Where Should You Buy Your Options? The Pricing of Exchange-Traded Certificates and OTC Derivatives in Germany

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In this article we study the pricing of exchange-traded Turbo Certificates and OTC retail derivatives. We investigate two main hypotheses. First, we expect that OTC products are comparatively more expensive due to imperfect competition and limits to arbitrage. Second, we test the life-cycle hypothesis by Wilkens, Erner, and Röder [2003] which states that issuers reduce overpricing during the life of the product. Results affirm the first hypothesis. The second is supported for some but not all OTC contracts under consideration. For Turbos Certificates evidence is weak. This indicates the existence of additional relevant factors other than the position in the product's life cycle driving issuer's pricing policy for derivatives.

After the collapse of the German stock market in general and the Neuer Markt for growth stocks in particular private investors were lacking attractive investment opportunities. Since no large returns could be expected from direct investments in common stocks risk hungry speculators shifted their attention to derivatives allowing them to bet on movements of stock prices, exchange rates, and other financial quantities. The growing demand for such structured financial products was met by banks which started to issue several investment products most of them under the label "certificates." Technically speaking these securities are simply obligations. Their product specifications are subject to individual terms and conditions. As such their designs range from plain index tracking instruments to exotic derivative products. In 2004, more than 25,000 contracts were traded in comparison to 3,400 four years earlier, representing an estimated trading volume of more than €100 billion.[1] The rapid growth of this market segment was fostered by more liberal regulatory rules concerning the sales of derivatives compared to other European countries or the U.S.

However, exchanges are no longer the only place to speculate on the movements of financial quantities. For instance, online bookmakers like Germany's Bluevex or UK's BetOnMarkets offer bets on the outcomes on capital markets. Most notably, ClickOptions, a 100% owned subsidiary of the French Société Générale has launched an internet site offering private investors the ability to buy derivatives "over the counter" (OTC). In October 2000, the company was founded by traders of Société Générale. It started its German operations in September 2002. The business concept is to sell presumably easy to understand standardized options contracts on the internet. This concept has been quite successful over the last years. According to the company ClickOptions had more than 10,000 customers in 2004 each of them executing on average 8 transactions per month.[2]

In this paper we investigate the pricing of exchange-traded structured products, so called "Turbo Certificates," and OTC derivatives offered by ClickOptions written on the

German stock market index DAX. Our study extends the existing literature in several directions: First, empirical evidence about the pricing of structured products is still limited. For the US Chen and Kensinger [1990], Chen and Sears [1990], and Baubois, Gastineau, and Purcell [1993] consider the pricing of equity-linked notes. The work by Wasserfallen and Schenk [1996] and Burth, Kraus, and Wohlwend [2001] covers structured products traded on the Swiss market. Wilkens, Erner, and Röder [2003] and Stoimenov and Wilkens [2005] analyze Discount Certificates, Reverse Convertibles, and other certificates in Germany. However, no empirical study has ever systematically examined Turbos which represent one of the most actively traded class of certificates.[3] Second, we study the pricing of ClickOptions' contracts employing a unique data set. To our knowledge there is no comparable investigation looking at retail derivatives offered OTC to private customers. We expect quotes for both exchange-traded Turbo Certificates as well as ClickOptions products to favor the seller of the product. Furthermore, we hypothesize that this overpricing is higher for ClickOptions due to evident limits to arbitrage and imperfect competition. Third, we explore the life cycle hypothesis proposed by Wilkens, Erner, and Röder [2003] in the context of Turbo Certificates and ClickOptions. Intuitively, this hypothesis states that dealers price derivatives according to the order flow. At the beginning of the products' lives investors tend to buy them. Thus, when dealers quote bid and ask prices, it is more likely that the next transaction will be a sell rather than a buy transaction. Hence, dealers set prices such that they exceed theoretically fair values. However, towards expiry "buy back risk" increases as investors tend to sell their products. Hence, dealers lower overpricing in order to decrease excess premiums when buying back derivatives.

In order to assess the pricing on both markets we employ the framework established by Black and Scholes [1973] (*BS*). Furthermore, for some ClickOptions contracts a static hedging argument is applicable. As expected, we find that derivatives in both markets tend to be overpriced with the overpricing of ClickOptions being much more severe. Furthermore, the life cycle hypothesis cannot be confirmed for Turbo Certificates and evidence for ClickOptions is weak. This indicates the existence of additional factors other than time to expiry that influence the pricing of structured products. The paper is organized as follows: The Market for Retail Derivatives describes the markets for retail derivatives under consideration and develops the hypotheses. ClickOptions and Turbo Certificates explains the products and their pricing. Data and Empirical Results address the data and the results.

THE MARKET FOR RETAIL DERIVATIVES

This section provides an overview of the market of retail derivatives and explores the applicability of option pricing theory on these products. Throughout this paper we will focus on the *BS* model due to its tractability and its wide acceptance in the derivatives' industry. More precisely, the following section describes the market for certificates as well as the design of the ClickOptions internet platform, and the section after that derives the hypothesis.

The Markets for Exchange-Traded Certificates and ClickOptions

Trading in certificates usually takes place on exchanges with the largest turnover being observed on the exchanges in Frankfurt and Stuttgart (EUWAX). Additionally, some issuers offer their clients off-market trading opportunities. Usually, issuers are obliged to act as market makers, i.e., they ensure liquid trading by continuously quoting prices for buying and selling their products.

ClickOptions are traded on each trading day from 9:10 A.M8. to 5:25 P.M. The trading hours are similar to those of the XETRA trading at Deutsche Börse (9:00 A.M. to 5:30 P.M.). During our observation period investors can sign up for free and buy contracts worth up to €30,000 with a maximum order size of €10,000 (€5,000 for beginners). Currently, ClickOptions offers contracts written on several underlyings including German and French single stocks as well as major stock indices. All contracts are short term with expiries not exceeding 6 weeks. During trading hours ClickOptions publishes prices at which the company is willing to sell the option contracts. An investor can buy one or more contracts by simply clicking on the offer. The equivalent value is debited directly from the deposit the investor holds with the company. Overall, buying and holding options is a very simple procedure and can be accomplished even by not so experienced market participants. In contrast, selling options is restricted. First of all, investors may not short sell (write uncovered) options. Second, options may only be sold in the same quantities as they were originally bought, i.e. partial sale of a position is not possible. More precisely, if 100 options are bought at one time they may only be sold in total at a later point in time. It is not possible to sell them in two batches of 50 contracts. Third, investors must sell their options at a discount of €4 from the current price published on the internet. Finally, no trading is possible in contracts with 3 days to expiry or less.[4]

