A Matrix-Based Lattice Model to Value Employee Stock Options

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Under the Revised FAS Statement No. 123 (FAS 123R)[1] introduced recently, U.S. public companies must expense all employee stock option (ESO) grants as of grant date, at *fair value* estimated "using option-pricing models adjusted for the unique characteristics of those instruments." FAS 123R notes that "lattice" models, that are typically used to price American-style traded options, may be better suited than "closed-form" models (such as variants of the Black-Scholes model) to accommodate various unique features of ESOs. A key feature of ESOs is the non-tradeable nature of such grants which results in early exercise, even when such early exercise would have been sub-optimal in the case of traded American-style options.

In lattice models, the entire set of future stock price paths can be described as a discrete lattice (or tree) and the possibility of early exercise may be evaluated at each node. In the case of traded options, if the option's intrinsic value at the node is less than its continuation value (the option's value if unexercised for another period) then early exercise would be irrational. An option holder who requires liquidity could sell the unexercised option to another investor. This logic, however, does not work in the case of *non-tradeable* ESOs. As a result, an employee may be forced to exercise an ESO early to meet her liquidity needs even when the ESO's intrinsic value is less than its continuation value at a particular node. Therefore, in contrast to the valuation of American style *traded* options, any ESO valuation model must address the employee's reasons (which could be idiosyncratic) for early exercise given the non-tradeable nature of such grants.

According to FAS 123R, such early exercise behavior could be a function of several factors including: (i) The underlying stock's price or the ESO's "moneyness" at a particular node on the lattice; (ii) the grant's remaining contractual life; (iii) employees' "historical exercise and post-vesting employment termination behavior observed for similar grants;" (iv) the stock's expected volatility; (v) blackout periods; and (vi) employees' ages, lengths of service, and home jurisdictions (domestic versus foreign).

Our paper makes two contributions. First, we develop an ESO valuation model that incorporates the various unobservable factors that could influence an employee's early exercise decision at each lattice node in a parsimonious fashion, through an estimated early exercise probability. This probability of early exercise of a vested and exercisable option at a particular node on the lattice is expressed as a two-dimensional matrix, i.e., as a function of the ESO's remaining vested life and its moneyness at that particular node. Thus, our valuation model incorporates many of the factors that FAS 123R explicitly recognizes as relevant to ESO valuation for accounting purposes.

Second, we describe how the early exercise matrix can be estimated, given sufficiently detailed data, and incorporated into our valuation model. To demonstrate the manner in which the early exercise matrix can be estimated and incorporated into the valuation model, we rely on a rich ESO dataset that has hitherto never been made public. The dataset contains the complete history of all ESOs awarded by two major technology companies (henceforth referred to as Companies A and B). The dataset for Company A tracks 4.34 billion ESOs granted by the company through 464,205 separate ESO "grants."[2] The dataset for Company B tracks 142 million ESOs granted by the company through 54,720 separate ESO "grants." To the best of our knowledge, this is the first model that incorporates an empirically estimated early exercise decision rule into the ESO valuation model that is consistent with FAS 123R guidelines.

The remainder of this paper consists of the following sections. In Section 2 we discuss the related literature on ESO valuation. In Section 3 we develop our matrix-based lattice model to value ESOs. In Section 4 we discuss our dataset in greater detail. In Section 5 we focus on the estimation of the early exercise matrix that is the central element of our valuation model. Section 6 describes the remaining relevant parameters for ESO valuation including dilution, cancellation, interest rate, dividend yield and volatility of stock return. Finally in Section 7 we implement our model to derive the value of a hypothetical ESO grant by each subject company. We compare these estimated values to those derived using alternative early exercise rules.

2.0 LITERATURE REVIEW

There are generally two alternative approaches taken in the literature to value ESOs: "utility-based" and "reduced-form" models. Utility-based models assume that the employee exercises her option according to a policy that maximizes her expected utility subject to hedging restrictions.[3] The benefit of a utility-based model is that it allows us to predict early exercise from first principles, given the choices presented to the employee at each point in time in the future. However, this benefit comes at a cost. Even when represented in simplistic forms, many of the factors that affect early exercise, such as the employee's degree of risk aversion or non-option wealth, are difficult to measure directly or to infer from publicly observable data.

Reduced-form models do not attempt to model the employee's optimal early exercise behavior from first principles as a function of unobservable parameters. Instead, in such models, early exercise is modeled directly as an exogenous factor. For instance, Hull and White [2004] assume that the employee will exercise her option whenever the stock price reaches (or exceeds) a certain multiple of the exercise price (a "moneyness" criterion). According to Carr and Linetsky [2000], the *ad hoc* early exercise criterion imposed in reduced form models serves as a proxy for "anything that may lead the employee to exercise his option early, including the desire for diversification or liquidity, and voluntary or involuntary employment termination."[4]

Reduced-form models are arguably simpler from a modeling perspective since they avoid explicitly modeling the employee's optimal early exercise policy. However, from a practical standpoint, reduced-form models can only be implemented if the employee's early exercise decision criterion can be empirically estimated accurately. Not surprisingly, there is no consensus in the academic literature regarding which class of ESO valuation models is superior. The main challenge in specifying a reduced form model is in finding an empirical exercise rule that is sufficiently flexible to describe the actual exercise behavior observed among the firm's employees.

Ours is a reduced form lattice model, in which the early exercise rule is empirically calibrated to the employee's past exercise behavior and not based on a utility maximization framework. Our early exercise rule predicts the fraction of outstanding exercisable options that will be exercised at each node of a lattice as a function of two observable parameters: the option's *moneyness* (as in Hull and White [2004]) as well as its *remaining vested life* at a particular node on the lattice. We later compare our results to that of the Hull and White [2004] model and the adjusted Black-Scholes model.

3.0 THE ESO VALUATION MODEL

ESO's Economic Value at Grant Date

We directly model the employee's early exercise decision criterion as a parameter, $P_{i,j}$, which represents the likelihood that the employee would exercise all "exercisable" (or vested and in-the-money) options that she has at date i and node j. $P_{i,j}$ (the early exercise criterion) is modeled as a function of the option's remaining contractual life as well as the stock-price to exercise-price multiple. The option's value at date i, node j is written as $C_{i,j}$. The equations describing the backward induction through the lattice are as follows:

At maturity, T, the option's value is:[5]

$$C_{T,j} = Max(S_{T,j} - X, 0) \tag{1}$$

For all preceding dates, $0 \leq i \leq N - 1$, if the option is vested (that is, $i > v$, where v denotes the vesting date), the value of the option at node j is determined as follows:

If $S_{i,j} - X > uC_{i+1,j+1} + (1 - u)C_{i+1,j}/r$ then:

$$C_{i,j} = S_{i,j} - X \tag{2}$$

If $0 < S_{i,j} - X \leq uC_{i+1,j+1} + (1 - u)C_{i+1,j}/r$ then:

$$C_{i,j} = (P_{i,j})(S_{i,j} - X) + (1 - P_{i,j}) \times \left(\frac{uC_{i+1,j+1} + (1 - u)C_{i+1,j}}{r}\right) \tag{3}$$

If $S_{i,j} - X \leq 0$ then:

$$C_{i,j} = (1 - q_{i,j})\left(\frac{uC_{i+1,j+1} + (1 - u)C_{i+1,j}}{r}\right) \tag{4}$$

If the option is unvested at $0 \leq i \leq N - 1$ (that is, if $i < v$):

$$C_{i,j} = (1 - q_{i,j})\left(\frac{uC_{i+1,j+1} + (1 - u)C_{i+1,j}}{r}\right) \tag{5}$$

The term r denotes one plus the risk free rate, u and $(1 - u)$ are the probabilities of up-jump and down-jump respectively on the binomial tree, and $q_{i,j}$ denotes the probability that the employee might voluntarily or involuntarily lose her job at date i and node j, when the option is out of the money. Note (4) indicates that if the employee does not lose her job (with probability $(1 - q_{i,j})$) when the option is out of the money then the option's value is simply the value derived from holding it for an additional period. The value of the ESO at the grant date is expressed as $C_{0,0}$. This value represents the estimated *ex ante economic* value of the ESO.

Two comments regarding Equations (2) and (3) of our model are in order at this juncture. First, both Equations (2) and (3) describe scenarios when the option is in-the-money. Equation (2) describes the case when the option is "deep" in-the-money (the ESO's intrinsic value is greater than its continuation value). Equation (3) also describes an in-the-money state, although the ESO's intrinsic value is now less than its continuation value. We assume that if the ESO is deep in-the-money [equation (2)] then the probability of early exercise $P_{i,j}$ equals 1, i.e., the employee will rationally exercise her option early (as she would in the case of traded options). In contrast to traded options, we allow the probability of early exercise to be greater than zero even when the option's intrinsic value is less than its continuation value [equation (3)]. Thus, as in Hull and White [2004], we treat the possibility of early exercise asymmetrically depending on how deep in-the-money the ESO is at a particular node on the lattice.

An alternative modeling structure would be to allow for the possibility that even when the option's intrinsic value exceeds its continuation value the employee may not rationally exercise her ESO early.[6] That is, we could assume that the employee's probability of early exercise may be less than 1 even in equation (2). Formally, if we permitted $P_{i,j}$ to be less than 1 even when the option's intrinsic value exceeds its continuation value then the right hand side of equation (2) would be identical to the RHS of equation (3), and a single equation (identical to equation (3) above) would then suffice to define the ESO's in-the-money value. By allowing for the possibility that employees may not optimally exercise their ESOs even if these were deep in the money, such an alternative model would result in even lower ESO values compared to the values we derive for dividend paying stocks.[7] Thus, our current modeling structure is conservative from an accounting perspective.

Second, it is important to note that an employee who (voluntarily or involuntarily) loses her job at date i can, and rationally should, exercise all vested and in-the-money ESOs that she holds as of that date. However, an employee may fail to act in this rational fashion, which would result in the cancellation of some of her vested in-the-money options when she loses her job. Since the cancellation rate $q_{i,j}$ is missing in equation (3) it may appear that we have disregarded this prospect of forfeiture of in-the-money options.[9] However, this is not the case since the empirically calibrated early exercise possibility, $P_{i,j}$, that appears on the RHS of equation (3) is based on the relative frequency of *all* options actually exercised by employees in the past (as a percentage of exercisable options) regardless of whether such exercises occurred due to job termination or not. In short, the early exercise probability modeled in equation (3) incorporates the prospect of job termination and the employee's decision to exercise (or not to exercise) all her vested in-the-money options at that time, which would otherwise be canceled shortly thereafter.

ESO's Accounting Value at Grant Date

The *ex ante* value of an unvested option to its holder, denoted by $C_{0,0}$ in equation (5), depends upon $q_{i,j}$, the probability of termination or resignation. This aspect of our model is identical to that of Hull and White [2004].

Although the above modeling perspective is appropriate in determining the *ex ante* economic value of the ESO at grant date, it is inconsistent with FAS 123R's implementation guidelines, which makes an important distinction between vested and unvested ESOs for estimating an ESOs' grant-date fair value for accounting purposes. FAS 123R recognizes that post-vesting restrictions (concerning sales and transfers) should be considered in "estimating the fair value of the instruments at the grant date."[9] However, FAS 123R states that:[10]

> In contrast, a restriction that stems from the forfeitability of instruments to which employees have not yet earned the right, such as the inability either to exercise a nonvested equity share option or to sell nonvested shares is not reflected in the fair value of the instruments at the grant date. Instead, those restrictions are taken into account by recognizing compensation cost only for awards for which employees render the requisite service.

