

Pricing and Hedging Mandatory Convertible Bonds

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This article examines the pricing and hedging of mandatory convertible bonds on the U.S. market using daily market prices for a period of 498 trading days, resulting in a sample of more than 14,600 daily price observations. We explore the pricing and hedging performance based on a simple contingent claims model. On average, the pricing errors are lower than those found for standard convertible bonds. An analysis of the hedging performance of the model indicates that it is useful for hedging as, on average, the hedging errors observed are relatively small and mostly not systematic.

We study the pricing and hedging of mandatory convertible bonds on the secondary market. So far, no such investigation has been conducted. The pricing of 40 mandatory convertibles is analyzed between May 2002 and April 2004, with a total of 498 trading days yielding a sample of 14,612 daily price observations. These securities were issued in the United States between October 2000 and July 2003 and mature between May 2004 and November 2006. We examine the U.S. market for mandatory convertibles because of the availability of accurate daily market prices, its large size compared with other markets, and the liquidity of many of its issues.

Mandatory convertibles are equity-linked hybrid securities and can be thought of as yield-enhanced equity. Mandatory convertibles pay higher dividends than common stock for a number of years and then mandatorily convert to common stock on a prespecified date. These securities are the most equity-like

of all convertible securities and, unlike normal convertibles, they provide little downside protection because mandatory convertibles usually have no fixed terminal value. Instead, the security will mandatorily convert into a variable number of shares at maturity. Mandatory convertibles have become popular in recent times: Chemmanur and Nandy [2003] report approximately \$20 billion issued in 2001, implying a market share of approximately 18% of the convertible bond market.

Despite the large size of international and especially U.S. mandatory convertible markets, very little research on the pricing of mandatory convertibles has been undertaken. Previous work includes Arzac [1997], who discusses the rationale for mandatory convertibles from the point of view of issuers as well as investors. He notes that mandatory convertibles allow highly leveraged (or temporarily troubled) companies to restructure their balance sheet by helping to control for the "asymmetric information" problem. Furthermore, he describes the main features and a straightforward valuation method. Chen et al. [1999] provide an analysis of one (Masco Tech) mandatory convertible for a time period of one year. They use the same valuation method as Arzac [1997] and show that mandatory convertibles are not very sensitive to changes in volatility, risk-free interest rate, and credit spread.

There are also a few articles dealing with the rationale of and stock market reactions to issuing mandatory convertibles: Huckins [1999] investigates the characteristics of firms issuing such bonds and the announcement effect on

stock prices. She finds that companies that issue mandatory convertibles tend to be highly leveraged. She also finds a neutral response of the stock market to announcements and issues of mandatory convertibles and attributes this effect to the mitigation of adverse selection costs compared with equity issues. Chemmanur and Nandy [2003] provide a theoretical model explaining the announcement effect. Furthermore, they show that in equilibrium the company issues straight debt, ordinary convertibles, or equity if the extent of asymmetric information is large but the probability of being in financial distress is relatively small. The company issues mandatory convertibles if the problem of asymmetric information is small but the probability of financial distress is high. Hegde and Krishnan [2003] test predictions of agency and asymmetric information theories to explain why some firms prefer to use mandatory convertibles as opposed to ordinary convertible securities. They find that the stock market reacts less negatively to a mandatory convertible issue than to an ordinary convertible issue.

This article is organized as follows. In the next section, we describe the structure and characteristics of a typical mandatory convertible and describe the pricing model. Then we give an overview of the data set and the methodology for the empirical analysis. The empirical results of the pricing study are reported and then we examine the hedging performance. The final section concludes.

PRICING MODEL

Description of Mandatory Convertible Bonds

In our study, we analyze the most common mandatory convertible bond structure. Mandatories pay a coupon, are subject to the full downside risk of the stock, and participate (partially) in the upside potential of the stock. Typically, there is a flat payoff zone between a lower strike price and an upper strike price where the payoff at maturity is fixed. These securities, referred to by various acronyms such as PIES, DECS, ACES, and PRIDES, therefore have characteristics of both common and preferred stock. For simplicity, we call the payoff structure described in this section simply "mandatory convertible" and use this term as a synonym for the acronyms given to these securities by the issuing investment banks.

Typically, a mandatory convertible pays a coupon in percentage of the par value that is 5–7 percentage points higher than the underlying common stock's dividend yield.

The annual coupon is usually paid quarterly. The securities are usually issued with a maturity of 3–5 years. At maturity, they are mandatorily converted into common stock. Often, they are call-protected for most or all of their life. A unique characteristic of mandatory convertibles is that the conversion ratio depends on the price of the underlying stock. The payoff (V_T) at maturity (T) is given by

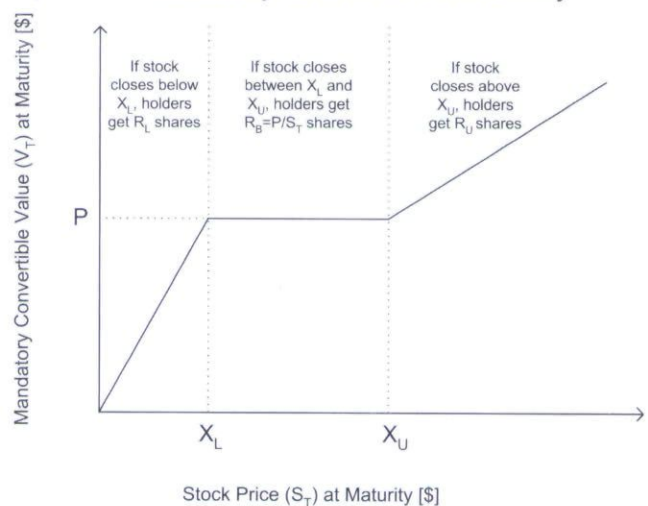
$$V_T = \begin{cases} S_T \cdot R_U & \text{if } S_T \geq X_U \\ S_T \cdot R_B = P & \text{if } X_L < S_T < X_U \\ S_T \cdot R_L & \text{if } S_T \leq X_L \end{cases} \quad (1)$$

At maturity, if the stock price has fallen below a lower strike price (X_L), investors usually receive a fixed lower ratio (R_L) of shares. If the stock price is between a lower (X_L) and an upper strike price (X_U), then investors receive stock worth as much as the par value (P); that is, the stock price (S_T) is multiplied by the variable ratio (R_B) between the lower and upper strike prices. This variable ratio is determined by dividing the par value (P) by the stock price at expiration (S_T). If the stock price is above the upper strike price (X_U), the investor receives a fixed upper ratio (R_U) of shares. Exhibit 1 illustrates the payoff profile of a mandatory convertible at maturity.

Note that the lower strike price multiplied by the lower ratio is equal to the upper strike price multiplied by the upper ratio and both are equal to the par value

EXHIBIT 1

Payoff of a Mandatory Convertible at Maturity



Notes: The payoff profile of a mandatory convertible at maturity T . X_L and X_U denote the lower and upper strike prices, R_L and R_U represent the lower and upper conversion ratios ($R_L > R_U$), and P denotes the par value of the security.

(P) of the mandatory convertible ($X_L \cdot R_L = X_U \cdot R_U = P$). The conversion ratio at the lower strike price is always higher than the ratio at the upper strike price. Therefore, if the stock increases from the lower strike price, an investor's participation is delayed until the upper strike price is reached. Above the upper strike price, the investor participates at a reduced rate equal to the upper conversion ratio, which is less than 1 (the reverse is true for $S_T < X_L$).

The Pricing Model

Arzac [1997] presents a simple valuation method, decomposing the securities into three basic components that are each readily valued individually: 1) the current value of the underlying common stock; 2) the fixed-income cash flow (present value of the coupon payments minus the present value of the dividend payments of the common stock); and 3) the stock options embedded in the security (R_U calls with strike price X_U minus R_L calls with strike price X_L). Chen et al. [1999] use the same valuation method.¹ Alternatively, the following equivalent decomposition can be used: 1) a fixed-income component with principle P paying the coupons I ; 2) minus R_L puts with strike price X_L ; 3) plus R_U calls with strike price X_U . For our analysis we use the second decomposition.²

The fair value or the (theoretical) model price of the mandatory convertible is the price at which it could be replicated theoretically. In the secondary market, the fair value of the mandatory convertible at time t (V_t) consists of the following components:³

- + value of a call with upper strike price multiplied by the upper ratio ($c_{t,u} \cdot R_u$)
- + value of a put with lower strike price multiplied by the lower ratio ($p_{t,L} \cdot R_L$)
- + present value of the (risk-free) par value⁴ ($PV_t(P)$)
- + present value of the (risky) coupon payments ($PV_t(I)$)

Therefore, on valuation date t , the mandatory convertible has a value of

$$V_t = c_{t,u} \cdot R_u - p_{t,L} \cdot R_L + PV_t(P) + PV_t(I) \quad (2)$$

The present value of the (riskless) par value is defined as

$$PV_t(P) = P \cdot e^{-(r_{t,\Delta T}) \cdot \Delta T} \quad (3)$$

and the present value of the (risky) coupon payment is defined as

$$PV_t(I) = \sum_{j=1}^m I \cdot e^{-(r_{t,t_j-t} + cs_t) \cdot (t_j - t)} \quad (4)$$