Hypotheses

The idealized *BS* assumptions about the capital markets are not matched in reality. In particular, they may seriously be doubted for the case of ClickOptions:

- *Frictionless Financial Markets:* In a frictionless capital market, investors can short sell overpriced options and replicate them with self-financing trading strategies. In the case of ClickOptions this correction mechanism is not present due to the no-short sales constraint. Clearly, the institutional setup of the German capital market is not completely in-line with the *BS* assumptions either. For instance, even financial intermediaries cannot trade at zero cost. However, the discrepancies are much smaller for exchange-traded options than for ClickOptions.
- *Perfect Competition:* Certainly, ClickOptions has created a product that is not offered by any other competitor/issuer on the derivatives market so far and can thus be seen as successful product differentiation. The concept of trading contracts OTC on the internet allows for direct interaction of the clients with the company, i.e. private customers are offered the opportunity to execute their trades directly by themselves. There is no need for a broker. Furthermore, the cash-settled ClickOptions represent presumably easy to understand bets on financial quantities which are—apart from a few exceptions—unmatched by exchange-traded securities.[5] Hence, arbitrage transactions between these two markets are impossible. In contrast, certificates traded on the stock exchange are more or less homogenous products. Product specifications for the different certificate classes do not vary substantially across banks. It seems reasonable to assume that banks have comparable know-how to design new financial contracts and to place them on the capital market. Thus, competition on the issuer market should eliminate (excess) profits at initial issuance of the product or during the trading period when the issuers act as market makers.
- *Rational Investors:* It is unclear whether private investors are skilled enough to correctly evaluate the derivative structures of ClickOptions. Certainly the same problem may in principle hold for certificates traded on the stock exchange. However, competition among issuers and other (rational) market participants should enforce reasonable prices.

Due to these limitations alone we expect that quoted derivatives prices deviate from theoretical *BS* prices which are ruled out by no-arbitrage considerations in a pure model world. Wilkens, Erner, and Röder [2003] observe in their analysis for the German market of Discount Certificates and Reverse Convertibles that quotes tend to favor the seller. They argue that quotes are mainly driven by market makers who generate premiums by selling products above their theoretically fair values. However, in this case market makers are also facing the risk of having to buy back certificates at high prices. Wilkens, Erner, and Röder [2003] develop and confirm the so called *life cycle hypothesis* which links surcharges to the anticipated buying/selling behavior of investors: in the beginning of the product's life issuers face excess demand. Hence, they tend to overprice their products. Towards the end though investors want to sell their products back. The observed overpricing is therefore reduced as the contract approaches expiry. This leads to the first two hypothesis:

H1: The quotes of exchange-traded certificates and ClickOptions contracts deviate from their theoretical *BS* prices in the direction favoring the seller of the product.

H2: Overpricing is reduced during the products' lives.

ClickOptions successfully created a market niche for OTC derivatives. This should lead to a less competitive pricing policy compared to the issuers of exchange-traded derivatives. Hence, we expect higher overpricing for ClickOptions. To make surcharges comparable we will look at the deviations from theoretical prices in percent. The third hypothesis is:

H3: Percentage-wise the premiums implicit in ClickOptions quotes are larger than those for exchange-traded certificates.

CLICKOPTIONS AND TURBO CERTIFICATES

The purpose of this section is to introduce the products discussed in this paper. First, we describe exchange-traded Turbo Certificates. Then we turn to the products offered by ClickOptions.

Turbo Certificates

Turbo Certificates are frequently also traded under the label *Hebelprodukte* (Leverage Certificates) on the

German capital market. Their name stems from their key feature—they enable the holder to participate more than proportionally in movements of the stock price. Two types of Turbo Certificates are *long* and *short* certificates. While *long* certificates are suited for speculation on price increases, *short* certificates are positively related to price decreases. A *long* certificate promises a payoff equal to that of a standard call option—usually multiplied by some exchange ratio c—provided that a certain down and out barrier is not hit until expiry T. This barrier is typically set equal to the strike price of the option and is monitored continuously on each trading day. Technically speaking, let H_{do} be the strike and down and out barrier. Furthermore, $S(T)$ and S_{min} define the terminal and minimum price of the underlying observed during the life of the certificate, respectively. Then the payoff function can be written as

$$Payoff_{long}(T) = c \cdot (S(T) - H_{do})^{+} 1_{S_{min} > H_{do}} \tag{1}$$

where $1_{\{\cdot\}}$ is the indicator function for the event $\{\cdot\}$. The closed-form solution for such a contract is well known.[6] If r is the risk-free rate of interest, σ is the stock price volatility, and $\Phi(\cdot)$ denotes the operator for the cumulative standard normal probability function, the price of a *long* Turbo Certificate is determined as

$$\begin{aligned} V(long) &= cS(0)\left[\Phi(d_1) - \left(\frac{H_{do}}{S(0)}\right)^{z_1}\Phi(d_3)\right] \\ &\quad - cH_{do}e^{-rT}\left[\Phi(d_2) - \left(\frac{H_{do}}{S(0)}\right)^{z_2}\Phi(d_4)\right] \\ d_1 &= \frac{ln\dfrac{S(0)}{H_{do}} + \left(r + \frac{1}{2}\cdot\sigma^2\right)\cdot T}{\sigma\cdot\sqrt{T}}, \quad d_2 = d_1 - \sigma\cdot\sqrt{T} \\ d_3 &= \frac{ln\dfrac{H_{do}}{S(0)} + \left(r + \frac{1}{2}\cdot\sigma^2\right)\cdot T}{\sigma\cdot\sqrt{T}}, \quad d_4 = d_3 - \sigma\cdot\sqrt{T} \\ z_1 &= 2\cdot\frac{r + \frac{1}{2}\cdot\sigma^2}{\sigma^2}, \quad z_2 = 2\cdot\frac{r - \frac{1}{2}\cdot\sigma^2}{\sigma^2} \end{aligned} \tag{2}$$

Similarly, a *short* Turbo Certificate pays off like a put option at expiry if the price of the underlying does not reach a constant up and out barrier H_{uo}. As with *long* certificates this barrier usually is identical to the strike price, i.e. the option payoff is given by

$$Payoff_{short}(T) = c \cdot (H_{uo} - S(T))^{+} 1_{S_{max} < H_{uo}} \tag{3}$$

where S_{max} denotes the maximum stock price before expiry. The arbitrage-free option value is then given by

$$\begin{aligned} V(short) &= cH_{do}e^{-rT}\left[\Phi(-d_2) - \left(\frac{H_{do}}{S(0)}\right)^{z_2}\Phi(-d_4)\right] \\ &\quad - cS(0)\left[\Phi(-d_1) - \left(\frac{H_{do}}{S(0)}\right)^{z_1}\Phi(-d_3)\right] \end{aligned} \tag{4}$$

where d_1, d_2, d_3, d_4, z_1, and z_2 are as in Equation (2) with H_{do} being replaced by H_{uo}.