Consistent with such regulatory language, equation (5) in our economic value model presented above should be amended to read as follows:

$$C_{i,j} = \left(\frac{uC_{i+1,j+1} + (1-u)C_{i+1,j}}{r} \right) \tag{6}$$

Thus, the ESOs fair value at grant date for accounting purposes (or the *ex ante* "accounting value") can be derived using the same set of equations (1)–(4) and by replacing (5) in the economic value model with (6) presented above.

From the preceding discussion it follows that the ESO's grant date economic and accounting values are both functions of the following parameters, each of which can be empirically estimated:

1. The early exercise probability, $P_{i,j}$.
2. The cancellation rate, $q_{i,j}$.
3. The dilution factor, λ.
4. Dividend yield, δ.
5. Risk-free rate, r.
6. The stock return volatility, σ.

Additionally, the ESO value depends on (i) the grant date stock price; (ii) exercise (strike) price, (iii) vesting schedule (time of initial vesting, time of subsequent vesting and corresponding vesting percent); (iv) black out periods; and (v) contractual life. These parameters are all known as of the grant date and do not need to be estimated.

4.0 THE ESO DATASET

Before discussing the manner in which we estimated the various model parameters, we provide some descriptive statistics about our dataset of ESO grants of Companies A and B, which we use to estimate empirical probabilities of early exercise.

The dataset contains the complete history of all ESOs awarded by Company A from its inception date in 1988 through June 2004, and the employees' subsequent exercise decisions regarding these awards. The dataset tracks 4.34 billion ESOs granted by the company through 464,205 separate ESO "grants." Through June 2004, employees had exercised 2.23 billion ESOs, through 192,474 separate early exercise decisions. Exhibits 1A and 2A provide some summary statistics regarding Company A's grants and exercises, respectively.

The dataset also contains the complete history of all ESOs awarded by Company B from its inception date in 1989 through January 2005, and the employees' subsequent exercise decisions regarding these awards. The dataset tracks 142 million ESOs granted by the company through 54,720 separate ESO "grants." Through January 2005, employees had exercised 60 million ESOs, through 58,743 separate early exercise decisions. Exhibits 1B and 2B provide some summary statistics regarding Company B's grants and exercises.

Grant Patterns for Company A

Exhibit 1A classifies the grants into cliff-vested and monthly-vested categories for company A. All ESOs awarded in a cliff-vested grant vest simultaneously on the vesting date. In contrast, only a certain fraction of the ESOs awarded through a monthly-vested grant vest on the initial vesting date. The remaining ESOs included in the grant then vest uniformly on a monthly basis over a specified number of subsequent months.[11] A relatively small

EXIHIBIT 1A

Summary Statistics of All Grants Issued by Company A from 1988–2004

Vesting	Cliff Vesting Grants			Monthly Vesting Grants			Other Vesting Schedule			Total		
Year	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)
1988	33	640,985,252	$ 0.0002	5	26,784,000	$ 0.0002				38	667,769,252	$ 0.0002
1989	159	398,954,228	$ 0.0045	15	31,536,000	$ 0.0026				174	430,490,228	$ 0.0044
1990	137	81,771,028	$ 0.07	22	8,769,600	$ 0.08				159	90,540,628	$ 0.07
1991	302	193,216,652	$ 0.20	90	35,094,096	$ 0.20				392	228,310,748	$ 0.20
1992	596	90,862,248	$ 0.49	186	16,572,240	$ 0.63				782	107,434,488	$ 0.51
1993	1,377	71,178,036	$ 0.97	489	25,352,402	$ 1.38	3	5,215	$ 6.22	1,869	96,535,653	$ 1.08
1994	3,041	123,167,808	$ 1.20	1,528	50,352,398	$ 1.51	47	25,059	$ 24.41	4,616	173,545,265	$ 1.29
1995	3,873	165,260,122	$ 2.78	2,678	95,852,400	$ 3.05	27	132,119	$ 3.73	6,578	261,244,641	$ 2.88
1996	5,914	138,721,644	$ 5.04	6,085	124,846,360	$ 5.52	153	616,058	$ 2.95	12,152	264,184,062	$ 5.26
1997	5,241	111,838,552	$ 5.30	8,872	181,725,376	$ 5.62	209	640,582	$ 4.88	14,322	294,204,510	$ 5.49
1998	342	6,010,650	$ 8.26	14,817	184,226,981	$ 11.45	566	1,539,686	$ 4.91	15,725	191,777,317	$ 11.30
1999	294	8,753,531	$ 34.65	28,949	237,260,971	$ 24.57	838	3,337,881	$ 9.53	30,081	249,352,383	$ 24.72
2000	241	3,802,698	$ 28.89	88,820	483,710,070	$ 50.99	440	2,809,141	$ 37.61	89,501	490,321,909	$ 50.74
2001	120	1,083,483	$ 3.86	89,325	235,993,959	$ 16.71	58	883,668	$ 0.53	89,503	237,961,110	$ 16.59
2002	6	44,321	$ 2.94	115,537	273,214,747	$ 15.11	157	872,992	$ 6.53	115,700	274,132,060	$ 15.08
2003	1,102	2,514,326	$ 16.25	76,724	259,106,879	$ 17.36	132	2,200,566	$ 5.08	77,958	263,821,771	$ 17.25
2004*	19	83,099	$ 20.74	4,636	18,037,808	$ 23.43				4,655	18,120,907	$ 23.42
Total	22,797	2,038,247,678	$ 1.26	438,778	2,288,436,287	$ 20.85	2,630	13,062,967	$ 12.8958	464,205	4,339,746,932	$ 11.63

The data provided below are categorized according to the following variables. *YEAR*: calendar year during which the company made a grant to an employee. *NUMBER OF GRANTS*: the total number of such grants made during a calendar year. (Note: Certain employees may have received multiple grants during the same calendar year). A grant consists of several employee stock options that vest according to a particular vesting schedule, i.e., the time interval between grant date and the date when the option becomes exercisable. *CLIFF VESTING GRANTS*: The vesting schedule is such that all options included in the grant vest simultaneously at a specified date. *MONTHLY VESTING GRANTS*: The vesting schedule is such that the majority of the options included in the grant vest proportionally in monthly intervals following the grant date. *OTHER VESTING SCHEDULES*: Grants with vesting schedules that are neither of the "cliff-" or "monthly-vesting" types. *AGGREGATE NUMBER OF ESOs GRANTED* indicates the total number of employee stock options included in the grants made during a calendar year. Grants were always designed to be at-the-money on grant date. *WEIGHTED AVERAGE STRIKE PRICE* provides the average stock price, averaged across all grant dates during the grant year, weighted by the number of ESOs granted on each grant dates.

EXHIBIT 1B

Summary Statistics of All Grants Issued by Company B from 1990–2005

Vesting	Cliff Vesting Grants			Monthly Vesting Grants		
Year	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)
1989	-	-	$ -	-	-	$ -
1990	-	-	$ -	-	-	$ -
1991	-	-	$ -	-	-	$ -
1992	-	-	$ -	-	-	$ -
1993	-	-	$ -	-	-	$ -
1994	-	-	$ -	180	899,778	$ 10.5421
1995	-	-	$ -	1,675	6,142,725	$ 13.1764
1996	15	9,765	$ 2.5405	4,161	10,140,375	$ 11.8832
1997	7	15,675	$ 10.2751	5,528	16,308,147	$ 8.5423
1998	-	-	$ -	2,774	9,017,796	$ 16.0637
1999	7	211,674	$ 21.6350	5,688	15,114,511	$ 27.7911
2000	13	1,685	$ 71.9200	5,581	9,892,016	$ 49.8476
2001	2	24,000	$ 31.5621	7,474	12,735,464	$ 35.2807
2002	26	73,614	$ 6.9300	3,109	5,295,249	$ 44.2245
2003	-	-	$ -	3,953	5,373,521	$ 43.9574
2004	-	-	$ -	4,050	5,624,042	$ 39.6995
2005	-	-	$ -	124	110,750	$ 40.2224
Total	70	336,413	$ 18.29	44,297	96,654,374	$ 26.44

Vesting	Annual Vesting Grants			Other Vesting Schedule			Total		
Year	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)	Number of Grants	Aggregate Number of ESOs Granted	Weighted Average Strike Price ($)
1989	-	-	$ -	65	1,184,910	$ 0.0218	65	1,184,910	$ 0.0218
1990	-	-	$ -	184	5,925,636	$ 0.1779	184	5,925,636	$ 0.1779
1991	-	-	$ -	124	2,413,671	$ 0.4999	124	2,413,671	$ 0.4999
1992	-	-	$ -	222	1,930,218	$ 1.3185	222	1,930,218	$ 1.3185
1993	-	-	$ -	733	7,811,418	$ 2.9496	733	7,811,418	$ 2.9496
1994	1,367	4,725,432	$ 7.4509	284	3,772,014	$ 5.7204	1,831	9,397,224	$ 7.0523
1995	2,493	4,005,366	$ 14.2974	8	41,826	$ 10.9941	4,176	10,189,917	$ 13.6081
1996	2,372	2,376,243	$ 11.1936	58	116,577	$ 10.8878	6,606	12,642,960	$ 11.7372
1997	1,576	3,206,868	$ 8.0718	83	3,216,930	$ 8.1498	7,194	22,747,620	$ 8.4217
1998	2	120,000	$ 15.0000	22	1,591,752	$ 16.3705	2,798	10,729,548	$ 16.0973
1999	406	1,008,446	$ 19.9135	9	428,703	$ 8.5356	6,110	16,763,334	$ 26.7470
2000	115	275,695	$ 43.0467	24	252,501	$ 26.1988	5,733	10,421,897	$ 49.0983
2001	-	-	$ -	25	148,264	$ 33.6283	7,501	12,907,728	$ 35.2548
2002	77	56,702	$ 6.9300	100	43,401	$ 25.8993	3,312	5,468,966	$ 43.1905
2003	1	150,000	$ 51.8900	-	-	$ -	3,954	5,523,521	$ 44.1728
2004	1	12,000	$ 42.7900	2	100,000	$ 42.7900	4,053	5,736,042	$ 39.7598
2005	-	-	$ -	-	-	$ -	124	110,750	$ 40.2224
Total	8,410	15,936,752	$ 11.76	1,943	28,977,821	$ 4.28	54,720	141,905,360	$ 20.25

See the notes to Exhibit 1A for variable description.

