Implied Volatility Indexes and Daily Value at Risk Models

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We assess the information content of the VXO, VXN, and VIX implied volatility indexes in a market risk framework. The empirical application focuses on the S&P 100, the Nasdaq 100, and the S&P 500 index performances in time periods that include both bull and bear markets and high- and low-volatility markets. The performance of the VaR models is evaluated using likelihood ratio, independence, and conditional coverage tests.

The results show that simple volatility forecasts based on the implied volatility indexes provide meaningful results when market risk must be quantified. Model performance does not deteriorate in challenging trading environments.

Forecasting volatility is one of the major success stories in quantitative finance, and volatility forecasting models have enjoyed tremendous success since the early 1980s. ARCH models attempt to forecast volatility on the basis of information given by past squared returns. Other simple techniques rely on rolling window estimations for the variance of the asset returns (see Alexander [2001]).

A growing trend advocates the use of implied volatility as the best estimate of future volatility. Under an option pricing model such as Black and Scholes [1973], the expected volatility of an asset over the life of an option is the volatility embedded in the price of the option. If call or put option prices are available, we can invert the Black and Scholes [1973] pricing formula in order to compute expected volatility over the life of the option from the observed market option prices. When all other option parameters are known, there is a one-to-one relationship between option prices and the underlying expected asset volatility. This yields the so-called implied volatility.

With the growing importance of modeling and prediction of asset volatility, the accuracy of implied volatility versus volatility forecasts based on historical returns is an important question in modern finance.

Our work differs from most research on implied volatility because we focus on the use of implied volatility in one-day value at risk models. We extend the typical analysis by evaluating how implied volatility indexes quantify market risk. Consequently, we do not ask whether implied volatility delivers unbiased and efficient forecasts of future realized volatility, but instead investigate whether options market data (such as given by the VXO, VXN, and VIX indexes) help us model the one-day VaR correctly. We also consider the RiskMetrics and GJR-GARCH volatility models in the horse race to assess VaR forecasts.

The historical analysis is for distinct subperiods that include 8/1/1994-5/30/1997 (low-volatility, bull market), 6/2/1997-3/31/2000 (high-volatility, bull market), and 4/3/2000-1/31/2003 (high-volatility, bear market). This allows us to test model performance in challenging trading environments and to look at stability over time. To quantify the VaR violations, we use statistical methods including likelihood ratio (LR) tests (i.e., tests based on the proportion of VaR violations), independence, and conditional coverage tests.

Our results show that the implied volatility indexes are useful inputs in VaR models because the number of VaR violations is correctly modeled in most cases, and the null hypotheses of independence (for the VaR violations) and conditional coverage are usually not rejected. Thus, the results show implied volatility indexes provide meaningful information for assessment of the underlying indexes' market risk. There are no real differences across the historical subperiods despite the very different market conditions—good news, as this shows the VaR models do not break down during difficult market conditions.

Our study also provides additional evidence that volatility computed from traded derivatives provides valuable inputs to market participants. This is particularly the case for managers of passive index funds, as the implied volatility pertaining to the index tracked can be directly fed into a market risk model.

I. LITERATURE REVIEW

Most studies of the so-called information content of implied volatility compare volatility forecasts based on implied volatility (which is by definition forward-looking) and historical volatility. The early research reviewed by Figlewski [1997] had to rely on somewhat crude datasets. More recent work uses improved databases of actively traded options. The empirical evidence as to which volatility forecast performs best is rather mixed, although a broad survey by Poon and Granger [2003] indicates that forecasts based on implied volatility tend to beat forecasts based on historical returns.

Day and Lewis [1992] compare the information content of implied volatility of call options on the S&P 100 index to GARCH-type conditional volatility, with mixed evidence. Xu and Taylor [1995] focus on the information efficiency of the PHLX currency options market. According to Jorion's [1995] study of foreign exchange data, implied volatility is an efficient but biased forecast of future volatility. Canina and Figlewski [1993] find almost no correlation between implied volatility and future realized volatility in their sample.

Christensen and Prabhala [1998] argue against the use of overlapping data, showing that, with non-overlapping data, implied volatility outperforms past volatility in forecasting future volatility and features a high information content. For the S&P 100 index and the VIX implied volatility index, Blair, Poon, and Taylor [2001] show that historical returns do not provide much incremental information beyond that in the VIX implied volatility index.

For three classes of assets (stock index, exchange rate, and oil), Martens and Zein [2004] show that implied volatility measures provide better volatility forecasts than daily GARCH models. Calibrating values using high-frequency intraday returns and long-memory models alters the outcome of the tests, however. For foreign exchange volatility and taking into account intraday returns, Neely [2002] argues that implied volatility is a biased estimator of future realized volatility and should consider volatility forecasts from econometric models.

Ederington and Guan [2002] examine the relevance of implied volatility forecasts using S&P 500 futures options data, concluding that "implied volatility has strong predictive power and generally subsumes the information in historical volatility" (p. 29). In Giot [2003], I compare the incremental information content of lagged implied volatility to GARCH models of conditional volatility for a collection of agricultural commodities traded on the New York Board of Trade; past squared returns only marginally improve the information provided by the lagged implied volatility.