The notation is as follows:

- P = par value of the mandatory convertible
- t = time of valuation
- T = maturity of the mandatory convertible
- $\Delta T = T - t$ = the time in years until maturity of the mandatory convertible
- V_t = value of the mandatory convertible at time t
- R_U = upper ratio
- R_L = lower ratio
- $c_{t,u}$ = price of a European call option with strike price X_U at time t
- $p_{t,L}$ = price of a European put option with strike price X_L at time t
- $PV_t(I)$ = present value of coupon payment of the mandatory convertible at time t
- $PV_t(P)$ = present value of par value at time t
- I = coupon for a given bond
- t_j = date of the j -th coupon payment, $j = 1, 2, \dots, m$, with $t \leq t_j \leq T$
- r_{t,t_j-t} = continuously compounded riskless interest rate at time t for the period $t_j - t$
- $r_{t,\Delta T}$ = continuously compounded riskless interest rate at time t for the period ΔT
- cs_t = credit spread of the mandatory convertible at time t

The Black and Scholes [1973] and Merton [1973] model can be applied to these options only if it can be ensured that early exercise is never optimal. Otherwise, numerical techniques such as binomial trees would have to be used. However, if the coupon payments exceed the (expected) dividend payments on the common stock (which is the case in our sample), then early exercise is never optimal. It is therefore viable to use the Black-Scholes-Merton option pricing formula for European options on shares paying a known dividend yield. The dividend yield at time t is expressed as a continuously compounded annual rate q_t . The value for a call $c_{t,k}$ and a put $p_{t,k}$ at time t for an option with strike price X_k is given by:

$$c_{t,k} = S_t \cdot e^{-q_t \cdot \Delta T} \cdot \Phi(d) - X_k \cdot e^{-r_{t,\Delta T} \cdot \Delta T} \cdot \Phi(d - \sigma_t \cdot \sqrt{\Delta T}) \quad (5)$$

$$p_{t,k} = X_k \cdot e^{-r_{t,\Delta T} \cdot \Delta T} \cdot \Phi(-d + \sigma_t \cdot \sqrt{\Delta T}) - S_t \cdot e^{-q_t \cdot \Delta T} \cdot \Phi(-d)$$

with

$$d = \left\{ \ln \left(\frac{S_t}{X_k} \right) + \left(r_{t,\Delta T} - q_t + \frac{\sigma_t^2}{2} \right) \cdot \Delta T \right\} / \sigma_t \cdot \sqrt{\Delta T}$$

where S_t is the market price of the underlying asset at time t , q_t the continuously compounded annual dividend yield at time t , σ_t the volatility for the underlying asset at

time t , and $\phi(\cdot)$ is the standard normal cumulative distribution function.

In our model, mandatory convertible prices depend on several parameters, namely, credit spreads, stock price volatilities, risk-free interest rates, and dividend yields. The price impact of these parameters varies greatly. Because mandatory convertibles are essentially yield-enhanced common stock, they are very sensitive to a change in the underlying stock (see the section on hedging performance). Dividend estimations can be critical, because future dividends of the underlying stock are uncertain and the coupon payments of the mandatory convertible are fixed. Owing to the offsetting nature of the embedded option spread, a change in volatility has only a minor effect on the mandatory convertible value (see the section "The pricing of mandatory convertible bonds" for a detailed analysis). Therefore, the impact of the volatility model on model prices is limited. A change in credit spread affects only the present value of the coupon payments and therefore also has a limited impact on prices. Finally, because mandatory convertibles are akin to stock rather than debt, they are not very sensitive to changes in interest rates.

DATA SET AND METHODOLOGY

Mandatory convertible bonds are not always issued by corporate debtors. There are also "synthetic" bonds created by investment banks in response to investor demand without the corporate's involvement. In those cases, the counterparty is the bank, not the corporate debtor. In this study, however, the counterparties of all mandatory convertibles analyzed are corporate debtors.

Our sample covers 46 mandatory convertible bonds. We exclude 4 callable⁵ securities and 2 bonds where companies have repurchased a large fraction of the outstanding securities. Therefore, we examine market prices of 40 U.S. mandatory convertibles from May 28, 2002, to April 22, 2004 (representing 498 trading days), issued in the United States between October 2000 and July 2003 and maturing between May 2004 and November 2006. The sample contains 14,612 data points. Mandatory convertible prices and stock prices were provided by Reuters and Datastream, respectively. For the riskless interest rate, we use the interpolated U.S. zero-curve yield calculated from swap rates for the respective maturity, provided by Datastream. Apart from directly observable input parameters, such as stock prices, prices of mandatory convertibles, and interest rates, the pricing model requires input parameters that have to be estimated and thus are a source of estimation error.

These variables include dividend yields, credit spreads, and volatilities. Information on current dividend yields at time t of the underlying common stock was provided by Datastream. For an appropriate default-adjusted interest rate ($r_{t,\Delta t} + cs_t$), we use the credit ratings provided by Moody's and the respective Moody's median bond spread data that provide average credit spreads (cs_t) for time t . Although a change in credit spreads can often have significant effects on the pricing of traditional convertible bonds, its impact here is limited because credit spreads affect only the present value of the coupon payments.

Because of the offsetting nature of the embedded option spread in mandatory convertibles, a change in volatility of the underlying asset has a minor effect on pricing. A popular approach to determine volatilities is the implied volatility concept. However, there are no liquid options data available for many of the firms analyzed in this study. Furthermore, liquid options tend to have shorter maturities than mandatory convertibles. Consequently, we estimate volatilities on a historical basis. The relevant volatility is calculated as the standard deviation of the returns of the past four years. Exhibit 6 in the next section shows that the results are robust with respect to this estimation window.

Most mandatory convertibles (39 out of 40) are American-style; that is, they allow exercise prior to maturity (i.e., settlement with the upper conversion ratio). However, in our sample, the coupon payments exceed the (expected) dividend payments on the common stock; early exercise is therefore never optimal. As a consequence, the early exercise feature does not have a price impact and therefore the valuation as a European-style security is justified.

Another feature of some mandatory convertibles is that the "settlement price" used to determine the number of shares (conversion ratio) to be received for each mandatory convertible is the average closing price during a (generally small) number of prespecified trading days immediately prior to maturity. This feature is similar to Asian options, where the value depends not on the value of the asset at expiration, but rather on the average value of a specified time period. The effect of averaging is to reduce the volatility of the option. When the averaging period is large in relation to the maturity of the option, the values of Asian options are significantly lower than those of the corresponding European options. In the case of mandatory convertibles, the averaging effect is small because the averaging periods are very short, and it is therefore neglected.

Exhibit 2 provides the characteristics of the mandatory convertibles: maturity, annual coupon, par value,

EXHIBIT 2

Characteristics of Mandatory Convertible Bonds

Name of Issuer	Maturity	Annual Coupon (%)	Par Value (\$)	Upper Ratio	Lower Ratio	Upper Strike (\$)	Lower Strike (\$)
AFFILIATED MANAGERS GROUP INC	17.11.2004	6	25	0.45	0.51	56.05	48.73
ALLIED WASTE INDUSTRIES INC	01.04.2006	6.25	50	4.94	6.02	10.13	8.30
AMEREN CORP	15.05.2005	9.75	25	0.54	0.63	46.61	39.50
AMERICAN ELECTRIC POWER CO INC	16.08.2005	9.25	50	1.02	1.22	49.08	40.90
ANTHEM INSURANCE COMPANIES INC	15.11.2004	6	50	1.14	1.39	43.92	36.00
BAXTER INTERNATIONAL INC	16.02.2006	7	50	1.40	1.74	35.69	28.78
CAPITAL ONE FINANCIAL CORP	17.05.2005	6.25	50	0.64	0.78	78.61	63.91
CENDANT CORP	17.08.2004	7.75	50	1.76	2.32	28.42	21.53
CENTURYTEL INC	15.05.2005	6.875	25	0.69	0.87	36.00	28.60
CHUBB CORP	16.08.2006	7	25	0.35	0.42	71.40	59.49
CITIZENS COMMUNICATIONS CO	17.08.2004	6.75	25	1.72	2.07	14.52	12.10
CONSTELLATION BRANDS INC	01.09.2006	5.75	25	0.73	0.89	34.16	28.00
CORNING INC	16.08.2005	7	100	50.81	62.50	1.97	1.60
DOMINION RESOURCES INC	15.05.2006	8.75	50	0.61	0.84	81.33	59.80
DOMINION RESOURCES INC	16.11.2004	9.5	50	0.82	0.98	61.20	51.00
DUKE ENERGY CORP	18.05.2004	8.25	25	0.53	0.64	47.56	38.98
DUKE ENERGY CORP	16.11.2004	8	25	0.53	0.62	47.55	40.13
ELECTRONIC DATA SYSTEMS CORP	17.08.2004	7.625	50	0.70	0.84	71.47	59.31
EQUITY SECURITY TRUST II	15.01.2005	6.25	22.5	0.98	1.19	23.07	18.91
GABELLI ASSET MANAGEMENT INC	17.02.2005	6.95	25	0.54	0.63	46.50	39.40
HARTFORD FINANCIAL SERVICES GROUP INC, THE	16.08.2006	7	50	0.88	1.10	56.88	45.50
HARTFORD FINANCIAL SERVICES GROUP INC, THE	16.11.2006	6	50	0.87	1.06	57.65	47.25
HOUSEHOLD INTERNATIONAL INC	15.02.2006	8.875	25	0.97	1.17	25.68	21.40
IMC GLOBAL INC	01.07.2006	7.5	50	6.44	7.86	7.76	6.36
KEYSPAN CORP	16.05.2005	8.75	50	1.18	1.42	42.36	35.30
MOTOROLA INC	16.11.2004	7	50	2.37	2.89	21.08	17.28
ONEOK INC	16.02.2006	8.5	25	1.21	1.45	20.63	17.19
PHELPS DODGE CORP	15.08.2005	6.75	100	2.08	2.50	48.00	40.00
PHOENIX COMPANIES INC, THE	16.02.2006	7.25	25	2.83	3.46	8.82	7.23
PRUDENTIAL FINANCIAL CAPITAL TRUST I	15.11.2004	6.75	50	1.47	1.82	34.10	27.50
SEMPRA ENERGY	17.05.2005	8.5	25	0.82	1.00	30.52	25.02
SOLETRON CORP	15.11.2004	7.25	25	2.16	2.55	11.58	9.81
SOUTHERN UNION CO	16.08.2006	5.75	50	2.56	3.13	19.52	16.00
ST PAUL COMPANIES INC	16.08.2005	9	50	1.72	2.07	29.04	24.20
TEMPLE-INLAND INC	17.05.2005	7.5	50	0.79	0.96	63.44	52.00
TOYS R US INC	16.08.2005	6.25	50	2.32	2.83	21.55	17.65
TXU CORP	16.05.2006	8.125	50	0.79	0.98	62.91	51.15
UNITED STATES STEEL CORP	15.06.2006	7	50	3.19	3.83	15.66	13.05
UNUMPROVIDENT CORP	15.05.2006	8.25	25	1.88	2.30	13.27	10.88
XEROX CORP	01.07.2006	6.25	100	8.13	9.76	12.30	10.25