EXHIBIT 2A

Company A's Employees' ESO Exercise Behavior from 1988–2004

Vesting	Cliff Vesting Grants				Monthly Vesting Grants			
Year	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)
1988	1	217,385,856	$ 0.0002	$ 0.0002				
1989	6	90,843,120	$ 0.0002	$ 0.0026	1	86,400	$ 0.0002	$ 0.0301
1990	160	206,797,552	$ 0.002	$ 0.084	16	15,914,884	$ 0.001	$ 0.103
1991	311	212,075,880	$ 0.007	$ 0.257	43	12,846,960	$ 0.018	$ 0.243
1992	575	171,237,364	$ 0.06	$ 0.66	133	16,579,820	$ 0.11	$ 0.68
1993	677	92,191,972	$ 0.16	$ 1.43	164	10,692,208	$ 0.11	$ 1.40
1994	744	68,137,928	$ 0.14	$ 1.80	189	13,824,600	$ 0.13	$ 1.87
1995	2,091	110,810,640	$ 0.39	$ 2.85	696	21,231,018	$ 0.53	$ 3.16
1996	3,496	88,844,506	$ 0.80	$ 5.81	1,434	23,393,856	$ 0.95	$ 5.68
1997	8,552	121,062,606	$ 1.70	$ 7.67	3,757	42,665,792	$ 2.13	$ 8.04
1998	12,179	103,647,826	$ 2.89	$ 14.03	9,107	68,399,338	$ 3.27	$ 15.00
1999	13,070	84,845,726	$ 3.49	$ 31.84	15,767	80,567,020	$ 4.58	$ 32.32
2000	11,114	67,815,978	$ 4.51	$ 60.16	23,601	88,478,321	$ 6.67	$ 48.15
2001	6,499	40,945,086	$ 4.79	$ 22.62	14,254	70,418,916	$ 5.19	$ 16.62
2002	1,553	12,439,435	$ 5.39	$ 15.39	5,908	22,619,209	$ 4.88	$ 12.16
2003	1,084	7,522,781	$ 4.55	$ 15.65	25,672	58,785,705	$ 9.29	$ 19.93
2004*	159	661,062	$ 2.96	$ 24.22	26,396	37,491,465	$ 10.58	$ 24.89
Total	62,271	1,697,265,318	$ 0.92	$ 6.88	127,138	583,995,512	$ 4.68	$ 20.62

Vesting	Other Vesting Schedule				Total			
Year	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)
1988					1	217,385,856	$ 0.0002	$ 0.0002
1989					7	90,929,520	$ 0.0002	$ 0.0027
1990					176	222,712,436	$ 0.002	$ 0.085
1991					354	224,922,840	$ 0.008	$ 0.256
1992					708	187,817,184	$ 0.07	$ 0.66
1993					841	102,884,180	$ 0.16	$ 1.42
1994					933	81,962,528	$ 0.14	$ 1.81
1995					2,787	132,041,658	$ 0.41	$ 2.90
1996					4,930	112,238,362	$ 0.83	$ 5.78
1997					12,309	163,728,398	$ 1.81	$ 7.76
1998	22	24,072	$ 7.13	$ 20.78	21,308	172,071,236	$ 3.04	$ 14.41
1999	195	459,968	$ 3.47	$ 17.82	29,032	165,872,714	$ 4.02	$ 32.03
2000	707	1,744,408	$ 3.83	$ 51.60	35,422	158,038,707	$ 5.72	$ 53.34
2001	1,142	1,224,965	$ 4.81	$ 22.21	21,895	112,588,967	$ 5.04	$ 18.87
2002	196	690,964	$ 2.15	$ 6.34	7,657	35,749,608	$ 5.00	$ 13.17
2003	573	775,076	$ 4.09	$ 17.79	27,329	67,083,562	$ 8.70	$ 19.43
2004*	230	374,026	$ 4.40	$ 24.64	26,785	38,526,553	$ 10.39	$ 24.87
Total	3,065	5,293,479	$ 3.90	$ 28.96	192,474	2,286,554,309	$ 1.89	$ 10.44

The data provided below are categorized according to the following variables. *YEAR*: the calendar year during which an employee exercised an ESO. *NUMBER OF EXERCISES*: Number of times ESOs were exercised during the calendar year. (Note: an "exercise" could include the decision to exercise several ESOs simultaneously). *AGGREGATE NUMBER OF ESOs EXERCISED* indicates the total number of employee stock options exercised during a calendar year. Grants were always designed to be at-the-money on grant date. *WEIGHTED AVERAGE STRIKE PRICE* provides the average strike price across all ESOs that were exercised during the calendar year, weighted by the number of ESOs exercised. *WEIGHTED AVERAGE STOCK PRICE* provides the average stock price, averaged across all exercise dates during the calendar year, weighted by the number of ESOs exercised on each date. See notes to Exhibit 1A for additional definition of the variables.

EXHIBIT 2B

Company B's Employees' ESO Exercise Behavior from 1990–2005

Vesting	Cliff Vesting Grants				Monthly Vesting Grants			
Year	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)
1990	-	-	$ -	$ -	-	-	$ -	$ -
1991	-	-	$ -	$ -	-	-	$ -	$ -
1992	-	-	$ -	$ -	-	-	$ -	$ -
1993	-	-	$ -	$ -	-	-	$ -	$ -
1994	-	-	$ -	$ -	-	-	$ -	$ -
1995	-	-	$ -	$ -	56	56,904	$ 10.33	$ 24.89
1996	4	2,346	$ 1.83	$ 13.09	154	285,918	$ 9.96	$ 16.90
1997	8	5,058	$ 5.73	$ 10.64	342	212,013	$ 9.11	$ 11.20
1998	4	2,433	$ 3.69	$ 17.83	3,599	4,037,967	$ 8.28	$ 18.21
1999	2	1,269	$ 2.77	$ 30.33	7,465	4,189,544	$ 10.44	$ 34.82
2000	14	121,674	$ 11.08	$ 66.35	7,642	4,005,901	$ 16.69	$ 54.17
2001	6	15,000	$ 8.08	$ 37.50	5,571	3,528,138	$ 18.55	$ 38.68
2002	45	75,788	$ 10.02	$ 45.39	10,534	5,169,045	$ 24.68	$ 46.89
2003	27	14,321	$ 29.22	$ 47.49	8,758	4,330,370	$ 27.43	$ 48.12
2004	5	4,900	$ 32.41	$ 41.08	3,816	2,685,776	$ 28.49	$ 44.87
2005	1	1,165	$ 6.93	$ 42.90	156	250,616	$ 27.10	$ 40.73
Total	116	243,954	$ 11.73	$ 42.45	48,093	28,752,192	$ 18.94	$ 40.45

Vesting	Annual Vesting Grants				Other Vesting Schedule			
Year	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)
1990	-	-	$ -	$ -	30	396,528	$ 0.1027	$ 0.4811
1991	-	-	$ -	$ -	22	86,022	$ 0.109	$ 0.500
1992	-	-	$ -	$ -	48	1,637,325	$ 0.161	$ 1.392
1993	-	-	$ -	$ -	528	2,313,738	$ 0.43	$ 5.86
1994	-	-	$ -	$ -	671	3,291,300	$ 0.63	$ 7.87
1995	324	246,006	$ 7.17	$ 20.53	1,160	3,670,821	$ 2.30	$ 13.83
1996	181	344,619	$ 7.63	$ 14.99	685	2,435,661	$ 2.32	$ 14.74
1997	710	658,482	$ 7.42	$ 10.63	594	2,508,393	$ 2.88	$ 9.85
1998	1,053	1,882,134	$ 7.71	$ 18.26	583	1,987,497	$ 4.84	$ 17.62
1999	955	1,521,348	$ 8.31	$ 32.66	327	1,934,899	$ 6.17	$ 29.54
2000	631	1,163,980	$ 11.43	$ 53.42	224	1,660,253	$ 9.63	$ 52.87
2001	296	288,685	$ 12.00	$ 38.54	144	928,118	$ 9.97	$ 38.52
2002	706	540,968	$ 13.88	$ 45.44	139	724,255	$ 11.98	$ 46.30
2003	207	172,861	$ 17.43	$ 47.86	125	244,445	$ 11.97	$ 47.84
2004	125	115,149	$ 9.40	$ 43.60	50	28,790	$ 17.46	$ 41.99
2005	13	7,318	$ 7.54	$ 40.06	3	1,323	$ 13.53	$ 40.54
Total	5,201	6,941,550	$ 9.34	$ 30.65	5,333	23,849,368	$ 3.51	$ 17.42

Vesting	Total			
Year	Number of Exercises	Aggregate Number of ESOs Exercised	Weighted Average Strike Price ($)	Weighted Average Stock Price on the Exercise Date ($)
1990	30	396,528	$ 0.1027	$ 0.4811
1991	22	86,022	$ 0.109	$ 0.500
1992	48	1,637,325	$ 0.161	$ 1.392
1993	528	2,313,738	$ 0.43	$ 5.86
1994	671	3,291,300	$ 0.63	$ 7.87
1995	1,540	3,973,731	$ 2.72	$ 14.41
1996	1,024	3,068,544	$ 3.63	$ 14.97
1997	1,654	3,383,946	$ 4.16	$ 10.09
1998	5,239	7,910,031	$ 7.28	$ 18.07
1999	8,749	7,647,060	$ 8.93	$ 33.05
2000	8,511	6,951,808	$ 14.03	$ 53.95
2001	6,017	4,759,941	$ 16.45	$ 38.64
2002	11,424	6,510,056	$ 22.20	$ 46.69
2003	9,117	4,761,997	$ 26.28	$ 48.10
2004	3,996	2,834,615	$ 27.61	$ 44.78
2005	173	260,422	$ 26.39	$ 40.72
Total	58,743	59,787,064	$ 11.64	$ 30.18

See the notes to Exhibit 2B for variable description.

portion of Company A's grants, classified as "Other Vesting Schedule," do not fall into either the cliff- or monthly-vesting categories.

Exhibit 1A reveals several patterns in the types of grants made by the subject company over time. In the early part of the firm's life (from 1988 to 1995) the company issued more cliff-vested grants than monthly-vested grants. There were 9,518 separate cliff-vested grants made over the 1988–1995 period compared to 5,013 monthly-vested grants over the same period. Moreover, the average size of cliff-vested grants (185,480 ESOs per grant) was significantly greater than the average size of cliff-vested grants (57,912 ESOs per grant) over the 1988–2005 period.

These grant trends changed over the subsequent period of Company A's life (1997 to 2004). The company issued far fewer cliff-vested grants (7,365) over this period compared to monthly-vested grants (427,680). However, the average size of cliff-vested grants made over this latter period (18,212 ESOs per grant) continued to be larger than the average size of monthly-vested grants (4,380 ESOs per grant).

Over the entire time period covered by our dataset, cliff-vested grants constituted only 4.9% of the total number of separate ESO grants awarded by Company As. However, the total number of ESOs included in cliff-vested and monthly-vested grants was roughly similar. Cliff- and monthly-vested grants included 47.3% and 52.7% of the 4.34 billion ESOs granted in total, respectively.

The company's grants were typically made at the money as of the grant date. Therefore, as the company's stock price rose meteorically from its IPO through the tech boom years of 1999 and 2000, so did the weighted average strike price of the ESOs granted over the years. The strike price declined after 2000 in the aftermath of the tech crash.

Exercise Behavior for Company A

Exhibit 2A provides the number of ESOs that were exercised in a particular year, the weighted average strike price of such exercised options and the weighted average price of the underlying stock at the time of exercise. Overall, across all exercised ESOs, the average strike price was \$1.89 and the stock price at the exercise date was nearly 13 times as high, at \$23.96.

Grant and Exercise Behavior for Company B

The Exhibits 1B and 2B show the grant and exercise behavior for company B. Company B had fewer cliff-vested options and more annual and monthly vesting grants. Exhibit 1B shows that the weighted average grant price of 54,720 grants encompassing 142 million ESOs was \$20.25. Exhibit 2B shows that overall, across all exercised ESOs, the average strike price was \$11.64 and the stock price at the exercise date was about 3.5 times as high, at \$40.80. The ratio of weighted average stock price to strike price (i.e., moneyness) differed with the vesting schedule. The weighted average moneyness at exercise was 2.08 for monthly vested options, 3.90 for annual vested options and 11.1 for other options.

It is important to note that Exhibits 2A and 2B do not separate ESOs by date of grant. That is, the options exercised in a given year are not classified by time elapsed since they were issued or by their remaining vested life.

Basic option pricing reasoning suggests that the decision to exercise an ESO depends on both the ESOs moneyness as well as the option's remaining vested life. Therefore, as we discuss in detail later, in estimating the early exercise probability matrix we further classify the exercise data discussed above by the option's remaining vested life at exercise date.