II. VOLATILITY FORECASTS AND THE QUANTIFICATION OF MARKET RISK

We first describe the VXO and VXN implied volatility indexes and then model the one-day VaR (considering the old VXN implied volatility index). We later discuss the changes the CBOE has made in construction of the implied volatility indexes.

VXO and VXN Implied Volatility Indexes

The VXO and the original VXN implied volatility indexes computed by the Chicago Board Options Exchange (CBOE) have enjoyed widespread acceptance in the community of both academics and practitioners. By construction, the VXO and VXN indexes are weighted averages of the implied volatilities computed from a total of eight call and put near-the-money, nearby-maturity, and second-nearby American option contracts on the underlying S&P 100 (VXO) or Nasdaq 100 (VXN) indexes. The weighting method is designed so that either index gives the implied volatility of a hypothetical at-the-money option with a constant maturity of 22 trading days to expiration.

In an efficient market where option prices reflect all available information, the level of these indexes reflect "the market's best assessment of the expected volatility of the underlying stock index over the remaining life of the option"(Whaley [2000, p. 12]), a life of 22 trading days, and the underlying index. Details regarding the construction of the VXO index are available in Whaley [1993], Fleming, Ostdiek, and Whaley [1995], or Whaley [2000].[1]

Implied Volatility Indexes as Inputs to VaR Models?

We want to assess the added value of the VXO/VXN (and later the VIX) to quantify short-term market risk. There are a wide range of implied volatilities (for different strikes, for different expiration dates), and one could argue that for assessing risk we should work with implied volatilities computed from deep out-of-the money or deep in-the-money options. Value at risk modeling is about extreme events (extreme negative returns for long VaR, extreme positive returns for short VaR), so the volatility of interest should be derived from the extreme outcomes in the option market.

While this is clearly an interesting research question, we do not deal with it, as we want to take into account the information given by the implied volatility indexes as such. These are by definition aggregates of many implied volatilities that do not come from deep in-the-money or out-of-the money options as far as the VXO and the original VXN are concerned. Another issue is maturity mismatch between the time horizon of the implied volatility indexes and the time horizon of the market risk forecast (one day). To address these potential problems, we consider linear transformations of the squared VXO/VXN measures in our VaR specifications, where all linear parameters are estimated along with the parameters of the probability density of returns.[2]

We also use a fat-tailed probability density for the returns. Most research on VaR modeling shows that the normal probability density is usually inadequate, even when volatility is modeled via GARCH. Because our volatility input is the VXO/VXN level at a given time (and hence is an average "option price"), there is a strong rationale for specifying a fat-tailed density for the index returns.

Therefore, at time t, VXO_t enters the model as $\omega + \eta\sigma^2_{imp,1,t}$, where ω and η are to be estimated along with the parameters of the fat-tailed probability density. For the S&P 100 index

$$\sigma_{imp,1,t} = \sqrt{\tfrac{1}{365}}\, VXO_t$$

while it is

$$\sigma_{imp,1,t} = \sqrt{\tfrac{1}{365}}\, VXN_t$$

for the Nasdaq 100 index. These two simple transformations scale the VXO/VXN indexes to the one-day level (given the way they are annualized by the CBOE). Thereafter they enter the model as squared terms with $\eta\sigma^2_{imp,1,t}$. Because we treat the implied volatility indexes as volatility information (and not as perfectly matched volatility forecasts), this approach deals indirectly with the fact that our fat-tailed probability density is not the density used in the computation of the VXO/VXN indexes.[3]

An alternative approach could rely on the availability of an extended dataset of implied volatilities for a full range of strikes at all times t, with a mapping between the implied volatility and the VaR level that is of interest. For example, forecasting a VaR level at 1% would use more extreme implied volatility than forecasting the VaR level at 5%.

VaR Models

The VaR at level α for a time horizon of h days is the nominal h-day loss that will not be exceeded in $100 \times \alpha$ portfolio realizations of 100. The literature on VaR models has grown remarkably since the mid-1990s, thanks to the popularity of the RiskMetrics VaR approach, originally developed at J.P. Morgan, and the risk-adjusted measures of capital adequacy enforced by the Basel Committee. A full review of VaR and risk models appears in Jorion [2000], and Dowd [2002] focuses on VaR and the modeling of market risk.

The definition of VaR refers to the statistical notion of a quantile of a probability distribution. More precisely, the VaR at time t for the forthcoming $[t + 1, t + h]$ time horizon is the expected quantile at the α level for the probability distribution of the h-day portfolio returns.