Turbo Certificates serve as a viable benchmark to ClickOptions for several reasons. First, Turbo Certificates have little sensitivity with respect to changes in *BS* volatility. This point is highlighted by Scholz, Ammann, and Baule [2003] and Baule, Scholz, and Wilkens [2004]. Therefore, the results should be robust to the specification of the (unobservable) volatility parameter. Second, Turbo Certificates are usually issued with expiries between one to three months. Similar to ClickOptions they can thus be seen as short-term bets on the underlying. Third, they belong to the class of barrier options and represent highly leveraged positions in the underlying—a feature common with several ClickOptions contracts.

ClickOptions Products

ClickOptions market their products as simple bets on future movements of financial quantities. The investor can express his or her views by investing money in different option contracts. If the investor's expectation is met then the payoff of the option is €100, otherwise it expires worthless. More precisely, eight different contracts are to be distinguished, which can be grouped into *plain vanilla* and *barrier* products.

The *plain vanilla* products include contracts sold under the names *EndHi*, *EndLo*, and *Lasso*. Their payoffs depend on the realization of the stock price at expiry T. More precisely, an *EndHi* option promises to pay €100 if the price of the underlying exceeds a predefined strike price X, i.e.,

$$Payoff_{EndHi}(T) = 100 \cdot 1_{S(T) \geq X} \tag{5}$$

Similarly, the holder of an *EndLo* option receives money if $S(T)$ is below a certain strike level

$$Payoff_{EndLo}(T) = 100 \cdot 1_{S(T) \leq X} \tag{6}$$

The payoff profiles of *EndHi* and *EndLo* options are identical to those of contracts usually referred to as "digital" or "binary" options in the literature. In the *BS* world, closed form solutions for these types of products are well known.[7] As such, the value of an *EndHi* option is

$$V_{EndHi} = 100 \cdot e^{-rT} \cdot \Phi(d)$$
$$d = \frac{ln\left(\frac{S(0)}{X}\right) + \left(r - \frac{1}{2}\cdot\sigma^2\right)\cdot T}{\sigma \cdot \sqrt{T}} \tag{7}$$

The value of an *EndLo* option can be calculated according to

$$V_{EndLo} = 100 \cdot e^{-rT} \cdot \Phi(-d) \tag{8}$$

where d is given in equation (7).

The holder of a *Lasso* option receives a €100 payment if the price of the underlying is between two strike levels at expiry. If H_L and H_U are the lower and upper strike respectively $(H_L < H_U)$ then the payoff of this contract can be written as

$$Payoff_{Lasso} = 100 \cdot 1_{H_L \le S(T) \le H_U} \tag{9}$$

This payoff could be replicated by entering a long position in an *EndHi* option with strike price equal to H_L and by short selling such a contract with strike H_U. To avoid arbitrage the option price must be equal to the net payment due in order to construct this portfolio.[8] According to Equation (7) the price of a *Lasso* option is

$$V_{Lasso} = 100 \cdot e^{-rT} \cdot (\Phi(d_L) - \Phi(d_U))$$
$$d_{(\cdot)} = \frac{ln\left(\frac{S(t)}{H_{(\cdot)}}\right) + \left(r - \frac{1}{2}\cdot\sigma^2\right)\cdot T}{\sigma \cdot \sqrt{T}} \tag{10}$$

In contrast to the vanilla options, the payoffs of the *barrier products* depend on the path taken by the stock price until expiry at time T. Generally, these products display high sensitivities towards movements of the underlying. For instance, a *HitHi* option pays off €100 on the first day the closing price of the underlying exceeds a constant barrier H_U. If the option does not hit this barrier until T then it expires worthless. More formally, let m be the number of monitoring points until the option's expiry. Then $\Delta = T/m = 1$ *trading day* is the time interval between each monitoring date and τ_U the first monitoring point for which the stock price is larger than the barrier, i.e., $\tau_U = \inf\{k > 0 : S(\Delta \cdot k) > H_U\}$, $k \in \mathbb{N}$. Then the payoff function of the option at time $t_i = i \cdot \Delta$, $i \in \{1, \ldots, m\}$, is given by

$$Payoff_{HitHi}(t_i) = 100 \cdot 1_{\tau_U = i} \tag{11}$$

Similarly, a *HitLo* option promises the holder to pay €100 the first time the underlying is smaller than a constant barrier $H_L < S(0)$. Define τ_L to be the first monitoring point for which this condition is fulfilled, i.e., $\tau_L = \inf\{k > 0 : S(\Delta \cdot k) < H_L\}$, $k \in \mathbb{N}$. The time t_i payoff follows as

$$Payoff_{HitLo}(t_i) = 100 \cdot 1_{\tau_L = i}. \tag{12}$$

While *HitHi* and *HitLo* represent bets that the closing price of the underlying reaches a certain level, *StayHi* and *StayLo* contracts pay off in the opposite events. More precisely, the holder of a *StayHi* option receives €100 provided that the underlying does not touch a constant barrier H_L before expiry T. Again the barrier is monitored only at the end of each trading day. The payoff at time T can thus be determined according to

$$Payoff_{StayHi}(T) = 100 \cdot 1_{\tau_L > m} \tag{13}$$

Accordingly, a *StayLo* option pays off if the stock price does not exceed a constant barrier H_U, i.e.,

$$Payoff_{StayLo}(T) = 100 \cdot 1_{\tau_U > m} \tag{14}$$

Finally, a *Tunnel* pays €100 if daily closing prices remain above a lower barrier H_L and below an upper barrier H_U until expiry T. Thus, the payoff is given by

$$Payoff_{Tunnel}(T) = 100 \cdot 1_{\tau_L > m,\ \tau_U > m} \tag{15}$$

Unlike contracts with continuously monitored barriers, like Turbo Certificates, products with discretely monitored barriers are difficult to price. Although Heynen and Kat [1996] derive closed-form solutions for this pricing problem, their formulas involve the calculation of the n-dimensional cumulative standard normal distribution, which is difficult to assess for $n \geq 5$. Moreover, it is well known that the monitoring frequencies might have substantial valuation effects.[9] Therefore, a naive application of the corresponding valuation formulas for options with continuously monitored barriers is not possible. In order to avoid numerical valuation procedures, approximation formulas for this pricing problem have been proposed and discussed by Broadie, Glasserman, and Kou [1997], Hörfelt [2003], and Kou [2003]. The problem with these approximation formulas is that they might cause substantial valuation errors as the stock price approaches one of the barriers or when the number of monitoring points is small. To achieve accurate pricing we apply a trinomial tree algorithm proposed by Broadie, Glasserman, and Kou [1999] for options with a single barrier.[10] According to the authors this procedure quickly achieves a "penny accuracy." The basic idea is to implement the stock price process on a trinomial lattice and value the derivative's payoffs by backwards induction. The high degree of accuracy comes from a judicious choice of the asset's jump width and the length of the time step. Thus, for the *HitHi*, *HitLo*, *StayHi*, and *StayLo* options the algorithm should result in precise results. For the *Tunnel* options, the trinomial method is not directly applicable due to the second barrier. We therefore price each *Tunnel* contract by twice implementing the algorithm such that the tree construction (choice of the asset's jump width and the time step) is optimal for either the simultaneously considered lower or upper barrier. The option is then assigned the mean price.