5.0 PARAMETER ESTIMATION

We focus our attention in this paper on estimating the key parameter of interest, *viz.*, the early exercise probability matrix. This estimation was conducted in the following steps.

Grant Cohorts

We began by "aggregating" the sample of each subject company's ESO grants, by treating all ESOs granted on the same date, with the same contractual terms (including vesting schedule and strike price) as a single "aggregated" grant or "grant cohort."

For both companies, our analysis includes only grants with defined vesting schedules.[12] Given the large number of grants issued by Company A, we restricted its sample further by requiring that grant cohorts were either cliff-vested or monthly-vested. These criteria helped us select grant characteristics that closely match the new grants offered by the company.

As a result of this grant-date aggregation, the ESOs for company A were compiled into 1,461 grant cohorts and those for Company B into 688 cohorts.

"Remaining Vested Life" Sub-intervals and "Moneyness" Ratios Within Sub-Intervals

In order to trace the early exercise decision by employees for all ESOs in a particular cohort we first defined the life of a particular grant cohort in terms of its Remaining Vested Life, *i*. The Remaining Vested Life of a grant at any calendar date was defined as $(T - \zeta)/(T - \nu_0)$, where T, ζ, and ν_0 denote the ESO's contractual maturity date, the calendar date on which the remaining vested life was being calculated, and the initial vesting date for the grant, respectively.[13] We categorized the grant cohort's total vested life into twenty equal sub-intervals, defined in terms of the grant's Remaining Vested Life i . Thus, each sub-interval spans 5% of the option's vested life. The first sub-interval $(i = 1)$ starts at the option's initial vesting date and ends at $i = 0.95$, which designates the beginning of the second sub-interval (which is labeled $i = 0.95$ for notational convenience). The second sub-interval ends at $i = 0.90$ which designates the beginning of the third sub-interval, and so on. The last such sub-interval, which begins at $i = 0.05$, spans the final 5% of the ESO's vested life. At the final date (which coincides with the maturity date, T), $i = 0$.

We tracked all ESOs belonging to a particular grant cohort across the 20 sub-intervals $i = 1, 0.95, \ldots 0$. We listed all the ESO exercises (across grant cohorts) that occurred within the same remaining vested life sub-interval *i,* and computed the "moneyness" ratio represented by the stock-price to exercise-price multiple at exercise date (S/X), corresponding to each observed exercise. We also recorded those cases where ESOs in a particular grant cohort were vested and exercisable (i.e., were in the money at some point during the sub-interval *i*) but were not exercised.

For each grant cohort, we recorded the minimum value of the company's stock price over a particular sub-interval *i*, denoted by S_i and the "moneyness" ratio, S_i/X, (or s_i for notational convenience) where X denotes the exercise price of all ESOs in the same grant cohort.[14] Since there were 1,461 grant cohorts for Company A, there were 1,461 s_i values calculated for each of the 20 sub-intervals, *i*. Within each sub-interval *i*, we then rank-ordered the s_i values and sub-divided this ranked set into ten sub-groups, where each subgroup contained 10% of all grant cohorts.

Thus, completing the three steps described above resulted in a defined matrix with 200 rows representing 20 time sub-intervals *i*, each of which is sub-divided into 10 columns denoting particular s_i sub-groups.

The Early Exercise Probability

Within each cell of this matrix we computed an early exercise probability, $p(i, s_i)$ for each of the grant cohort included in this cell. This ratio is defined as the number of ESOs belonging to a particular grant cohort that were exercised within sub-interval *i* divided by the number of vested shares in that particular grant cohort with remaining vested life *i*.[15]

Since there are 10 rows in the matrix, each denoting a particular s_i sub-group, each cell in the matrix (holding the sub-interval *i* constant) includes 10% of all grants ranked by their respective moneyness ratios, s_i. With 1,461 grant cohorts for Company A, each cell in the matrix would typically contain 140 to 150 calculated $p(i, s_i)$ values. However, for more recent grants, all sub-intervals *i* may not have yet occurred. As a result, the cells corresponding to lower values of sub-interval *i* may have fewer $p(i, s_i)$ observations.[16]

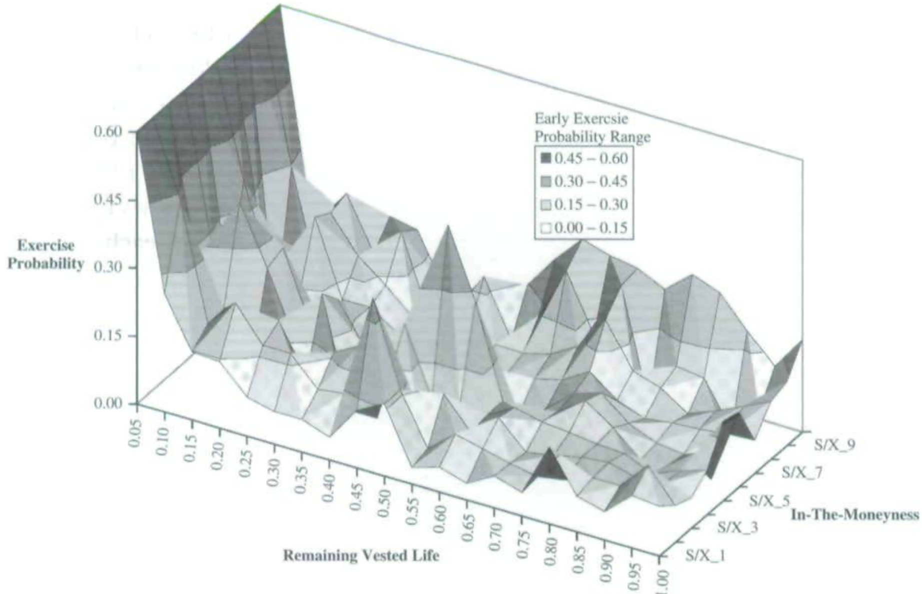
We averaged across all the $p(i, s_i)$ values within a particular matrix cell (i.e., for the same values of *i* and s_i, weighing each $p(i, s_i)$ value by that grant cohort's "relative size," where "size" is defined by the number of ESOs belonging to a particular grant cohort that were vested and exercisable during a particular sub-interval *i*.[17]) We define this average $p(i, s_i)$ value as $P(i, s_i)$.

The estimated early exercise probability matrix for Company A and Company B are graphically shown in Exhibits 3A and 3B, respectively. The underlying data for these Exhibits (the complete early exercise probability matrices for Companies A and B) are provided in Exhibits 4A and 4B, respectively. These matrices consist of approximately 200 rows and 4 columns. The first column lists the Remaining Vested Life, which is further sub-divided into moneyness deciles. The next two columns of the matrix report the lower and the upper bounds of each moneyness decile.[18] The last column reports the average early exercise probability, $P(i, s_i)$, for the particular moneyness decile.

The early exercise probability matrices derived for Companies A and B reveals several interesting features concerning employees' ESO exercise behavior (see Exhibits 3A and 3B, respectively). In particular, our analysis reveals the following:

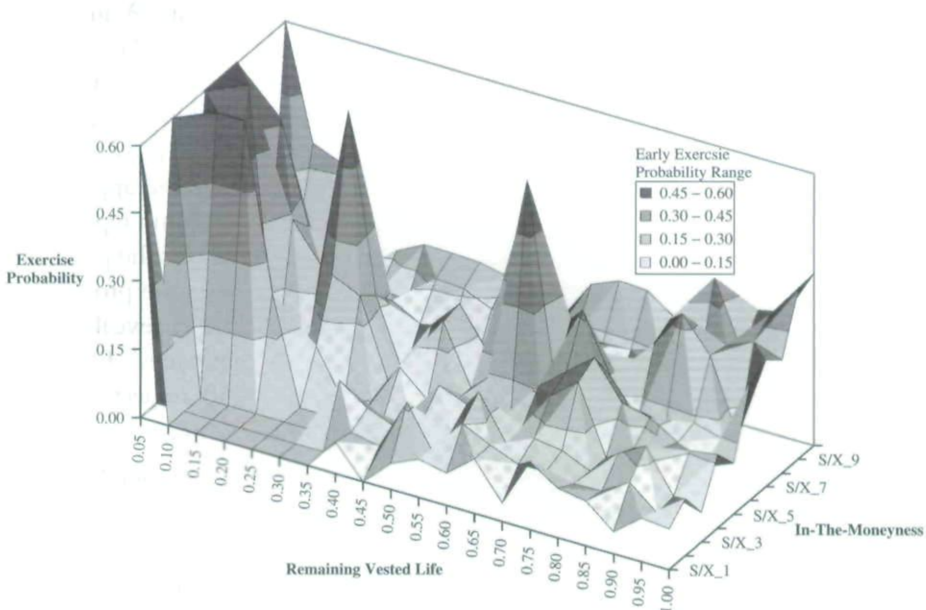
1. There is considerable early exercise behavior associated with ESOs given the non-tradeable and unhedgeable nature of such grants.
2. There is a spurt of early exercise immediately upon initial vesting. As shown in Exhibit 3A and 3B, the early exercise frequency $P(i, s_i)$, demon-

Exhibit 3A
Early Exercise Probability Matrix for Company A



REMAINING VESTED LIFE is defined at any calendar date ζ as $(T - \zeta)/(T - \nu_0)$, where T, ζ, and ν_0 denote the ESO's contractual maturity date, the calendar date on which the Remaining Vested Life was being calculated, and the initial vesting date for the grant, respectively. Remaining Vested Life is divided into 20 equal intervals, ranging from 0.05 (near maturity) to 1.0 (at initial vesting date). For each Remaining Vested Life interval, the moneyness ratio (S/X) was divided into deciles, S/X_1 (lowest), S/X_2, ...S/X_10 (highest). *EARLY EXERCISE PROBABILITY* is the ratio of ESOs exercised to the number of vested and exercisable ESOs available at a given Remaining Vested Life and the specified S/X decile. The data used for this diagram was reported in Exhibit 3A.

Exhibit 3B
Early Exercise Probability Matrix for Company B



See notes to Exhibit 3A for the description of variables.