Suppose the dataset consists of a collection of daily demeaned returns r_t. Given a conditional volatility specification g_t such that $r_t = \sqrt{g_t}\,\varepsilon_t$, and a probability density for the residuals ε_t, the parametric one-day VaR at time t is given by:[4]

$$VaR_t = Z_\alpha \sqrt{g_t} \quad (1)$$

Z_α is the quantile at $100 \times \alpha\%$ of the standardized probability distribution. As in Giot and Laurent [2003], we focus on the skewed Student-t distribution, which allows for excess kurtosis and skewness in returns.[5]

The innovation process ε follows a standardized skewed Student density if:

$$f(\varepsilon|\xi,\upsilon) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}} sp\left[\xi(s\varepsilon+m)|\upsilon\right] & \text{if} \quad \varepsilon < -\frac{m}{s} \\ \frac{2}{\xi+\frac{1}{\xi}} sp\left[(s\varepsilon+m)/\xi|\upsilon\right] & \text{if} \quad \varepsilon \geq -\frac{m}{s} \end{cases} \quad (2)$$

where ξ is the asymmetry coefficient; υ accounts for the tail thickness; and $p(.\,|\,\upsilon)$ is the symmetric Student density with unit variance. The asymmetry coefficient, which is positive, is defined so that the ratio of the probability masses above and below the mean is equal to

$$\frac{\Pr(\varepsilon \geq 0|\xi)}{\Pr(\varepsilon < 0|\xi)} = \xi^2$$

The sign of $\ln(\xi)$ indicates the direction of the skewness: If $\ln(\xi) > 0$ (< 0); the third moment is positive (negative), and the density is skewed to the right (left). Coefficients m and s^2 are, respectively, the mean and the variance of the non-standardized skewed Student. Note that this probability density encompasses the Student density ($\xi = 1$) and the normal density ($\xi = 1$ and $\upsilon = \infty$).

To fully characterize the VaR model, g_t must be specified. In our framework, the implied volatility indexes are the key inputs. We use a linear transformation of the observed squared implied volatility index, $g_t = \omega + \eta\sigma^2_{imp,1,t-1}$. The volatility input is directly proportional to (but not equal to) the implied volatility observed the day before, scaled to the daily level.

Following typical practice in the implied volatility literature, we conduct a horse race by introducing competing models. We use the GJR-GARCH model of Glosten, Jagannathan, and Runkle [1993], which predicts the VaR using historical returns. As in Day and Lewis [1992] or Blair, Poon, and Taylor [2001], the lagged implied volatility can also be included in the GARCH model, yielding a third model. We also consider the RiskMetrics model much favored by market practitioners.

To estimate:

$$VaR_t = Z_\alpha \sqrt{g_t}$$

the four models define g_t as follows:

Model 1. Lagged implied volatility:

$$g_t = \omega + \eta\,\sigma^2_{imp,1,t-1} \quad (3)$$

Model 2. RiskMetrics:

$$g_t = 0.04 r^2_{t-1} + 0.96 g_{t-1} \quad (4)$$

Model 3. GJR-GARCH:

$$g_t = \omega + \alpha_1 r^2_{t-1} + \alpha_n r^2_{t-1} d_{t-1} + \delta_1 g_{t-1} \quad (5)$$

where d_{t-1} is a dummy variable equal to 1 if $r_{t-1} < 0$.

Model 4: GJR-GARCH with lagged implied volatility:

$$g_t = \omega + \alpha_1 r^2_{t-1} + \alpha_n r^2_{t-1} d_{t-1} + \delta_1 g_{t-1} + \eta\,\sigma^2_{imp,1,t-1} \quad (6)$$

In all four cases, Z_α is the quantile at the $100 \times \alpha\%$ level from the skewed Student density as given in Equation (2). Note that the second model, the RiskMetrics volatility specification, is enhanced by use of the skewed fat-tailed probability density.

Backtesting the VaR Models

To backtest the VaR results, we first use the Kupiec [1995] likelihood ratio test. Given the ex post date $t + 1$ observed returns $\{r_{t+1}\}$ and ex ante at date t forecasted $\{VaR_t\}$, the empirical failure rate $\hat{f}$ is given by the number of returns lower than the VaR. If the VaR model is correctly specified, this proportion must be equal to α in expected value.

More precisely, Kupiec [1995] shows that in the binomial framework the hypothesis H_0: $f = \alpha$ against H_1: $f \neq \alpha$ can be tested with the LR statistic:

$$LR = -2\ln\left(\alpha^{T-N}(1-\alpha)^N\right) + 2\ln\left([1-(N/T)]^{T-N}(N/T)^N\right)$$

where f is the theoretical failure rate, T is the total number of observations, and N is the number of VaR violations. Under the null hypothesis that $f = \alpha$ is the true failure rate, the LR test statistic is asymptotically distributed as $\chi^2(1)$.

We also use the independence and conditional coverage tests suggested by Christoffersen [1998]. The Kupiec

[1995] LR test assesses only the equality between the proportion of VaR violations and the expected α level (referred to as the unconditional coverage in Christoffersen [1998]). In a risk management framework, it is of paramount importance that VaR violations be uncorrelated over time, which prompts independence and conditional coverage tests based on the evaluation of interval forecasts.

Using the same notation as Christoffersen [1998], we define the indicator sequence of VaR violations as $\{I_t\}$, where I_t is a dummy variable equal to 1 if there is a VaR violation at time t (i.e., r_t is lower than VaR_t) and 0 if there is no VaR violation at time t. If $\pi_{i,j}$ is the transition probability for two successive I_t dummy variables, i.e., $\pi_{i,j} = P(I_t = j \mid I_{t-1} = i)$, then the likelihood function for the sequence of I_t is equal to:

$$L_1 = \pi_{0,0}^{n_{0,0}} \pi_{0,1}^{n_{0,1}} \pi_{1,0}^{n_{1,0}} \pi_{1,1}^{n_{1,1}} \tag{7}$$

where $n_{i,j}$ is the number of observations with value i followed by value j.

