A New Procedure for Pricing Parisian Options

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Parisian options extend barrier options in that their covenant depends on the time the underlying spends beyond a given threshold. By their nature, they are hard to price and hedge, although some quite involved methods are available.

Parisian options can be valued by four different methods: Monte Carlo simulations, lattices, partial differential equations, and inverse Laplace transforms. We develop a new inverse Laplace transform method that is quick and appropriate to the problem. Our method can also be used to treat other financial problems that require inversion of a Laplace transform.

Parisian options, the brainchild of Chesney, Jeanblanc, and Yor [1997], are extensions of barrier options. A barrier option is activated or deactivated as soon as a threshold has been reached, but activation of a Parisian option depends on the time spent above or below the barrier. A down-and-in call, for example, is activated when the underlying spends more than a specified time strictly below the threshold.

If the activation depends only on the total time spent beyond the barrier, the option is known as a *cumulative Parisian option*. These options can be priced more easily than standard Parisian options; Hugonnier [1999] develops a formula that can be evaluated by means of quadratures.

Parisian options can be *calls* or *puts*; they can be *up* or *down*; and they can also be *in* or *out*. When the surveillance period, or window, is of null length, this is a standard barrier option. Conversely, when the window is longer than the maturity, we have either a standard option or a product with zero value.

Several equalities and inequalities can be established. For instance, a standard down-and out-call is cheaper than a Parisian down-and out-call, which itself is cheaper than a standard call—all common parameters equal. As with barrier options, the simple result is that the sum of an *in* Parisian option and an *out* Parisian option is a standard option.

Parisian options can be valued by simulation. In this case, there is a bias linked to the choice of discretization in the Monte Carlo algorithm. Andersen and Brotherton-Ratcliffe [1996] discuss the treatment of biases in exotic option pricing. Valuation can also use lattices; see for this method Avellaneda and Wu [1999] and Costabile [2002]. Haber, Schönbucher, and Wilmott [1999] use a partial differential equation approach to price these options, as do Stokes and Zhu [1999], who develop a finite-element method.

All these approaches share one main drawback: They are slow to yield precise results. Another approach that allows for the quick pricing of Parisian options is based on a Laplace inversion of the formula given by Chesney, Jeanblanc, and Yor [1997]. The first numerical study in this direction is Chesney, Cornwall, Jeanblanc, Kentwell, and Yor [1997]. Hartley [2000] uses the Abate and Whitt [1995] method to compute Parisian option values by inverse Laplace transform. We provide a new and very simple method to compute an inverse Laplace transform, and we apply this method to pricing Parisian options and deriving their Greeks.

A simple and easy to implement method should be of general interest to derivatives users. Our approach can be used any time the desired price or Greek is known through its Laplace transform or when this Laplace transform is a real (not complex) function.

Examples of such products are Asian options. Geman and Yor [1993] give a formula for the Laplace transform of an Asian option price. Each time a highly path-dependent derivative depends on the movement of the underlying, the Laplace transform of a complicated Brownian time or of a Brownian function appears—and inversion of this Laplace transform is required to obtain the price and sensitivities of the product.

Chesney, Geman, Jeanblanc, and Yor [1997] develop the mathematical tools to help price Asian, Parisian, and related options. Our method is applicable in this case as long as the function to be inverted is real, as is the case with Asian options, but our method would not work when the function to be inverted is complex (as often happens when dealing with the optimal structure of the firm; see, for instance, Hilberink and Rogers [2002], who work with Lévy processes).

Parisian options were created to reduce the incentive for market manipulation that arises with barrier options. While they are interesting as market instruments, Parisian options also serve as tools in modeling a variety of diverse situations. De Giuli, Maggi, and Paris [2003], and Bernard, Le Courtois, and Quittard-Pinon [2005] apply the Parisian methodology to value bank deposit guarantees; bank deposit insurance, when banks join in a consortium (i.e., a coinsurance vehicle), is typically an exotic option of the Parisian type. As soon as monitoring is involved, Parisian covenants arise naturally.