EXHIBIT 4A

Early Exercise Probability Matrix for Company A

No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)
1	1	1.00	1.27	11.05%	41	0.8	1.00	1.11	17.10%	81	0.6	1.00	2.04	5.53%	121	0.4	1.00	1.55	5.13%	161	0.2	1.00	3.48	14.59%
2	1	1.27	1.39	8.19%	42	0.8	1.11	2.35	6.72%	82	0.6	2.04	2.18	9.74%	122	0.4	1.55	2.01	11.57%	162	0.2	3.48	3.76	24.56%
3	1	1.39	1.68	6.84%	43	0.8	2.35	3.28	8.69%	83	0.6	2.18	3.05	7.01%	123	0.4	2.01	2.46	19.14%	163	0.2	3.76	4.07	7.49%
4	1	1.68	1.86	9.88%	44	0.8	3.28	3.57	8.60%	84	0.6	3.05	3.57	7.81%	124	0.4	2.46	3.61	7.59%	164	0.2	4.07	5.87	26.04%
5	1	1.86	2.03	17.59%	45	0.8	3.57	4.07	9.16%	85	0.6	3.57	3.99	7.93%	125	0.4	3.61	4.59	8.18%	165	0.2	5.87	9.17	27.48%
6	1	2.03	2.13	16.64%	46	0.8	4.07	4.40	7.04%	86	0.6	3.99	4.17	1.95%	126	0.4	4.59	6.26	9.85%	166	0.2	9.17	11.75	13.63%
7	1	2.13	2.15	7.37%	47	0.8	4.40	5.14	5.00%	87	0.6	4.17	6.45	6.62%	127	0.4	6.26	7.22	6.74%	167	0.2	11.75	11.88	28.17%
8	1	2.15	2.29	12.82%	48	0.8	5.14	8.85	8.67%	88	0.6	6.45	9.55	8.89%	128	0.4	7.22	8.00	4.07%	168	0.2	11.88	13.48	26.04%
9	1	2.29	2.56	13.96%	49	0.8	8.85	9.28	10.35%	89	0.6	9.55	10.83	24.31%	129	0.4	8.00	10000	10.13%	169	0.2	13.48	15.53	27.37%
10	1	2.56	10000	20.27%	50	0.8	9.28	10000	27.62%	90	0.6	10.83	10000	28.31%	130					170	0.2	15.53	10000	19.74%
11	0.95	1.00	1.21	11.70%	51	0.75	1.00	2.24	5.26%	91	0.55	1.00	1.03	3.68%	131	0.35	1.00	1.77	7.18%	171	0.15	1.00	5.02	14.87%
12	0.95	1.21	1.44	15.29%	52	0.75	2.24	3.33	5.73%	92	0.55	1.03	1.68	4.87%	132	0.35	1.77	2.14	10.76%	172	0.15	5.02	5.04	18.20%
13	0.95	1.44	1.61	8.19%	53	0.75	3.33	4.07	7.43%	93	0.55	1.68	2.45	13.80%	133	0.35	2.14	3.39	15.73%	173	0.15	5.04	6.36	18.38%
14	0.95	1.61	1.69	7.96%	54	0.75	4.07	4.38	9.36%	94	0.55	2.45	2.69	7.54%	134	0.35	3.39	4.57	10.68%	174	0.15	6.36	9.46	38.93%
15	0.95	1.69	2.12	8.88%	55	0.75	4.38	5.23	13.30%	95	0.55	2.69	2.74	28.10%	135	0.35	4.57	5.53	8.27%	175	0.15	9.46	13.23	15.98%
16	0.95	2.12	2.66	7.01%	56	0.75	5.23	5.29	11.90%	96	0.55	2.74	3.72	10.25%	136	0.35	5.53	7.76	8.08%	176	0.15	13.23	15.02	16.67%
17	0.95	2.66	2.72	9.95%	57	0.75	5.29	6.77	7.29%	97	0.55	3.72	4.61	5.96%	137	0.35	7.76	8.31	8.19%	177	0.15	15.02	16.47	17.49%
18	0.95	2.72	3.16	5.74%	58	0.75	6.77	9.78	10.66%	98	0.55	4.61	5.38	7.37%	138	0.35	8.31	9.13	5.31%	178	0.15	16.47	17.84	15.33%
19	0.95	3.16	10000	12.57%	59	0.75	9.78	10000	22.55%	99	0.55	5.38	16	12.74%	139	0.35	9.13	10000	8.29%	179	0.15	17.84	20	5.71%
20					60					100	0.55	16.229	10000	0.220297637	140					180	0.15	19.5297	10000	0.200020763
21	0.9	1.00	1.75	6.36%	61	0.7	1.00	2.22	7.18%	101	0.5	1.00	1.24	16.00%	141	0.3	1.00	2.26	7.14%	181	0.1	1.00	6	26.09%
22	0.9	1.75	1.97	9.05%	62	0.7	2.22	3.11	6.84%	102	0.5	1.24	1.28	15.93%	142	0.3	2.26	2.28	16.77%	182	0.1	5.89	6.36	43.60%
23	0.9	1.97	2.21	7.77%	63	0.7	3.11	5.02	6.30%	103	0.5	1.28	2.02	9.95%	143	0.3	2.28	3.73	8.52%	183	0.1	6.36	7.77	27.61%
24	0.9	2.21	2.48	5.66%	64	0.7	5.02	5.46	9.86%	104	0.5	2.02	2.20	7.70%	144	0.3	3.73	4.28	23.95%	184	0.1	7.77	11.41	27.52%
25	0.9	2.48	3.09	4.30%	65	0.7	5.46	6.17	9.18%	105	0.5	2.20	2.44	43.22%	145	0.3	4.28	4.34	19.68%	185	0.1	11.41	16.56	16.17%
26	0.9	3.09	3.52	3.80%	66	0.7	6.17	7.44	9.97%	106	0.5	2.44	3.52	15.16%	146	0.3	4.34	7.13	12.00%	186	0.1	16.56	17.93	35.74%
27	0.9	3.52	4.12	6.27%	67	0.7	7.44	8.44	11.81%	107	0.5	3.52	4.95	5.52%	147	0.3	7.13	8.84	11.86%	187	0.1	17.93	18.75	22.96%
28	0.9	4.12	4.60	7.22%	68	0.7	8.44	9.00	20.23%	108	0.5	4.95	5.81	7.57%	148	0.3	8.84	9.68	13.62%	188	0.1	18.75	22.23	29.78%
29	0.9	4.60	10000	18.70%	69	0.7	9.00	9.30	24.68%	109	0.5	5.81	11.72	5.62%	149	0.3	9.68	10.97	22.31%	189	0.1	22.23	10000	20.61%
30					70	0.7	9.30236	10000	0.254430759	110	0.5	11.719	10000	0.145496908	150	0.3	10.9654	10000	0.197066122	190				
31	0.85	1.00	1.84	8.01%	71	0.65	1.00	1.09	3.75%	111	0.45	1.00	1.59	12.74%	151	0.25	1.00	2.76	8.56%	191	0.05	1.00	7.97	103.77%
32	0.85	1.84	2.18	9.02%	72	0.65	1.09	2.47	8.37%	112	0.45	1.59	1.60	34.59%	152	0.25	2.76	2.87	16.85%	192	0.05	7.97	10	103.14%
33	0.85	2.18	2.43	9.29%	73	0.65	2.47	2.64	12.12%	113	0.45	1.60	1.66	13.10%	153	0.25	2.87	3.16	19.34%	193	0.05	9.89	14.54	102.22%
34	0.85	2.43	2.60	8.28%	74	0.65	2.64	2.95	11.29%	114	0.45	1.66	2.00	22.70%	154	0.25	3.16	4.75	12.20%	194	0.05	14.54	18.58	102.01%
35	0.85	2.60	3.55	3.80%	75	0.65	2.95	4.09	5.74%	115	0.45	2.00	2.32	5.71%	155	0.25	4.75	8.23	12.21%	195	0.05	18.58	19.78	101.76%
36	0.85	3.55	3.73	4.55%	76	0.65	4.09	5.65	5.05%	116	0.45	2.32	3.93	10.09%	156	0.25	8.23	8.57	8.10%	196	0.05	19.78	20.07	103.89%
37	0.85	3.73	4.60	5.81%	77	0.65	5.65	8.38	8.16%	117	0.45	3.93	5.13	12.32%	157	0.25	8.57	10.86	11.75%	197	0.05	20.07	22.43	106.08%
38	0.85	4.60	5.69	4.80%	78	0.65	8.38	9.10	12.48%	118	0.45	5.13	6.16	7.79%	158	0.25	10.86	12.05	18.34%	198	0.05	22.43	26.92	106.64%
39	0.85	5.69	5.91	9.53%	79	0.65	9.10	10.97	12.09%	119	0.45	6.16	6.66	17.81%	159	0.25	12.05	14.15	15.75%	199	0.05	26.92	46.22	101.00%
40	0.85	5.91	10000	25.38%	80	0.65	10.97	10000	26.48%	120	0.45	6.66	10000	13.50%	160	0.25	14.15	10000	20.90%	200	0.05	46.22	10000	100.35%

REMAINING VESTED LIFE denotes the option's Remaining Vested Life at exercise date expressed as a percentage of the option's maximum vested life, (the period spanning the first vesting date through the option's contractual expiration date). That is, Remaining Vested Life at any calendar date is defined as, $(T - \zeta)/(T - v_0)$ where T, ζ, and v_0 denote the ESO's contractual maturity date, the calendar date on which the Remaining Vested Life was being calculated, and the initial vesting date for the grant, respectively. Remaining Vested Life is divided into 20 equal intervals, ranging from 0.05 to 1.0. For each Remaining Vested Life interval, the moneyness ratio (S/X) was divided into deciles. S/X (UPPER BOUND) denotes the maximum moneyness ratio observed within a given decile of S/X at a particular Fractional Time. S/X (LOWER BOUND) denotes the minimum moneyness ratio observed within a given decile of S/X at a particular Fractional Time. *EARLY EXERCISE PROBABILITY* is the ratio of ESOs exercised to the number of vested and exercisable ESOs available at a given Remaining Vested Life and the specified S/X decile.

