_j \theta_f \overline{R}_j + \theta_f R_f$$

$$\sigma_W = \sum_j \theta_j \beta_{jp} \sigma_p$$

$$S_W = \sum_j \theta_j \gamma_{jp} S_p$$

$$K_W = \sum_j \theta_j \delta_{jp} K_p \tag{1}$$

where

$\bar{R}_j$ = expected return on risky asset $j+1$

R_f = return on risk-free asset + 1.

The corresponding coefficients $(\theta_j,\ \theta_f)$ denote the investor's holdings in the risky and risk-free asset, respectively. β_{jp}, γ_{jp}, δ_{jp} are measures of covariance, coskewness, and cokurtosis of the risky asset j with the investor's portfolio p and given by

$$\beta_{jp} = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_p - \bar{R}_p\right)\right] / \sigma_p^2$$

$$\gamma_{jp} = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_p - \bar{R}_p\right)^2\right] / S_p^3$$

$$\delta_{jp} = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_p - \bar{R}_p\right)^3\right] / K_p^4 \tag{2}$$

.The standard deviation, skewness, and kurtosis of the investor's holding in risky assets are given by σ_p, S_p, and K_p shown below

$$\sigma_p = \left[E\left(\tilde{R}_p - \bar{R}_p\right)^2\right]^{1/2}$$

$$S_p = \left[E\left(\tilde{R}_p - \bar{R}_p\right)^3\right]^{1/3}$$

$$K_p = \left[E\left(\tilde{R}_p - \bar{R}_p\right)^4\right]^{1/4}. \tag{3}$$

To optimize the investor's end-of-period wealth, the first-order conditions from the following Lagrangian are determined

$$L = \zeta\left(\bar{W}, \sigma_W, S_W, K_W\right) - \lambda\left(\sum_j \theta_j + \theta_f - W_0\right). \tag{4}$$

W_0 is the investor's beginning wealth. The partial derivatives with respect to θ_j and θ_f, after grouping and subtracting like terms, give the following first-order conditions

$$\bar{R}_j - R_f = -\left(\frac{\zeta_{\sigma_W}}{\zeta_{\bar{W}}}\right)\beta_{jp}\sigma_p - \left(\frac{\zeta_{S_W}}{\zeta_{\bar{W}}}\right)\gamma_{jp}S_p - \left(\frac{\zeta_{K_W}}{\zeta_{\bar{W}}}\right)\delta_{jp}K_p \quad \forall j \tag{5}$$

where the terms in brackets in Eq. (5) are the investor's marginal rates of substitution between expected wealth and standard deviation, skewness, and kurtosis, respectively. To restrict the search for optimal portfolios, we assume (like Kraus and Litzenberger, 1976) that all investors have risk tolerance functions that are linear functions of wealth, with the same cautiousness, denoted by a. Such an assumption ensures that in equilibrium, the optimal holdings of each investor will be equal to the market risk portfolio. Therefore, for markets to clear, the excess return on the jth asset will be

$$\bar{R}_j - R_f = a_1 \beta_j + a_2 \gamma_j + a_3 \delta_j. \tag{6}$$

Hence, we replace the investor's portfolio with the market portfolio and have

$$\beta_j = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_m - \bar{R}_m\right)\right] / \sigma_m^2$$
$$\gamma_j = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_m - \bar{R}_m\right)^2\right] / S_m^3$$
$$\delta_j = E\left[\left(\tilde{R}_j - \bar{R}_j\right)\left(\tilde{R}_m - \bar{R}_m\right)^3\right] / K_m^4. \tag{7}$$

This is simplified to

$$\beta_j = \sigma_{jm} / \sigma_m^2$$
$$\gamma_j = S_{jmm} / S_m^3$$
$$\delta_j = K_{jmmm} / K_m^4. \tag{8}$$

Using the partial derivatives for the marginal rates of substitution, we write the coefficients as

$$a_1 = \left(\frac{\partial \bar{W}}{\partial \sigma_W} \right) \sigma_m$$
$$a_2 = \left(\frac{\partial \bar{W}}{\partial S_W} \right) S_m$$
$$a_3 = \left(\frac{\partial \bar{W}}{\partial K_W} \right) K_m \tag{9}$$

.This leads us to the empirical form of the model, which for time t is outlined below

$$\tilde{r}_j = a_0 + a_1 \tilde{\beta}_j + a_2 \tilde{\gamma}_j + a_3 \tilde{\delta}_j + e_j \tag{10}$$

where the term on the left-hand side designates the excess return, and the variables on the right-hand side represent measures of covariance, coskewness, and cokurtosis. Importantly, in this model, it is systematic risk that contributes to an asset's return. We also conduct regressions that incorporate only the second and third moments. The regression form is given below

$$\tilde{r}_j = a_0 + a_1 \tilde{\beta}_j + a_2 \tilde{\gamma}_j + e_j \tag{11}$$

where the term on the left-hand side designates the excess return, and the variables on the right-hand side represent measures of covariance and coskewness. To check the traditional form of the CAPM, we perform the following regression

$$\tilde{r}_j = a_0 + a_1 \tilde{\beta}_j + e_j \tag{12}$$

where the term on the left-hand side designates the excess return, and the term on the right-hand side represents a measure of covariance.

4. Some econometric issues and research design

To ensure the estimates from the regressions are precise, we adopt several procedures. The excess return is deflated by the risk-free rate to ensure the resulting values come from an independent and identically distributed (IID) random variable. To proxy for the risk-free rate, we use the return on Treasury bills. Other procedures address several criticisms of methodology related to risk premia noted in the literature.

4.1. The market proxy

The use of a proxy for the market portfolio always raises debate. Many suspect the results are sensitive to the market proxy used, and these fears are not altogether unsubstantiated, particularly so for futures markets. Carter et al. (1983) criticize an earlier study of risk premia in futures markets conducted by Dusak (1973) suggesting that her model is misspecified. Criticism is directed toward the particular market proxy employed by Dusak, the S&P 500 cash index. They suggest that a better proxy for the market should include some weighting in commodities. Carter et al. (1983) use the return on a combination of the S&P 500 cash index and a futures proxy, the Dow Jones Commodity futures index, in equal weights, for the return on the market and find a significant risk premium. However, Baxter et al. (1985), using different weights for the S&P 500 cash index (93.7%) and Dow Jones Commodity futures index (6.3%), fail to detect a risk premium. Such evidence suggests that the proxy for the market plays a role in studies of this nature in futures markets. We address the issue of an appropriate index in several ways. Futures on six market indexes, the S&P 500, the New York Composite, the KC Value Line, the CRB, the Major Market, and Dollar indexes, and three other indexes are used as proxies for the market. Of the other three indexes, two are constructed by a weighting procedure. Specifically, one is constructed by weighting futures returns on each of the six indexes equally (1/6 each, i.e., an equally weighted futures index) and the second is constructed by weighting 50% of the total equally in five index futures returns (the S&P 500 index futures are excluded) and the remaining 50% is weighted in the S&P 500 cash index returns. The ninth index is an all-equity index, the S&P 500 cash index. We employ such a wide range of proxies with the intention of addressing criticisms that question the composition and weighting of components in a market index for futures. While the equally weighted futures index is representative of the role futures might play in a market portfolio, the absence of equity in this index might lead to criticisms. To address this a blended cash/futures index is constructed. Finally, the S&P 500 cash index is offered to address arguments that point out that the net positions of futures contracts should sum to zero, and that a better proxy for the futures market should be an all-equity index. The upshot of the use of so many proxies is that robustness is built into the results, facilitating proper interpretations.