The maximized likelihood function is then equal to:

$$\widehat{L}_1 = \left[\frac{n_{0,0}}{n_{0,0}+n_{0,1}}\right]^{n_{0,0}} \left[\frac{n_{0,1}}{n_{0,0}+n_{0,1}}\right]^{n_{0,1}} \times \left[\frac{n_{1,0}}{n_{1,0}+n_{1,1}}\right]^{n_{1,0}} \left[\frac{n_{1,1}}{n_{1,0}+n_{1,1}}\right]^{n_{1,1}} \tag{8}$$

If the $\{I_t\}$ sequence (the sequence of VaR violations) is (first-order) independent, then $\pi_{0,0} = 1 - \pi$, $\pi_{0,1} = \pi$, $\pi_{1,0} = 1 - \pi$, and $\pi_{1,1} = \pi$. This gives the likelihood under the null of first-order independence:

$$L_2 = (1-\pi)^{n_{0,0}+n_{1,0}} \pi^{n_{0,1}+n_{1,1}} \tag{9}$$

which is then estimated as:

$$\widehat{L}_2 = \left[1 - \frac{n_{0,1}+n_{1,1}}{n_{0,0}+n_{1,0}+n_{0,1}+n_{1,1}}\right]^{n_{0,0}+n_{1,0}} \times \left[\frac{n_{0,1}+n_{1,1}}{n_{0,0}+n_{1,0}+n_{0,1}+n_{1,1}}\right]^{n_{0,1}+n_{1,1}} \tag{10}$$

The LR test statistic for (first-order) independence in the VaR violations is:

$$LR_{ind} = -2(ln(\widehat{L}_2) - ln(\widehat{L}_1)) \sim \chi^2(1) \tag{11}$$

If we condition on the first observation in the test for unconditional coverage, the LR statistic for conditional coverage (i.e., the joint hypothesis of unconditional coverage and independence) is:

$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2) \tag{12}$$

where LR_{uc} is the LR statistic for unconditional coverage (computed above for the Kupiec test). See additional details and proofs in Christoffersen [1998].

III. EMPIRICAL RESULTS

We use the VaR methodology and the tests as detailed above to assess the VaR forecasts for the S&P 100 and Nasdaq 100 indexes. Exhibit 1 provides descriptive information for the full period and three subperiods. Subperiod 1 (8/1/1994-5/30/1997, for the S&P 100 index) represents a bull market with rather low volatility. Subperiod 2 (6/2/1997-3/31/2000) was also a favorable time period for market participants with long positions (the mean daily return μ_d in subperiod 1 is not statistically different from the mean daily return in subperiod 2, but exhibited significantly greater volatility than in subperiod 1). Finally, subperiod 3 (4/3/2000-3/31/2003) represents a clear bear market with high volatility (in this case μ_d is statistically different from μ_d in subperiods 1 and 2, and the volatility is also significantly

EXHIBIT 1
Descriptive Data

	S&P 100 Index				VXO Index		
	Start	End	μ_d	σ_a	Start	End	Mean
Aug. 1, 1994 - Jan. 31, 2003	213.93	432.57	0.033%	14.37%	10.27	35.78	23.33
Aug. 1, 1994 - May 30, 1997	213.93	413.35	0.093%	8.56%	10.27	22.12	15.84
Jun. 2, 1997 - Mar. 31, 2000	413.47	815.06	0.095%	15.49%	22.12	27.21	25.56
Apr. 3, 2000 - Jan. 31, 2003	820.62	432.57	-0.089%	19.09%	25.66	35.78	28.63
	Nasdaq 100 Index				VXN Index		
	Start	End	μ_d	σ_a	Start	End	Mean
Jun. 2, 1997 - Jan. 31, 2003	958.69	983.05	0.0018%	33.25%	33.24	46.81	48.04
Jun. 2, 1997 - Mar. 31, 2000	958.69	4397.84	0.21%	26.24%	33.24	61.56	38.98
Apr. 3, 2000 - Jan. 31, 2003	4077.02	983.05	-0.21%	40.31%	64.5	46.81	57.17

μ_d: Mean daily return. σ_a: mean annualized return volatility.