EXHIBIT 4 B

Early Exercise Probability Matrix for Company B

No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)	No.	Remaining Vested Life	S/X (Lower Bound)	S/X (Upper Bound)	Early Exercise Proabability (Mean)
1	1	1.00	1.06	9.13%	41	0.8	1.00	1.12	10.99%	81	0.6	1.00	1.57	7.27%	121	0.4	1.00	2.58	7.06%	161	0.2	1.00	2.52	0.00%
2	1	1.06	1.08	12.76%	42	0.8	1.12	1.17	8.94%	82	0.6	1.57	1.84	15.12%	122	0.4	2.58	2.83	6.08%	162	0.2	2.52	2.95	0.00%
3	1	1.08	1.17	6.59%	43	0.8	1.17	1.29	8.66%	83	0.6	1.84	2.65	19.03%	123	0.4	2.83	3.41	8.44%	163	0.2	2.95	3.62	0.00%
4	1	1.17	1.22	16.37%	44	0.8	1.29	1.44	8.54%	84	0.6	2.65	2.93	8.81%	124	0.4	3.41	3.81	3.22%	164	0.2	3.62	3.80	23.35%
5	1	1.22	1.34	11.40%	45	0.8	1.44	1.62	17.59%	85	0.6	2.93	3.26	7.09%	125	0.4	3.81	4.03	11.27%	165	0.2	3.80	3.94	0.00%
6	1	1.34	1.42	25.75%	46	0.8	1.62	1.95	10.89%	86	0.6	3.26	3.94	56.57%	126	0.4	4.03	4.34	4.73%	166	0.2	3.94	4.25	0.00%
7	1	1.42	1.55	28.03%	47	0.8	1.95	2.32	24.47%	87	0.6	3.94	4.33	10.92%	127	0.4	4.34	5.01	14.42%	167	0.2	4.25	4.99	20.57%
8	1	1.55	1.60	24.94%	48	0.8	2.32	3.00	26.93%	88	0.6	4.33	4.58	9.37%	128	0.4	5.01	5.47	5.11%	168	0.2	4.99	5.40	5.74%
9	1	1.60	1.86	37.06%	49	0.8	3.00	4.46	24.66%	89	0.6	4.58	5.22	13.23%	129	0.4	5.47	5.97	16.25%	169	0.2	5.40	6.02	12.25%
10	1	1.86	10000	37.89%	50	0.8	4.46	10000	19.18%	90	0.6	5.22	10000	21.26%	130	0.4	5.97	10000	18.94%	170	0.2	6.02	10000	11.03%
11	0.95	1.00	1.08	9.70%	51	0.75	1.00	1.15	15.13%	91	0.55	1.00	1.54	8.52%	131	0.35	1.00	2.76	0.00%	171	0.15	1.00	2.61	0.00%
12	0.95	1.08	1.13	7.69%	52	0.75	1.15	1.34	9.91%	92	0.55	1.54	2.11	6.23%	132	0.35	2.76	3.12	0.00%	172	0.15	2.61	2.63	0.00%
13	0.95	1.13	1.15	8.42%	53	0.75	1.34	1.50	11.41%	93	0.55	2.11	2.74	8.35%	133	0.35	3.12	3.92	12.61%	173	0.15	2.63	2.85	0.00%
14	0.95	1.15	1.22	15.42%	54	0.75	1.50	1.68	23.45%	94	0.55	2.74	3.04	11.44%	134	0.35	3.92	3.96	0.00%	174	0.15	2.85	3.77	100.00%
15	0.95	1.22	1.42	8.91%	55	0.75	1.68	2.11	25.79%	95	0.55	3.04	3.28	8.37%	135	0.35	3.96	4.27	2.13%	175	0.15	3.77	3.91	13.49%
16	0.95	1.42	1.58	12.11%	56	0.75	2.11	2.60	18.71%	96	0.55	3.28	3.76	12.70%	136	0.35	4.27	4.45	3.62%	176	0.15	3.91	4.25	0.00%
17	0.95	1.58	1.84	9.48%	57	0.75	2.60	3.03	11.60%	97	0.55	3.76	4.23	38.88%	137	0.35	4.45	5.28	1.58%	177	0.15	4.25	5.16	28.78%
18	0.95	1.84	2.15	27.15%	58	0.75	3.03	3.71	15.90%	98	0.55	4.23	4.78	12.60%	138	0.35	5.28	5.49	9.82%	178	0.15	5.16	5.44	24.14%
19	0.95	2.15	2.85	30.12%	59	0.75	3.71	5.31	15.25%	99	0.55	4.78	5.33	9.15%	139	0.35	5.49	5.92	16.64%	179	0.15	5.44	5.99	9.61%
20	0.95	2.85	10000	27.59%	60	0.75	5.31	10000	19.27%	100	0.55	5.33	10000	12.64%	140	0.35	5.92	10000	19.23%	180	0.15	5.99	10000	32.26%
21	0.9	1.00	1.11	4.97%	61	0.7	1.00	1.25	4.13%	101	0.5	1.00	1.61	5.82%	141	0.3	1.00	2.85	0.00%	181	0.1	1.00	2.73	0.00%
22	0.9	1.11	1.18	12.78%	62	0.7	1.25	1.57	9.67%	102	0.5	1.61	2.57	16.23%	142	0.3	2.85	3.14	0.00%	182	0.1	2.73	2.97	0.00%
23	0.9	1.18	1.30	17.78%	63	0.7	1.57	1.66	12.33%	103	0.5	2.57	2.87	9.55%	143	0.3	3.14	3.95	0.00%	183	0.1	2.97	3.81	0.00%
24	0.9	1.30	1.33	14.10%	64	0.7	1.66	2.01	18.63%	104	0.5	2.87	3.19	13.86%	144	0.3	3.95	4.12	0.00%	184	0.1	3.81	3.91	100.00%
25	0.9	1.33	1.36	12.44%	65	0.7	2.01	2.64	20.96%	105	0.5	3.19	3.59	7.37%	145	0.3	4.12	4.37	0.00%	185	0.1	3.91	3.94	0.00%
26	0.9	1.36	1.51	13.71%	66	0.7	2.64	2.99	31.51%	106	0.5	3.59	3.93	7.15%	146	0.3	4.37	4.57	20.37%	186	0.1	3.94	4.30	100.00%
27	0.9	1.51	1.82	25.48%	67	0.7	2.99	3.78	19.18%	107	0.5	3.93	4.38	12.14%	147	0.3	4.57	5.44	17.29%	187	0.1	4.30	5.40	11.83%
28	0.9	1.82	2.44	24.56%	68	0.7	3.78	4.52	5.44%	108	0.5	4.38	5.05	3.56%	148	0.3	5.44	5.53	13.24%	188	0.1	5.40	6.11	46.51%
29	0.9	2.44	3.22	19.98%	69	0.7	4.52	5.41	13.46%	109	0.5	5.05	5.49	16.76%	149	0.3	5.53	6.05	10.33%	189	0.1	6.11	6.55	15.09%
30	0.9	3.22	10000	26.34%	70	0.7	5.41	10000	23.50%	110	0.5	5.49	10000	15.37%	150	0.3	6.05	10000	19.35%	190	0.1	6.55	10000	34.78%
31	0.85	1.00	1.10	9.90%	71	0.65	1.00	1.54	12.13%	111	0.45	1.00	2.34	0.00%	151	0.25	1.00	2.78	0.00%	191	0.05	1.00	3.67	100.00%
32	0.85	1.10	1.16	9.10%	72	0.65	1.54	1.62	12.13%	112	0.45	2.34	2.68	4.87%	152	0.25	2.78	2.99	0.00%	192	0.05	3.67	3.87	0.00%
33	0.85	1.16	1.26	14.26%	73	0.65	1.62	1.87	9.39%	113	0.45	2.68	3.08	6.01%	153	0.25	2.99	3.68	0.00%	193	0.05	3.87	4.09	100.00%
34	0.85	1.26	1.41	12.45%	74	0.65	1.87	2.69	10.14%	114	0.45	3.08	3.60	2.55%	154	0.25	3.68	3.86	0.00%	194	0.05	4.09	4.15	0.00%
35	0.85	1.41	1.51	16.17%	75	0.65	2.69	3.15	15.90%	115	0.45	3.60	3.89	5.81%	155	0.25	3.86	4.21	10.53%	195	0.05	4.15	5.41	60.23%
36	0.85	1.51	1.68	11.89%	76	0.65	3.15	3.60	14.27%	116	0.45	3.89	4.33	19.94%	156	0.25	4.21	4.42	23.05%	196	0.05	5.41	5.49	100.00%
37	0.85	1.68	2.17	32.92%	77	0.65	3.60	3.91	24.50%	117	0.45	4.33	4.92	20.87%	157	0.25	4.42	5.16	56.56%	197	0.05	5.49	5.55	100.00%
38	0.85	2.17	3.06	15.77%	78	0.65	3.91	4.53	16.39%	118	0.45	4.92	5.44	4.47%	158	0.25	5.16	5.51	5.96%	198	0.05	5.55	5.60	0.00%
39	0.85	3.06	3.70	34.66%	79	0.65	4.53	5.31	13.21%	119	0.45	5.44	5.98	13.59%	159	0.25	5.51	5.93	10.62%	199	0.05	5.60	10000	100.00%
40	0.85	3.70	10000	25.07%	80	0.65	5.31	10000	23.81%	120	0.45	5.98	10000	17.69%	160	0.25	5.93	10000	16.18%	200				

See notes to Exhibit 4A for description of the variables.

strates a "smirk" effect. After a burst of early exercise immediately following initial vesting, the early exercise frequency remains stable for much of the remaining vested life. This is consistent with conventional wisdom that employees are more likely to exercise ESOs to satisfy initial pent-up liquidity demands. However, close to expiration, the early exercise frequency once again increases sharply.

(3) At any given sub-interval i, the early exercise probability is generally greater the higher the option's moneyness ratio s_i.

(4) The upper and lower bounds of the moneyness deciles differ across Companies A and B. For instance, holding the Remaining Vested Life constant at 0.05, the lower bound of the first moneyness decile is 1 and that of the last moneyness decile is 46 in Company A's case. In contrast, this range of lower bounds is 1 to 5.6 for Company B. Such a result is consistent with the fact that Company A's meteoric stock price increase over the sample period was far greater than that of Company B over the same period.

(5) The large range observed in the moneyness ratio confirms that a simple ad hoc specification such as assuming all options would be exercised if the moneyness ratio exceeded a single exogenously specified value, as Hull and White [2004] postulate, may not be realistic. In contrast, our matrix approach covers the entire spectrum of observed moneyness ratios.

(6) Moreover, the relationship between early exercise and moneyness is clearly non-linear as shown in Exhibits 3A and 3B. Hence, an attempt to estimate the likelihood of early exercise as a function of the ESO's moneyness using a regression model may not prove effective.

A final comment is in order regarding the manner in which we have modeled the early exercise probability. The early exercise probability matrix is a parsimonious representation of the factors that may influence the employee's early exercise decision. In such a matrix formulation early exercises are modeled as a function of two variables alone: the option's moneyness and Remaining Vested Life at the exercise date. However, such a parsimonious representation may appear overly simplistic. Therefore, to examine the significance of other factors that could influence early exercise, we also ran a (piecewise linear) probit regression to jointly determine the influence of four independent variables on the odds of early exercise.[19] The independent variables included in this probit regression were[20]

1. the stock's "implied" volatility at the date of exercise;
2. the risk-free interest rate at the exercise date;[21]
3. the option's moneyness at the exercise date;
4. and the option's Remaining Vested Life, i.[22]

The main results of our probit regression analysis were as follows:[23]

1. Neither *volatility nor the interest rate* was statistically significantly correlated to the early exercise probability, even at the 90% confidence level. Both of these variables were negatively related to the early exercise probability. This is consistent with intuition because the continuation value of the ESO would increase (and the probability of early exercise decline) as the stock's volatility or the risk-free interest rate increased.
2. The *moneyness* variable was highly significant and positively correlated to the odds of early exercise probability (Probit), as intuition would suggest. That is, the odds of early exercise increase with an increase in the option's intrinsic value at the exercise date node.[24]
3. The remaining life (i) was related to the odds of early exercise in a non-linear manner. For options in their early life, the relationship is positive and significant. For options with little life remaining, the relationship is negative and significant. For options with intermediate life the relationship was statistically insignificant but negative.[25]

The non-linear relationship between Remaining Vested Life and early exercise probability is intuitive. Over a grant's early life, as the grant vests gradually (typically in monthly installments over 5 years in our dataset), employees tend to exercise the initially vested options to meet liquidity needs, knowing that they will continue to receive additional grants in the near future.

Similarly, holding other factors constant, employees have a tendency to exercise all in-the-money options regardless of how deeply in the money such options may be if their remaining life is very short. That is, when the remaining life is short, the odds of early exercise should be significantly correlated to the option's remaining life and such a correlation should be *negative* (i.e., one would expect to see the probability of early exercise to be higher for options with relatively little remaining life). Our results confirm this hypothesis.

For options of intermediate life, the net impact of longer life on the odds of exercise may be ambiguous. On the one hand, liquidity demands would tend to increase the probability of early exercise (as discussed above). On the other hand, the continuation value of an option is an increasing function of its remaining life. This would tend to reduce the probability of exercise for intermediate life options because the employee no longer expects to get fresh options to replace the existing ones.

The results of our probit regression analysis confirmed that by modeling early exercise parsimoniously in the form of a two-dimensional matrix we have not ignored the impact of other variables, such as volatility or interest rates that may influence early exercise. However, we should caution that such additional variables may have a significant impact on early exercise in the exercise histories of other companies. A multivariate probit regression framework may then be more suitable to calibrate employees' likelihood of early exercise than the matrix approach implemented here.

6.0 ESTIMATION OF OTHER PARAMETERS

Before implementing our valuation methodology described above, we briefly discuss the manner in which the other relevant parameters in our model could be estimated.

Cancellation rate, $q_{i,j}$: Employees forfeit their ESOs when their employment is terminated. Typically, unvested options are cancelled immediately upon termination[26] and employees have a 90-day grace period to exercise all their vested options or they are cancelled. The subject companies tracked their respective cancellation rates on a monthly basis. We estimated the monthly cancellation rate to be 0.21% for Company A. This rate was assumed to be constant for the entire life of the grant for company A. For company B, due to the availability of detailed data on cancellation, we measured the cancellation rate as a function of Remaining Vested Life interval. This monthly cancellation rate ranged from 0.20% to 1.20% in the initial stages of the grant and decreased to 0% at the later stages of the grant.