4.2. Returns calculation and contract rollovers

Returns on the futures instruments are calculated using the log form of returns. The return on a given commodity in any month M is given by

$$\text{Return} = \log\left(\frac{\text{Closing price}_M}{\text{Opening price}_M}\right). \tag{13}$$

To avoid jumps in futures prices due to contract expiration, the expiring contract is rolled over into the next-to-expire contract on the first day in the month of expiration. Theory relates investors ex-ante required rates of return to ex-ante beliefs concerning the behavior of prices (Kraus and Litzenberger, 1976). For the four-moment model, this implies investors will form subjective beliefs concerning ex-ante standard deviation (variance), as well as ex-ante skewness and kurtosis. Observing investors' subjective beliefs are not possible, so we represent these expectations by calculating the second, third, and fourth moments using proxies.

4.3. The two-pass methodology, Shanken's critique, and useless factors

The procedure we employ is a variant of the method used by Kraus and Litzenberger (1976) and is essentially a two-pass methodology that allows for time variation in the coefficient estimates. For instance, to get estimates of β, γ, and δ for commodity j in year Y, we calculate the estimates for the preceding 5-year (60-month) period on a monthly basis using the following equations

$$\begin{aligned}
\tilde{\beta}_j &= \left[\sum_t \left(\tilde{R}_{jt} - \bar{R}_{jt}\right)\left(\tilde{R}_{mt} - \bar{R}_{mt}\right)\right] \Big/ \sum_t \left[\left(R_{mt} - \bar{R}_{mt}\right)^2\right] \\
\tilde{\gamma}_j &= \left[\sum_t \left(\tilde{R}_{jt} - \bar{R}_{jt}\right)\left(\tilde{R}_{mt} - \bar{R}_{mt}\right)^2\right] \Big/ \sum_t \left[\left(R_{mt} - \bar{R}_{mt}\right)^3\right] \\
\tilde{\delta}_j &= \left[\sum_t \left(\tilde{R}_{jt} - \bar{R}_{jt}\right)\left(\tilde{R}_{mt} - \bar{R}_{mt}\right)^3\right] \Big/ \sum_t \left[\left(R_{mt} - \bar{R}_{mt}\right)^4\right].
\end{aligned} \tag{14}$$

These estimates are then used in monthly regression for commodity j in year Y.[1] The data we use begins in June 1982 and ends in December 1996. Our first set of estimates for β, γ, and δ, calculated using the months of July 1982 to May 1987 (a total of 60 months), is used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This procedure attempts to proxy as closely as possible investors' ex-ante beliefs. A total of 115 regressions are conducted. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors.

The two-pass methodology has some drawbacks. The problem of measurement error, or errors-in-variables, can be encountered. Shanken (1992) notes that this estimation error cannot be ignored even if the estimation error disappears in the limit, i.e., in large samples. What is important, he suggests, is the rate of convergence, reflected by the asymptotic variance of the second-pass estimator. Shanken recommends MLE estimation for small-sample bias in the second-pass cross-sectional regression estimate that may be induced by measurement error. We conduct additional tests using MLE procedures to address this problem. Kan and Zhang (1999) examine the performance of the two-pass method in the presence of useless factors. A useless factor defined by them to be a factor that is independent of asset returns if included will often be priced more than it should. They also note that this misspecification bias exacerbates when the number of time-series observations increases. Thus, we employ specification tests to examine the efficacy of our model.

4.4. Data spanning

Some misgivings concerning the span of the data set are legitimate. Tests of this nature in equity markets typically sample data over 20 years or more. Badrinath and Chatterjee (1988) use data that spans the period July 1962–December 1985, while Singleton and Wingender (1986) use data over the period 1961–1980. Exceptions do exist, especially for studies that examine futures and

[1] Kraus and Litzenberger (1976) use a grouping procedure initiated by Fama and Macbeth (1973). The estimates for the moments could incorporate measurement errors. Kraus and Litzenberger (1976) mitigate this problem by first ranking the stocks on the basis of their beta and gamma values and then by forming portfolios before the cross-sectional regressions are conducted. However, as Junkus (1991) points out, forming portfolios in futures markets is not possible because of the limited number of futures available.

options markets, given the limitations of data (spanning) availability in these markets. Ehrardt et al. (1987) employ settlement prices on wheat, corn, and soybeans from 1972 to 1980. Junkus (1991) uses 10 years of data and Cotner (1991) uses 5 years of data. We use data spanning 15 years for many of our tests. Further, robustness is built into the estimates by sampling data from a wide range of instruments, from commodities to interest rates, and by using nine different proxies for the market.

Given the earlier explanations, we can predict, a priori, the expected relationship between the excess return and the coefficients of our measures of systematic risk. We expect the coefficients to display the following signs: $a_1 > 0$, a_2 to exhibit a sign opposite to market skewness (i.e., if positive skewness exists, a negative coefficient is expected), and $a_3 > 0$. The rational behind these expectations is straightforward. Theory suggests that in general investors dislike variance and kurtosis and therefore will need to be compensated for the additional risk undertaken. However, investors will prefer positive skewness and therefore will be willing to accept a lower return.

5. Description of data and summary statistics

Monthly returns for 28 futures are sampled.[2] Data from the interest rate markets (Treasury bonds, Treasury notes, Eurodollars), commodity markets (soybean, soybean oil, soymeal, wheat, corn, cocoa, oats, rapeseed oil, sugar, coffee, orange juice, lumber, feeder cattle, live cattle, live hogs, pork bellies, heating oil), metal markets (gold, silver, and platinum), and the currency markets (British pound, Canadian dollars, Deutsche mark, Japanese yen, Swiss francs) are used. The first month sampled is June 1982 and the last month sampled is December 1996, a total of 175 months. Data for futures on the six indexes are also sampled. For three of the six indexes, the CRB, Dollar, and Major Market indexes, we have data from January 1987. In these instances, the number of regressions is reduced. For example, with the CRB index as the market proxy only 60 regressions can be conducted, whereas with the indexes with data beginning in June 1982, 115 regressions are possible.[3] When we construct a weighted index from the different indexes, data for the weighted index begins January 1987. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used.