EXHIBIT 2

VaR Results for S&P 100 Index (8/1/1994-1/31/2003)

	Lagged implied volatility (VXO)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
$\hat{f}$	**11.54**	5.37	1.17	10.84	**6.26**	1.03
I	0.59	0.32	-	0.51	0.82	-
CC	0.06	0.45	-	0.39	0.03	-
	RiskMetrics					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
$\hat{f}$	11.30	**6.17**	**1.54**	10.93	**6.26**	1.03
I	0.33	0.31	0.10	0.44	0.88	-
CC	0.09	0.03	0.02	0.30	0.04	-
	GJR-GARCH(1,1)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
$\hat{f}$	**11.86**	5.65	1.21	**11.63**	5.74	**0.56**
I	0.56	0.73	-	0.53	0.19	-
CC	0.02	0.37	-	0.04	0.13	-
	GJR-GARCH(1,1) + lagged implied volatility (VXO)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
$\hat{f}$	**11.44**	5.93	1.12	10.70	5.70	0.75
I	0.12	0.54	-	0.42	0.68	-
CC	0.03	0.13	-	0.44	0.32	-

Empirical failure rates for left quantile (LQ) and right quantile (RQ) at 10%, 5%, and 1%. Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

higher). Similar results are observed for the Nasdaq 100 index, for which we consider only two subperiods.

We construct one-day-ahead out-of-sample volatility forecasts for the four models using rolling estimations of the volatility and density specifications for the VaR models according to Equations (3), (4), (5), and (6). All models are evaluated in a purely out-of-sample framework.

In this setting, some initial returns need to be put aside to estimate the first set of parameters. For the Nasdaq 100 index, this implies that the full period is shorter than the corresponding full period for the S&P 100 index, as the first two years are used to get the first estimates of the parameters. Therefore the full backtesting period for the Nasdaq 100 index is 6/2/1997–1/31/2003, and we have two distinct subperiods: 6/2/1997–3/31/2000 and 4/3/2000–1/31/2003. For the S&P 100 index, the VXO data start on January 2, 1986, so that we have the three subperiods noted.

All models are reestimated on a weekly basis as the analysis is rolled forward. At the end of this procedure, we have one-day-ahead out-of-sample VaR forecasts for 8/1/1994-1/31/2003 (S&P 100 index) and 6/2/1997-1/31/2003 (Nasdaq 100 index). The VaR forecasts $\{VaR_t\}$, pertaining to the returns defined on $[t, t+1]$, can then be backtested against the observed returns $\{r_t + 1\}$.[6]

We first compute the empirical failure rates and Kupiec LR tests for the right and left quantiles at 10%, 5%, and 1%. The left quantile is relevant for a trader who is long the index (trading losses occur on the left side of the returns density), while the right quantile is relevant for a trader who is short the index (trading losses occur on the right side of the returns density). Second, we compute the independence and conditional coverage tests, using Equations (11) and (12).

Exhibit 2 provides the empirical results for the four models (S&P 100 index, full period). Exhibits 3 and 4 provide S&P 100 index results for the three subperiods. Exhibits 5 and 6 report Nasdaq 100 index full period and subperiod results.

The failure rates and Kupiec LR tests indicate relatively few differences among the models, although the GJR-GARCH model with lagged implied volatility seems to exhibit the best performance, while the performance of the GJR-GARCH and RiskMetrics models is somewhat disappointing. It is worth noting that the basic implied volatility only model is no worse than the more sophisticated GARCH models.

Bear in mind that the implied volatility and RiskMetrics models are used in conjunction with the skewed Student density. Cruder versions of both models estimated with the normal density (i.e., the standard RiskMetrics model) yield very poor results. There are no real differences with regard to the subperiods.

Note in model 4 results that coefficients α_n and η are strongly significant, while coefficient δ_1 drops considerably from the value it took (close to 1) when the lagged implied volatility was not included. To illustrate, we report in Exhibit 7 some selected coefficients for models 1 and 4. To save space, we report only mean coefficients (averaged across all estimations when the rolling estimations are performed) and their standard deviations for the probability density, and selected parameters of the volatility specification. These results show that the fitted probability density is always fat-tailed (υ is rather low) and in most cases negatively skewed ($\ln(\xi)$ is often significantly negative).

Note that the implied volatility coefficient η is strongly significant in both types of models. As in Day and Lewis [1992] and Blair, Poon, and Taylor [2001], this shows that implied volatility does provide important incremental information with respect to past squared returns (the GARCH effects are weaker when lagged implied volatility is included). Note, however, that α_n, the coefficient on lagged squared returns when these returns are negative, is still very significant. Thus, while lagged implied

EXHIBIT 3

VaR Results for S&P 100 Index (8/1/1994-5/30/1997)

	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
Lagged implied volatility (VXO)						
$\hat{f}$	9.36	4.89	1.26	12.15	6.42	0.70
I	0.90	0.82	-	0.39	0.98	-
CC	0.84	0.97	-	0.15	0.24	-
RiskMetrics						
$\hat{f}$	10.20	6.42	**1.96**	**13.41**	**7.82**	1.26
I	0.54	0.54	0.03	0.31	0.42	-
CC	0.81	0.20	0.01	0.01	0	-
GJR-GARCH(1,1)						
$\hat{f}$	9.08	5.45	1.40	**12.99**	6.56	0.42
I	0.36	0.93	-	0.97	0.96	-
CC	0.47	0.86	-	0.04	0.18	-
GJR-GARCH(1,1) + lagged implied volatility (VXO)						
$\hat{f}$	8.52	5.03	1.12	11.60	5.87	0.70
I	0.26	0.89	-	0.59	0.74	-
CC	0.21	0.99	-	0.01	0.55	-

Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

volatility does provide important information, some additional volatility information can be incorporated in the model via the lagged (negative-only) squared returns.