Dilution rate: Companies may issue additional shares to honor ESO exercises by its employees. As a result, the prospect of future ESO exercises by other employees dilutes the per share value of the company's stock holding other factors constant (see Rubinstein [1995]). Based on quarterly data of additional shares issued to finance acquisition and to honor ESO exercises, we estimated the average annual dilution rate (net of the dilutive effect of acquisitions) for Company A to be 2.71% and 3.77% for Company B.

The dividend yield: Neither of the subject companies currently pay dividends or are expected to do so in the foreseeable future. Therefore, we estimate the future dividend yield to be zero.[27]

The risk free rate: Based on our analysis of the early exercise behavior of the subject companies employees observed on past ESO grants, we found that grants are typically exercised over the first five years of their lives.[28] Correspondingly, we set the risk-free rate at the observed 5-year Treasury rate as of the specific grant date.

Expected stock return volatility: There is a large body of literature that discusses alternative approaches to forecasting volatility. Broadly speaking, the vast academic literature on forecasting volatility[29] can be divided into two strands (a) forecasts based on an analysis of the stock price history ("time series" or "historical volatility (HV)" approach); (b) forecasts based on actual option prices ("implied volatility" or IV approach).[30] FAS 123R does not specify a preferred method that companies should adopt to forecast volatility, although both FAS 123R and Staff Accounting Bulletin 107 (SAB 107) released by the SEC on March 29, 2005, provides guidance on this issue.

HV methods (such as standard deviation of historical returns and moving average models) consider time series of past returns to estimate the volatility observed over a historical sample period. In addition, autoregressive models such as GARCH consider changes in volatility estimates over time to predict future volatility. As a result, instead of simply providing a point estimate of future volatility, these models forecast the time varying path of future volatility. This path is characterized by an unconditional long-term volatility forecast and a series of short-term conditional forecasts.

However, even such autoregressive methods are "backward looking" (i.e., based on time series of past returns). In contrast, as SAB 107 notes, "implied volatility can be useful in estimating expected volatility because it is generally reflective of both historical volatility and expectations of how future volatility will differ from historical volatility." If options markets were efficient then implied volatility would "encompass" all relevant information about future volatility, and the HV forecast would be redundant.

Several studies have considered such "forecast encompassing" tests[31] and found that IV forecasts do not encompass historical forecasts in the short-term (e.g., one-day forecasting horizon), suggesting that either the option market is inefficient or that the option model (based on which the implied volatility was derived) is mis-specified.[32] Longer-term forecasts (one to three months) appear to indicate that option markets are efficient (or that implied volatility forecasts outperform historical estimates). However, since traded options are typically no more than 3 years in maturity, while an ESO's contractual life could be significantly longer, the implied volatilities derived from traded option values may not accurately forecast longer-term volatility necessary to value ESOs.

Given these problems associated with estimating volatility using either the HV or the IV approach, SAB 107 indicates that the issuing firm could use some combination of the two approaches to derive an expected future volatility to value ESOs. There is a large literature on combining forecasts and firms could consequently choose a variety of reasonable ways to determine a combined volatility forecast. One reasonable approach would be to take the arithmetic average of an IV and an HV estimate as of the grant date, according to Diebold and Lopez [1995] and Poon and Granger [2003]. Accordingly, we use an equally weighted average of IV and HV estimates as the volatility parameter estimate in our hypothetical ESO valuation analysis discussed below.

7.0 ESO VALUATION USING OUR CALIBRATED MODEL

We next use these $P(i, s_i)$ values to calibrate our lattice-based reduced form model to value ESOs. We do so by first mapping each node on the binomial lattice (i,j) into a corresponding node defined in terms of the ESO's Remaining Vested Life, i, and its moneyness multiple, s_i. We next calibrate the lattice model by estimating the early exercise likelihood at each node to be the observed relative frequency of early exercise actually observed at that node in the past, $P(i, s_i)$, or, equivalently, $P(i, j)$, in terms of the standard binomial lattice.

We compare the estimated value for a stylized ESO grant that would be derived using our reduced form calibrated model developed in this paper to the value that would be derived for the same grant using three alternative approaches, *viz.*, the standard Black-Scholes model, an adjusted Black-Scholes model, and the Hull and White [2004] approach.

The stylized ESO grant is assumed to consist of 100 options, which are at the money on June 1, 2005, the hypothetical grant date. The stock price (and the exercise price) is set at \$100. The vesting schedule is assumed to be 25% cliff vested at the end of the first 12 months, followed by monthly vesting of the remaining 75% of the grant over the next 36 months.

The ESO's contractual life is assumed to be 8 years. The adjusted Black-Scholes model incorporates the differences in the ESOs and the market-traded options by substituting the contractual maturity of the ESO with an "expected life" estimate. Standard and Poors' *ExecuComp* dataset provides the estimated value for ESOs granted to executives of several public companies, using such an adjusted Black-Scholes model in which the expected life is assumed to equal 70% of the option's contractual maturity. Accordingly, in computing the ESO's adjusted Black-Scholes values we assume that their expected life equals 5.6 years or 70% of the assumed contractual life. The risk-free 5-year Treasury interest rate (as of June 1, 2005) was 4%. On June 1, 2005, the estimated historical volatility, using five years of monthly data, was 52% and 47% for companies A and B, respectively. According to *OptionMetrics*, the implied volatilities based on 2-year LEAP prices were 22% and 26% on June 1, 2005, for Companies A and B, respectively.[33] Thus, the equally weighted average of historical and implied volatilities for Companies A and B on June 1, 2005, were estimated to be 37% and 36%, respectively. We use such average volatility estimates in deriving the value of our stylized ESO grants.[34] The annual dilution rates for companies A and B were estimated to be 2.74% and 3.77%, respectively. The monthly cancellation rate for company A was estimated to be 0.21%. For company B, due to the availability of detailed data on cancellation, we measured the cancellation rate as a function of the Remaining Vested Life interval. This monthly cancellation rate ranged from 0.20% to 1.20% in the initial stages of the grant and decreased to 0% at the later stages of the grant.

Exhibit 5 summarizes our valuation results. The standard Black-Scholes values (per ESO) for companies A and B were estimated at \$49.55 and \$48.74, respectively. Similarly, the adjusted Black-Scholes values, derived using the expected life estimate of 5.6 years, were calculated to be \$41.37 and \$40.63 for Companies A and B, respectively. Note that these Black-Scholes model-based ESO values do not take into account the properties of an ESO, such as non-transferability, non-hedgeability, and forfeiture. Thus, these Black-Scholes model-based ESO

Exhibit 5
Stylized ESO Valuation

Model	Hypothetical ESO Value		Hypothetical ESO Value (as percentage of Black-Scholes Value)	
	Company A	Company B	Company A	Company B
Standard Black-Scholes	$ 49.55	$ 48.74	100%	100%
Adjusted Black-Scholes	$ 41.37	$ 40.63	83%	83%
Hull and White Accounting value	$ 39.05	$ 31.08	79%	64%
Matrix-Based Model Accounting Value	$ 35.36	$ 30.13	71%	62%
Hull and White Economic value	$ 34.17	$ 22.83	69%	47%
Matrix-Based Model Economic Value	$ 31.54	$ 22.29	64%	46%

This numerical example assumes that the subject companies (A and B) each issued a hypothetical ESO grant that consists of 100 options, and are at-the-money on June 1, 2005, the hypothetical grant date. The vesting schedule is assumed to be 25% cliff vested at the end of first 12 months followed by monthly vesting of the remaining 75% of the grant for the next 36 months. The ESO's contractual life is assumed to be 8 years. The expected life of the options is assumed to be 70% of the contractual life or 5.6 years. The stock price (and the exercise price) on grant date is assumed to be $100. The risk-free interest rate parameter is set at 4%, which was the observed 5-year Treasury interest rate as of June 1, 2005). The volatility estimates for company A and company B were assumed to be 37% and 36%, respectively. These estimates were the equally-weight historical and implied volatility estimates for these two companies on June 1, 2005. The annual dilution rates for companies A and B were estimated to be 2.74% and 3.77%, respectively. The monthly cancellation rate for company A was estimated to be 0.21%. For company B, due to the availability of detailed data on cancellation, we measured the cancellation rate as a function of Remaining Vested Life interval. This monthly cancellation rate ranged from 0.20% to 1.20% in the initial stages of the grant and decreased to 0% at the later stages of the grant. In the Hull and White models, early exercise occurs with probability one if the option's moneyness ratio at any node on the binomial tree exceeds 5 for Company A and 2 for Company B. The binomial tree used in our analysis has 216 steps. The early exercise matrices used to evaluate the stylized ESO grants issued by these two companies using our Matrix-Based model are as shown in Exhibits 3A and 3B, respectively. Two versions of the Hull and White and Matrix-Based models are analyzed. The economic value version considers the impact of cancellation prior to vesting while the accounting value version ignores this impact as specified by FAS 123R.

values ignore the estimated cancellation, dilution or early exercise parameters of companies A and B. The ESO values in either the standard or adjusted Black-Scholes models differ across the subject companies *only* due to the difference in the companies' estimated volatilities.

We next consider the valuation of the same ESOs using richer ESO valuation models that do explicitly consider the unique features of an ESO. In this connection we contrast the Hull and White based model with the results derived using our own matrix-based reduced form model. Moreover, as discussed earlier, each of these two models can be modified to exclude the impact of cancellation or forfeiture in the pre-vesting period as the accounting rules specified in FAS 123R mandate. Thus, Exhibit 5 reports the economic value as well as the accounting value estimated for the stylized ESO grants issued by Companies A and B using either the Hull and White [2004] model or our own model.

Recall that the Hull and White [2004] model assumes that early exercise occurs with zero probability if the option's moneyness ratio lies below an exogenously specified level. If instead the option's moneyness ratio exceeds the exogenously specified level then Hull and White assume that the employee will exercise all her vested and exercisable options for certain. In contrast to the Hull and White [2004] approach discussed above, our reduced form calibrated model does not assume that the early exercise probability is either zero or one at each node of the binomial tree. Instead the early exercise probability is modeled as a function of moneyness as well as the option's Remaining Vested Life and can take on intermediate values between zero and one as shown in the Exhibit 4.

Hull and White [2004] do not provide any empirical estimates or guidance as to how such a threshold value for the moneyness ratio ought to be set. It could be set in an arbitrary manner (FAS 123R mentions 200% as a possible threshold) or based on historical experience. In our stylized example the threshold values of the moneyness ratios were assumed to be 5 for company A and 2 for company B. These estimates were derived as follows. As shown in Exhibit 2A, for Company A, the weighted average strike price and the weighted average stock price at the exercise date for all monthly–vested grants that had been exercised were $4.68 and $23.48, respectively. Thus, the average stock price at exercise date was 5 times the average strike price. Similarly, for Company B, the weighted average strike price and the weighted average stock price at the exercise date for all monthly vested grants that had been exercised were $18.94 and $39.37, respectively (See Exhibit 2B). Thus, the average stock price at exercise date was 2 times the average strike price.

According to FAS 123R, the likelihood of option cancellation during the pre-vesting period should not be included in the accounting valuation of the ESO at grant date. Accordingly, we have computed such accounting values for the hypothetical grants by companies A and B. Based on the Hull and White approach, the ESOs' *accounting* value for company A is $39.05 and for Company B is $31.38. Based on our matrix-based model, the ESOs' *accounting* values for each company would be even lower: $35.36 for company A and $30.13 for Company B.