DATA

The study covers the (randomly chosen) sample period from December 11, 2003 until January 23, 2004. During each trading day pricing data of Turbo Certificates and ClickOptions' products were obtained through the internet pages www.onvista.de and www.clickoptions.de. We include only contracts written on the German stock market index DAX, which represents the 30 largest German stock companies in terms of trading volume and market capitalization. The choice of this underlying is motivated by the fact that the DAX certainly belongs to one of the most actively monitored financial entities by German private investors. This observation seems also to be reflected by issuers since by far the most certificates are written on the DAX. Furthermore, DAX futures are liquidly traded contracts on the EUREX, enabling issuers to hedge at low cost. In contrast to price indices like the S&P 500 or the EUROSTOXX 50, the DAX is a performance index based on the assumption that dividend payments are re-invested. Therefore, the *BS* model is directly applicable. There is no need to adjust for potential dividend payments.

Summary statistics for the data collected on Turbo Certificates is provided in Exhibit 1. Altogether 4,690 quotes on 364 Turbo Certificates were obtained. Only a few contracts were excluded. For instance, some contracts specify different strikes and barriers committing the issuer to a redemption payment as soon as the barrier is touched. This affects, e.g., all contracts offered by Germany's DZ Bank (14 contracts, 331 quotes). The reasoning behind the exclusion is that the amount of the redemption payment cannot be determined exactly in advance and as such might induce biases in the valuation. Furthermore, in order to avoid distortion due to market micro structure effects potentially arising from trading in "penny" certificates, contracts with prices less than €0.35 were not taken into account. Finally, we eliminated some obviously wrong quotes from the data set. In total we ended up with 4,187 quotes on 334 contracts. Interestingly, there are many more *long* than *short* certificates. One possible explanation for this phenomenon might be that investors were generally more bullish at that time. Indeed during the sample period we observed that the DAX increased from 3,842 points on December 11, 2003, to 4,142 points on January 23, 2004 (7.6% increase).

We use the mid-prices calculated as the arithmetic average of bid and ask quotes. Note that bid-ask spreads are very thin for Turbo Certificates. They rarely exceed €0.02. This is 0.4% of the mean certificate price (€4.6). However, the bid-ask spread becomes (relatively speaking) more important for certificates which are close to being knocked-out and which therefore trade at low prices. The mean initial distances of the barriers to the DAX of *long* and *short* certificates are 439 and 193 points, respectively. Hence, on average, *long* certificates are more expensive since they are deeper in the money. Furthermore, the range of barriers is also greater for *long* (2,000–2,685 points) than for *short* certificates (3,890–4,685 points). The largest issuer of Turbo Certificates in the sample is HSBC Trinkaus & Burkhardt followed by Citigroup and Deutsche Bank as well as Dresdner Bank. The average time to expiry is 47 calendar days or 33 trading days. In the following we will calculate the maturity of each contract based on actual trading days with one year being assumed to have 257 trading days. This corresponds to the number of trading days in Germany for the year 2004. For Turbo Certificates the choice of the day-count method has only a marginal effect on the results stated below. However, in order

EXHIBIT 1
Summary Statistics Turbo Certificates

	Contracts	Mean	Std.	Min.	Median	Max.
All	334	4.64	3.52	0.38	3.67	18.71
Long	228	5.51	3.58	0.41	4.61	18.41
Short	106	1.97	1.21	0.38	1.64	6.88
Citigroup [CIT]	54	3.20	1.90	0.40	2.90	8.69
Commerzbank [CBK]	45	3.09	1.74	0.41	2.92	7.79
Deutsche Bank [DBK]	46	3.51	2.49	0.42	2.88	11.66
Dresdner Bank [DRB]	39	7.06	4.55	0.50	6.09	17.68
HSBC Tr. & B. [HSB]	77	5.73	3.69	0.38	5.05	15.00
Lang & Schwartz [LAS]	34	2.64	1.58	0.41	2.32	7.46
Sal. Oppenheim [OPP]	39	5.00	3.60	0.44	4.14	18.71

The Exhibit shows summary statistics for the Turbo Certificates analyzed in the sample period (December 11, 2003, until January 23, 2004). The column "Contracts" refers to the total number of different option contracts considered. The other columns relate to mid quotes expressed in €.

to price options with discretely monitored barriers, as those offered by ClickOptions, using trading days appears to be more appropriate.

ClickOptions data is summarized in Exhibit 2. During the sample period 3,428 quotes for 425 different contracts were observed. After adjusting for some unrealistically high quotes, 3,281 quotes for 393 contracts remained in the data set. The vast majority of contracts considered incorporate barrier features since the *HitHi*, *HitLo StayHi*, *StayLo*, and *Tunnel* options account for a total of 302 contracts. The average expiry is 24 calendar days or 16 trading days. Similar to Turbo Certificates we consider mid quotes. As stated above, the spread is fixed at €4 regardless of price or product. This is 8.5% of the average price (€46.57) and therefore considerably more than for Turbo Certificates. The magnitude of the bid-ask spread might partially be attributed to the fact that ClickOptions does not require any commissions for executing trades or custody account administration. For small order volumes savings in commissions and spreads might offset each other. However, for large trades these bid-offer spreads represent a substantial cost.

In order to apply the *BS* model we need estimates for the interest rates and the volatility parameter. As relevant interest rate we assign the respective EURIBOR interpolated for each expiry date. The determination of the unobservable volatility parameter is less clear cut. We chose the VDAX which is published by Deutsche Börse. The VDAX is an index designed to measure the expected DAX volatility for the next 45 calender days. It is obtained by calculating the implied volatilities of plain vanilla at-the-money options on the DAX traded at the EUREX through inversion of the *BS* formula. This choice is plausible because these options are liquidly traded and should therefore reflect market expectations correctly.[11] EURIBOR and VDAX data are obtained through Datastream.

