Factor Copulas: *External Defaults*

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In this article we address a fundamental problem of the standard one factor Gaussian copula model. In this model a default event of one of the names will have a large impact on the default probability of the surviving names, through a shock in available information on the common factor. Moreover this effect is larger for sudden defaults and is also present in many extensions presented in the literature.

In this article an extended model is presented. An extra idiosyncratic term is introduced which models external default risk. Here one can think of default events due to fraud or legal issues, for example. The most noticeable default events over the last years have been Enron, Worldcom and Parmalat. These did not default due to bad market conditions; their default events had an external cause. The occurrence of such default events does not increase the available information on the common factor driving correlated defaults.

In addition it is shown that this extended model provides a better fit to quotes observed in the synthetic CDO market, since this model requires two additional parameters with which one can control the shape of the so-called base correlation skew.

Over the last decade, the growth in the credit derivatives market has been enormous. First, the market for credit default swaps on single names has become very liquid. This growth was followed by the rapid development of the synthetic CDO market over the last couple of years. Following this large growth in liquidity there has been a need for models which can be calibrated to market quotes. Amongst practitioners, the one factor Gaussian factor copula has become standard. In order to allow for calibration an ad-hoc method has become popular, the base correlation approach presented in McGinty, Beinstein, Ahluwalia and Watts [2004]. This method can not be regarded as a model explaining the observed market quotes, rather as a fix of the standard one factor Gaussian copula model, much like implied volatilities in the equity derivatives markets. In order to explain the market implied correlation structure, some extensions to the standard one factor Gaussian model have been presented in the literature.

Apart from the fact that it cannot explain observed market quotes, one of the fundamental problems of the one factor Gaussian copula model is that a default event for a certain name generates a shock in information about the common factor which subsequently has a large influence on the default probabilities of correlated surviving names. However, the default event of Parmalat, for instance, did not reveal any information about the state of the economy and therefore did not result in large jumps in CDS rates of surviving names. This fundamental problem in the model becomes even more apparent for a default event close to the initial date used in the model. This problem of unrealistically high default correlation effects for instantaneous defaults has also been discussed in Rogge and Schönbucher [2003]. We show that this behavior is present in many of the factor copula models known in the literature.

By proposing a simple extension to the standard model we achieve two goals: first the undesirable correlation effect of sudden defaults is not present in the extended model. Second, the model is capable of explaining, to a large extend, the observed market quotes for standard tranches.

The outline of this article is as follows. In the following section we present a review of the standard one factor Gaussian copula model with focus on the unrealistic correlation effect of sudden defaults and the poor fit to observed market quotes. Here the base correlation method due to McGinty, Beinstein, Ahluwalia and Watts [2004] is also discussed. The section *External Defaults* introduces the external defaults model and explains how this model handles the problem of instantaneous defaults and the fit to the base correlation skew observed in the market. In the section *Numerical Results* results of two tests are presented. First, the correlation effect of a sudden default is compared between the standard one factor Gaussian copula model and the external defaults model. Second, it is shown that the external defaults model can provide a good fit to the quotes of synthetic CDOs. Finally, the last section concludes this article.

FACTOR COPULA

The one factor Gaussian copula is a special case of the more general Gaussian copula presented in Li [2000]. Consider modelling default events for a basket of M names. The default event of a single name is modelled by means of a latent standard normal variable, X_i. Correlation is imposed by correlating these latent variables, which under the one factor Gaussian copula is done by means of a single common factor, Y and idiosyncratic terms ξ_i.

$$X_i = \rho_i Y + \sqrt{1-\rho_i^2}\,\xi_i, i = 1, \ldots, M \tag{1}$$

All ξ_i and Y are i.i.d. standard normal variables. The parameter ρ_i determines the correlation of the latent variable with the common factor. Often this correlation parameter is chosen to be equal for all names.