In determining the economic value of an ESO at grant date, an employee would, however, take into consideration the likelihood of option cancellation during the pre-vesting period. We have therefore, also computed the economic values of the stylized ESO grants using the Hull and White model and our own model. Based on the Hull and White approach, the ESOs' *economic* values for company A is $34.17 and for Company B is $22.83. Based on our model, the ESOs' *economic* values for company A is estimated to be $31.54 and $22.29 for Company B. Thus, while the likelihood of pre-vesting cancellation has a small impact on Company A's grant-date ESO value, it has a considerably larger impact on the economic value of the ESO grant by Company B.

The differences in ESO valuation due to modeling differences are apparent when the estimated ESO values are reported as a percentage of the standard Black-Scholes value. As is evident from Exhibit 5, an ad hoc adjustment of the ESO's contractual life to reflect an expected life that is 70% of the contractual life reduces the ESO's value by 17% relative to the Black-Scholes value. However, according to the Hull and White approach, the grant date value of the ESO granted by Company A is properly estimated at 79% of the standard Black-Scholes value. In contrast, according to our matrix-based model, the same option should be valued at 71% of the standard Black-Scholes value on the grant date. Thus, the disparity between the Hull and White approach and our more finely calibrated approach is substantial, and amounts to about 11% of the grant's value.[35] In contrast, the disparity between the Hull and White and our own approach is smaller in the case of Company B.

The result follows from the fact that Company A's stock price had witnessed a much sharper increase in value relative to Company B. As a result, Company A's average moneyness at exercise date was 5 compared to that of Company B, which was 2. Accordingly, in the Hull and White approach, the ESO is assumed to go unexercised even when the option is in the money but with a moneyness ratio of less than 5. In contrast, our model calibrates early exercise more finely and incorporates the likelihood of early exercise at lower moneyness ratios. As a result, the average exercise date in our approach is less than that of the Hull and White approach. Correspondingly, our model yields a lower (more properly calibrated) ESO value compared to the Hull and White approach. This disparity is less in the case of Company B, since the threshold level for early exercise assumed under the Hull and White approach in that case is much lower at 2.

Our analysis also reveals that the economic value of the ESOs is significantly less than the accounting value per FAS 123R's stipulations. In the case of Company B, the accounting value would be 62% of the Black-Scholes value while the economic value would be 46% of the Black-Scholes value.

In short, our model indicates that ESO values are significantly lower than that of traded options (priced using the Black-Scholes formula).[36] There are two possible interpretations of this result.

Our model is a reduced-form one in which early exercise is calibrated to *observed* early exercise behavior of employees, and not derived from a utility maximization framework. Hence, our result may be interpreted as evidence that employees rationally exercise their ESOs much earlier than they might otherwise have done in the case of traded options given their risk preferences and these non-traded options' unique characteristics (such as vesting period, potential cancellation and blackout periods during which time even vested options cannot be exercised). Alternatively, the significantly lower valuation of ESOs relative to that of traded options may have a behavioral explanation, namely that employees inefficiently exercise their ESOs too early and leave money on the table.[37]

CONCLUSION

The reduced-form calibrated lattice model presented in this paper describes a methodology of valuing ESOs. The central feature of our model is the estimation of an early exercise matrix, based on the past history of ESO exercise behavior displayed on past grants by all the company's employees. This early exercise probability matrix yields the probability that an exercisable ESO will be exercised at a particular node on a binomial lattice. Thus, early exercise probability is modeled as a function of time remaining to exercise and moneyness parameters.

Our empirical analysis, which is based on the complete early exercise behavior of two large technology companies and involves more that four billion ESOs, reveals that ESO valuation approaches permitted by FAS 123R, such as simply adjusting the expected life of the ESO in the Black-Scholes model or assuming that early exercise is certain if the option's moneyness ratio exceeds some threshold level, are likely to lead to severe valuation biases.

ENDNOTES

[1]Statement of Financial Accounting Standards No. 123, Share-Based Payment (revised 2004), No. 263-C, December 2004.

[2]It is helpful to clarify some terminology at this juncture. An ESO is the option to purchase one share at a specified strike price, which is typically the company's stock price on the date of grant. By "grant" we refer to a particular set of employee stock options awarded to a specific employee. Thus, for instance, if an employee received 100 ESOs with the same contractual terms on January 15, 2005, then these options would be classified together as a single grant. If the same employee received two sets of options with somewhat different contractual terms, the two sets would be treated as two different ESO grants even if both sets of options were granted on the same date. Similarly, if two different employees received ESO grants on the same date with identical terms, then each employee's award would be termed a separate "grant." As we discuss later, grants made on the same date with identical contractual terms but made to different employees may be combined for calibration purposes into "aggregated" grants.

[3]Examples of utility-based models include Rubinstein [1995], Huddart [1995] and Marcus and Kulatilaka [1994].

[4]For other examples of reduced form ESO valuation models, see Hull and White [2004] and Carpenter [1998].

[5]The intrinsic values are net of dilution (See Rubinstein [1995]).

[6]We thank the referees for raising this point.

[7]Early exercise may be rational for traded options only if the stock pays dividends. Otherwise, the option's intrinsic value cannot exceed its continuation value. We note that neither of the companies included in our empirical analysis pay dividends. Therefore, the alternative modeling distinction discussed above would not affect these companies' ESO valuation using our calibrated matrix model.

[8]We thank the referees for raising this point.

[9]FAS123R paragraph A.3.

[10]FAS123R paragraph A.4.

[11]For instance, a typical monthly-vested schedule would stipulate an initial vesting of 20% of all ESOs granted one year from the grant date, with the remaining 80% vesting on a monthly basis over the subsequent 48 months.

[12]About 1,843 individual grants out of 54,720 for Company B did not have a defined vesting schedule available for company B and were hence not included in the calculations.

[13]In the case of monthly-vested grants, this vesting date refers to the date when the first allocation of grants was vested.

[14]Although all grant cohorts contained ESOs written on the same company's shares, their remaining vested life sub-intervals do not coincide in terms of calendar time and hence the minimum stock prices recorded over such sub-intervals would not be identical across grant cohorts. For instance, consider two 10-year option grants (that were immediately vested upon grant). Suppose the first grant was made on January 1, 2000 and the second was made on January 1, 2001. Then, the first grant's sub-interval $i = 1$ would span its first six months of vested life (from January 1, 2000 through June 30, 2000), while the second grant's first sub-interval $i = 1$ would span its first six months of vested life (from January 1, 2001 through June 30, 2001). It is clear from this example that the stock's minimum price recorded during the first grant's sub-interval $i = 1$ would not coincide with the stock's minimum price recorded during the second grant's sub-interval $i = 1$.

[15]Since there are 10 columns in the matrix, each denoting a particular s_i sub-group, going across a particular row of the matrix, each cell would include 10% of all grants rank-ordered by their respective moneyness ratios, s_i.

[16]However, even for lower values of i, each cell in the matrix would contain 10% of all grant cohorts for that sub-interval i (i.e., holding the sub-interval row, i, constant).

[17]An ESO is considered exercisable within a sub-interval i if the minimum stock price during the calendar period spanning the grant's sub-interval i was higher than the ESO's exercise price.

[18]Thus, by definition, the lower S/X bound for the first moneyness decile is 1.0. Similarly, the upper bound for the last moneyness decile is unbounded (empirically set at 10,000 in our early exercise matrices).

[19]We thank the referees for raising this issue.

[20]Our analysis was based on grant data from one of the two companies in our dataset. An ESO grant to an individual employee on a particular date is referred to as a "individual grant." Such an individual grant consists of a specified number of employee stock options that vest over time. Our probit analysis was based on a sample of 414,310 individual grants that contained 1,671,776,932 options granted. These individual grants were aggregated into 113 "aggregate" grants based on the grant date, grant price, expiration date, and vesting schedule. The selected individual grants satisfied the following criteria: 1) grants were made on or after 1/1/1996, during which period implied volatility data for the company was available; 2) the aggregate grant consisted of at least 100 individual grant holders, 3) the aggregate grant consisted of at least 1,000,000 options granted. The resulting sample covers 81% of all individual grants, and 65% of the total shares granted, by the company over the January 1, 1996–June 30, 2005 period. The final sample used in our probit analysis contained 1,905 observations of the ratio of exercised options as a percentage of vested and exercisable options (that were vested and in the money), and which satisfied the above selection criteria. The implied volatility was "matched" to the extent feasible with the ESO's remaining life as of the

exercise date. That is, the implied volatility was taken from the traded option on the company's stock that had a remaining life that was nearest to that of the ESO's remaining life at exercise date. Since traded options' maturities are generally 2 years or less while that of ESOs may be considerably longer, in those instances where the remaining ESO life was greater than 2 years, we used the implied volatility from the longest-lived traded options at the time. The implied volatility data was obtained from *OptionMetrics,* the best-known vendor of such data.

[21]The risk-free interest rate was the constant maturity Treasury rate available from the Federal Reserve as of the ESO's exercise date. Such rates were available for 1-, 3-, 5-, 7- and 10-year maturities. Accordingly, we used the risk-free rate quoted as of the exercise date for the maturity that was nearest to the ESO's remaining life at the time.

[22]To run piecewise-linear regressions, we defined three tranches for the remaining time to maturity. The first tranche was defined to include remaining vested life, i was 1–0.50 (the option's early life during which period the vesting is not complete). In the second tranche, i was 0.05–0.5, and in the last tranche, i was 0.05 or lower.

[23]Results are available from the authors upon request.

[24]The same effect also tends to increase the continuation value of the option, but relatively less so.

[25]This is consistent with the hypothesis that the continuation value increases with the option's remaining life, which tends to reduce the odds of early exercise.

[26]The cancellation of such unvested options has no repercussions in either our valuation model or in the booking of ESO expenses from an accounting perspective.

[27]Although we make such an assumption for the purpose of our stylized ESO valuation exercise in this paper we should point out that it could also plausibly be assumed that the subject company may start paying dividends at some future date, before the ESO being valued expires.

[28]We estimate the expected term as the average period by which ESOs were exercised. We based this analysis on a subsample of all grants issued by the subject companies. In particular, we considered only those grants that had expired. Hence, for such grants we had a complete history of all exercise decisions.

[29]See Poon and Granger [2003] for a detailed review on the subject.

[30]The IV is the value of the volatility parameter in some theoretical option valuation formula (such as the Black-Scholes or the Cox, Ross and Rubinstein model) that would make the theoretical price equal to the actual observed option price.

[31]See Christensen and Prabhala [1998], Lamoureux and Lastrapes [1993], Canina and Figlewski [1993] and Day and Lewis [1992].

[32]However, such studies do not consider very long run forecasts of several years that would span ESOs' contractual lives.

[33]OptionMetrics, a subscriber-based data vendor, was used for each of the subject companies. (See OptionMetrics [2005] for a detailed discussion of how the vendor calculates implied volatilities.)

[34]Clearly, alternative volatility estimates would result in different estimated ESO values. However, since the focus of this paper is on the calibration of early exercise probability and not on volatility estimation, we have abstracted from analyzing at length the numerous alternative ways in which volatility estimates could be derived and their corresponding impact on ESO valuation.

[35]$(71-79)/71 = 11\%$.

[36]This conclusion is also consistent with the results drawn based on Hull and White's [2004] model.

[37]We thank the referees for raising this point.

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