[2] The data are obtained from Tick Data Incorporated (TDI). TDI compiles its database from official time and sales records. Any missing data points are filled in using real-time data feeds. The data are screened by TDI for missticks, gaps, and missing days.

[3] The number of regressions for each market index are 115 for the New York Composite, 115 for the KC Value Line, 115 for the S&P 500, and 60 each for the CRB Index, the Major Market Index, and the Dollar Index.

Table 1 provides characteristics of the sample. Relatively high variance characterizes futures returns on sugar, coffee, and pork bellies. In contrast, the currencies and interest rate instruments display low variance. Only five instruments (17.24%) are negatively skewed. Such evidence, although preliminary, suggests that the coefficient of systematic skewness is more likely to exhibit negative values than positive values. Like others, we find strong evidence of leptokurtosis in futures returns. Oats and corn, in particular, tend to be highly leptokurtic. Notice that futures displaying high variance do not necessarily exhibit leptokurtosis. Among the indexes, the KCValue Line, Major Market, New York Composite, and S&P 500 futures indexes display high levels of leptokurtosis. For all futures we compute β, γ, δ values (see Eq. (14)). If any pattern exists in these values, it is not readily apparent.[4]

6. Empirical results

6.1. Results from empirical tests

The regression procedure uses nine proxies for the market, six nonweighted indexes, two weighted indexes, and an all-equity index. For the nonweighted indexes, we describe in detail only the results for the S&P 500 index futures, as the results with the other nonweighted indexes are very similar. We also describe the results for the weighted indexes, because of dissimilarities in their construction, and the results for the equity index. The results from the two-pass regressions for the six nonweighted proxies are reported in Table 2.

For the S&P 500 index futures, the estimates of β, γ, and δ are the averages from 115 monthly regressions. The coefficient estimates from the regressions are then tested for significance by calculating t-values.[5] The R^2 value from each regression is then averaged across the regressions and these averages are also reported. This methodology follows the procedure adopted by Kraus and Litzenberger (1976) and by Junkus (1991). The estimates in Panel A suggest that the coefficient of the variance term in the two-moment test is significant and that investors price systematic risk. The coefficient is positive (0.364) and highly significant ($t = 9.45$). Assuming that the traditional form of the CAPM is misspecified, a second regression, which includes a coskewness term, is then performed. The coefficients from this regression are also significant and in the directions

[4] The computed β, γ, δ values are not reported for the sake of brevity, but are available from the authors on request.

[5] The t-values are equal to the mean value of the coefficient divided by its standard deviation. This tests whether the coefficient value is significantly different from zero.

hypothesized. Harvey and Siddique's (2000) finding that coskewness is priced in equity markets can now be extended to futures markets. The beta coefficient is again positive (0.347) and highly significant ($t = 10.48$) while the coskewness

Table 1
Summary statistics: monthly data for the futures (June 1982–December 1996)

Panel A

Futures	Mean $\times 10^4$	Variance $\times 10^4$	Skewness	Excess kurtosis
Soybean	−45.37	32.56	0.05	4.84***
Soybean oil	−24.58	50.35	0.74***	3.72***
Soymeal	−13.96	35.43	0.31	1.77*
Wheat	−1.94	33.87	0.26	1.70*
Corn	−36.69	40.32	0.76***	7.46***
Cocoa	−49.70	52.95	0.44*	0.78
Oats	−149.04	80.92	2.09***	16.39***
Rapeseed oil	−37.48	42.43	0.59**	4.59***
Sugar	−59.96	142.21	0.76***	2.80**
Coffee	4.09	105.98	2.72***	2.74**
Orange juice	−4.56	61.02	0.42*	2.80**
Lumber	−13.06	80.27	0.20	1.05
Feeder cattle	21.68	13.92	−0.21	1.02
Live cattle	55.64	17.21	0.10	0.06
Live hog	75.39	39.05	0.01	0.31
Pork bellies	−110.04	108.68	−0.06	1.10
Heating oil	49.27	79.96	−0.51*	4.00***
British pound	27.01	11.65	0.13	2.26**
Canadian dollar	6.70	1.65	−0.49*	1.39
Deutsche Mark	17.82	11.43	0.19	0.02
Japanese Yen	29.32	11.42	0.33	0.60
Swiss Francs	8.93	13.57	0.07	0.03
Eurodollars	13.14	0.30	0.78***	2.56***
TNotes	44.52	5.21	0.09	0.03
TBonds	57.72	10.72	0.16	0.13
Gold	−30.93	17.95	−0.16	3.14***
Platinum	−26.11	46.19	0.54*	3.12***
Silver	−61.79	55.92	0.08	3.00***

Panel B

Indexes	Mean $\times 10^4$	Variance $\times 10^4$	Skewness	Excess kurtosis
KC Valueline	69.87	23.01	1.84***	11.38***
NY Composite	76.34	16.61	1.11***	7.17***
S&P 500 Futures Index	77.90	15.91	1.27***	6.87***
CRB Index@	5.62	5.24	0.21	1.10
Dollar Index@	−33.91	7.00	0.56*	0.30
S&P 500 Cash Index	101.99	16.40	−0.66***	3.78***
Major Market Index@	85.84	17.32	1.55***	8.74***
Risk-Free Rate Treasury Bill	9.28	0.24	1.36***	5.49***

term is negative (-0.155) and significant $(t = -6.34)$. The reader will note that our results are more in conformity with the findings of Raynauld and Tessier (1984) and Chang (1985) in futures markets than with the results from Junkus' (1991) study. Raynauld and Tessier (1984), for instance, find evidence of significant risk premia in corn, wheat and oats markets. Differences between our results and Junkus' may reflect the differences in sampling periods and the breadth of the samples used as ours is a broader (wider) sample. Our results for the two-moment model are also at odds with the results found in earlier studies that employ a two-moment CAPM (see, e.g., Dusak, 1973; Bodie and Rosansky, 1980). We find significant coefficients on the β variable in the two-moment regression. We attribute differences between our results and the results from earlier studies that use a two-moment CAPM type framework to 'learning'. The early studies in futures markets, conducted in the 1970s and early 1980s, sampled data when the futures markets were embryonic. Such samples may have been characterized by 'learning', and consequently, mispricing may have been more common in those periods. We sample prices from futures markets that have had time to season. Typically, we sample data from the middle to late 1980s and early 1990s.