For name i, let τ_i denote the default time and $p_i(t)$ the default probability up to time t. The following relation between the default time and the latent variable guarantees that the default distribution for name i is maintained:

$$\begin{aligned} \tau_i \le t &\Leftrightarrow X_i \le \chi_i(t) \\ \chi_i(t) &= \Phi^{-1}(p_i(t)) \end{aligned} \tag{2}$$

This copula setup allows one to separate correlation modelling from the modelling of the marginal default distributions. The strength of the one factor copula model lies in the fact that one can condition on the common factor, after which all remaining sources of risk become independent. Equipped with these independent conditional default probabilities one can determine the conditional loss distribution of the entire basket. For instance, one can apply the technique described in Andersen, Sidenius and Basu [2003]. After the conditional distributions have been determined, one can integrate out over the common factor to obtain the unconditional loss distribution of the entire basket. Finally one can easily determine prices for tranche default swaps. More details on pricing can be found in Laurent and Gregory [2003] or Hull and White [2004].

Instantaneous Defaults

A major drawback of the one factor Gaussian copula framework is the limited complexity of the correlation structure. This is reflected in the way default events affect the default probability of surviving names (see Schönbucher and Schubert [2001]).

This correlation effect is especially relevant for default events in the near future. Consider the case of a default event of a certain name, which does not have an extraordinarily large initial CDS spread. This sudden default event implies that its latent variable has been exceptionally small. Since the common factor and the idiosyncratic factor are linearly combined using normal distributions, by far the most likely scenario is one where both these values have a very low realization. Thus the sudden default provides a large amount of information about the realization of the common factor. This has dramatic consequences for the remaining names, as these will suddenly become much more likely to default in the immediate future as well and spreads should increase dramatically. This problem is most visible for sudden defaults, but any default event will directly result in a shock in available information on the common factor.

Base Correlations

A second major drawback of the standard one factor Gaussian approach is of a more practical nature. Due to the increased liquidity in synthetic CDO tranches, quotes have become readily available. This has created the need for a model which can be calibrated to these quotes. A first approach to tackle this problem is to determine the compound correlation for each single tranche observed in the market. This is defined as the correlation which, when used in the one factor Gaussian copula model with the assumption that ρ_i are equal to some fixed ρ, gives a fair premium equal to the observed market quote.

If synthetic CDO markets can be explained by the one factor Gaussian copula model with uniform correlations, one should observe that the compound correlations are equal for every single tranche on a certain basket. However, this is not the case as the compound correlations, when plotted against the detachment level, show a figure which resembles a smile. In contrast to implied volatilities in the Equity derivatives market, one can not just use interpolation methods to obtain correlations required for pricing other tranches. The problem here is that these compound correlations are a function of both the attachment point as well as the detachment point. McGinty, Beinstein, Ahluwalia and Watts [2004] write the expected loss on a certain tranche as the difference between the expected loss of two equity tranches. Using this relation in combination with the one factor Gaussian copula model allows one to determine implied correlations for equity tranches. The resulting correlations are widely known as the base correlations. In practice these usually turn out to be upward sloping in detachment level. In Exhibit 1 an example of a compound correlation smile as well as a base correlation skew are plotted, based on market data of the 1st of November 2006. The five standard tranches on the DJ CDX index with a five year maturity are used in order to generate these results.

The compound correlation for the equity tranche is around 14%, after which it decreases to a level close to 6.5% for the junior mezzanine tranche. For more senior tranches it increases again from roughly 14% to 18% to reach a level of 31% for the super senior tranche. Further, one can observe a close to linear relationship between base correlations and attachment level. By construction, the base correlation for the equity tranche equals the compound correlation. The base correlations increase to roughly 66% for the most senior tranche. The fact that the lines are not horizontal clearly illustrates the model's inability to fit the market with a single correlation ρ.

The base correlation approach cannot be seen as a proper model, but merely as a method to visualize market quotes in terms of correlations. It does not allow one to determine prices on bespoke baskets, without the use of mapping methods. Difficulties will increase even further when prices for more complex credit derivatives are required, such as a CDO of CDOs, or CDO[2].