Real options and contingent claims-based default theory are further illustrations. Gauthier [2002] was able to solve certain real option problems by resorting to Parisian options. He uses a case study of project valuation when there may be a delay between an investment decision and its implementation. The implementation time is in this context conditional on the Parisian monitoring of the decision variable (e.g., eurodollar exchange rate, oil price) by managers. François and Morellec [2004] use Parisian times to model some U.S. bankruptcy Chapter 11 provisions and to build an ad hoc default framework.

It thus appears Parisian options are both intrinsically interesting and also useful instruments in financial markets and corporate finance.

I. PARISIAN OPTIONS

We briefly review general analytical results that are useful in obtaining Parisian option prices in terms of inverse Laplace transforms. We present the formulas related to a down-and-in call and to a down-and-out put option. We give the Greeks of down-and-in call options, with a formula for the gamma that we believe is new to the literature.

General Analytics

We adopt the standard context of a perfect and complete market. Q is the risk-neutral measure, r the instantaneous risk-free rate, and S the underlying stock modeled by the diffusion:

$$\frac{dS_t}{S_t} = (r-q)dt + \sigma dz_t$$

where q is the continuous dividend rate, σ is the volatility, and z is a Q-Brownian motion. Upon integration, one obtains:

$$S_t = xe^{\sigma(mt+z_t)}$$

where

$$m = \frac{1}{\sigma}\left(r - q - \frac{\sigma^2}{2}\right)$$

and

$$x = S_0$$

To specify a Parisian option, one needs to define some additional variables. Let T be the maturity of the option, and K its strike price. L is the barrier level, and D the activation or deactivation window. The product is activated or deactivated if the underlying spends more than D beyond the threshold L.

In our applications, we mainly study down Parisian options (this is not restrictive); we observe the time spent *below* the barrier. To do so, we need the functional $g_t^L(S)$, which is the last time before t that the process S reaches the value L. In other words:

$$g_t^L(S) = \sup\{s \leq t | S_s = L\}$$

We then define $G^-_{D,L}(S)$, the first time the underlying S remains more than D below the barrier L, by:

$$G^-_{D,L}(S) = \inf\{t > 0 \mid (t - g^L_t(S))1_{S_t \leq L} \geq D\}$$

Exhibit 1 illustrates a sample excursion path according to these definitions. We now have the the main definitions useful for the description of Parisian options.

Down-and-In Parisian Call Option Formula

By standard arbitrage arguments, the price of a down-and-in Parisian call option can be expressed as:

$$C^d_i = e^{-rT} E_Q \left[\left(xe^{\sigma(mT+z_T)} - K \right)^+ 1_{G^-_{D,L}(S)<T} \right]$$

Let $\beta(x) = \frac{1}{\sigma} \ln \frac{K}{x}$ and $b = \frac{1}{\sigma} \ln \frac{L}{x}$. Chesney, Jeanblanc, and Yor [1997] show that:

If K > L:

$$C^d_i = e^{-(r+\frac{1}{2}m^2)T} \int_{\beta(x)}^{+\infty} e^{my} \, (xe^{\sigma y} - K)\, h_1(T, y) dy$$

And if $K \leq L$:

$$C^d_i = e^{-(r+\frac{1}{2}m^2)T} \left(\int_{\beta(x)}^{b} e^{my} \, (xe^{\sigma y} - K)\, h_2(T, y) dy + \int_{b}^{+\infty} e^{my} \, (xe^{\sigma y} - K)\, h_1(T, y) dy \right) \quad (1)$$