From an economic point of view, there are no key differences among the models. This lends credence to the assertion that (perhaps surprisingly) sophisticated volatility models may not really be needed in VaR applications. This is the conclusion of Berkowitz and O'Brien [2002], who show that simple GARCH univariate models can adequately model the VaR of a global financial institution. In the same vein, Giot and Laurent [2004] show that the choice of the proper density is of paramount importance, and that using sophisticated volatility measures computed from intradaily data (realized volatility) does not enhance out-of-sample VaR forecasts.

The P-values for the independence test (rows labeled I in the exhibits) show that the null hypothesis of first-order independence is almost never rejected. This is important, as it means there are no statistically significant successive VaR violations. In other words, adapting the VaR level according to the rules given above ensures that repeated failures can be avoided. Note that when $\alpha = 1\%$ many times the test cannot be used, because $n_{1,1} = 0$; i.e., there are no successive VaR violations. These latter cases should of course be viewed favorably, even if we cannot report a test statistic.

The P-values for the conditional coverage test (rows

EXHIBIT 4

VaR Results for S&P 100 Index (6/2/1997-3/31/2000 and 4/3/2000-1/31/2003)

	6/2/1997 - 3/31/2000						4/3/2000 - 1/31/2003					
	L=10%	L=5%	L=1%	R=10%	R=5%	R=1%	L=10%	L=5%	L=1%	R=10%	R=5%	R=1%
Lagged implied volatility (VXO)												
$\hat{f}$	11.75	**6.99**	1.15	11.75	6.43	1.40	**13.66**	4.51	0.85	8.73	5.77	1.27
I	0.14	-	-	0.75	0.53	-	0.93	0.23	-	0.79	0.30	-
CC	0.10	-	-	0.29	0.19	-	0.01	0.41	-	0.51	0.38	-
RiskMetrics												
$\hat{f}$	10.49	6.57	1.68	10.63	6.15	0.70	**13.24**	5.77	0.85	8.73	4.79	1.27
I	0.96	0.14	-	0.57	0.21	-	0.42	0.03	-	0.84	0.58	-
CC	0.90	0.06	-	0.85	0.18	-	0.02	0.06	-	0.51	0.83	-
GJR-GARCH(1,1)												
$\hat{f}$	11.19	6.29	1.68	**13.43**	6.29	0.56	**15.35**	5.21	0.70	8.31	4.37	0.70
I	0.46	0.19	-	0.73	0.19	-	0.42	0.45	-	0.96	-	-
CC	0.43	0.13	-	0.01	0.13	-	0	0.72	-	0.31	0.59	-
GJR-GARCH(1,1) + lagged implied volatility (VXO)												
$\hat{f}$	10.63	6.57	1.40	11.33	5.87	0.70	**15.21**	6.48	0.85	8.87	5.21	0.85
I	0.08	0.48	-	0.77	0.73	-	0.47	0.52	-	0.53	0.45	-
CC	0.18	0.14	-	0.48	0.54	-	0	0.18	-	0.49	0.72	-

Empirical failure rates for left quantile (LQ) and right quantile (RQ) at 10%, 5%, and 1%. Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

Exhibit 5

VaR Results for Nasdaq 100 Index
(6/2/1997-1/31/2003)

	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
Lagged implied volatility (VXN)						
$\hat{f}$	**12.69**	5.05	0.77	10.03	5.47	1.26
I	0.14	0.33	-	0.63	0.20	-
CC	0	0.62	-	0.89	0.31	-
RiskMetrics						
$\hat{f}$	**13.11**	6.03	0.63	10.24	5.40	0.91
I	0.74	0.56	0.10	0.78	0.67	-
CC	0	0.19	0.02	0.92	0.72	-
GJR-GARCH(1,1)						
$\hat{f}$	**13.46**	6.10	0.91	**11.92**	**7.01**	**1.61**
I	0.17	0.75	-	0.40	0.25	-
CC	0	0.17	-	0.04	0	-
GJR-GARCH(1,1) + lagged implied volatility (VXN)						
$\hat{f}$	**12.48**	4.98	0.63	9.75	5.75	1.12
I	0.04	0.35	-	0.47	0.29	-
CC	0	0.65	-	0.74	0.26	-

Empirical failure rates for left quantile (LQ) and right quantile (RQ) at 10%, 5%, and 1%. Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

labeled CC in the exhibits) show that the null hypothesis of conditional coverage is usually not rejected. This indicates that model performance is quite stable over time and does not deteriorate in turbulent markets.

For example, all four models perform well (except at the 10% level, left tail) in the third subperiod (4/3/2000-1/31/2003), although these were not easy times for market participants. While the failure rate tests are mandatory under the Basel Committee requirements, we thus also have the interesting result that VaR violations are not correlated.