Notes: This exhibit provides the basic characteristics of the mandatory convertibles in our sample. Maturity is the due date of the security (dd.mm.yyyy). The annual coupon is the percentage of the par value paid per year. The par value is the lower strike multiplied by the lower ratio or the upper strike price multiplied by the upper ratio. The upper ratio and the lower ratio are the conversion ratios that specify the number of shares the mandatory convertible is converted into depending on the final stock price. The upper and lower strike prices determine the offsetting option spread and the characteristic payoff profile of the mandatory convertibles.

EXHIBIT 3

Overview of Average Input Parameters for the Pricing Model

Name of Issuer	Average time to maturity (years)	Average 4-year historical volatility	Average dividend yield	Average credit spread (bps)	Average moneyness
AFFILIATED MANAGERS GROUP INC	1.55	46%	0.41%	583	0.80
ALLIED WASTE INDUSTRIES INC	2.50	58%	0.00%	490	1.16
AMEREN CORP	1.75	22%	5.78%	110	1.00
AMERICAN ELECTRIC POWER CO INC	2.10	37%	6.43%	158	0.69
ANTHEM INSURANCE COMPANIES INC	1.36	32%	0.00%	137	1.63
BAXTER INTERNATIONAL INC	2.53	38%	2.00%	112	0.94
CAPITAL ONE FINANCIAL CORP	1.97	60%	0.24%	171	0.73
CENDANT CORP	1.08	50%	0.12%	121	0.79
CENTURYTEL INC	1.84	37%	0.72%	136	0.99
CHUBB CORP	2.77	33%	2.33%	68	1.00
CITIZENS COMMUNICATIONS CO	1.01	43%	0.00%	132	0.94
CONSTELLATION BRANDS INC	2.77	35%	0.00%	316	1.00
CORNING INC	2.21	84%	0.00%	397	3.66
DOMINION RESOURCES INC	3.06	28%	4.32%	137	0.96
DOMINION RESOURCES INC	1.54	28%	4.32%	139	1.01
DUKE ENERGY CORP	1.04	37%	5.54%	152	0.51
DUKE ENERGY CORP	1.16	38%	5.54%	113	0.46
ELECTRONIC DATA SYSTEMS CORP	1.29	61%	2.85%	127	0.39
EQUITY SECURITY TRUST II	1.53	61%	0.00%	113	0.96
GABELLI ASSET MANAGEMENT INC	1.80	36%	0.09%	153	0.86
HARTFORD FINANCIAL SERVICES GROUP INC, THE	2.82	39%	2.14%	97	1.04
HARTFORD FINANCIAL SERVICES GROUP INC, THE	3.42	39%	2.14%	113	0.98
HOUSEHOLD INTERNATIONAL INC	2.37	16%	6.42%	69	1.53
IMC GLOBAL INC	2.64	46%	0.84%	618	1.23
KEYSPAN CORP	1.66	24%	4.96%	107	0.98
MOTOROLA INC	1.33	62%	1.45%	136	0.66
ONEOK INC	2.47	29%	3.32%	115	1.02
PHELPS DODGE CORP	2.27	40%	0.00%	177	1.05
PHOENIX COMPANIES INC, THE	2.48	46%	1.54%	161	1.22
PRUDENTIAL FINANCIAL CAPITAL TRUST I	1.34	27%	0.90%	117	1.10
SEMPRA ENERGY	1.66	32%	3.85%	112	1.00
SOLECTRON CORP	1.54	71%	0.00%	510	0.47
SOUTHERN UNION CO	2.79	71%	0.00%	134	1.00
ST PAUL COMPANIES INC	2.22	37%	3.10%	117	1.23
TEMPLE-INLAND INC	1.84	34%	2.69%	158	0.91
TOYS R US INC	2.10	47%	0.00%	176	0.68
TXU CORP	2.86	44%	3.38%	273	0.41
UNITED STATES STEEL CORP	2.55	43%	1.17%	326	1.77
UNUMPROVIDENT CORP	2.58	57%	2.75%	137	1.10
XEROX CORP	2.64	72%	0.00%	619	1.06
Mean	2.1	43%	2.0%	203	1.02

Notes: This exhibit provides some characteristics of our sample within the observed time window. The average time to maturity indicates where the securities are within their life cycle. The average four-year historical volatility describes the standard deviation of the continuously compounded returns of the underlying shares. The average dividend yield describes the average dividend divided by current stock price. Moody's median bond spread data give the credit spreads. The moneyness is defined as follows: moneyness = 1 if the current stock price is between the lower and the upper strike prices; moneyness <1 if the current stock price is below the lower strike price; moneyness >1 if the current stock price is above the upper strike price.

upper ratio, lower ratio, and upper and lower strike prices. The securities mature between May 18, 2004, and November 16, 2006. All bonds pay quarterly coupons. The average annual coupon rate is 7.38% of the face value. Exhibit 3 shows further average characteristics of our sample. The average time to maturity is approximately 2 years (average maturities over all data points), the average 4-year historical volatility is 43%, the average dividend yield of the underlying shares is 2.0%, the average credit spread is 203 bp and the average moneyness⁶ is 1.02 (i.e., the average security is slightly “in-the-money”: the stock price is slightly above the upper strike price).

For each of the mandatory convertibles, our database provides daily price quotes ($V_{l,t}^{\text{market}}$, $l = 1, 2, \dots, L$; $t = 1, 2, \dots, N_l$, where $N_l \leq 498$ denotes the number of records for the l -th bond and $L = 40$). To look at the relative price differences between market prices ($V_{l,t}^{\text{market}}$) and model prices ($V_{l,t}^{\text{model}}$), percentage pricing errors ($E_{l,t}$) at time t for a mandatory convertible bond l are defined as

$$E_{l,t} = \frac{V_{l,t}^{\text{market}} - V_{l,t}^{\text{model}}}{V_{l,t}^{\text{model}}} \quad (6)$$

The model price ($V_{l,t}^{\text{model}}$) of the mandatory convertible l is obtained using Equation (2) at time t . For each mandatory convertible, we calculate the mean pricing error (arithmetic mean) of each security, defined as

$$E_l = \frac{1}{N_l} \sum_{t=1}^{N_l} E_{l,t} \quad (7)$$

If the pricing errors in Equation (7) are positive, model prices tend to underestimate market prices and if they are negative, model prices tend to overestimate market prices.

Furthermore, for each mandatory convertible bond we calculate the root mean squared error (RMSE), defined as:

$$\text{RMSE}_l = \sqrt{\frac{1}{N_l} \sum_{t=1}^{N_l} (E_{l,t})^2} \quad (8)$$

THE PRICING OF MANDATORY CONVERTIBLE BONDS

Overview

The observed mandatory convertible prices on the U.S. market are compared with theoretical prices obtained

by the pricing model proposed earlier. Exhibit 4 shows an overview of pricing errors for all securities in the sample. The exhibit provides the mean percentage pricing errors (E_l), the root mean squared error (RMSE_{*l*}), the mean absolute error (MAE_{*l*}), and the standard deviation ($\sigma_{E,l}$) of the pricing errors for each issue l . A negative value of the mean pricing error indicates an observed underpricing; that is, the observed market price is below the theoretical value. In other words, the pricing model overprices the market. The *mean over all mandatory convertibles* in Exhibit 4 is defined as the average parameter of all 40 securities (e.g., the equally-weighted mean of the mean pricing error for each bond), whereas the results for the *mean over all data points* are calculated over the entire sample of data points, weighting each of the 14,612 data points equally.