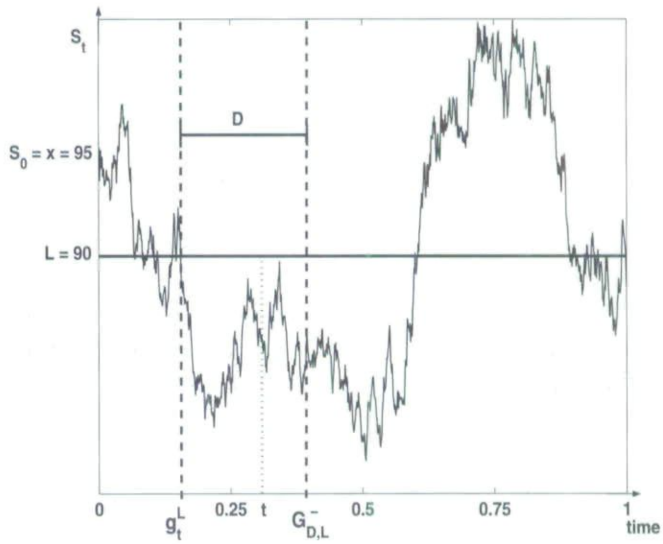
provided x is higher than L.

To compute these prices, one needs to know $h_1(T, y)$ and $h_2(T, y)$. The difficulty here is that it is impossible to obtain a closed-form expression either for h_1 or for h_2; both functions display quite complicated forms. For the sake of example, we can recall the expression of h_1:

$$h_1(t, y) = \int_{-\infty}^{\infty} E_{\bar{Q}} \left[1_{G^-_{D,L} \leq T} \frac{\exp\left(-\frac{(y-s)^2}{2(t - G^-_{D,L})} \right)}{\sqrt{2\pi(t - G^-_{D,L})}} \right] \phi(ds)$$

where $\bar{Q}$ is defined on $\mathcal{F}_T$ by the Radon-Nikodym density:

EXHIBIT 1
Excursion Below *L*



$$\frac{d\bar{Q}}{dQ} = \exp\left(-mz_T - \frac{1}{2} m^2 T \right)$$

and ϕ is the law of $W(G^-_{D,L})$, with $W(t) = z_t + mt$ a $\bar{Q}$-Brownian motion.

The two functions $\hat{h}_1(T, y)$ and $\hat{h}_2(T, y)$ are both described through Laplace transforms in T that we denote by $\hat{h}_1$ and $\hat{h}_2$. One shows that:

$$\widehat{h}_1(\lambda, y) = \frac{e^{(2b-y)\sqrt{2\lambda}} \, \Psi(-\sqrt{2\lambda D})}{\sqrt{2\lambda} \, \Psi(\sqrt{2\lambda D})}$$

and

$$\widehat{h}_2(\lambda, y) = \frac{e^{y\sqrt{2\lambda}}}{\sqrt{2\lambda}\Psi(\sqrt{2\lambda D})} + \frac{\sqrt{2\pi D} e^{\lambda D}}{\Psi(\sqrt{2\lambda D})} \times \left[e^{y\sqrt{2\lambda}} \left(\mathcal{N}\left(-\sqrt{2\lambda D} - \frac{y-b}{\sqrt{D}} \right) - \mathcal{N}\left(-\sqrt{2\lambda D} \right) \right) - e^{(2b-y)\sqrt{2\lambda}} \mathcal{N}\left(-\sqrt{2\lambda D} + \frac{y-b}{\sqrt{D}} \right) \right]$$

where $\Psi(z) = 1 + z\sqrt{2\pi} e^{\frac{z^2}{2}} \mathcal{N}(z)$, and $\mathcal{N}$ is the Gaussian cumulative distribution function.

From the pricing formula of a Parisian call such as in Equation (1), we see that computations require two steps. First, one has to invert the Laplace transforms $\hat{h}_1$ and $\hat{h}_2$ with respect to their first argument to obtain h_1 and h_2; second, one should perform a numerical integration. The first step is the crucial one. To Laplace-invert a function is not typically easy, but we can provide a new method accomplishing this first step with less difficulty.