Following the work of Dittmar (1999) and others who examine the importance of cokurtosis, we conduct a third regression that incorporates a term for covari-

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Notes to Table 1:

Notes: For all instruments, other than three indexes, the sample covers the period June 1982–December 1996, a total of 175 months. The @ indicates a sample period beginning January 1987 and ending December 1996. Mean, variance, skewness, and excess kurtosis are calculated for the 28 futures, seven indexes, and for the risk-free rate (Treasury bill rate). The contracts are rolled into the next to expire contract on the first day of the expiration month. To compute skewness and excess kurtosis and to test for significance we use

$$\text{Skewness(Sk)} = \frac{N^2}{(N-1)(N-2)} \frac{m_3}{s^3},$$

$$\text{Excess kurtosis(Ku)} = \frac{N^2}{(N-1)(N-2)(N-3)} \frac{(N+1)m_4 - 3(N-1)m_2^2}{s^4},$$

$$\text{Sk} = 0 \quad z = \text{Sk}\left[\frac{(N-1)(N-2)}{6N}\right]^{1/2},$$

$$\text{Ku} = 0 \quad z = \text{Ku}\left[\frac{(N-1)(N-2)(N-3)}{24N(N+1)}\right]^{1/2},$$

where

$$m_k = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^k.$$

Positive values for skewness and excess kurtosis indicate positive skewness and leptokurtosis (long tails), respectively, whereas negative values for skewness and excess kurtosis indicate negative skewness and platykutosis (short tails). The equations can be obtained from the RATS handbook, pp. 14–238 (1996). The *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 2
Two-pass regressions with nonweighted indexes as market proxies

Regression	Constant	Coefficient second moment	Coefficient third moment	Coefficient fourth moment	Average R^2	No. of regressions
Panel A: S&P 500 index futures (June 1982 – December 1996)						
Two moment	0.041 (0.52)	0.364 (9.45)	–	–	0.07	115
Three moment	−0.049 (−0.98)	0.347 (10.48)	−0.155 (−6.34)	–	0.13	115
Four moment	0.068 (1.29)	0.502 (11.59)	−0.320 (−6.56)	0.629 (11.81)	0.22	115
Panel B: New York Composite index futures (June 1982 – December 1996)						
Two moment	0.036 (0.44)	0.310 (8.87)	–	–	0.07	115
Three moment	−0.064 (−1.15)	0.300 (9.55)	−0.147 (−5.87)	–	0.12	115
Four moment	0.041 (1.02)	0.489 (10.86)	−0.298 (−5.18)	0.597 (10.75)	0.20	115
Panel C: KC Value Line index futures (June 1982 – December 1996)						
Two moment	0.021 (0.11)	0.191 (6.12)	–	–	0.05	115
Three moment	0.028 (0.65)	0.187 (5.85)	−0.084 (−4.05)	–	0.10	115
Four moment	0.005 (0.02)	0.286 (6.74)	−0.115 (−3.84)	0.427 (6.54)	0.16	115
Panel D: CRB index futures (January 1987 – December 1996)						
Two moment	−0.046 (−0.28)	0.201 (7.14)	–	–	0.07	60
Three moment	−0.019 (−0.74)	0.193 (6.14)	−0.066 (−3.75)	–	0.11	60
Four moment	0.043 (0.37)	0.301 (7.13)	−0.186 (−4.22)	0.486 (7.89)	0.21	60
Panel E: Major Market index futures (January 1987 – December 1996)						
Two moment	0.059 (0.73)	0.420 (11.21)	–	–	0.08	60
Three moment	−0.066 (−1.41)	0.389 (11.86)	−0.166 (−7.05)	–	0.14	60
Four moment	0.085 (1.36)	0.534 (12.74)	−0.327 (−7.12)	0.689 (13.14)	0.23	60
Panel F: Dollar index futures (January 1987 – December 1996)						
Two moment	0.014 (0.12)	0.254 (8.63)	–	–	0.08	60
Three moment	0.033 (1.04)	0.215 (7.02)	−0.072 (−4.12)	–	0.13	60
Four moment	−0.014 (−0.11)	0.287 (6.85)	−0.171 (−3.58)	0.397 (5.65)	0.18	60

ance, coskewness and cokurtosis. Scott and Horvath (1980) show (their Theorem 2) that with consistent risk aversion, strict consistency of moment preference, and positive preference for positive skewness, a negative preference for the fourth statistical moment results (i.e., $U^3(w) > 0$ imply $U^4(w) < 0$ for all values of w). Our sample is characterized by positive skewness, and the results of the three-moment regression confirm that investors show preference for positive skewness (the negative coefficient a_2 in the second regression in Panel A of Table 2). Thus, we expect the fourth moment to exhibit a positive coefficient. The regression results confirm our expectations. Cokurtosis is disliked. The coefficient value for cokurtosis is positive (0.629) and highly significant ($t = 11.81$). These results for futures markets parallel Dittmar's (1999) findings that cokurtosis is priced in equity markets. Importantly, the average R^2 from the regressions increases as the number of moments in the regressions increases. It is not easy provide a direct benchmark with which to compare our results, although candidates do exist. Junkus' (1991) results and results from a few other studies have already been compared. Some guidance can also be obtained by examining Bessembinder's and Chan's (1992) results with currencies, metals, and commodity futures. They investigate the forecasting power of selected variables in these futures markets. The variables chosen by Bessembinder and Chan, lagged differences in bond yields, lagged equity dividend yields, and lagged three-month Treasury bill yields, are known to have forecast power in equity and bond markets.

Bessembinder and Chan (1992) find results consistent with time-varying risk premia in futures markets. Part of their analysis involves predictive regressions. The R^2 they obtain from these regressions range from 2.8% to 7%, and average around 4 1/2% (across the 12 markets they examine). These values are less than the values we obtain with either the three- or four-moment models. Our results in Table 2 show that the average R^2 values range from 0.10 to 0.14 for the

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Notes to Table 2:

Notes: The data for all instruments, other than three indexes, begin in June 1982 and end in December 1996. Our first set of estimates for β, γ, and δ is calculated using the months of July 1982–May 1987 (a total of 60 months). These estimates are then used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This ensures that the estimates are updated and proxy as closely as possible investors ex-ante beliefs. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors. Data for three of the six indexes start after June 1982. For the CRB, Dollar, and Major Market indexes, we have data from January 1987. In these instances the number of regressions is reduced. For example, with the CRB index as the market proxy only 60 regressions can be conducted, whereas with the indexes with data beginning in June 1982, 115 regressions are possible. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used. t-values are given in parentheses.

three-moment regressions, and from 0.16 to 0.23 for the four-moment regressions, and average 0.12 and 0.20, respectively. This leads us to infer that the cross-sectional variation captured by our model includes the risk premia detected by Bessembinder and Chan (1992). However, such comparison must be undertaken with caution, given that Bessembinder and Chan (1992) investigate the predictive power of instrumental variables.