Factor Copula Extensions

In order to better fit to the observed market prices and to be able to price more complicated credit derivatives, different extensions to the standard factor copula model have been proposed. Andersen, Sidenius and Basu [2003] use correlated Student t variables to model the latent variables. Closely related is the approach in Hull and White [2004], where both the common factor as well as the idiosyncratic terms are modelled by means of Student t-distributions.

Andersen and Sidenius [2004] extend the model by allowing for random recovery. Another extension proposed in this article is to allow the factor weights ρ to depend on the state of the economy, or the common factor, Y. An intuitively appealing feature of this model is that one can model high correlations when the economy is in a bad state and low correlations when the common factor is high.

However, all these extensions do not solve the problem of unrealistic correlation effects in case of default events in the immediate future. As the common factor is linearly combined with idiosyncratic risk factors, sudden defaults are most likely caused by both terms having a small realization.

EXTERNAL DEFAULTS

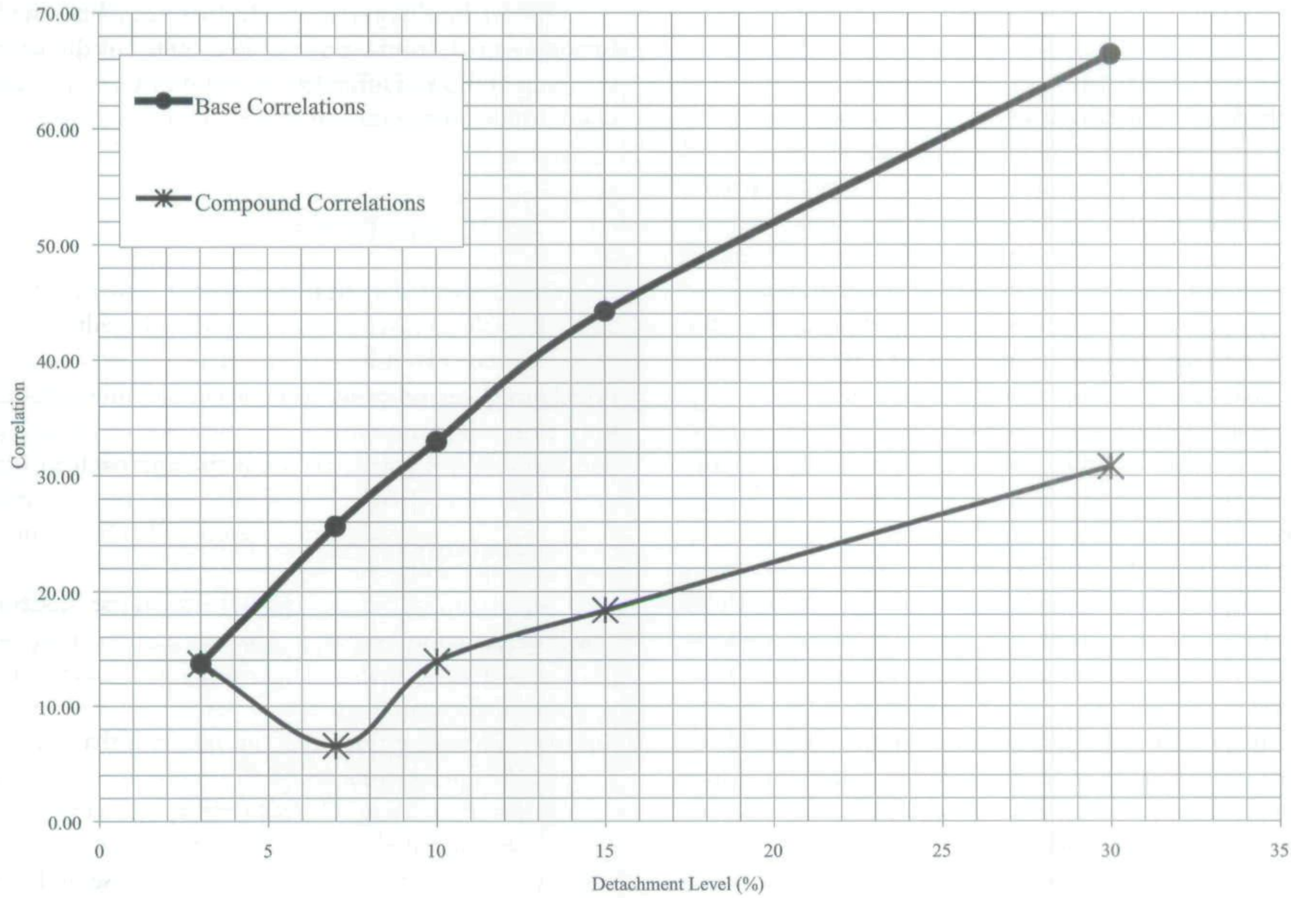
In our proposed model we introduce an external latent variable for each reference entity in addition to the usual latent variable, and a default event can be caused by either of these latent variables. The model is specified as follows:

$$
\begin{aligned}
X_i &= \rho_i Y + \sqrt{1-\rho_i^2}\,\xi_{i,1} \\
Z_i &= \mu_i + \sigma_i \xi_{i,2} \\
\tau_i \le t &\Leftrightarrow \min\{X_i, Z_i\} \le \chi_i(t)
\end{aligned}
\tag{3}
$$

Exhibit 1

Compound and Base Correlations Plotted Against the Detachment Level of the Tranches

Quotes on standard tranches on the DJ CDX basket are used from the 1st of November 2006.



Throughout the article it is assumed that the random variables, Y, $\xi_{i,1}$ and $\xi_{i,2}$, have independent standard normal distributions. Results for different distributions can be derived in a straightforward manner.

The latent variable X_i is modelled as in the standard one factor Gaussian model, Equation (1). The external latent variable for reference entity i is denoted by Z_i and depends on a standardized random variable $\xi_{i,2}$. The parameters μ and σ allow for shifting and scaling.

A default event occurs when either of the two latent variables is smaller than some time dependent threshold, $\chi_i(t)$. As in the standard one factor Gaussian copula, one should calibrate these thresholds such that the model reprices CDSs to market quotes.

One can interpret the term X_i as a factor modelling market risk, which depends on the state of the economy, Y. The term Z_i can be interpreted as modelling external default risk, not caused by standard economic behavior of the firm, such as default due to fraud. Thus, by using the minimum of two factors, a default event caused by external factors does not lead to the increase in information on the common factor. Thus on one hand, the model generates more independence at the short end of CDO capital structure by allowing short term default events to occur without there being a bad market scenario. This mainly affects equity tranches. On the other hand, default correlation is larger when there are multiple default events, as the most likely scenario under this model would be one with a bad state of the market. The latter mainly affects the more senior tranches. This is exactly the kind of behavior which is reflected in the market for CDOs, due to the upward sloping nature of the base correlation skew.

The parameters μ_i and σ_i model the likelihood of a default being caused by external events. Large values for σ_i correspond to a large likelihood of short term defaults having an external cause. Large values for μ_i decrease the likelihood of sudden defaults due to external causes. With this model, one can avoid the problem in the standard factor copula, where short term defaults have extreme effects on the surviving names. One can choose parameters such that the external latent variable, Z_i, has much fatter tails than the market latent variable X_i. In this case a sudden default event has likely been caused by the external variable and thus does not give much information about the state of the economy, Y.

Furthermore, when considering the model as stated in Equation (3), one can observe that implied default correlation will always be smaller than that of the corresponding standard factor copula model, as the inclusion of the external default risk factor will bring correlation down. Thus the base correlation for any detachment level will always be smaller than ρ^2. This important observation can be used when estimating model parameters to observed market quotes.