Down-and-Out Put Option Formula

Down-and-in Parisian put options are theoretically priced by:

$$P_i^d = e^{-rT} E_Q\left[\left(K - xe^{\sigma(mT+z_T)}\right)^+ 1_{G_{D,L}^-(S)<T}\right]$$

This formula can be developed straightforwardly using similar arguments as for down-and-in calls. If $K > L$:

$$P_i^d = e^{-(r+\frac{1}{2}m^2)T}\left(\int_{-\infty}^{b} e^{my}(K - xe^{\sigma y})h_2(T,y)dy + \int_{b}^{\beta(x)} e^{my}(K - xe^{\sigma y})h_1(T,y)dy\right)$$

and if $K \leq L$:

$$P_i^d = e^{-(r+\frac{1}{2}m^2)T}\int_{-\infty}^{\beta(x)} e^{my}(K - xe^{\sigma y})h_2(T,y)dy$$

when the initial asset value x is higher than L.

Thanks to a parity relationship, one can also price down-and-out Parisian puts:

$$P_o^d = P(x,T) - P_i^d$$

where $P(x, T)$ is the Black and Scholes price of a standard put option.

Parisian Digital Options

In the pricing of Parisian digital options, we have a valuation formula. We use the same notation. $G_{D,L}^-(S)$ is the first time the underlying S remains more than D below the barrier L.

A Parisian digital option allows the holder of the option to receive at maturity a prespecified amount M, provided the underlying satisfies a standard Parisian condition. In other words, one has:

$$E_Q\left(Me^{-rT}1_{G_{D,L}^-(S)\leq T}\right) \quad \text{or} \quad E_Q\left(Me^{-rT}1_{G_{D,L}^-(S)\geq T}\right)$$

The first expression describes a down-and-in digital option, and the second one a down-and-out option. These formulas can be simplified:

$$Me^{-rT}Q\left(G_{D,L}^-(S) \leq T\right)$$

or:

$$Me^{-rT}Q\left(G_{D,L}^-(S) \geq T\right)$$

By computing these probabilities, we can derive Parisian digital option prices. Following a proof similar to one given in Chesney, Jeanblanc, and Yor [1997], we obtain when $S_0 \geq L$ and $b \geq 0$:

$$Q\left(G_{D,L}^-(S) \leq T\right) = e^{-\frac{m^2T}{2}}\left(\int_{-\infty}^{b} h_2(T,y)e^{my}dy + \int_{b}^{+\infty} h_1(T,y)e^{my}dy\right) \tag{2}$$

where $h_1(T, y)$ and $h_2(T, y)$ are both known through their Laplace transforms $\hat{h}_1$ and $\hat{h}_2$.

Down-and-in Parisian digital options are priced by solving Equation (2), which can be achieved, here again, by means of an inverse Laplace transform. Down-and-out, up-and-in, and up-and-out Parisian digital options can be valued similarly.

Delta of a Down-and-In Call

We want to compute the delta of a down-and-in Parisian call option, namely, $\Delta = \frac{\partial}{\partial x}\left(C_i^d\right)$. For this, we need to take the derivative of the down-and-in call price formula (1). The functions h_1 and h_2, are known only through their Laplace transforms, however; we cannot obtain straight away the delta of the option. By reversing the order of the integration and derivative operators, we can give a closed formula for the Laplace transform $\hat{\Delta}$ of the delta. Provided $x \geq L$ and $K > L$, one has:

$$\hat{\Delta}(\lambda) = \frac{-\Psi(-\sqrt{2\mu}D)}{\Psi(\sqrt{2\mu}D)} \times \left(\frac{K}{x}\right)^{\frac{m+\sigma-\sqrt{2\mu}}{\sigma}} \frac{e^{2b\sqrt{2\mu}}(m+\sqrt{2\mu})}{(m+\sigma-\sqrt{2\mu})(m-\sqrt{2\mu})\sqrt{2\mu}} \tag{3}$$

where μ is given by: $\mu = \lambda + (m^2/2) + r$.

If we apply a Laplace inversion method to $\hat{\Delta}$, the Δ

can be computed as a function of the maturity. In other words, Parisian Greeks can be computed the same way as Parisian prices, as long as we can derive formulas such as Equation (3).