EMPIRICAL RESULTS

This section reports the empirical results. The first subsection deals with Turbo Certificates. The second subsection considers the pricing of ClickOptions.

Turbo Certificates

For each quote record in our data set of *long* and *short* Turbo Certificates we apply Equations (2) and (4) to obtain the theoretical model prices V_i^{Model}, where the subindex *i* counts the records. These prices can be compared to the quoted market prices V_i^{Market}. We distinguish between absolute and relative valuation errors ERR_i^{abs} and ERR_i^{rel}. Absolute valuation errors are computed as the difference between model and market prices $ERR_i^{abs} = V_i^{Model} - V_i^{Market}$. They indicate the valuation errors in € from an investor's perspective. If the error is negative then the model price is smaller than the market price. Investing in the product is unfavorable from a theoretical standpoint. The relative valuation errors are obtained by dividing the absolute pricing errors by the theoretical model prices, i.e. $ERR_i^{rel} = ERR_i^{abs}/V_i^{Model}$. Intuitively,

EXHIBIT 2
Summary Statistics ClickOptions

	Contracts	Mean	Std.	Min.	Median	Max.
All	393	46.57	27.76	3.00	45.63	94.98
EndHi	34	57.41	24.84	3.81	57.42	94.98
EndLo	30	31.48	21.93	3.00	29.83	85.29
HitHi	58	34.23	23.47	3.11	28.49	91.67
HitLo	63	24.86	22.56	3.00	16.09	87.05
StayHi	54	68.42	20.69	17.65	73.44	94.97
StayLo	54	67.21	21.65	16.33	72.45	94.97
Lasso	27	51.32	22.02	3.23	51.12	94.43
Tunnel	73	58.76	23.11	10.64	58.03	94.94

The Exhibit shows summary statistics for the ClickOptions' contracts analyzed in the sample period (December 11, 2003, until January 23, 2004). The column "Contracts" refers to the total number of different option contracts considered. The other columns relate to mid quotes expressed in €.

ERR_i^{rel} represents the relative severity in percent of the pricing errors as a percentage of the theoretical model prices. Again, a negative sign means that in a *BS* world the product would be too expensive.

Exhibit 3 shows summary statistics for the absolute pricing errors as well as the medians of the relative valuation errors. We report medians instead of means because the distribution of the relative valuation contains many extreme values. This can be explained by the fact that for small theoretical prices even minor absolute pricing errors of a few € cents might induce large relative errors. Therefore, we feel that the median is a more informative statistic to characterize the "typical" relative pricing error. Finally, the Exhibit shows the "Hits," i.e., the number of quotes for which the theoretical model prices are within bid-ask spreads.

As indicated, the mean and median absolute pricing error are both €–9.3 cents respectively, i.e. market prices are too expensive compared to theoretical model prices. The absolute pricing errors transform into relative errors with a median of –2.3%. This number compares to Wilkens, Erner, and Röder [2003] who estimate that Reverse Convertibles and Discount Certificates display relative valuation average errors of–3.04% and –4.29%.[12] Looking at the minimum and the maximum absolute pricing error it becomes obvious that pricing errors can be substantial. Interestingly, there are some quotes which strongly favor the buyer of a Turbo Certificate with absolute errors being as large as €17.2 cents. The number of quotes that could be explained by the *BS* model is fairly low. Only 222 hits are observed which corresponds to a hit rate (= number of hits divided by number of quotes) of 5.3%.

Looking at *long* and *short* certificates separately reveals that mean absolute pricing errors are more or less the same for both products. A small difference of roughly €2 cents is observed for the medians (*long* €–8.8 cents, *short* €–10.6 cents). The relative pricing errors for *long* and *short* certificates are −1.7% and −5.7% respectively. Thus, percentage-wise pricing errors are more severe for *short* certificates. This can be explained by the fact that although observed prices for *short* certificates are on average smaller than those of *long* certificates absolute pricing errors are the same for both asset classes. Turning to the hits we find that 198 model prices are within bid-ask spreads for *long* certificates while there are only 24 hits for *short* certificates. This corresponds to hit rates of 6.3% and 2.3% respectively. For individual issuers absolute pricing errors are similar across most banks. The only exception is Dresdner Bank with a comparatively low overpricing of €4.6 cents. Similar observations are made for the medians of the relative valuation errors. Furthermore, we have also checked the pricing errors on a day by day basis. To save space the results are not reported here in detail. Nevertheless, we found that they are more or less stable for single trading days.

In the next step we verify the life cycle hypothesis of Wilkens, Erner, and Röder [2003]. This hypothesis states that in the beginning of a product's life overpricing should be more severe than at its end because dealers price according to the order flow: Initially, banks face excess demand and hence prices exceed theoretical fair values.

EXHIBIT 3
Pricing Errors Turbo Certificates

	Quotes	Mean	Std.	Min	Median	Max	Median %	Hits
All	4187	–9.3	7.6	–57.0	–9.3	17.2	–2.3%	222
Long	3158	–9.1	7.5	–38.9	–8.8	11.1	–1.7%	198
Short	1029	–9.8	8.1	–57.0	–10.6	17.2	–5.7%	24
CIT	609	–10.9	7.1	–39.0	–10.6	14.5	–3.4%	17
CBK	426	–10.1	8.4	–57.0	–9.0	17.2	–3.0%	18
DBK	590	–10.1	7.8	–37.4	–10.3	14.4	–3.3%	30
DRB	649	–4.6	7.4	–31.5	–3.8	13.3	–0.5%	84
HSB	1083	–9.9	6.8	–36.2	–9.5	15.3	–1.8%	47
LAS	363	–11.4	6.1	–37.6	–11.1	12.4	–4.7%	4
OPP	467	–8.9	7.7	–38.8	–7.7	15.3	–1.9%	22

The Exhibit shows summary statistics of the pricing errors of Turbo Certificates. All columns refer to absolute pricing errors (theoretical price minus quoted price) in € cent (€0.01) except for "Median %" and "Hits." "Median %" shows median relative pricing errors (absolute pricing error/theoretical value). The column "Hits" reports the number of observations for which the model price is within the quoted bid-ask spread.