In further analysis we will be interested in default probabilities, both conditional on the realization of the common factor as well as unconditionally. In order to simplify notation, the term $\kappa_i(t)$ is introduced to denote the probability of the external default latent variable being smaller than the default threshold at time t for name i:

$$\kappa_i(t) \equiv Pr(Z_i \leq \chi_i(t)) = \Phi\left(\frac{\chi_i(t) - \mu_i}{\sigma_i}\right) \tag{4}$$

Using this notation one can determine the probability of a default event of name i before time t as follows:

$$\begin{aligned} Pr(\tau_i \leq t) &\equiv Pr(\min\{X_i, Z_i\} \leq \chi_i(t)) \\ &= 1 - Pr(X_i > \chi_i(t), Z_i > \chi_i(t)) \\ &= 1 - Pr(X_i > \chi_i(t)) \cdot Pr(Z_i > \chi_i(t)) \\ &= (1 - \kappa_i(t)) \cdot \Phi(\chi_i(t)) + \kappa_i(t) \end{aligned} \tag{5}$$

Here we have used the independence between the market risk variable X_i and the external risk variable Z_i. A similar multiplication of the survival probabilities has been used in Tavares, Nguyen, Chapovsky and Vaysburd [2004]. By using this equation one can easily determine the threshold levels $\chi_i(t)$, such that the model matches market observed CDS quotes.

Once the thresholds are determined, it is straightforward to determine the conditional default probabilities, i.e., the probability of default conditional on the realization of the common factor.

$$\begin{aligned} Pr(\tau_i \leq t \mid Y = y) = (1 - \kappa_i(t)) \\ \cdot \Phi\left(\frac{\chi_i(t) - \rho_i y}{\sqrt{1-\rho_i^2}}\right) + \kappa_i(t) \end{aligned} \tag{6}$$

Pricing multi-name credit derivatives under this model follows along the same lines as the pricing routine for the standard one factor Gaussian copula model.

Probability of External Default

To gain further insight into the model it is useful to consider the probability that a default event is caused by the external default risk factor. We are thus interested in:

$$\begin{aligned} Pr(Z \leq X \mid \min\{X, Z\} \leq \chi) \\ = \frac{Pr(Z \leq X, \min\{X, Z\} \leq \chi)}{Pr(\min\{X, Z\} \leq \chi)} \end{aligned} \tag{7}$$

Note that for brevity both the dependence on the particular name and time have been removed. The denominator of the expression above is the default probability for threshold χ and has been presented in Equation (5). Evaluating the numerator is what remains. Again the independence of the factors is used, which allows the multivariate distribution to be written as the product of the marginals.

$$\begin{aligned} &Pr(Z \leq X, \min\{X, Z\} \leq \chi) \\ &= \int_{-\infty}^{\infty} \phi(x) \int_{-\infty}^{\min\{x,\chi\}} \phi_{\mu,\sigma}(z)\,dz\,dx \\ &= \int_{-\infty}^{\chi} \phi(x) \int_{-\infty}^{x} \phi_{\mu,\sigma}(z)\,dz\,dx + \int_{\chi}^{\infty} \phi(x) \int_{-\infty}^{\chi} \phi_{\mu,\sigma}(z)\,dz\,dx \end{aligned}$$

$$= \int_{-\infty}^{\chi} \phi(x) \cdot \Phi\left(\frac{x-\mu}{\sigma}\right) dx + \int_{\chi}^{\infty} \phi(x) \cdot \Phi\left(\frac{\chi-\mu}{\sigma}\right) dx$$

$$= \Phi_2\left(\frac{-\frac{\mu}{\sigma}}{\sqrt{1+\frac{1}{\sigma^2}}}, \chi; \frac{-\frac{1}{\sigma}}{\sqrt{1+\frac{1}{\sigma^2}}}\right) + \Phi\left(\frac{\chi-\mu}{\sigma}\right) \cdot (1-\Phi(\chi)) \tag{8}$$

Here Φ_2 denotes the bivariate cumulative normal distribution function. The final equation is based on Andersen and Sidenius [2004]. Equation (7) is useful in making parameter choices. For instance one might want to fix the probability that a default event within a certain period has an external cause.