To see if our sample yields results similar to Bessembinder and Chan (1992), we conduct the same predictive GMM tests that they conduct. These tests are performed for 10 of the 12 futures that they examine (excluding cotton and copper, for which we have no data). Remarkably, our results tend to be very similar to theirs for the 10 futures.[6] Thus, our preceding inferences relating to risk premia (i.e., we capture the risk premia detected by Bessembinder and Chan, 1992 with our model) appears to be reasonable.

We are impressed by the consistency of the results across the different market proxies. For example, consider the results of the four-moment regressions with the CRB index. Covariance, coskewness, and cokurtosis are all priced. The coefficient values for covariance, coskewness, and cokurtosis are 0.301, -0.186, 0.486 and the corresponding t-values are 7.13, -4.22, and 7.89, respectively. The reader will also note that the average R^2 from the regressions with the CRB index increase as the number of moments in the regressions increases. Such consistency provides compelling evidence that investors price higher moments in valuing futures. We now turn to the results obtained with the weighted market proxies and the equity proxy.

6.2. Results from empirical tests with the weighted market proxies and the all-equity market proxy

Prior studies that combine futures and equity in constructing market indexes motivate this part of the analysis (see, e.g., Carter et al., 1983; Baxter et al., 1985). We conduct three series of two-pass regression tests in this part of the analysis. The first series employs the market proxy composed of equal weights in each of the six index futures returns (1/6 each). The second series employs the market index constructed by weighting 50% of the total equally in five index futures returns (weighted 1/10 each, S&P 500 excluded) and the remaining 50% is weighted in the S&P 500 cash index returns. The third series uses the S&P 500 cash index as the market proxy.

The results from these tests are reported in Table 3. The results are consistent with the previous findings. For example, in the four-moment regression with the equally weighted futures index, the coefficients for β, γ, and δ are 0.471, -0.219 and 0.599, respectively, and significant. The signs are in the expected

[6] For brevity, the results of these tests are not reported, but are available from the authors on request.

Table 3
Two-pass regressions with weighted indexes and an all-equity index as market proxies

Regression	Constant	Coefficient second moment	Coefficient third moment	Coefficient fourth moment	Average R^2	No. of regressions
Panel A: Equally weighted futures index (January 1987 – December 1996)						
Two moment	0.004 (0.32)	0.234 (5.52)	–	–	0.06	60
Three moment	−0.998 (−1.90)	0.259 (4.84)	−0.157 (−3.86)	–	0.12	60
Four moment	0.527 (1.49)	0.471 (2.85)	−0.219 (−3.57)	0.599 (4.52)	0.17	60
Panel B: Weighted futures (1 / 10 each, total 50%) and 50% in S&P 500 cash index (January 1987 – December 1996)						
Two moment	−0.104 (−0.96)	0.217 (5.86)	–	–	0.05	60
Three moment	−0.194 (−0.95)	0.226 (4.93)	−0.149 (−2.73)	–	0.11	60
Four moment	0.201 (1.60)	0.349 (4.62)	−0.177 (−2.75)	0.464 (3.21)	0.15	60
Panel C: S&P 500 cash equity index (June 1982 – December 1996)						
Two moment	0.445 (2.80)	0.205 (6.39)	–	–	0.05	115
Three moment	−0.341 (−1.75)	0.214 (5.15)	−0.132 (−2.28)	–	0.11	115
Four moment	0.273 (1.46)	0.314 (6.02)	−0.146 (−2.02)	0.441 (2.97)	0.14	115

Notes: The data for all instruments, other than three indexes, begin in June 1982 and end in December 1996. Our first set of estimates for β, γ, and δ is calculated using the months of July 1982–May 1987 (a total of 60 months). These estimates are then used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This ensures that the estimates are updated and proxy as closely as possible investors ex-ante beliefs. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors. Data for three of the six indexes start after June 1982. For the CRB, Dollar, and Major Market indexes, we have data from January 1987. Thus, the weighted indexes, that include weights from these indexes, have fewer regressions. For example, with the weighted futures index as the market proxy only 60 regressions can be conducted. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used. The t-values are given in parentheses.

directions, suggesting that investors in futures markets prefer coskewness and dislike covariance and cokurtosis. Thus, irrespective of the weighting of the cash index (from 0% to 100%) we find significant risk premia.

6.3. Skewness and kurtosis in cash commodities and futures markets

To give the reader an idea of the relative importance of coskewness and cokurtosis in futures and cash markets, we construct a subsample of futures and cash prices for thirteen commodities. This sample is restricted to these commodities as we do not have cash prices for the remaining instruments. The values for skewness and kurtosis for the commodities and their cash values, shown in Table 4, indicate larger values for the futures than for the cash prices. This is consistent with many studies that have found pronounced nonlinearities in futures distributions (Stevenson and Bear, 1970; Dusak, 1973; Hudson et al., 1987). Such casual evidence prompts us to suspect that higher moments are relatively more important in modeling futures returns than in modeling cash returns.

We conduct two-pass regression tests for the futures subsample and corresponding cash subsample. Results from these tests are shown in Table 5. For these tests, given their comparative nature, we employ the weighted indexes and the cash index. The results for the subsample of futures are found to be consistent with the results shown earlier. Investors in futures markets prefer coskewness and dislike

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Notes to Table 4:

Notes: The sample covers the period June 1982–December 1996, a total of 175 months. Mean, variance, skewness, and excess kurtosis are calculated for returns on the 13 futures and corresponding cash prices. The contracts are rolled into the next to expire contract on the first day of the expiration month. To compute skewness and excess kurtosis and to test for significance we use

$$\text{Skewness}(\text{Sk}) = \frac{N^2}{(N-1)(N-2)}\,\frac{m_3}{s^3},$$

$$\text{Excess kurtosis}(\text{Ku}) = \frac{N^2}{(N-1)(N-2)(N-3)}\,\frac{(N+1)m_4 - 3(N-1)m_2^2}{s^4},$$

$$\text{Sk} = 0 \quad z = \text{Sk}\left[\frac{(N-1)(N-2)}{6N}\right]^{1/2},$$

$$\text{Ku} = 0 \quad z = \text{Ku}\left[\frac{(N-1)(N-2)(N-3)}{24N(N+1)}\right]^{1/2},$$

where

$$m_k - \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^k.$$

Positive values for skewness and excess kurtosis indicate positive skewness and leptokurtosis (long tails), respectively, whereas negative values for skewness and excess kurtosis indicate negative skewness and platykutosis (short tails). The equations can be obtained from the RATS handbook, pp. 14–238 (1996). The *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 4
Summary statistics

Panel A: Subsample of monthly data for the futures prices (June 1982–December 1996)