Exhibit 4 shows the results of the pricing study. The mean RMSE is 3.58%; the mean standard deviation 2.34%. On average, the mandatory convertibles are slightly overpriced by 0.27% (i.e., the pricing model underestimates market prices). Although not negligible, the pricing errors are smaller than those reported in similar studies on standard U.S. convertible bonds.⁷ We will analyze these results in two respects: first, we explore the possible driving factors of the pricing errors and, second (in the next section), we analyze the hedging performance of the valuation model.

Exhibit 5 shows the frequency distribution of the pricing errors for the whole sample of 14,612 data points. Of the pricing errors, 42% are positive; that is, the market price is above the model price. The vast majority of values for $E_{l,t}$ are concentrated around zero, whereas the distribution has a mean of 0.33% and a median of -0.65% with a standard deviation of 4.45%.

In Exhibit 6 pricing errors are shown for a changing rolling volatility estimation window ranging from 0.25 to 5 years. The pricing error distributions are very similar and indicate that the results are robust with respect to the changing volatility window used in the analysis. Tests for differences in the mean show that mean pricing errors are not statistically different from each other.

Analysis of Pricing Errors

This subsection explores the driving factors for the pricing errors. First, we show a breakdown of the pricing error by moneyness of the option and, second, pricing errors are regressed against the factors that enter the valuation model.

EXHIBIT 4

Pricing Overview

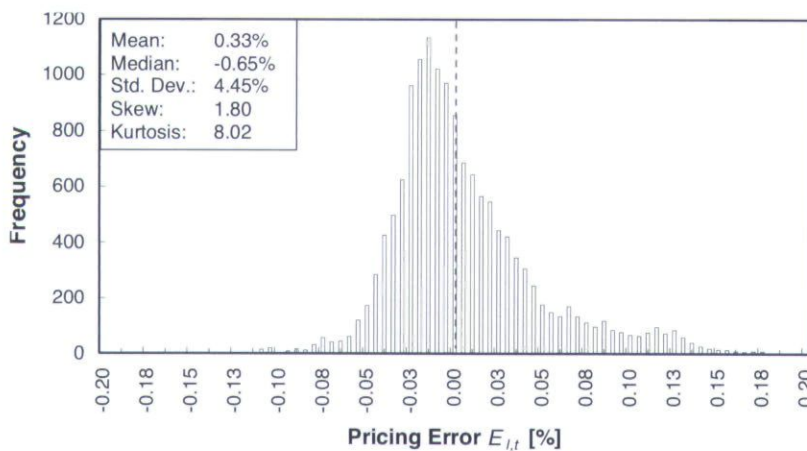
Name of Mandatory Convertible Bond	Data Points	Mean percentage error	Root mean squared error	Mean absolute error	Std. Dev.
	(N_l)	(E_l)	$(RMSE_l)$	(MAE_l)	$(\sigma_{E,l})$
AFFILIATED MANAGERS GROUP INC 17.11.2004 6%	498	-0.53% *	2.16%	1.70%	2.10%
ALLIED WASTE INDUSTRIES INC 01.04.2006 6.25%	274	-2.01% *	3.07%	2.37%	2.32%
AMEREN CORP 15.05.2005 9.75%	345	1.28% *	3.22%	2.59%	2.96%
AMERICAN ELECTRIC POWER CO INC 16.08.2005 9.25%	394	8.48% *	9.52%	8.50%	4.32%
ANTHEM INSURANCE COMPANIES INC 15.11.2004 6%	403	-0.43% *	1.05%	0.77%	0.96%
BAXTER INTERNATIONAL INC 16.02.2006 7%	351	1.86% *	3.11%	2.81%	2.50%
CAPITAL ONE FINANCIAL CORP 17.05.2005 6.25%	456	-0.74% *	2.21%	1.58%	2.09%
CENDANT CORP 17.08.2004 7.75%	389	5.51% *	5.87%	5.51%	2.01%
CENTURYTEL INC 15.05.2005 6.875%	394	-1.90% *	2.63%	2.31%	1.83%
CHUBB CORP 16.08.2006 7%	220	-0.98% *	1.40%	1.13%	1.01%
CITIZENS COMMUNICATIONS CO 17.08.2004 6.75%	353	-0.03%	2.84%	2.43%	2.84%
CONSTELLATION BRANDS INC 01.09.2006 5.75%	194	2.73% *	3.75%	3.36%	2.58%
CORNING INC 16.08.2005 7%	450	-0.95% *	1.86%	1.39%	1.60%
DOMINION RESOURCES INC 15.05.2006 8.75%	498	0.16%	4.37%	3.65%	4.37%
DOMINION RESOURCES INC 16.11.2004 9.5%	498	-1.28% *	1.91%	1.41%	1.42%
DUKE ENERGY CORP 18.05.2004 8.25%	498	7.76% *	11.37%	8.16%	8.31%
DUKE ENERGY CORP 16.11.2004 8%	300	8.14% *	9.48%	8.20%	4.86%
ELECTRONIC DATA SYSTEMS CORP 17.08.2004 7.625%	498	1.03% *	2.50%	1.86%	2.28%
EQUITY SECURITY TRUST II 15.01.2005 6.25%	405	-2.97% *	3.30%	2.99%	1.44%
GABELLI ASSET MANAGEMENT INC 17.02.2005 6.95%	498	-3.90% *	5.86%	5.08%	4.38%
HARTFORD FINANCIAL SERVICES GROUP INC, THE 16.08.2006 7%	241	-2.33% *	2.46%	2.34%	0.78%
HARTFORD FINANCIAL SERVICES GROUP INC, THE 16.11.2006 6%	421	-2.06% *	2.29%	2.10%	1.00%
HOUSEHOLD INTERNATIONAL INC 15.02.2006 8.875%	272	-1.03% *	1.84%	1.50%	1.52%
IMC GLOBAL INC 01.07.2006 7.5%	216	-2.42% *	3.37%	2.61%	2.36%
KEYSPAN CORP 16.05.2005 8.75%	300	2.62% *	3.04%	2.67%	1.55%
MOTOROLA INC 16.11.2004 7%	388	5.81% *	6.38%	5.82%	2.65%
ONEOK INC 16.02.2006 8.5%	323	-2.66% *	3.05%	2.72%	1.48%
PHELPS DODGE CORP 15.08.2005 6.75%	484	-3.23% *	3.75%	3.46%	1.92%
PHOENIX COMPANIES INC, THE 16.02.2006 7.25%	328	-3.04% *	3.62%	3.14%	1.96%
PRUDENTIAL FINANCIAL CAPITAL TRUST I 15.11.2004 6.75%	394	-1.05% *	1.87%	1.57%	1.54%
SEMPRA ENERGY 17.05.2005 8.5%	299	-1.94% *	2.23%	1.94%	1.11%
SOLETRON CORP 15.11.2004 7.25%	498	0.04%	3.14%	2.60%	3.15%
SOUTHERN UNION CO 16.08.2006 5.75%	229	1.08% *	1.70%	1.54%	1.31%
ST PAUL COMPANIES INC 16.08.2005 9%	454	-2.57% *	2.85%	2.58%	1.24%
TEMPLE-INLAND INC 17.05.2005 7.5%	393	0.79% *	1.81%	1.52%	1.63%
TOYS R US INC 16.08.2005 6.25%	394	0.76% *	2.59%	2.12%	2.48%
TXU CORP 16.05.2006 8.125%	394	6.51% *	8.70%	7.48%	5.77%
UNITED STATES STEEL CORP 15.06.2006 7%	195	-2.18% *	2.80%	2.35%	1.75%
UNUMPROVIDENT CORP 15.05.2006 8.25%	254	-2.29% *	2.62%	2.37%	1.27%
XEROX CORP 01.07.2006 6.25%	219	-1.28% *	1.67%	1.38%	1.08%
Mean over all Mandatory Convertibles (40 Bonds)	14,612	0.27% *	3.58%	3.04%	2.34%
Mean over all Data Points (14,612 Data Points)	14,612	0.33% *	4.50%	3.10%	4.45%

*Denotes significance level of 1% for rejection of the null hypothesis that model and market prices are equal in the mean.

Notes: Data points (N_l) indicates the number of days for which model prices are computed. The root mean squared error (RMSE) for each security l is defined as $RMSE_l = (1 / N_l \cdot \sum_{t=1}^{N_l} (E_{l,t})^2)^{1/2}$. The mean absolute error (MAE) for security l is defined as $MAE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} |E_{l,t}|$. Std. dev. denotes the standard deviation of the pricing errors.

EXHIBIT 5

Distribution of Pricing Errors for 14,612 Data Points of the Whole Sample



Note: The distribution is right-skewed with a skew of 1.80 and leptokurtic with a kurtosis of 8.02.

EXHIBIT 6

Robustness of the Model with Respect to Volatility

Volatility window (years)	0.25	1	2	3	4	5
Mean Pricing Error	0.22% *	0.49% *	0.43% *	0.36% *	0.33% *	0.28% *
Std. Dev.	4.28%	4.46%	4.56%	4.47%	4.45%	4.43%
Median	-0.45%	-0.43%	-0.64%	-0.63%	-0.65%	-0.63%
Skewness	2.01	1.94	1.89	1.84	1.80	1.79
Kurtosis	9.97	8.56	7.66	8.01	8.02	8.11

*Denotes significance level of 1%.