Large Portfolio Limit

An approximation which can be used in case of a large number of names in the portfolio is the large homogenous pool approximation presented in Vasicek [1991]. In case one considers a pool with an infinite number of equivalent names, there is no remaining uncertainty once the realization of the common factor is known, due to the law of large numbers. Thus, after conditioning on the realization of the common factor, one knows exactly the fraction of names of the portfolio which have defaulted. In order to determine this for the model with external default events we follow along the same lines as in Schönbucher [2003]. Let $L(t)$ denote the fraction of names of the portfolio having defaulted before time t. The distribution of this loss fraction can be written as follows:

$$\begin{aligned} Pr\,(L(t) \le l) &= \int_{-\infty}^{\infty} Pr\,(L(t) \le l \mid Y = y)\, \phi(y) dy \\ &= \int_{-\infty}^{\infty} I\left(\kappa(t) + (1-\kappa(t)) \cdot \Phi\left(\frac{\chi(t) - \rho \cdot y}{\sqrt{1-\rho^2}}\right) \le l\right) \phi(y) dy \\ &= \int_{y^*}^{\infty} 1 \cdot \phi(y) dy = \Phi(-y^*) \end{aligned} \tag{9}$$

Here $I(\cdot)$ denotes the indicator function. What remains is to determine the level y^* at which the conditional default probability exactly matches the loss fraction l, which is just a matter of inverting. The resulting distribution is thus given as:

$$Pr\,(L \le l) = \Phi\left(\frac{\sqrt{1-\rho^2} \cdot \Phi^{-1}\left(\frac{l-\kappa(t)}{1-\kappa(t)}\right) - \Phi^{-1}\left(\frac{p(t)-\kappa(t)}{1-\kappa(t)}\right)}{\rho}\right) \tag{10}$$

One can note the close resemblance with the large homogenous pool result for the standard one factor Gaussian copula model. Both l and $p(t)$ have been scaled using the probability of an external default event. Note that this formula should be applied for $l \le \kappa(t)$, as it is zero elsewhere. No matter what the realization of the common factor will be, the number of default events is bounded from below by $\kappa(t)$ due to external default risk.

NUMERICAL RESULTS

In this section the standard one factor Gaussian copula model is compared with the external defaults model. Two aspects of the model are considered. First a comparison between the standard one-factor Gaussian copula and the external defaults model is made, where we consider the effect of a sudden default on the default probability of a correlated surviving name. A second test considers the model's ability to fit to quotes observed in the synthetic CDO market.

Instantaneous Defaults

We consider a simple basket consisting of two names, both having a recovery rate of 40% and a CDS premium of $50BP$. The marginal default distributions are modelled by means of a constant default intensity for both names, approximately $83BP$.[1] We condition on default of the second name before time t and survival of the first name up to time t, and investigate the probability of the first name to default within the following year, thus up to $T \equiv t + 1Y$. This probability can be determined as follows:

$$Pr\,(\tau_1 \le T \mid \tau_1 > t, \tau_2 \le t) = \frac{Pr\,(t < \tau_1 \le T, \tau_2 \le t)}{Pr\,(\tau_1 > t, \tau_2 \le t)} \tag{11}$$

A closed form formula can be derived by applying the results in the appendix of Andersen and Sidenius [2004].

We consider this probability for different times of default, t. This allows us to see the implications of sudden defaults on surviving names under external defaults model. Exhibit 2 displays three different curves. One gives the results of the standard factor Gaussian copula, while the other curves result from the external defaults model, under different parameter choices. In one case the external default risk factor, Z, has been given a standard normal distribution. In the other case its distribution is normal with mean 20 and standard deviation 10. In all cases the copula correlation $\rho_1 \cdot \rho_2$ has been set to 20%.