IV. NEW VIX

The CBOE in 2003 introduced a new VIX index, whose underlying stock index is now the S&P 500 index. While the basic idea of the VIX has not changed, as this new implied volatility index still delivers volatility forecasts at the typical 30 calendar-day (or 22 trading-day) horizon, a few important changes must be noted.[7]

First, the underlying stock index is no longer the S&P 100 index but rather the S&P 500 index. This change addresses investors' concern that the S&P 500 index is the standard U.S. benchmark; hence it became inappro-

Exhibit 6

VaR Results for Nasdaq 100 Index (6/2/1997-3/31/2000 and 4/3/2000-1/31/2003)

	6/2/1997 - 3/31/2000						4/3/2000 - 1/31/2003					
	L=10%	L=5%	L=1%	R=10%	R=5%	R=1%	L=10%	L=5%	L=1%	R=10%	R=5%	R=1%
Lagged implied volatility (VXN)												
$\hat{f}$	11.45	4.19	1.12	10.75	5.73	1.12	**14.08**	5.92	**0.28**	9.44	5.35	1.27
I	0.36	-	-	0.91	0.67	-	0.34	0.76	-	0.48	0.19	-
CC	0.29	-	-	0.79	0.62	-	0	0.62	-	0.69	0.39	-
RiskMetrics												
$\hat{f}$	11.73	5.73	0.84	11.31	6.15	0.70	**14.51**	6.20	0.42	9.01	4.65	1.13
I	0.96	0.67	-	0.40	0.63	-	0.72	0.22	-	0.59	0.27	-
CC	0.31	0.62	-	0.36	0.35	-	0	0.21	-	0.59	0.49	-
GJR-GARCH(1,1)												
$\hat{f}$	**13.55**	6.28	1.26	**13.83**	**8.24**	1.82	**13.38**	5.92	0.56	10.14	5.63	1.41
I	0.48	0.92	-	0.23	0.66	-	0.22	0.71	-	0.90	0.02	-
CC	0.01	0.31	-	0	0	-	0.01	0.60	-	0.98	0.06	-
GJR-GARCH(1,1) + lagged implied volatility (VXN)												
$\hat{f}$	11.03	4.33	0.84	10.61	6.01	0.84	**13.94**	6.06	**0.28**	9.58	5.35	1.41
I	0.28	-	-	0.98	0.69	-	0.23	0.71	-	0.53	0.19	-
CC	0.36	-	-	0.86	0.45	-	0	0.51	-	0.77	0.39	-

Empirical failure rates for left quantile (LQ) and right quantile (RQ) at 10%, 5%, and 1%. Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

EXHIBIT 7

Rolling Estimations: Selected Estimated Coefficients

	Lagged implied volatility—Model 1				
	υ	$ln(\xi)$	η		
OEX: 8/1/1994 - 1/31/2003	9.30 (4.29)	-0.07 (0.06)	0.50 (0.09)		
OEX: 8/1/1994 - 5/30/1997	5.40 (0.82)	0 (0.02)	0.42 (0.05)		
OEX: 6/2/1997 - 3/31/2000	8.20 (1.72)	-0.10 (0.05)	0.48 (0.04)		
OEX: 4/3/2000 - 1/31/2003	14.49 (3.74)	-0.09 (0.04)	0.60 (0.04)		
SPX: 1/3/2000 - 12/31/2004	11.49 (3.86)	-0.07 (0.04)	0.60 (0.04)		
	GJR-GARCH(1,1) + Lagged implied volatility—Model 4				
	υ	$ln(\xi)$	η	δ_1	α_n
OEX: 8/1/1994 - 1/31/2003	5.67 (0.62)	-0.03 (0.03)	0.27 (0.16)	0.47 (0.27)	0.09 (0.06)
OEX: 8/1/1994 - 5/30/1997	4.98 (0.06)	-0.01 (0.01)	0.47 (0.01)	0.13 (0.01)	0.03 (0.01)
OEX: 6/2/1997 - 3/31/2000	6.08 (0.05)	-0.02 (0.01)	0.21 (0.03)	0.56 (0.05)	0.09 (0.02)
OEX: 4/3/2000 - 1/31/2003	7.90 (0.05)	-0.07 (0.01)	0.10 (0.02)	0.75 (0.04)	0.16 (0.01)
SPX: 1/3/2000 - 12/31/2004	8.72 (0.10)	-0.10 (0.03)	0.07 (0.02)	0.81 (0.04)	0.18 (0.02)

EXHIBIT 8

VaR Results for New VIX Index
(1/3/2000-12/31/2004)

	Lagged implied volatility (new VIX)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
f	**12.26**	5.25	0.80	10.11	5.89	1.11
I	0.12	0.79	-	0.36	0.21	-
CC	0.01	0.88	-	0.65	0.17	-
	RiskMetrics					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
f	**12.58**	5.57	0.72	9.87	5.89	1.19
I	0.41	0.13	-	0.46	0.75	-
CC	0.01	0.21	-	0.76	0.35	-
	GJR-GARCH(1,1)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
f	11.23	4.46	0.64	10.03	4.86	0.96
I	0.53	0.72	-	0.84	0.53	-
CC	0.33	0.56	-	0.98	0.80	-
	GJR-GARCH(1,1) + lagged implied volatility (new VIX)					
	LQ=10%	LQ=5%	LQ=1%	RQ=10%	RQ=5%	RQ=1%
f	**12.26**	5.10	0.64	10.27	5.89	0.88
I	0.81	0.88	-	0.48	0.75	-
CC	0.03	0.98	-	0.74	0.35	-

Boldface indicates empirical failure rate is significantly different (LR test or unconditional coverage) from the theoretical value. I and CC give P-values for the independence and conditional coverage tests, respectively.

priate to anchor the VIX with respect to the less actively tracked S&P 100 index (even though the S&P 100 index and the S&P 500 index are strongly correlated).