Notes: This exhibit shows pricing statistics for different rolling window lengths used for volatility estimation. As can be seen, the estimation window has a very limited impact on pricing errors. Std. dev. denotes the standard deviation of the pricing errors.

We define the moneyness ($M_{l,t}$) of the mandatory convertible l on day $t = 1, 2, \dots, N_l$ as

$$M_{l,t} = \begin{cases} \frac{S_{l,t}}{X_{l,L}} & \text{if } S_{l,t} < X_{l,L} \\ 1 & \text{if } X_{l,L} \leq S_{l,t} \leq X_{l,U} \\ \frac{S_{l,t}}{X_{l,U}} & \text{if } S_{l,t} > X_{l,U} \end{cases} \quad (9)$$

The univariate relationship between the moneyness ($M_{l,t}$) and the pricing errors ($E_{l,t}$) is illustrated in Exhibits 7 and 8. The mean pricing error for at-the-money ($M_{l,t} = 1$; i.e., the current stock price is between the lower and the upper strike price) mandatory convertibles is slightly below zero (-0.61%). If the securities are out-of-the-money ($M_{l,t} < 1$; i.e., the current stock price is below the lower strike price), mean pricing errors tend to be positive (1.22% and 4.25%) and model prices underestimate market values. On the contrary, if mandatory convertibles move in-the-money ($M_{l,t} > 1$; i.e., the current stock price is above the upper strike price), the model prices tend to overestimate the market values (mean pricing errors of -1.90% and -0.87%). RMSE and standard deviation of pricing errors decrease remarkably as the bonds move in-the-money.

The pricing formula in Equation (2) is mainly sensitive to the current stock price, the coupon payments, and the expected dividend payments if the mandatory convertible is deeply out-of-the-money or deeply in-the-money. The only uncertain cash flow is the expected dividend payment. If investors expected higher dividend yields in the future than the current yield (i.e., the model input), market prices would be lower than model prices (see Equation 2). Thus, for deeply in-the-money securities (i.e., stock prices have risen in the past), investors perhaps expect higher dividend yields in the future. The same argument can be made in the opposite direction for mean market prices above mean model prices (positive mean pricing errors). Alternatively, the pricing error pattern might also be attributable to the smile effect.⁸

The high RMSE and standard deviation for out-of-the-money bonds may be an indication that future dividends are more uncertain if the underlying common stock has performed poorly. The increased RMSE for deeply out-of-the-money bonds can perhaps also be attributed to a lack of liquidity for these bonds. However, the insight from such a univariate analysis is limited because of the relatively small sample size and the multicollinearity among the explanatory factors.

EXHIBIT 7

Pricing Error for Different Moneyness Classes

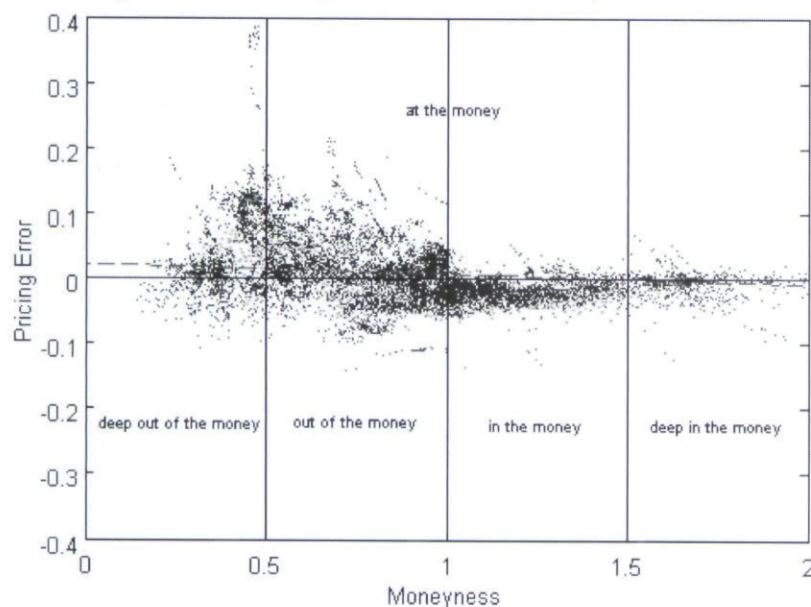
Moneyness, $M_{i,t}$	Mean pricing error	RMSE	Std. Dev.	Data Points
deep out of the money (0 - 0.49)	4.25% *	8.10%	6.90%	1712
out of the money (0.50 - 0.99)	1.22% *	4.89%	4.74%	4994
at the money (1.00)	-0.61% *	3.00%	2.94%	4199
in the money (1.01 - 1.50)	-1.90% *	2.51%	1.64%	2615
deep in the money (>1.50)	-0.87% *	2.07%	1.87%	1092

*Denotes significance level of 1%.

Notes: Mean pricing error states the extent to which market prices are, on average, above model prices for a given moneyness class. RMSE is the root mean squared error of the observations in the respective class. Std. dev. is the standard deviation of the observations in the respective class. Data points is the number of records in the category.

EXHIBIT 8

Percentage Pricing Error of Each Daily Observed Market Price with Respect to the Moneyness of the Mandatory Convertible



Note: Assuming a linear dependence (the dashed line represent a linear trend), the pricing error decreases with the moneyness.

To analyze whether pricing errors are systematic with respect to the input parameters, they are jointly regressed against the dividend yield (q), the annual coupon (I), the moneyness (M), the maturity (ΔT), the credit spread (κ), and the volatility (σ). The regression is specified as follows:

$$E_{i,t} = \alpha_0 + \alpha_1 q_{i,t} + \alpha_2 I_{i,t} + \alpha_3 M_{i,t} + \alpha_4 \Delta T_{i,t} + \alpha_5 \kappa_{i,t} + \alpha_6 \sigma_{i,t} + \varepsilon_{i,t} \quad (10)$$

where $\alpha_i \in \mathbb{R}$, $i = 0, 1, 2, \dots, 6$; $l = 1, 2, \dots, 40$; $t = 1, 2, \dots, N_l$, where $N_l \leq 498$. Correlation analysis shows that the dividend yield, the coupon, the volatility, and the credit spread are rather highly correlated. As a result of these correlations, we run the regressions in Equation (10) excluding the highly correlated regressors. The estimated standard errors are adjusted for autocorrelation and heteroskedasticity following Newey and West [1987], who develop a variance-covariance estimator that is consistent in the presence of both heteroskedasticity and autocorrelation. The truncation lag parameter represents the number of autocorrelations used in evaluating the dynamics of the OLS residuals and is set following the suggestion of Newey and West [1987].⁹ The results of the regressions are given in Exhibit 9.

Regressions (1) to (6) in Exhibit 9 show the individual regressions for the six independent variables. In regression (1) the constant α_0 is significantly negative and the coefficient $\alpha_1 = 0.706$ is significantly positive. The scaling of the parameter q (%) implies an increase of 0.706% of the pricing error (e.g., from 1% to 1.706%) for an increase of the dividend yield of 1% (e.g., from 2% to 3%). Therefore, pricing errors increase if dividend yields increase, indicating that market participants perhaps expect lower dividends for the future if dividends are high.

The same argument holds for regression (2), where the pricing errors increase with the coupon payments (the coefficient of 0.871 implies that pricing errors increase by 0.871% for a coupon increase of 1%) owing to the large correlation between the coupon and the dividend yield (correlation coefficient of 0.8). Firms with higher divi-

dividend yields on their common stock issue mandatory convertibles with higher coupon payments. In our sample, the mean "income advantage" (coupon payments in percent minus dividend yield in percent) is about 5.4% for the whole sample, with a small standard deviation of 1.4%. Therefore, firms with high dividends tend to issue manda-

EXHIBIT 9

Regression Analysis of Factors Influencing Pricing Errors

Reg #	CONSTANT α_0	DIVIDEND YIELD α_1	COUPON α_2	MONEYNESS α_3	MATURITY α_4	CREDITSPREAD α_5	VOLATILITY α_6	Adj. R^2
(1)	-0.011 (-7.76) *	0.706 (10.42) *						11.24%
(2)	-0.061 (-8.18) *		0.871 (8.04) *					4.57%
(3)	0.017 (5.90) *			-0.014 (-5.83) *				4.26%
(4)	0.038 (9.99) *				-0.017 (-10.55) *			8.85%
(5)	0.010 (5.58) *					-0.352 (-7.31) *		1.44%
(6)	0.004 (1.55)						-0.003 (-0.48)	0.00%
(7)	0.048 (9.20) *			-0.014 (-6.73) *	-0.017 (-10.72) *		0.007 (1.35)	12.95%
(8)	0.053 (11.40) *			-0.013 (-6.20) *	-0.016 (-10.42) *	-0.173 (-3.86) *		13.22%
(9)	-0.007 (-0.84)		0.746 (7.23) *	-0.013 (-6.10) *	-0.016 (-10.51) *			16.21%
(10)	0.035 (9.16) *	0.672 (10.73) *		-0.009 (-5.00) *	-0.018 (-11.36) *			22.64%

*Denotes significance level of 1%.