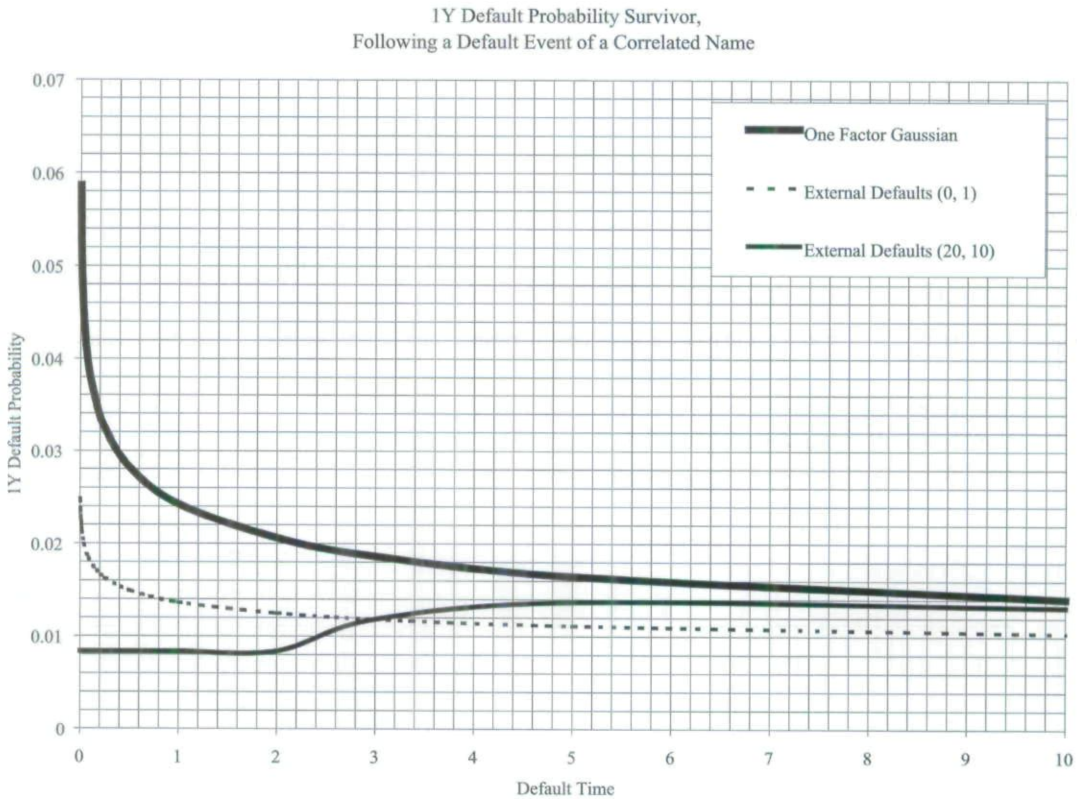
From the exhibit one can observe totally different correlation effects. The standard copula model behaves as expected where a sudden default event causes a large shock in the default probability of the surviving name. In the limit, when $t \downarrow 0$, the conditional default probability of the surviving name will tend to 1, i.e., certain default. When the default event occurs further in the future, the effect of the default event on the surviving name reduces.

For the external defaults model the effect largely depends on the distribution of the external default risk factor, Z. When choosing a standard normal distribution for Z, the results look similar to those of the standard factor Gaussian copula, but with a far lower shock. Here one should note that the distributions of both the market risk factor, X, and that of the external default risk factor, Z, are equal. Thus an instantaneous default event is equally likely to be caused by the market risk factor or by the external risk factor.

In case Z has a normal distribution with mean 20 and standard deviation 10, the behavior is totally different. This is caused by the fat tails of this distribution, compared to the standard normal distribution. One can observe that the default probability of the surviving name is hardly

EXHIBIT 2

Conditional Probability of Name 1 Defaulting before Time t +1Y, Given that it has Survived up to Time t and Name 2 has Defaulted before Time t



affected by a default event of the other name within one year. After this period the effect slowly increases. This can be explained by the fact that, under these settings, a sudden default event can be attributed to an external cause. Intuitively this latter behavior is preferred. Both names have an initial CDS spread of only $50BP$ and are thus not expected to default in the near future. If, however, a default event would occur within a year, it is not likely that the state of the economy has dramatically worsened. An external default event is much more likely and this will not affect the default probability of the surviving name. This behavior is reflected in the exhibit.