Towards the end of the product's life investors tend to sell their products and hence overpricing is reduced. If the i^{th} quote is for a *long* certificate define the variables $MONEY_i$ and $TIME_i$ as $MONEY_i = 0.001 \cdot (DAX_i - H_i)$ and $TIME_i = 0.01 \cdot DAYS_i$ respectively, where $DAYS_i$ denotes the number of trading days until expiry and H_i is the relevant barrier. In the case of *short* certificates we set $MONEY_i = 0.001 \cdot (H_i - DAX_i)$. We run the OLS regression

$$ERR_i^{abs} = CONST + \beta_1 \cdot MONEY_i + \beta_2 \cdot TIME_i + \varepsilon_i \tag{16}$$

which tests the influence of moneyness and time to expiry on absolute valuation errors. Intuitively β_1 represents the changes in the valuation error per 1,000 points the DAX moves away from the barrier, i.e. a 1,000 point increase for a *long* certificate and a 1,000 point decrease for a *short* certificate. In other words, β_1 captures the possible systematic effect of the knock-out risk on absolute pricing errors. The coefficient β_2 characterizes the change in absolute pricing errors per 100 additional trading days. According to the life cycle hypothesis we expect a negative value for β_2, i.e. the longer the time to expiry the greater is the initial loss of the investor. The estimation equation (16) is a modified version of Wilkens, Erner, and Röder [2003]. First, instead of absolute pricing errors they use relative pricing errors. However, as already stated above, relative valuation errors can reach extreme values especially for low moneyness values. Therefore, a linear regression seems not to be appropriate. Second, they measure the position in the life cycle as the product's relative age defined as time since issuance divided by the initial time to expiry. For the case of Turbo Certificates we feel though that considering the remaining trading days is sufficient because initial times to expiry are similar to each other (one to three months).[13] Third, Wilkens, Erner, and Röder [2003] also consider the size of the spread in their regression. This is not applicable here since the spread is more or less invariant at a level of €2 cents.

Exhibit 4 shows the regression results. First, we perform a regression on the whole data set. On this aggregate level the life cycle hypothesis cannot be confirmed. The β_2 coefficient is positive though insignificant. Furthermore, we regress *long* and *short* certificates separately. For *long* certificates the coefficient β_2 is significantly positive leading again to a rejection of the life cycle hypothesis. Per 100 additional trading days the surcharge on theoretical prices is reduced by €2.1 cents according to this estimation. In contrast for *short* certificates the β_2 coefficient is significantly negative which in turn supports the hypothesis. Finally, we consider regressions on a single issuer level. The results are mostly in line with the life cycle hypothesis with nearly all β_2 coefficients being significantly negative. For the influence of the moneyness measured by β_1 no clear cut conclusion concerning significance or direction can be drawn from the analysis.

When interpreting these results we have to acknowledge that the corresponding adjusted coefficients of deter-

EXHIBIT 4
Regression Results Turbo Certificates

	CONST	t-value	β_1	t-value	β_2	t-value	R^2_{adj}
All	-11.0^{***}	−40.57	3.6^{***}	11.13	0.4	0.64	0.03
Long	-12.1^{***}	−40.25	4.3^{***}	11.78	2.1^{***}	3.47	0.05
Short	-6.7^{***}	−9.47	-4.2^{**}	−1.98	-6.3^{***}	−4.19	0.03
CIT	-7.3^{***}	−7.91	-4.9^{***}	−3.19	-8.8^{***}	−3.89	0.02
CBK	-10.8^{***}	−4.85	4.2	1.06	−2.7	−0.48	0.01
DBK	-6.4^{***}	−6.98	−1.2	−0.90	-13.0^{***}	−5.66	0.04
DRB	-2.3^{***}	−3.07	1.6^{**}	2.26	-7.1^{***}	−4.81	0.03
HSB	-10.6^{***}	−19.75	1.1^{*}	1.94	0.4	0.36	0.00
LAS	-10.5^{***}	−11.07	1.2	0.64	-3.2^{**}	−1.99	0.01
OPP	-7.1^{***}	−7.03	4.7^{***}	5.1	-8.1^{***}	−5.06	0.10

The Exhibit shows the parameter estimates of the linear regression model $ERR_i = CONST + \beta_1 \cdot MONEY_i + \beta_2 \cdot TIME_i + \varepsilon_i$ for the Turbo Certificates. The asterisks denote statistical significance at the ***99% level, **95% level, *90% level. Standard errors are adjusted according to White (1980). R^2_{adj} is the adjusted coefficient of determination.

mination R^2_{adj} are disappointingly low indicating limited explanatory power of the variables $MONEY_i$ and $TIME_i$. For the whole data set the R^2_{adj} is just 0.03 and does not exceed 0.10 in any regression on the subsets. This suggests that there must be additional factors to explain observed overpricing. First, we have to recognize that Turbo Certificates are economically different from those products analyzed by Wilkens, Erner, and Röder [2003]. In contrast to Reverse Convertibles and Discount Certificates, Turbo Certificates are subject to knock-out risk. Once the certificate is knocked-out it becomes worthless and trading activities are terminated immediately making any further price quotations according to the life cycle obsolete. To put it differently the risk of having to buy back the security is reduced. Second, due to high leverage Turbo Certificates are very risky securities. Thus, we expect that holding periods for these certificates should be much shorter than for Discount Certificates or Reverse Convertibles. Therefore, the trading behavior of investors might be more closely related to the short term market situation rather than time to expiry as predicted by the life cycle hypothesis.

Click Options

Before we investigate the pricing policy within the *BS* framework we note that some ClickOptions contracts can be bundled today such that their payoff will be €100. This static hedge enables us to create synthetic zero-coupon bonds.[14] More precisely consider the following combinations:

- *EndHi/EndLo*: This portfolio consists of an *EndHi* and an *EndLo* option with identical expiry and strike price. It pays off €100 regardless of the stock price at expiry.[15]
- *EndHi/EndLo/Lasso*: This portfolio consists of an *EndHi*, an *EndLo*, and a *Lasso* option. The strikes of the *EndHi* and *EndLo* contracts equal the upper and lower strikes of the *Lasso*, respectively. All contracts have the same expiry ensuring that the portfolio pays off €100.
- *StayHi/HitLo*: This portfolio consists of a *StayHi* and a *HitLo* option with the same expiry dates. The barriers of the two options are equal to each other. Regardless of the path taken by the underlying stock, the portfolio always pays off €100.[16]
- *StayLo/HitHi*: This portfolio is constructed analogously to the *StayHi/HitLo* portfolio.
- *HitHi/HitLo/Tunnel*: This portfolio consists of a *HitHi*, a *HitLo*, and a *Tunnel* option. The barriers of the *HitHi* and *HitLo* contracts equal the upper and lower barriers of the *Tunnel*, respectively. If all options have the same expiries the portfolio pays off €100 whatever path the underlying takes.