Futures	Mean $\times 10^4$	Variance $\times 10^4$	Skewness	Excess kurtosis
Soyabean	−45.37	32.56	0.05	4.84***
Soybean oil	−24.58	50.35	0.74***	3.72***
Soymeal	−13.96	35.43	0.31	1.77*
Wheat	−1.94	33.87	0.26	1.70*
Corn	−36.69	40.32	0.76***	7.46***
Oats	−149.04	80.92	2.09***	16.39***
Sugar	−59.96	142.21	0.76***	2.80***
Coffee	4.09	105.98	2.72***	2.74***
Feeder cattle	21.68	13.92	−0.21	1.02
Live cattle	55.64	17.21	0.10	0.06
Live hog	75.39	39.05	0.01	0.31
Pork bellies	−110.04	108.68	−0.06	1.10
Heating oil	49.27	79.96	−0.51*	4.00***

Panel B:Subsample of monthly data for corresponding cash prices (June 1982–December 1996)

Cash	Mean $\times 10^4$	Variance $\times 10^4$	Skewness	Excess kurtosis
Soyabean	7.96	33.91	0.17	2.66***
Soybean oil	6.67	54.22	0.42**	2.33***
Soymeal	15.15	41.66	0.08	0.75
Wheat	8.57	49.51	−0.25	1.02
Corn	1.14	54.86	−0.08	3.56***
Oats	−3.40	69.94	0.73***	6.84***
Sugar	25.76	139.91	0.63***	2.29**
Coffee	−2.47	101.49	0.51*	1.92*
Feeder cattle	2.77	17.93	0.17	−0.40
Live cattle	−1.27	19.91	−0.16	1.12
Live hog	−3.32	68.54	0.08	0.36
Pork bellies	−3.79	233.92	0.14	1.19
Heating oil	−12.74	133.31	−0.66**	3.78***

covariance and cokurtosis. For the cash sample, we find the signs of the coefficients to be in the directions hypothesized. However, the cokurtosis terms are not significant at conventional levels, while the coskewness terms are marginally significant. It appears that commodity cash prices, unlike equity prices, are adequately explained by a three-moment model.

6.4. Measurement errors

Following Shanken's critique, we conduct maximum likelihood estimation (MLE) to examine if measurement errors in our results lead to misleading inferences. These tests are conducted with the full sample of futures (28), and as

Table 5
Two-pass regressions with weighted indexes and an all-equity index as market proxies for futures and matching cash subsample

Regression	Constant	Coefficient second moment	Coefficient third moment	Coefficient fourth moment	Average R^2	No. of regressions
Futures subsample						
Panel A: Equally-weighted futures index (January 1987–December 1996)						
Two moment	0.127 (0.64)	0.249 (6.41)	–	–	0.06	60
Three moment	−0.394 (−1.41)	0.277 (5.01)	−0.139 (−3.08)	–	0.12	60
Four moment	0.622 (1.04)	0.463 (2.67)	−0.194 (−3.29)	0.612 (5.23)	0.18	60
Panel B: Weighted futures (1 / 10 each, total 50%) and 50% in S&P 500 cash index (January 1987–December 1996)						
Two moment	−0.086 (−0.83)	0.235 (5.99)	–	–	0.05	60
Three moment	−0.179 (−1.09)	0.251 (4.84)	−0.133 (−3.15)	–	0.12	60
Four moment	0.004 (0.02)	0.432 (4.49)	−0.163 (−2.79)	0.529 (4.17)	0.16	60
Panel C: S&P 500 cash equity index (June 1982–December 1996)						
Two moment	−0.214 (−1.63)	0.229 (5.87)	–	–	0.05	115
Three moment	−0.169 (−0.94)	0.238 (4.79)	−0.129 (−2.86)	–	0.12	115
Four moment	0.035 (0.12)	0.407 (5.93)	−0.141 (−2.35)	0.512 (3.28)	0.15	115
Matching cash sample						
Panel D: Equally-weighted futures index (January 1987–December 1996)						
Two moment	0.267 (1.81)	0.368 (3.82)	–	–	0.06	60
Three moment	−0.208 (−1.47)	0.273 (3.01)	−0.079 (−1.79)	–	0.09	60
Four moment	0.191 (1.62)	0.237 (3.29)	−0.084 (−1.72)	0.225 (1.40)	0.12	60
Panel E: Weighted futures (1 / 10 each, total 50%) and 50% in S&P 500 cash index (January 1987–December 1996)						
Two moment	0.349 (2.05)	0.273 (3.55)	–	–	0.05	60
Three moment	−0.366 (−1.84)	0.245 (2.75)	−0.055 (−1.73)	–	0.08	60
Four moment	0.243 (1.68)	0.198 (2.79)	−0.066 (−1.66)	0.174 (1.35)	0.11	60
Panel F: Panel C: S&P 500 cash index (June 1982–December 1996)						
Two moment	0.397 (2.62)	0.244 (3.37)	–	–	0.04	115
Three moment	−0.444 (−2.03)	0.218 (2.64)	−0.048 (−1.71)	–	0.08	115
Four moment	0.425 (1.95)	0.193 (2.09)	−0.037 (−1.69)	0.130 (1.22)	0.11	115

market proxies we use the S&P 500 cash and futures indexes plus the weighted indexes. In general, the MLE tests maximize the likelihood function given by

$$\max_{a} \sum_{t=1}^{T} f(r_t, \beta, \gamma, \delta, a). \tag{15}$$

The procedure is similar to the two-pass regressions. From the MLE procedures we do not obtain R^2 values, so only comparisons of the coefficients are possible.

Notes to Table 5:

Notes: The data for all instruments, other than three indexes, begin in June 1982 and end in December 1996. Our first set of estimates for β, γ, and δ is calculated using the months of July 1982–May 1987 (a total of 60 months). These estimates are then used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This ensures that the estimates are updated and proxy as closely as possible investors ex-ante beliefs. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors. Data for three of the six indexes start after June 1982. For the CRB, Dollar, and Major Market indexes, we have data from January 1987. Thus, the weighted indexes, that include weights from these indexes, have fewer regressions. For example, with the weighted futures index as the market proxy only 60 regressions can be conducted. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used. t-values are given in parentheses.

Notes to Table 6:

Notes: The two-pass methodology has some drawbacks. The problem of measurement error, or errors-in-variables, can be encountered. Shanken's recommends MLE estimation for small-sample bias in the second-pass cross-sectional regression estimates that may be induced by measurement error. Thus, we conduct additional tests using MLE procedures to address this problem. These tests are conducted with the full sample of futures (28) and as market proxies we use the S&P 500 cash and futures indexes as well as the weighted indexes. In general, the MLE tests maximize the likelihood function given by

$$\max_{a} \sum_{t=1}^{T} f(r_t, \beta, \gamma, \delta, a).$$

The procedure is similar to the two-pass regressions. The data for all instruments, other than three indexes, begin in June 1982 and end in December 1996. Our first set of estimates for β, γ, and δ is calculated using the months of July 1982–May 1987 (a total of 60 months). These estimates are then used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This ensures that the estimates are updated and proxy as closely as possible investors ex-ante beliefs. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors. Data for three of the six indexes start after June 1982. Thus, the weighted indexes that include weights from these indexes have fewer regressions. For example, with the weighted futures index as the market proxy only 60 regressions can be conducted. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used. The t-values are given in parentheses.