When taking a closer look at the model in Equation (3), one can conclude that in case σ is strictly larger than 1 an instantaneous default, i.e., a default before time $t \downarrow 0$, is caused by an external shock and thus has no influence whatsoever on the default probability of the surviving names. Thus with $\sigma > 1$, all graphs as in Exhibit 2 will start at $83BP$. The behavior for larger values for t depends on the correlation and the parameters for the external risk factor, μ and σ.

Fitting to the Synthetic CDO Market

In order to investigate the performance of the external defaults model, it is calibrated to market quotes on synthetic CDO tranches of the 1st of November 2006. Calibration is performed by minimizing the sum of squared errors between model premia and market quotes. This approach is applied to both the standard one factor Gaussian copula as well as the external defaults model. In the former case the correlation parameter is used for calibration, while in the latter model two parameters are used, ρ and μ. The scaling parameter for the external factor, σ, is set to 1.[2]

In Exhibit 3 the results of the calibration test are presented. In the first column one can observe the different tranches. The second column shows the corresponding market quotes. The remaining two columns present the calibration result of the one factor Gaussian copula and the external defaults model. From the results one can observe that the one factor Gaussian copula is not capable of providing a reasonable fit to the quotes observed in the market. In contrast, the external defaults model provides a much better fit. Here one should keep in mind that the external defaults model used here has one additional parameter compared to the one factor Gaussian copula.

CONCLUSION

Over the last years the market for synthetic CDO tranches has undergone a rapid development together with the research efforts in order to value these products

EXHIBIT 3

Analysis of Calibration Performance of Both the One Factor Gaussian Copula as well as the External Defaults Model

Results for the equity tranche are given in upfront percentages, where the coupon equals 500 basispoints. For the remaining four tranches results are given in basispoints.

Tranche	Market Quote	One factor Gaussian	External Defaults
0 – 3	23.3	27.0	25.7
3 – 7	86.0	88.8	86.0
7 – 10	18.0	3.3	18.9
10 – 15	6.5	0.1	6.2
15 – 30	3.3	0.0	0.8

consistently with the observed market quotes. A widely used model for imposing default correlation is the one-factor Gaussian copula approach, and calibration is done using the base correlation method.

In this article we propose an alternative model which addresses two major weaknesses of the standard one factor Gaussian copula. First the proposed model tackles the highly undesirable correlation effects of the one factor Gaussian copula in case of sudden default events. Second, it fits much better to market quotes.

Due to increased liquidity of CDO tranches and the need to value more complex credit derivatives, it is of crucial importance to use a model which is able to explain the observed quotes. Although the base correlation method allows one to calibrate to market quotes, it cannot be considered a proper model and numerous assumptions are required in case one wants to apply it for pricing more complex derivatives.

The alternative model is simple and maintains a large degree of analytical tractability. It has two default drives, one incorporating market risk while the other can be regarded as a source of external risk.

It is shown that the lack of external default risk in the standard model provides a plausible explanation for the base correlation skew observed in the market. Due to the inclusion of external causes for default events, the model allows for more independence for low subordination levels. The results of a simple test presented in this article show that the external defaults model, in its simplest form, provides a much better fit to the market. This model can consequently be used for modelling more complex credit derivatives, such as a CDO of CDOs or forward starting CDOs.

ENDNOTES

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[1]The relationship between a CDS spread π, the recovery rate R and default intensity λ, is approximately given by $\lambda \approx \pi \cdot (1 - R)$.

[2]It was found that allowing both μ and σ to vary during calibration did not improve results much.

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