Altogether, 196 records of synthetic zero coupon bonds could be constructed from our data set. Exhibit 5 reports the pricing errors of these portfolios relative to the theoretical prices which are implied by the interest rate data. They turn out to be substantial. If all contracts are taken together the mean absolute pricing error is €–9.75 which compares to theoretical prices being slightly below €100. Furthermore, for the *EndHi/EndLo/Lasso* combination the mean absolute pricing error amounts to €–15.26 while for the *EndHi/EndLo* and the *StayHi/HitLo* portfolios it is given by €–8.28 and €–9.63 respectively. For each portfolio class the standard deviations of the absolute pricing errors are remarkably low pointing towards a distinct pricing strategy. Interestingly, only 6 records can be constructed for the *StayHi/HitLo* and none for the *StayLo/HitHi* and the *HitHi/HitLo/Tunnel*. Thus, only very limited information is revealed about the pricing policy of *HitHi*, *HitLo*, *StayHi*, *StayLo*, and *Tunnel* options.

Pricing ClickOptions in the *BS* framework yields the results shown in Exhibit 6. In most cases observed prices exceed theoretical option values by far. Exceptions are the *HitHi* and the *HitLo* options with mean absolute pricing errors of €1.30 and €1.15. Nevertheless, this underpricing is well below €2 which is one half of the bid-ask spread. The absolute valuation errors are the most severe for the products *StayHi*, *StayLo*, *Lasso*, and *Tunnel*. Their mean absolute pricing errors range from €–10.36 to €–15.89 compared with an average of €–6.16 for the whole sample. Nevertheless, for these contracts we also observe absolute pricing errors as large as €31.15 which imply substantial discounts on theoretical prices. However, these are single outliers. Percentage-wise pricing errors are more severe than in the case f exchange-traded Turbo Certificates. The median relative valuation error for the whole set of ClickOptions is −13.0%, i.e. the products are quoted 13.0% above their theoretical values. For the *Lasso* and the *Tunnel* options we even find median relative valuation errors of −29.9% and −37.4%. Certainly, this high overpricing of ClickOptions relative to exchange-traded products is only feasible because there is no kind of arbitrage possible between both markets under consideration. Finally, 724 of 3,281 quotes are within the bid-ask spread which corresponds to a hit rate of 22.1%. For the contracts *EndLo* and *HitHi* this number even amounts to 46.5% and 45.8%. When comparing these numbers to Turbo Certificates we have to keep in mind though that bid-ask spreads for these securities are much smaller.

In order to test the life cycle hypothesis we run similar regressions as in section `Turbo Certificates'. The variables $TIME_i$ and $MONEY_i$ are determined as before for *EndHi*, *EndLo*, *StayHi*, and *StayLo* options.[17] Since *HitHi* and *HitLo* options can be interpreted as being "out of the money" unless the underlying has not hit the barrier we set $MONEY_i = 0.001 \cdot (DAX_i - H_i)$ and $MONEY_i = 0.001 \cdot (H_i - DAX_i)$ respectively. This definition consistently ensures that $MONEY_i$ increases as the DAX approaches the barrier and the option becomes more valuable. Furthermore, for the *Lasso* and *Tunnel* options a meaningful variable cannot be defined. Therefore, for these contracts the estimation of the β_1 variable is omitted.

The regression results are shown in Exhibit 7. For the whole data set the life cycle hypothesis is supported with the β_2 coefficient being significantly negative at the 99% confidence level. More precisely, the surcharge on theoretical prices increases by €5.61 per extra 100 trading days. On a single-contract level most β_2 coefficients also turn out to be negative although they are statistically significant only for *HitHi*, *Lasso*, and *Tunnel* options. Especially for *Lassos* and *Tunnels* the effect is pronounced with

EXHIBIT 5
Deviations from Zero-Coupon Bond Prices

	Quotes	Mean	Std.	Min	Median	Max	Median %
All	195	–9.75	2.88	–15.86	–8.53	–5.83	–8.6%
EndHi/EndLo	149	–8.28	0.58	–10.17	–8.37	–5.83	–8.4%
EndHi/EndLo/Lasso	40	–15.26	0.67	–15.86	–15.48	–13.30	–15.6%
StayHi/HitLo	6	–9.63	0.44	–10.17	–9.48	–9.05	–9.5%
StayLo/HitHi	0						
HitHi/HitLo/Tunnel	0						

The Exhibit shows summary statistics of the absolute pricing errors (theoretical price—quoted price) from the zero coupon prices which can be constructed synthetically by investing in portfolios of options. All deviations are reported in € except for "Median %," which shows median relative pricing errors (absolute pricing error/theoretical price).

EXHIBIT 6
Pricing Errors ClickOptions

	Quotes	Mean	Std.	Min	Median	Max	Median %	Hits
All	3281	–6.16	8.18	–41.08	–5.07	31.15	–13.0%	724
EndHi	307	–4.89	3.79	–12.41	–5.32	4.33	–9.4%	81
EndLo	344	–1.56	2.81	–9.22	–1.44	4.94	–6.2%	160
HitHi	498	1.30	4.35	–41.08	1.34	10.18	4.3%	228
HitLo	619	1.15	3.62	–15.07	0.76	11.30	3.1%	232
StayHi	280	–11.37	5.00	–19.84	–11.34	31.15	–22.0%	3
StayLo	309	–10.36	4.69	–19.02	–10.32	28.06	–19.7%	0
Lasso	312	–11.05	4.92	–23.28	–11.27	3.83	–29.9%	17
Tunnel	612	–15.89	6.59	–35.19	–15.54	18.52	–37.4%	3

The Exhibit shows summary statistics of the pricing errors of ClickOptions' products. All columns refer to absolute pricing errors (theoretical price—quoted price) in € except for "Median %" and "Hits." "Median %" shows median relative pricing errors (absolute pricing error/theoretical price). The column "Hits" reports the number of observations for which the model price is within the quoted bid-ask spread.

EXHIBIT 7
Regression Results ClickOptions

	CONST	t-value	β_1	t-value	β_2	t-value	R^2
All	–5.29***	–19.47			–5.61***	–3.24	0.00
EndHi	–4.42***	–15.41	–27.5***	–23.18	–0.83	–0.58	0.71
EndLo	–3.01***	–7.46	–11.09***	–7.88	1.66	–0.91	0.20
HitHi	–1.75***	–2.69	–9.86***	–3.77	–8.25***	3.85	0.08
HitLo	0.65	1.44	9.67***	8.87	17.20***	10.62	0.28
StayHi	–9.76***	–14.86	–14.84**	–2.29	2.09	0.44	0.04
StayLo	–7.79***	–8.95	–12.59***	–3.61	–5.02	–1.62	0.08
Lasso	–7.32***	–13.31			–26.51***	–9.18	0.20
Tunnel	–9.17***	–15.33			–43.50***	–12.58	0.26

The Exhibit shows the parameter estimates of the linear regression model $ERR_i = CONST + \beta_1 \cdot MONEY_i + \beta_2 \cdot TIME_i + \varepsilon_i$ for the contracts offered by ClickOptions. For the contracts "Lasso" and "Tunnel" as well as the whole data set the regression model reduces to $ERR_i = CONST + \beta_2 \cdot TIME_i + \varepsilon_i$. The asterisks denote statistical significance at the ***99% level, **95% level, and *90% level. Standard errors are adjusted according to White (1980). R^2_{adj} is the adjusted coefficient of determination.