Table 6
Maximum likelihood estimation (MLE)

Regression	Constant	Coefficient second moment	Coefficient third moment	Coefficient fourth moment	No. of regressions
Panel A: Equally weighted futures index (January 1987 – December 1996)					
Two moment	0.058 (0.94)	0.227 (4.77)	–	–	60
Three moment	−2.107 (−3.24)	0.266 (5.12)	−0.148 (−2.97)	–	60
Four moment	0.624 (1.68)	0.466 (2.94)	−0.201 (−2.85)	0.563 (4.21)	60
Panel B: Weighted futures (1/10 each, total 50%) and 50% in S&P 500 cash index (January 1987 – December 1996)					
Two moment	−0.168 (−1.73)	0.231 (6.28)	–	–	60
Three moment	−0.714 (−1.86)	0.218 (4.65)	−0.127 (−2.12)	–	60
Four moment	0.394 (1.89)	0.363 (4.98)	−0.164 (−2.19)	0.411 (2.83)	60
Panel C: S&P 500 cash index (June 1982 – December 1996)					
Two moment	0.621 (3.45)	0.195 (4.71)	–	–	115
Three moment	−0.492 (−2.06)	0.183 (3.27)	−0.126 (−2.02)	–	115
Four moment	0.341 (1.79)	0.327 (6.84)	−0.134 (−1.89)	0.372 (2.43)	115
Panel D: S&P 500 futures index (June 1982 – December 1996)					
Two moment	0.049 (0.65)	0.355 (8.64)	–	–	115
Three moment	−0.069 (−1.17)	0.336 (9.06)	−0.149 (−5.89)	–	115
Four moment	0.084 (1.39)	0.497 (10.14)	−0.314 (−6.23)	0.618 (10.65)	115

Consider the results from these tests, presented in Table 6, with the S&P 500 cash index. The β, γ, and δ coefficients from the MLE tests are consistent with the results for the cash index reported for the two-pass regressions in Table 3. However, closer inspection of the results reveals a slight downward bias in the value of the coefficients with the MLE procedures, suggesting some degree of measurement error. Such errors are minor. For instance, in the four-moment regression, the coefficient values of β, γ, and δ in the MLE estimation are 0.327, -0.134, and 0.372, respectively, and significant, whereas in the two-pass estimation the corresponding values for β, γ, and δ, reported in Table 3, are 0.314, -0.146, 0.441, respectively, and significant.

6.5. The effects of the October 1987 crash on the results

It is also possible that the results can be induced by the effects of the October 1987 crash. To check whether the results are driven by the October 1987 crash, we perform three series of regressions using the S&P 500 futures index as the market proxy. The first procedure excludes data for October 1987, the second excludes data for October 1987 and November 1987, and the third excludes data for October 1987, November 1987, and December 1987, thus ensuring any residual effects from the October crash are accounted for. The results, reported in Table 7, are identical to the results found before.

6.6. Collinearity and specification checks

The nature of the regressions suggests collinearity in the model. To check this, we perform regressions of the fourth moment on the second and third moments. We also conduct specification checks on the four-moment model.[7]

The results from these tests confirm the presence of collinearity between the moments. This in itself is not alarming, since it confirms the strength of the coefficient estimates from the two-pass estimation. The existence of collinearity between the moments leads to substantially high standard errors in the regression parameters, consequently, reducing the power of the significance tests. Thus, the significance of the estimates found in our regressions make this even more impressive.[8] We also perform a specification check using a regression error specification test (RESET). These tests use the S&P 500 index futures as the

[7] For brevity, these results are not reported, but are available from the authors on request.

[8] The presence of multicollinearity may not always require correcting, especially in this case. The average R^2 values increase as the number of regressors (moments) are added, and the coefficients remain significant. Thus, the presence of collinearity does not seem to prevent proper inferences from being drawn. This view is reinforced not only by the results that show that the signs of the coefficients remain constant but also by the evidence that indicates the results seem robust to the market proxy used. Furthermore, the computation of the estimates of the three moments, β, γ, and δ, are independent of the computation of the excess returns.

Table 7
Two-pass regressions with S&P 500 futures index (June 1982–December 1996)

Regression	Constant	Coefficient second moment	Coefficient third moment	Coefficient fourth moment	Average R^2	No. of regressions
Panel A: October 1987 excluded						
Two moment	0.039 (0.51)	0.325 (8.49)	–	–	0.07	114
Three moment	−0.052 (−1.04)	0.326 (9.63)	−0.139 (−5.92)	–	0.12	114
Four moment	0.057 (1.10)	0.481 (10.36)	−0.297 (−6.02)	0.611 (10.71)	0.20	114
Panel B: October and November 1987 excluded						
Two moment	0.037 (0.46)	0.319 (7.90)	–	–	0.07	113
Three moment	−0.048 (−0.97)	0.322 (8.65)	−0.131 (−5.63)	–	0.11	113
Four moment	0.052 (0.94)	0.459 (9.38)	−0.286 (−5.77)	0.589 (9.32)	0.19	113
Panel C: October, November, and December 1987 excluded						
Two moment	0.035 (0.44)	0.308 (7.71)	–	–	0.07	112
Three moment	−0.043 (−0.92)	0.320 (8.41)	−0.125 (−5.39)	–	0.11	112
Four moment	0.049 (0.91)	0.446 (9.05)	−0.277 (−5.32)	0.575 (9.04)	0.19	112

Notes: It is possible that the results can be induced by the effects of the October 1987 crash. To check whether the results are driven by the October 1987 crash we perform three series of regressions using the S&P 500 index futures as the market proxy. The first procedure excludes data for October 1987, the second excludes data for October 1987 and November 1987, and the third excludes data for October 1987, November 1987, and December 1987. This ensures any residual effects from the October crash are accounted for. The data begin in June 1982 and end in December 1996. Our first set of estimates for β, γ, and δ is calculated using the months of July 1982–May 1987 (a total of 60 months). These estimates are then used in monthly regressions for the period June 1987–May 1988. The first 12 months used to estimate β, γ, and δ, June 1982–May 1983, are then discarded and the next 12 months, June 1987–May 1988, are included in the calculation of the next set of β, γ, and δ estimates. These estimates are then used in the regressions for June 1988–May 1989. This process is repeated until the regression for the last month is performed. This ensures that the estimates are updated and proxy as closely as possible investors ex-ante beliefs. The coefficient estimates of the three moments obtained from the regressions are then cumulated and tested against priors. To proxy for the risk-free return, the return on the Treasury bill future (plus unity) is used. The t-values are given in parentheses.

market portfolio, and tests are performed for each month, from June 1987 to December 1996. The test looks for residual nonlinearities in the specification. At the 5% level of significance, we find evidence of neglected nonlinearities in only 24 instances (out of a total of 115). The evidence seems to suggest that the model is well specified.