The second change is that the new VIX is computed according to a formula that no longer relies on an option pricing model (it is thus model-free) but is based on options over a wide range of strike prices. The new calculations are fully explained in the CBOE technical document, "The New CBOE Volatility Index—VIX," available at the CBOE website. The original VIX was based on at-the-money options. Because the new VIX also takes into account information from out-of-the money options, the volatility forecast should better reflect the information content of the options markets.

The CBOE has conveniently recomputed the new VIX using historical data, so data are freely available over an extended period.

Is the new VIX a meaningful input in the VaR framework we have detailed? While we leave to future researchers the assessment of the quality of the new VIX volatility forecasts, a simple regression of the 22-day realized volatility (RV) against the new VIX with non-overlapping data over 1990-2003 reveals that the joint hypothesis H0: $\alpha = 0$ and $\beta = 1$, where $RV_k = \alpha + \beta VIX_k + \delta RV_{k-1} + \varepsilon_k$, is not rejected (using a Wald test). This confirms the empirical findings presented in the CBOE technical document on the new VIX measure.

Besides, because we use a linear function of the implied volatility index (either the VXO, VXN, or VIX) with *estimated* coefficients in the VaR models, the scale change in the VIX should not have a meaningful impact on the VaR performance of the models. Indeed, the method is flexible as, along with the linear function of the implied volatility index, the skewness and kurtosis parameters are estimated to improve performance in the tails. This flexibility, with the fact that the VIX seems to forecast the expected volatility quite well, should lead to adequate VaR modeling.

Exhibit 8 presents the VaR results for our four models when the dataset is the S&P 500 stock index returns and the new VIX measure is the implied volatility input. These are rolling estimations (with a fixed five-year window), and the out-of-sample VaR performance is assessed over 2000-2003.

The empirical results and estimated coefficients are similar to those we have discussed for the VXO and VXN indexes. The empirically observed failure rates are (in all cases but one) not statistically different from expected, and the VaR violations are independent.

V. CONCLUSION

Our study has dealt with the information content of the VXO, VXN, and VIX implied volatility indexes when these are used in very different markets in a daily value at risk model. We also estimate the models with the new VIX index. The objective is to test model performance in challenging trading environments and to look at model stability through time. To assess the performance of the VaR estimates, we use likelihood ratio, independence, and conditional coverage tests.

The statistical tests show that implied volatility indexes provide meaningful volatility information in VaR models, as the number of VaR violations is not significantly different from the target value in most cases, and the null hypotheses of independence and conditional coverage are usually not rejected. We see that performance does not deteriorate under challenging market conditions, and is stable through time. This analysis broadly supports the hypothesis that basic market-based volatility forecasts combined with a flexible probability density are adequate inputs to risk models.

ENDNOTES

The author is grateful to Carol Alexander, Luc Bauwens, Peter Christoffersen, Sébastien Laurent, an anonymous referee, and seminar participants at CORE, Université Catholique de Louvain, the ISMA Center at the University of Reading, and the University of Namur who offered helpful comments.

[1]Note that by construction these implied volatility indexes take into account early exercise and dividend payment features. They do not use as inputs market prices from options that are not actively traded (thus avoiding the problem of stale quotes for deep out-of-the-money or in-the-money options).

[2]Parameters must be squared, because the VXO/VXN are expressed in standard deviation units; i.e., they are not expressed in terms of variances. Our VaR model uses variances as inputs.

[3]The linear relationship is also suggested by Lamoureux and Lastrapes [1993], in the absence of the correct option pricing model (which we clearly do not have here). I'd like to thank the referee for pointing that out.

[4]In practice, daily returns can exhibit some weak autocorrelation. This usually involves fitting an AR, MA, or ARMA structure on the r_t prior to the VaR analysis. We assume the r_t are the demeaned and uncorrelated returns (or residuals from an ARMA analysis).

[5]Combining a GARCH volatility specification, Giot and Laurent [2003] show that the VaR model provides excellent out-of-sample performance.

[6]Rolling estimations mean that we drop the oldest data points when the model is reestimated. The lengths of the rolling windows are three years for models 1 and 2 and eight years for models 3 and 4. For models 3 and 4, a longer rolling window is needed to ensure convergence when the GARCH models are estimated in GAUSS (a maximum-likelihood framework).

[7]The CBOE renamed the original VIX index the VXO index and kept the VIX name for the new VIX. The VXN index construction was also changed, yielding an original VXN index (which we use because of data availability) and a current VXN index. Readers should exercise caution, as studies of the VIX made prior to 2003 should now be interpreted as studies of the VXO index.

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