Gamma of a Down-and-In Call

It is possible to take the derivative of $\hat{\Delta}$ to obtain the Laplace transform of the gamma with respect to maturity. In other words, it is possible to compute $\hat{\gamma}$:

$$\hat{\gamma}(\lambda) = \frac{\Psi(-\sqrt{2\mu D})}{\Psi(\sqrt{2\mu D})} \frac{e^{2b\sqrt{2\mu}}\,(m+\sigma+\sqrt{2\mu})\,(m+\sqrt{2\mu})}{(m+\sigma-\sqrt{2\mu})\,(m-\sqrt{2\mu})\,\sqrt{2\mu}} \times \frac{1}{\sigma x}\left(\frac{K}{x}\right)^{\frac{m+\sigma-\sqrt{2\mu}}{\sigma}} \quad (4)$$

Then, because we have a closed expression for $\hat{\gamma}$, we can compute $\hat{\gamma}$ by Laplace inversion. To tackle all main Parisian valuation and hedging problems, it is therefore necessary to compute numerical Laplace inverses.

Finally, note that the formulas for the Greeks require quite long calculations despite the simplicity of the method (proofs are available on request). The formula for the delta corrects an error in Chesney, Jeanblanc, and Yor [1997], while the formula for the gamma appears to be new.

II. APPROXIMATION METHOD

Knowledge of the inverse Laplace transforms h_1 and h_2 is crucial because performing quadratures on slightly modified versions of them yields Parisian prices and Greeks. Inverting a Laplace function exactly is in general not possible. A few functions have explicit inverses, but many do not.

The Laplace function λ^α has for an inverse $t^{-1-\alpha}\Gamma(\alpha + 1)$ when $\alpha > -1$. The exact inverses of $\exp(\alpha\lambda)$, $\alpha \in \mathbb{R}$, $\exp(\alpha\lambda^2)$, $\alpha < 0$, and of several trigonometric functions are also known. One can invert rational functions of negative degree, and some fractions written in terms of exponential or trigonometric functions (see Bateman et al. [1954]). Thanks to the linearity property of Laplace transforms, we are also able to invert linear combinations of them exactly, but functions like $\hat{h}_1$ and $\hat{h}_2$ do not have known explicit inverses.

Knowing that the plots of the Laplace transforms $\hat{h}_1(\lambda, y)$ and $\hat{h}_2(\lambda, y)$ along λ strongly resemble the plots of functions such as $1/\lambda^\kappa$ (for some $\kappa > 0$), we can derive a simple algorithm. Indeed, functions like $1/\lambda^\kappa$ admit known inverse Laplace transforms. By approximating $\hat{h}_1$ with a linear combination of such fractional functions, one can estimate h_1 as the inverse Laplace transform of the linear combination. Estimation of h_2 follows from the same method.

First, fit $\hat{h}_1(\lambda, y)$ and $\hat{h}_2(\lambda, y)$, for given values of y, by a linear combination of fractional functions such as:

$$\lambda \to \sum_{i=1}^{N} \alpha_i \frac{1}{\lambda^i}$$

In other words, we need to find the optimal parameters α_i that make the linear combination of fractional functions fit $\hat{h}_1$ and $\hat{h}_2$ as closely as possible (standard routines do this straightforwardly).

The inverse of this linear combination can be easily derived by a linearity argument and admits the expression:

$$t \to \sum_{i=1}^{N} \alpha_i \frac{t^{i-1}}{(i-1)!}$$

This last formula finally provides an approximation for h_1 and h_2 as functions of t. In the process of Parisian pricing, one needs to integrate these functions with respect to y; each time the quadrature calls for a value of h_1 or h_2, an inverse Laplace transform is required.

In our applications, we will rely on a slightly refined version of the algorithm. Using fractional powers, we approximate $\hat{h}_1(\lambda, y)$ and $\hat{h}_2(\lambda, y)$ by:

$$\lambda \to \sum_{i=1}^{N} \alpha_i \frac{1}{\lambda^{i/j}}$$

where j is set, for example, to equal 8.

This type of linear combination admits for an inverse:

$$t \to \sum_{i=1}^{N} \alpha_i \frac{t^{i/j-1}}{\Gamma(i/j)}$$

yielding a good approximation of the desired Laplace inverse function.