7. Summary and conclusions

In this study we examine the importance of coskewness and cokurtosis in explaining futures returns. Studies in futures markets that examine risk premia using the traditional CAPM framework often fail to detect significant risk premia in futures prices. Research that relies on models that depart from a traditional framework find mixed results. Given the current state of the literature, we attempt to provide insight into the return-generating process in futures markets by using a four-moment CAPM model.

Although it is almost impossible to know the exact form of the return generating relationship, the preponderance of the evidence from this study provides strong support for the inclusion of terms that represent coskewness and cokurtosis. In the regressions where the excess return on the future is employed as the dependent variable and regressed against the proxies for the second, third, and fourth moments, we find that the explanatory power increases as terms for coskewness and cokurtosis are included. To illustrate, when we employ S&P 500 index futures as the market proxy with the full sample of 28 futures, the average R^2 values increase from 0.07 for the two-moment regression to 0.13 for the three-moment regression and to 0.22 for the four-moment regression. Dispute over the appropriate market proxy for futures prompts us to use nine different market proxies, from weighted to nonweighted futures indexes. Additionally, we also employ an all-equity index as a market proxy. The results are robust to the market proxy used. Criticisms of the two-pass methodology are addressed by using corrective techniques. The corrections applied do not change the results. We also check the specification of the model and find the model to be well specified. The overall results confirm that coskewness and cokurtosis are important in explaining the return-generating process in futures markets. Of practical importance is the implication for risk computation on futures instruments. These calculations must account for the pricing of coskewness and cokurtosis.

References

Arditti, F., Levy, H., 1975. Portfolio efficiency analysis in three moments: the multiperiod case. Journal of Finance 30, 797–809.

Badrinath, S., Chatterjee, S., 1988. On measuring skewness and elongation in common stock return distributions: the case of the market index. Journal of Business 61, 451–472.

Basu, S., 1977. The investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis. Journal of Finance 32, 663–682.
Baxter, J., Conine, T.E., Tamarkin, N., 1985. On commodity risk premiums: additional evidence. Journal of Futures Markets 5, 121–135.
Bessembinder, H., Chan, K., 1992. Time-varying risk premia and forecastable returns in futures markets. Journal of Financial Economics 32, 169–193.
Black, F., 1972. Capital market equilibrium with restricted borrowing. Journal of Business 45, 444–454.
Black, F., Jensen, M., Scholes, M., 1972. The capital asset pricing model: some empirical tests. In: Jensen, M.C. (Ed.), Studies in the Theory of Capital Markets. Praeger, New York, pp. 79–124.
Bodie, Z., Rosansky, J., 1980. Risk and return in commodity futures. Financial Analysts Journal 36, 27–39.
Carter, C.A., Rausser, G.C., Schmitz, A., 1983. Efficient asset portfolios and the theory of normal backwardation. Journal of Political Economy 91, 319–331.
Chang, F.C., 1985. Returns to speculators and the theory of normal backwardation. Journal of Finance 40, 193–208.
Chen, N.F., Roll, R., Ross, S., 1986. Economic forces and the stock market. Journal of Business 59, 383–403.
Cotner, J.S., 1991. Index option pricing: do investors pay for skewness? Journal of Futures Markets 2, 1–8.
Dittmar, R., 1999. Nonlinear pricing kernels, kurtosis preference, and evidence from the cross-section of equity returns, Working Paper, Indiana University.
Dusak, K., 1973. Futures trading and investor returns: an investigation of commodity market risk premia. Journal of Political Economy 81, 1387–1406.
Ehrardt, M.C., Jordan, J.V., Walkling, R.A., 1987. An application of the arbitrage pricing theory to futures markets: tests of normal backwardation. Journal of Futures Markets 7, 21–34.
Fama, E., French, K., 1992. The cross-section of expected returns. Journal of Finance 59, 427–465.
Fama, E., French, K., 1993. Common risk factors in the returns of stocks and bonds. Journal of Financial Economics 33, 3–56.
Fama, E., Macbeth, J., 1973. Risk, return, and equilibrium empirical tests. Journal of Political Economy 81, 607–636.
Fang, H., Lai, T.-Y., 1997. Co-kurtosis and capital asset pricing. The Financial Review 32, 293–307.
Friend, I., Blume, M., 1970. Measurement of portfolio performance under uncertainty. American Economic Review 1970, 561–575 September.
Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. Journal of Finance 55, 1263–1295.
Hudson, M.A., Leuthold, R.A., Sarassora, G., 1987. Commodity futures price changes: recent evidence for wheat, soybean, and live cattle. Journal of Futures Markets 7, 287–301.
Junkus, J., 1991. Systematic skewness in futures contracts. Journal of Futures Markets 11, 9–24.
Kan, R., Zhang, C., 1999. Two-pass tests of asset pricing models with useless factors. Journal of Finance 54, 203–235.
Kraus, F., Litzenberger, R.H., 1976. Skewness preference and the valuation of risk assets. Journal of Finance 31, 1085–1100.
Levy, H., 1969. A utility function depending on the first three moments: comment. Journal of Finance 24, 715–721.
Lim, K.-G., 1989. A new test of the three_moment capital asset pricing model. Journal of Financial and Quantitative Analysis 24, 205–216.
Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 13–37.
Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica 768–783.
Raynauld, J., Tessier, J., 1984. Risk premiums in futures markets: an empirical investigation. Journal of Futures Markets 4, 189–211.

Scott, R.C., Horvath, P.A., 1980. On the direction of preference for moments of higher order than the variance. Journal of Finance 35, 915–919.

Shanken, J., 1992. On the estimation of beta pricing models. Review of Financial Studies 5, 1–34.

Sharpe, W., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. Journal of Finance 19, 425–442.

Singleton, J., Wingender, J., 1986. Skewness persistence in common stock returns. Journal of Financial and Quantitative Analysis 21, 335–341.

Stevenson, R., Bear, R.A., 1970. Commodity futures: trends or random walks? Journal of Finance 25, 65–81.

Tukey, J.W., 1977. Exploratory Data Analysis. Addison-Wesley, Reading, MA.