β_2 being estimated as €–26.51 and €–43.40 respectively. The life cycle hypothesis is clearly rejected for *HitHi* contracts. For *EndLo* contracts a positive though insignificant β_2 coefficient is obtained. Furthermore, except for the *HitLo* options all β_1 coefficients are significantly negative. Thus, surcharges increase as moneyness and thereby theoretical option prices increase. This observation is consistent with Wilkens, Erner, and Röder [2003]. Comparing the R^2_{adj} to those of the Turbo Certificates we find that at least on a single product level the explanatory power of the regressions is higher. For the *EndHi* options we even observe an R^2_{adj} of 0.71 although β_2 is not significant. For *Lasso* and *Tunnel* contracts R^2_{adj} it is 0.20 and 0.26 respectively. For the whole data set however R^2_{adj} is close to 0. Therefore, the life cycle hypothesis can only be confirmed overall for some single products at best. This observation might partially be explained through the presence of substantial bid-offer spreads. Furthermore, similar to issuers of Turbo Certificates, ClickOptions can keep the total initial surcharge if the option is held until maturity or knocks-out in the case of *StayLo*, *StayHi*, and *Tunnel* contracts. We also have to keep in mind that ClickOptions imposes constraints on the sale of the products since it is not possible to partially offset positions. This also reduces buy-back risk.

CONCLUSION

This research investigates the pricing of exchange-traded Turbo Certificates and OTC option contracts offered by ClickOptions. We find that both types of contracts exceed their theoretical prices. The median surcharge for the ClickOptions' products is 13.0% which compares to 2.3% for Turbo Certificates. Thus, the results support hypothesis H1 and H3. Turning to the life cycle hypothesis H2 results are mixed. At most this hypothesis can be confirmed on an single dealer level for Turbo Certificates and for some ClickOptions contracts. In general, the low explanatory power of the regressions points towards the existence of factors other than time to expiry to explain observed overpricing.

This study might be extended in several directions. First, an interesting question for future research might be to further investigate the pricing patterns of Turbo Certificates which could not fully be resolved in this paper. We argue that investors' anticipated purchase/sales behavior might be related to the short-term market situation. In a subsequent study it appears to be reasonable to consider intraday data of both Turbo Certificates and its underlying to explore this issue in greater detail. Second, we cover only DAX options here. As discussed above the DAX belongs to the most closely monitored financial entities in the German market. Examining different underlyings would show whether their choice also influences overpricing. Wilkens, Erner, and Röder [2003] find substantial differences in the case of Discount Certificates and Reverse Convertibles on DAX and NEMAX[18] companies. Third, we assume the *BS* model in our study. Its application is justified by its wide acceptance and application in practice. In the literature several extensions have been proposed, including stochastic volatility and jumps.[19] In a recent article Pan [2002] proposes "implied-state GMM" estimations to calibrate these models using S&P 500 index options. A broader application of this approach to the market of structured products might bring about insights on the robustness of our results.

ENDNOTES

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[1]Sources http://www.onvista.de, http://boerse.ard.de.

[2]Source: "Hebelprodukte: Spielkameraden," in: *New Investor,* No. 3, 2004, Download: http://www.new-investor.de/0304/news/artikel3.html.

[3]Following Discount Certificates, Turbo Certificates are the most actively traded certificates on the German market.

[4]This rule applies to our entire observation period. Recently, the no-trade period was slightly relaxed to 2 business days before expiry. However, this minor change should have no effect on our general conclusions.

[5]For instance Commerzbank offers a few so called "Touch Down Certificates" which resemble *EndHi* and *EndLo* contracts issued by ClickOptions. The size of these issues of certificates is negligible compared to the total market of certificates.

[6]The *long* Turbo Certificate can be identified as a special case of a down-and-out barrier call contract, which was originally discussed by Merton [1973]. Rubinstein and Reiner [1991] derive valuation formulas for a broad range of barrier contracts.

[7]See the textbook of Hull [2003].

[8]Of course it would also be possible to construct a *Lasso* as a spread between two *EndLo* contracts.

[9]This was first observed by Heynen and Kat [1994] for options with barriers, which are monitored continuously but only during a fraction of the contract's lifetime.

[10]Broadie, Glasserman, and Kou [1999] refer to this method as *enhanced trinomial method* in contrast to the *basic trinomial method* proposed by Ritchken [1995] or the *binomial method* considered by Boyle and Lau [1994].

[11]A slight inconsistency is introduced at this point because the VDAX is computed based on calendar days rather than trading days. However, the impact should be negligible.

[12]In contrast to Turbo Certificates, the prices of Reverse Convertibles and Discount Certificates do not approach zero (except for some extreme theoretical cases). Therefore, the average relative valuation error is certainly more informative than in the case of Turbo Certificates.

[13]In contrast, the maturities of Discount Certificates and Reverse Convertibles are usually several years.

[14]Credit risk issues can safely be neglected at this point because especially in the light of the results discussed below they should have only minor effects.

[15]If the stock price at expiry is exactly equal to the common strike price then the portfolio pays off €200. However, for a continuous stock price distribution, such an event has zero probability and therefore does not influence the valuation. In reality stock prices are of course not continuously distributed. It is reasonable to assume though that the probability of exactly hitting a particular stock price level is small enough to be neglected for valuation purposes. Similar considerations hold for the combination *EndHi/EndLo/Lasso*.

[16]At this point we neglect the specification that the *HitLo* contract pays off €100 as soon as the barrier is hit while the *StayHi* contract pays off at expiry. However, given the short maturity of the contracts and the low interest rates this slight inconsistency should have only minor effects on the portfolio value. Similar limitations apply to the portfolios *StayLo/HitHi* and *HitHi/HitLo/Tunnel*.

[17]The moneyness variables for *EndHi* and *StayHi* options is determined according to *long* certificates. For *StayLo* and *EndLo* their computation corresponds to the case of *short* certificates.

[18]The NEMAX was the major stock market index for Germany's Neuer Markt.

[19]Models with stochastic volatility have been proposed by Hull and White [1987] and Heston [1993]. Merton [1976] discusses a jump-diffusion model. Examples for models with both jumps and stochastic volatility include Bates [1996, 2000], Bakshi, Cao, and Chen [1997], and Pan [2002].

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