Exhibit 2 illustrates the method. The plain line is the function to be inverted ($\hat{h}_1(\lambda, 1)$, for instance) and the dotted line the approximating function. By increasing the number of terms in the latter, the two curves become indistinguishable—a good fit of the Laplace transform

EXHIBIT 2
Approximating Laplace Transform of Parisian DIC

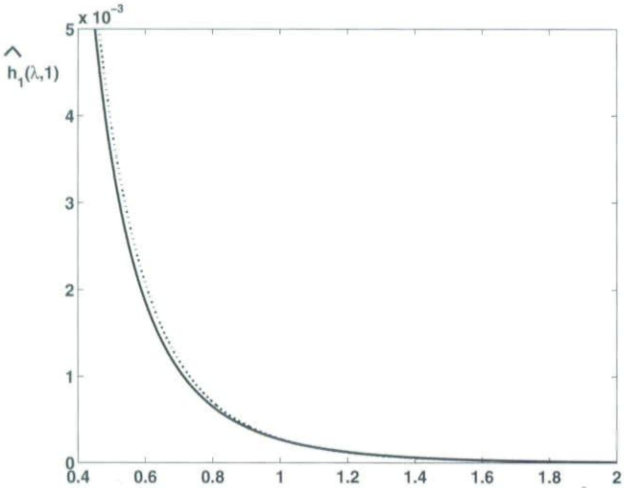


EXHIBIT 3
Parameters

K	L	σ	r	q	T
95	90	20%	5%	0	1 year

$\hat{h}_1(\lambda, 1)$, meaning a good estimation of the searched inverse Laplace transform $h_1(t, 1)$. The method works well and has been checked by means of Monte Carlo simulations.

Our method can be summed up as follows. We know the Laplace transforms $\hat{h}_1$ and $\hat{h}_1$ of h_1 and h_2. These Laplace transforms are fitted by linear combinations of simple fractional functions. In the process, a set of coefficients α_i are determined. Because the inverse Laplace transform of a simple fractional function is known, the inverse Laplace transforms h_1 and h_2 of $\hat{h}_1$ and $\hat{h}_1$ can be computed as linear combinations of fractional function Laplace inverses, with the α_i as the coefficients needed for the numerical implementation. To compute Parisian option prices, quadratures on simple functions of h_1 and h_2 then suffice.

III. NUMERICAL RESULTS

We now apply the numerical method to the valuation and hedging of Parisian options.

Parisian Option Valuation

The values for the parameters are shown in Exhibit 3. D and S_0 are not held fixed—we are going to study the dependence of Parisian prices with respect to them.

Exhibit 4 shows the Parisian down-and-in call (DIC) prices for S_0 varying between 90 and 100, and D ranging between one day and three months. Computations are extremely rapid and yield four- to five-digit accuracy in less than a tenth of a second on a standard computer with a simple Matlab program.

The degree of accuracy has been checked by means of simulations (where the discretization step needs to be set extremely small, implying a very long computation time). It has also been checked by comparing the results obtained using a more complex Laplace inverse method such as those of Abate and Whitt [1995] or Weeks [1966].

Exhibit 5 plots the dependence of Parisian DICs on S_0 and D. When the window D is narrow, the probability of activation is high, so the price is high. Similarly, the higher the initial value S_0 of the underlying, the lower the activation probability, and thus the lower the call price. The monotonicity apparent in the plot makes the analysis relatively easy (one can guess, for example, that deltas are going to be negative), but we will see later that this type of analysis is not always so simple.