Notes: Pricing errors (E) are regressed against the mandatory convertible's dividend yield as a percentage (q), the annual coupon as a percentage (I), the moneyiness (M), the maturity ΔT (in years), the credit spread as a percentage (cs), and the historical four-year volatility (σ):

$$E_{i,t} = \alpha_0 + \alpha_1 q_{i,t} + \alpha_2 I_{i,t} + \alpha_3 M_{i,t} + \alpha_4 \Delta T_{i,t} + \alpha_5 cs_{i,t} + \alpha_6 \sigma_{i,t} + \varepsilon_{i,t}$$

Standard errors are adjusted for autocorrelation and heteroskedasticity according to Newey and West [1987]. Adj. R^2 denotes the value of the adjusted R^2 (coefficient of determination). In regressions (1) to (6), each regressor is estimated separately. In regressions (7) to (10), we omit the highly correlated regressors. Numbers in parentheses are t-values.

tory convertibles with high coupons to achieve a sufficient level of income advantage for the bond. So, the dividend yield and coupon effect can be attributed to the same pricing error source.

In regression (3), the moneyiness coefficient is significantly negative, showing that pricing errors decrease if the securities move in-the-money. This analysis supports the findings earlier (Exhibit 7), where in-the-money or deeply in-the-money securities had a strong stock price appreciation in the past and therefore dividend payments are perhaps expected to increase.

Regression (4) shows a significantly negative maturity coefficient ($\alpha_4 = -0.017$), implying a pricing errors decrease for larger maturities (an increase of the maturity of 1 year results in a decrease of 1.7% of the pricing error). Because we use a continuously compounded dividend

yield instead of discrete dividend payments in the valuation model, short maturities tend to overestimate the dividend payments (the overestimation increases if the period between last dividend payment date and the maturity increases).

In regression (5), the coefficient for the credit spread is significantly negative. As we explained in discussing our model, the influence of the credit spread on mandatory convertible pricing is fairly small because, other than for traditional convertibles, only the coupon payments need to be discounted by the credit-spread-adjusted interest rate. Owing to the negative correlation between credit spreads and dividend yields, this result is an indication that the credit spread coefficient might have the same error source as the dividend yield coefficient. In our sample, mandatory convertibles with higher dividends

tend to have better credit ratings and therefore lower credit spreads.

Finally, regression (6) confirms the results of the previous section that the volatility is not a source of systematic pricing errors.

The multivariate regressions (7) to (10) in Exhibit 9 show that the coefficients remain similar and that the statistical significance does not change, confirming the results of the univariate analysis in regressions (1) to (6). All coefficients, except α_6 for the volatility, are significant at the 1% level. However, the full model in Equation (10) is not tested because of the multicollinearity among several factors. The coefficient (α_1) for dividend yields has an adj. R^2 of approximately 11%, and the coefficient (α_4) for maturity has an adj. R^2 of approximately 8%, whereas the four other parameters have an adj. R^2 of less than 5%. These results indicate that dividends can explain pricing errors better than the other candidate parameters. The adj. R^2 can be further increased substantially in regressions (7) to (10) by using non-correlated regressors. Regression (10) has the highest adj. R^2 of approximately 23%, combining the dividend yield, the moneyness, and the maturity of the mandatory convertibles.

THE HEDGING OF MANDATORY CONVERTIBLE BONDS

In practice, hedging and understanding and managing the risk exposure of mandatory convertible bonds is just as relevant as pricing. Therefore, besides pricing, a second measure to assess a contingent claims model is its hedging performance. In this section we analyze how accurately our model predicts price changes in addition to price levels. In particular, we analyze whether the delta from the model gives a good indication of how much the market value of the bond's price (V_t^{market}) changes when the underlying stock price S changes.

We use delta-hedging to insulate a mandatory convertible bond position from the effects of stochastic changes in the price of the underlying asset.¹⁰ Delta is the partial derivative of the mandatory convertible bond model price (V_t^{model}) with respect to a change in the price of the underlying stock S . The definition of delta δ_t at time t is¹¹

$$\begin{aligned}\delta_t &= \frac{\partial V_t^{\text{model}}}{\partial S_t} = \frac{\partial c_{t,U}}{\partial S_t} \cdot R_U - \frac{\partial p_{t,L}}{\partial S_t} \cdot R_L \\ &= \{e^{-q_t \cdot \Delta T} \cdot \phi(d_U)\} \cdot R_U - \{e^{-q_t \cdot \Delta T} [\phi(d_L) - 1]\} \cdot R_L\end{aligned}\quad (11)$$

with

$$V_t^{\text{model}} = c_{t,U} \cdot R_U - p_{t,L} \cdot R_L + P \cdot e^{-(r_{t,\Delta T}) \cdot \Delta T} + \sum_{j=1}^m I \cdot e^{-(r_{t,\Delta T} + \alpha_5) \cdot (t_j - t)}$$

and

$$d_k = \left\{ \ln \left(\frac{S_t}{X_k} \right) + \left(r_{t,\Delta T} - q_t + \frac{\sigma_t^2}{2} \right) \cdot \Delta T \right\} / \sigma_t \cdot \sqrt{\Delta T}$$

We examine the hedging performance by setting up delta-neutral positions. The position is assumed to be held for one trading day (later we extend the analysis with a rebalancing period of five trading days). Ignoring the holding period expected return on the position ($B_t - B_{t-1} = \Delta B_t \approx 0$), which is very small for such a short holding period, we set up the following hedge position:

$$\begin{aligned}S_{t-1} \cdot \delta_{t-1} + B_{t-1} &= V_{t-1}^{\text{market}} \text{ (at time } t-1) \\ S_t \cdot \delta_{t-1} + B_t + \varepsilon_t^{\text{market}} &= V_t^{\text{market}} \text{ (at time } t)\end{aligned}\quad (12)$$

$$\begin{aligned}\Rightarrow (S_t - S_{t-1}) \cdot \delta_{t-1} + (B_t - B_{t-1}) + \varepsilon_t^{\text{market}} &= V_t^{\text{market}} - V_{t-1}^{\text{market}} \\ \Leftrightarrow \underbrace{(\Delta S_t) \cdot \delta_{t-1}}_{\equiv \Delta H_t} + \underbrace{(\Delta B_t)}_{\approx 0} + \varepsilon_t^{\text{market}} &= \Delta V_t^{\text{market}}\end{aligned}\quad (13)$$

The parameters at time t are defined as follows: V_t^{market} is the market value of the bond's price, $\varepsilon_t^{\text{market}}$ is the hedging error in dollar terms, B_t is the money market account at time t , and $\Delta H_t \equiv \Delta S_t \cdot \delta_{t-1}$ is the hedge position. The hedging error in dollar terms ($\varepsilon_t^{\text{market}} \approx \Delta V_t^{\text{market}} - \Delta H_t$) is the difference between the bond's price change and the change in the hedge position. To compare the different mandatory convertible bonds in their hedge performance, we use the hedging error as a percentage $\hat{\varepsilon}_t^{\text{market}}$ of the market's bond price V_t^{market} :

$$\hat{\varepsilon}_t^{\text{market}} \approx \frac{\Delta V_t^{\text{market}} - \Delta H_t}{V_t^{\text{market}}} = \frac{\varepsilon_t^{\text{market}}}{V_t^{\text{market}}}\quad (14)$$

Hedging performance is measured by mean percentage hedging error, root mean squared error, and mean absolute error. Exhibit 10 presents the hedging overview for all securities. Overall, delta hedges based on the hedge of the market price position (V_t^{market}) show an RMSE of approximately 1.14%. The mean percentage hedging error is -0.01% and not significantly different from zero. Furthermore, none of the mean percentage hedging errors of the individual securities is significantly different from zero.

Next, we analyze the following three issues: 1) we compare the actual market hedge shown earlier with a

EXHIBIT 10

Hedging Performance: Hedge against the Market with a Rebalancing Period of One Trading Day

Name of Mandatory Convertible Bond	Data Points	Mean percentage hedging error	Root mean squared error	Mean absolute error	Std. Dev.
	(N_l)	($MPHE_l$)	($RMSE_l$)	(MAE_l)	(σ_l)
AFFILIATED MANAGERS GROUP INC 17.11.2004 6%	497	-0.02%	0.98%	0.67%	0.98%
ALLIED WASTE INDUSTRIES INC 01.04.2006 6.25%	273	-0.03%	1.38%	1.00%	1.38%
AMEREN CORP 15.05.2005 9.75%	344	-0.01%	1.30%	0.59%	1.31%
AMERICAN ELECTRIC POWER CO INC 16.08.2005 9.25%	393	0.02%	1.23%	0.82%	1.23%
ANTHEM INSURANCE COMPANIES INC 15.11.2004 6%	402	-0.01%	0.71%	0.52%	0.71%
BAXTER INTERNATIONAL INC 16.02.2006 7%	350	0.00%	0.93%	0.66%	0.93%
CAPITAL ONE FINANCIAL CORP 17.05.2005 6.25%	455	-0.03%	1.23%	0.89%	1.23%
CENDANT CORP 17.08.2004 7.75%	388	-0.04%	0.76%	0.53%	0.76%
CENTURYTEL INC 15.05.2005 6.875%	393	-0.01%	1.00%	0.72%	1.01%
CHUBB CORP 16.08.2006 7%	219	0.01%	0.76%	0.56%	0.76%
CITIZENS COMMUNICATIONS CO 17.08.2004 6.75%	352	-0.01%	1.17%	0.71%	1.18%
CONSTELLATION BRANDS INC 01.09.2006 5.75%	193	0.03%	0.95%	0.61%	0.95%
CORNING INC 16.08.2005 7%	449	-0.02%	2.06%	1.43%	2.06%
DOMINION RESOURCES INC 15.05.2006 8.75%	497	0.00%	1.19%	0.51%	1.19%
DOMINION RESOURCES INC 16.11.2004 9.5%	497	-0.02%	1.16%	0.59%	1.16%
DUKE ENERGY CORP 18.05.2004 8.25%	497	-0.08%	2.34%	1.01%	2.34%
DUKE ENERGY CORP 16.11.2004 8%	299	-0.03%	0.96%	0.65%	0.97%
ELECTRONIC DATA SYSTEMS CORP 17.08.2004 7.625%	497	-0.06%	1.42%	0.87%	1.42%
EQUITY SECURITY TRUST II 15.01.2005 6.25%	404	-0.02%	1.16%	0.86%	1.16%
GABELLI ASSET MANAGEMENT INC 17.02.2005 6.95%	497	0.00%	1.14%	0.78%	1.14%
HARTFORD FINANCIAL SERVICES GROUP INC, THE 16.08.2006 7%	240	0.00%	0.68%	0.49%	0.68%
HARTFORD FINANCIAL SERVICES GROUP INC, THE 16.11.2006 6%	420	-0.01%	0.72%	0.54%	0.72%
HOUSEHOLD INTERNATIONAL INC 15.02.2006 8.875%	271	0.03%	0.86%	0.61%	0.86%
IMC GLOBAL INC 01.07.2006 7.5%	215	-0.01%	1.76%	1.30%	1.76%
KEYSPAN CORP 16.05.2005 8.75%	299	0.00%	0.59%	0.39%	0.59%
MOTOROLA INC 16.11.2004 7%	387	-0.03%	1.20%	0.84%	1.20%
ONEOK INC 16.02.2006 8.5%	322	0.01%	0.70%	0.52%	0.71%
PHELPS DODGE CORP 15.08.2005 6.75%	483	-0.02%	1.27%	0.84%	1.27%
PHOENIX COMPANIES INC, THE 16.02.2006 7.25%	327	-0.01%	1.23%	0.89%	1.23%
PRUDENTIAL FINANCIAL CAPITAL TRUST I 15.11.2004 6.75%	393	-0.02%	0.86%	0.61%	0.86%
SEMPRA ENERGY 17.05.2005 8.5%	298	-0.01%	0.66%	0.46%	0.66%
SOLETRON CORP 15.11.2004 7.25%	497	-0.05%	1.97%	1.33%	1.97%
SOUTHERN UNION CO 16.08.2006 5.75%	228	-0.01%	0.82%	0.61%	0.83%
ST PAUL COMPANIES INC 16.08.2005 9%	453	-0.01%	0.98%	0.73%	0.99%
TEMPLE-INLAND INC 17.05.2005 7.5%	392	-0.02%	0.94%	0.64%	0.94%
TOYS R US INC 16.08.2005 6.25%	393	-0.03%	1.36%	0.92%	1.36%
TXU CORP 16.05.2006 8.125%	393	0.01%	1.06%	0.66%	1.06%
UNITED STATES STEEL CORP 15.06.2006 7%	194	-0.03%	1.62%	1.12%	1.62%
UNUMPROVIDENT CORP 15.05.2006 8.25%	253	0.00%	1.50%	1.17%	1.50%
XEROX CORP 01.07.2006 6.25%	218	-0.01%	1.13%	0.87%	1.13%
Mean over all Mandatory Convertibles (40 Bonds)	14,572	-0.01%	1.14%	0.76%	1.14%
Mean over all Data Points (14,572 Data Points)	14,572	-0.02%	1.24%	0.77%	1.24%

Notes: The hedging errors as percentages are defined as $\hat{\epsilon}_t^{\text{market}} \approx (\Delta V_t^{\text{market}} - \Delta H_t) / V_t^{\text{market}}$. Data points (N_l) indicates the number of days for which prices are available. The mean percentage hedging error (MPHE) for each security l is defined as $MPHE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} \hat{\epsilon}_{lt}^{\text{market}}$. All values for the mean percentage hedging error are not significantly different from zero. The root mean squared error (RMSE) for each security l is defined as $RMSE_l = (1 / N_l \cdot \sum_{t=1}^{N_l} (\hat{\epsilon}_{lt}^{\text{market}})^2)^{1/2}$. The mean absolute error (MAE) for security l is defined as $MAE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} |\hat{\epsilon}_{lt}^{\text{market}}|$. Std. dev. denotes the standard deviation of the hedging errors.

EXHIBIT 11

Hedging Performance: Hedge against the Model with a Rebalancing Period of One Trading Day

	Data Points (N_l)	Mean percentage hedging error ($MPHE_l$)	Root mean squared error ($RMSE_l$)	Mean absolute error (MAE_l)	Std. Dev. (σ_l)
Mean over all Mandatory Convertibles (40 Bonds)	14,572	-0.01%	0.27%	0.07%	0.27%
Mean over all Data Points (14,572 Data Points)	14,572	-0.02%*	0.33%	0.08%	0.33%

*Denotes significance level of 1% for rejection of the null hypothesis that the mean percentage hedging errors are zero in the mean.

Notes: The hedging errors as percentages are defined as $\hat{\epsilon}_t^{\text{model}} \approx (\Delta V_t^{\text{model}} - \Delta H_t) / V_t^{\text{model}}$. Data points (N_l) indicates the number of days for which prices are available. The mean percentage hedging error (MPHE) for each security l is defined as

$MPHE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} \hat{\epsilon}_t^{\text{model}}$. The root mean squared error (RMSE) for each security l is defined as

$RMSE_l = (1 / N_l \cdot \sum_{t=1}^{N_l} (\hat{\epsilon}_t^{\text{model}})^2)^{1/2}$. The mean absolute error (MAE) for security l is defined as $MAE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} |\hat{\epsilon}_t^{\text{model}}|$. Std. dev. denotes the standard deviation of the hedging errors.

hypothetical delta hedge against the model (V_t^{model}). Such a comparison is used to determine to what extent the RMSE is caused by the delta-hedging methodology (because of the linear approximation of a non-linear relationship, simple delta-hedging always generates hedging errors even if the valuation model is perfect) and to what extent it is caused by the model, either by systematic misspecification or by unsystematic noise. 2) We explore whether there are systematic driving factors of the hedging errors with respect to the input parameters of the pricing model. 3) We analyze the hedging errors with a rebalancing period of five instead of one trading days to determine the influence of the rebalancing period on the hedging errors.

The hypothetical hedge against the model is equal to the hedge against the market, except that we use V_t^{model} instead of V_t^{market} for our hedge position. The hedging error of a perfect model is $\epsilon_t^{\text{model}} = \Delta V_t^{\text{model}} - \Delta H_t$ and the hedging error as a percentage of the model price at time t is

$$\hat{\epsilon}_t^{\text{model}} \approx \frac{(V_t^{\text{model}} - V_{t-1}^{\text{model}}) - \Delta H_t}{V_t^{\text{model}}} = \frac{\Delta V_t^{\text{model}} - \Delta H_t}{V_t^{\text{model}}} = \frac{\epsilon_t^{\text{model}}}{V_t^{\text{model}}} \quad (15)$$

Exhibit 11 presents the hedging results of the hedge against the model. Overall, delta hedges based on the hedge against the model show an RMSE of approximately 0.27%. The mean percentage hedging error is -0.01%.

The RMSE over all data points (14,572) is 0.33%, and the mean hedging error is -0.02% and significantly different from zero. The negative mean hedging errors are caused by the delta hedging methodology because the mean daily return of all data points is positive (+0.1%). On average, the positive jumps of the underlying stock prices result in a slightly overestimated hedge position ΔH_t and slightly negative hedging errors. For a rebalancing period of one trading day, the results of Exhibits 10 and 11 indicate that the hedging errors of the hedge against the market (RMSE of 1.14%) are caused by systematic errors and/or by market noise, given that the hedging error of a perfect model is much smaller (RMSE of 0.27%).

Therefore, we explore next whether there are any systematic driving factors of the hedging errors with respect to the input parameters of the pricing model or whether the hedging errors are only unsystematic. Exhibit 12 presents the hedging errors classified by the dividend yield, the coupon payments, the moneyness, the maturity, and the credit spread. Exhibit 12 shows that the mean percentage hedging errors are slightly negative, but none is significantly different from zero. The negative mean errors can be attributed to the delta-hedging methodology and the (on average) increasing stock prices, as seen earlier in the hedge against the model (Exhibit 11). Exhibit 12 does not show any obvious systematic hedging errors measured in terms of mean errors or standard deviations with respect to the input parameters.