EXHIBIT 4
Parisian Down-and-In Call Prices

$S_0 \backslash D$	1 day	1 week	2 weeks	1 month	3 months
90	5.7094	3.9888	3.0890	1.9786	0.5953
92	4.6966	3.2329	2.4776	1.5586	0.4458
94	3.8520	2.6123	1.9810	1.2237	0.3326
96	3.1502	2.1045	1.5792	0.9579	0.2473
98	2.5690	1.6906	1.2553	0.7476	0.1832
100	2.0893	1.3543	0.9951	0.5819	0.1353

EXHIBIT 5
Parisian Down-and-In Call Prices

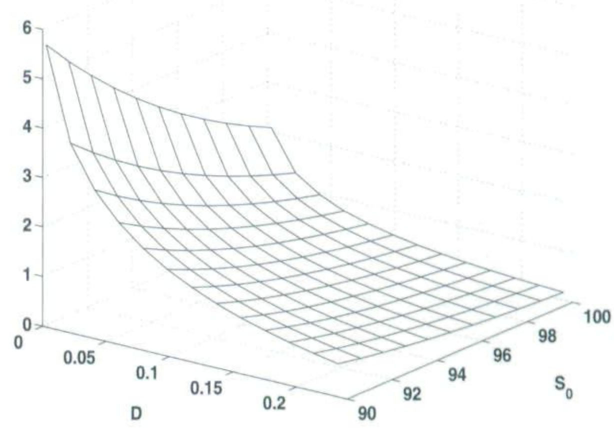


Exhibit 6
Parisian Down-and-Out Put Prices

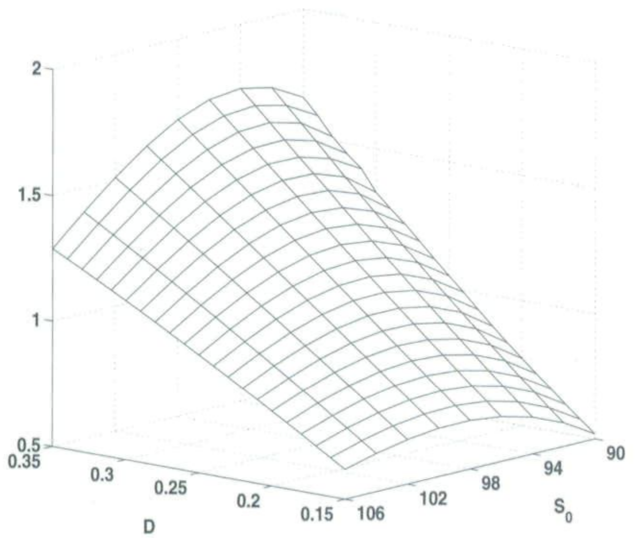


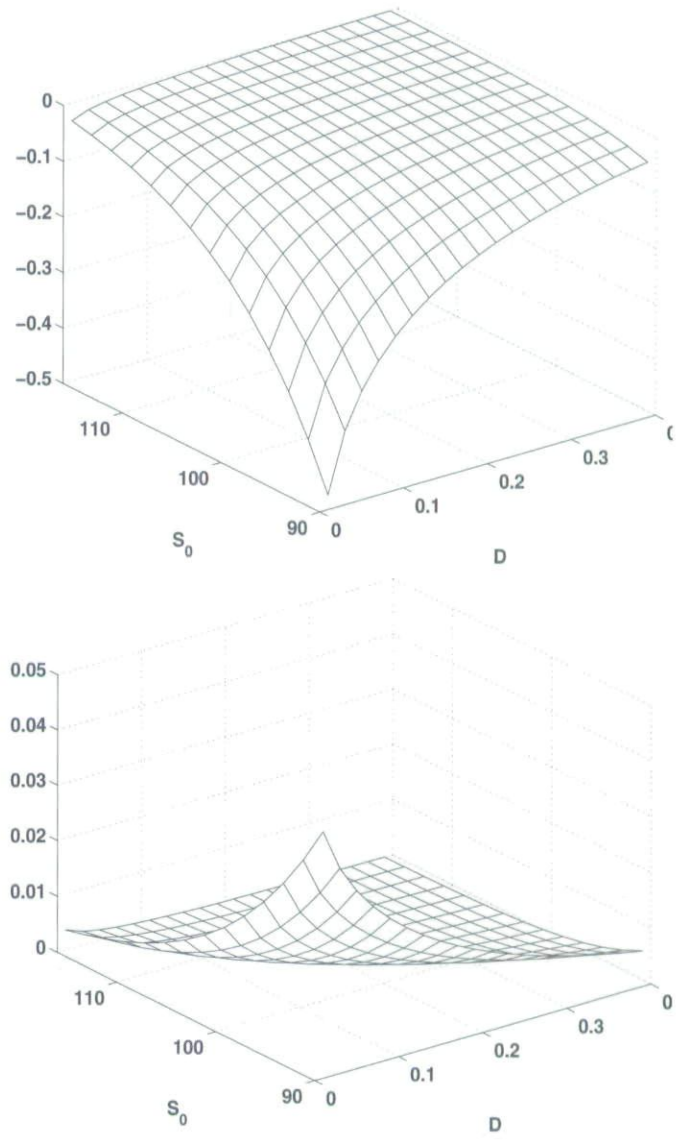
Exhibit 7
DIC Deltas

$S_0\backslash D$	1 day	1 week	2 weeks	1 month	3 months
90	-0.55309	-0.41592	-0.33818	-0.23448	-0.08554
92	-0.46207	-0.34215	-0.27518	-0.18724	-0.06490
94	-0.38465	-0.28040	-0.22306	-0.14893	-0.04901
96	-0.31906	-0.22896	-0.18013	-0.11801	-0.03686
98	-0.26375	-0.18629	-0.14495	-0.09317	-0.02761
100	-0.21729	-0.15106	-0.11624	-0.07331	-0.02060

Exhibit 8
DIC Gammas

$S_0\backslash D$	1 day	1 week	2 weeks	1 month	3 months
90	0.04922	0.04020	0.03452	0.02613	0.01171
92	0.04196	0.03372	0.02863	0.02125	0.00903
94	0.03561	0.02816	0.02363	0.01719	0.00693
96	0.03010	0.02341	0.01941	0.01384	0.00529
98	0.02533	0.01937	0.01588	0.01109	0.00402
100	0.02123	0.01596	0.01293	0.00885	0.00303

Exhibit 9
DIC Deltas and Gammas



Our next example concerns Parisian down-and-out put or DOP options. We graph the DOP prices in Exhibit 6 along S_0 and D. First, note that for given values of S_0 DOP prices increase with D. Indeed, when the window D is wide, the probability of deactivation is low, implying higher put prices.

The analysis is a bit more complicated when D is held fixed and we consider dependence with respect to S_0. In this case, one observes a maximum on the curve; when S_0 declines, the DOP price starts rising before dropping. When S_0 is high and starts to drop, the main driving factor explaining the increase in price is the nature of the option. Indeed, because this option is a put, its value increases with lower values of S_0.