EXHIBIT 12

Analysis of the Hedging Errors: Hedge against the Market with a Rebalancing Period of One Trading Day

Dividend Yield (%)	Mean Percentage Hedging Error (MPHE)	Std. Dev.	Datapoints
0.00 - 0.49	-0.02%	1.31%	5609
0.50 - 1.99	-0.01%	1.14%	2671
2.00 - 3.24	-0.01%	0.99%	2318
3.25 - 5.49	-0.02%	1.08%	3002
≥5.50	-0.03%	1.83%	1007
Coupon			
≤6.99%	-0.01%	1.08%	5615
7.00% - 7.99%	-0.03%	1.40%	4166
≥8.00%	-0.01%	1.25%	4828
Moneyness			
deep out of the money (0 - 0.49)	-0.07%	1.80%	1711
out of the money (0.50 - 0.99)	0.00%	1.15%	4993
at the money (1.00)	-0.01%	1.01%	4198
in the money (1.01 - 1.50)	-0.02%	1.13%	2614
deep in the money (>1.50)	-0.03%	1.52%	1091
Maturity (Years)			
0.00 - 0.99	-0.03%	1.52%	1629
1.00 - 1.49	-0.02%	1.11%	2525
1.50 - 1.99	-0.02%	1.14%	3054
2.00 - 2.49	0.00%	1.25%	3348
> 2.50	-0.01%	1.25%	4051
Credit Spread (bps)			
0 - 129	-0.02%	1.11%	6530
130 - 159	-0.01%	1.03%	2442
160 - 299	-0.01%	1.31%	3163
> 300	-0.02%	1.59%	2473

Notes: The hedging errors as percentages are defined as $\hat{\epsilon}_t^{\text{market}} \approx (\Delta V_t^{\text{market}} - \Delta H_t) / V_t^{\text{market}}$. Data points (N_l) indicates the number of days for which prices are available. The mean percentage hedging error (MPHE) for each security l is defined as $\text{MPHE}_l = 1 / N_l \cdot \sum_{t=1}^{N_l} \hat{\epsilon}_{lt}^{\text{market}}$. Std. dev. denotes the standard deviation of the hedging errors. All mean percentage hedging errors are not significantly different from zero.

Exhibit 13 shows the overview of the hedging performance with a rebalancing period of 5 trading days for the hedges against the market and the model. Both the mean percentage hedging errors (-0.23% of the market hedge and -0.21% of the model hedge) and the root mean squared errors (3.14% of the market hedge and 3.08% of the model hedge) are almost equal.¹² The RMSEs of 3.14% and 3.08% show that for longer rebalancing periods, the hedge against the market is almost equivalent to the hedge against the model, indicating that the hedging error is almost entirely caused by the delta-hedging methodology and that the effect of the model error is negligible. It therefore seems that the hedging error observed when rebalancing took place daily is mostly

unsystematic, possibly caused by market noise such as jumps between bid and ask prices. As the rebalancing period is extended, the effect of the market noise decreases and the effect of the hedging methodology increases. As a consequence, the hedging errors (RMSE) of the hedge against the market converge in size to the hedging errors of the hedge against the model.

Although systematic errors of factors not investigated cannot be excluded, our results indicate that for a rebalancing period of one trading day, the difference of the hedging errors of the hedge against the market (RMSE of 1.14%) and of the hedge against the model (RMSE of 0.27%) consists, to a substantial extent, of unsystematic market noise possibly attributable to limited market liquidity.

EXHIBIT 13

Hedging Overview: Hedge against the Market and Model with a Rebalancing Period of Five Trading Days

	Data Points (N_t)	Mean percentage hedging error ($MPHE_t$)	Root mean squared error ($RMSE_t$)	Mean absolute error (MAE_t)	Std. Dev. (σ_t)
Hedge against the market					
Mean over all Mandatory Convertibles (40 Bonds)	2,930	-0.23%	3.14%	2.30%	3.14%
Mean over all Data Points (2,930 Data Points)	2,930	-0.18% *	3.57%	2.37%	3.57%
Hedge against the model					
Mean over all Mandatory Convertibles (40 Bonds)	2,930	-0.21%	3.08%	2.25%	3.08%
Mean over all Data Points (2,930 Data Points)	2,930	-0.17% *	3.56%	2.34%	3.56%

*Denotes significance level of 1% for rejection of the null hypothesis that MPHE are zero.

Notes: The hedging errors as percentages are defined as $\hat{\varepsilon}_t^{\text{market/model}} \approx (\Delta V_t^{\text{market/model}} - \Delta H_t) / V_t^{\text{market/model}}$ for the hedge against the market and the hedge against the model, respectively. Data points (N_t) indicates the number of days for which prices are available. The mean percentage hedging error (MPHE) for each security l is defined as $MPHE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} \hat{\varepsilon}_{lt}^{\text{market/model}}$. The root mean squared error (RMSE) for each security l is defined as $RMSE_l = (1 / N_l \cdot \sum_{t=1}^{N_l} (\hat{\varepsilon}_{lt}^{\text{market/model}})^2)^{1/2}$. The mean absolute error (MAE) for security l is defined as $MAE_l = 1 / N_l \cdot \sum_{t=1}^{N_l} |\hat{\varepsilon}_{lt}^{\text{market/model}}|$. Std. dev. denotes the standard deviation of the hedging errors.

CONCLUSIONS

A relatively large proportion of convertible bonds are currently issued as mandatory convertible bonds. We perform the first empirical analysis of the pricing and hedging of mandatory convertible bonds on the U.S. market. The analysis uses daily market prices for 40 bonds for a period of 498 trading days employing a sample of more than 14,600 daily price observations. The simple pricing model used in the analysis produces lower pricing errors than observed in pricing studies of standard convertible bonds. The results are robust with respect to the method of volatility estimation. Furthermore, the hedging analysis shows that the model is useful for hedging. The hedging errors tend to be small and unsystematic.

ENDNOTES

We would like to thank Bernd Brommundt, Michael Genser, Stephan M. Kessler, Axel Kind, Michael Verhofen,

Rico von Wyss, the editor Stephen Figlewski, and an anonymous referee for their helpful comments. Financial support from the Swiss National Science Foundation (SNF) is gratefully acknowledged.

¹They also adjust for dilution in their valuation model. However, dilution in share prices should be considered when mandatory convertibles are issued only. Afterwards, the dilution in shares of the issuing company is already anticipated and the effects have already been accounted for in the current common stock price. Therefore, valuing mandatory convertibles after the primary issue does not require an adjustment for dilution.

²As an alternative to the simple component model used in this article, a multidimensional lattice model within a framework such as the one proposed by Das and Sundaram [2004] could be used. Such a model would provide more flexibility and allows for modeling correlation between different risk factors. However, the choice of the component model, where the components are valued by a standard option pricing model, seems justified because of the simple structure of the mandatory convertible bonds analyzed in this study. For example, a credit model affects only the value of the stream of coupon

payments, not the basic mandatory structure itself. As a consequence, a correlation model for equity, interest rates, and credit spreads is likely to have a very limited price impact.

³Note that V_t gives the dirty price of the mandatory convertible (i.e., including accrued interest) in monetary units, whereas during the exchange trade clean prices are quoted in general.

⁴In combination with the options, the par value determines the payoff that is settled by the delivery of stock at maturity. As the firm is always able to deliver its stock—perhaps worthless stock in case of bankruptcy—to the holder of the security, the par value is essentially riskless.

⁵A *hard* call feature allows the issuer of a mandatory convertible to redeem the security before maturity. In only four issues provisional or *soft* call features exist. In that case the company can force conversion into common stock prior to maturity if the stock price is deeply in-the-money (e.g., 150% of the upper strike price) for a predefined period of consecutive trading days (e.g., 20 out of 30). This “qualifying period” introduces a path-dependent feature not considered in the analysis. Because the bond price approaches the intrinsic value for securities deeply in-the-money, this option feature is expected to have a minor effect on prices and is neglected.

⁶See Equation (9) and the section “The pricing of mandatory convertible bonds” for a definition.

⁷See, e.g., Ammann et al. [2005], who report a mean RMSE of 6.18% in their study of U.S. convertible bonds, and King [1986], who reports a standard deviation of more than 7.20% in his pricing study of U.S. convertible bonds.

⁸The pattern of the pricing error versus moneyness might perhaps also be explained by fat-tailed return distributions for the underlying stock. Model prices of the bonds are too low for (deeply) out-of-the-money options and too high for (deeply) in-the-money options in the presence of a symmetric option smile. An out-of-the-money mandatory convertible bond contains a short put (out-of-the-money) and a long call (more deeply out-of-the-money). Therefore, the model underprices the bond because the underpricing of the call is greater than the underpricing of the put. Conversely, for an in-the-money mandatory convertible bond, the short put is more deeply in-the-money than the long call. Therefore, the model overprices the bond because the underpricing of the put is greater than the underpricing of the call. The observed pattern of the pricing errors is consistent with this explanation (Exhibit 7).

⁹Following Newey and West [1987] and current practice, the truncation lag is set to $\text{truncate}[4(K/100)^{2/9}] = 12$, with $K = 14,612$ (number of data points).

¹⁰See Figlewski [2002] for an analysis of hedging performance.

¹¹All parameters are defined as in our description of the pricing model.

¹²The null hypothesis of identical RMSE cannot be rejected if tested according to Diebold and Mariano [1995].

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