Yet there is a value of S_0 at which the DOP does not increase any longer, but instead starts to drop. This is because for low values of S_0 the *out* covenant of the option predominates; the lower the S_0, the higher the deactivation probability, and thus the lower the put price.

Why are DIC prices monotonic in S_0 when DOP prices are not? In the first instance, the product needs to be activated before really coming into being; therefore the only important factor is the probability of activation, which is monotonic in S_0. In the second instance, the option is activated from the beginning. A trade-off between the put and the out characteristics helps explain the behavior of DOP prices, which are not monotonic in S_0.

This behavior is not common, and it can reasonably be argued that a down-and-out Parisian put is not the most natural instrument available to market investors. A standard non-Parisian down-and-out put with rebate would be a simpler and more obvious instrument. Yet Parisian down-and-out puts are useful as soon as the observation of a critical determinant (the assets of a firm, for instance) is involved. See Bernard, Le Courtois, and Quittard-Pinon [2005] for an overview of the application of Parisian down-and-out puts to the pricing of bank deposit guarantees.

Parisian Option Hedging

The valuation of Parisian options is hard enough, but not as hard as computation of their Greeks. What is important to keep in mind is that the Laplace transforms of Parisian Greeks can be written explicitly. That is, Parisian Greeks can be computed by inverse Laplace transformation of a known analytical formula, just like Parisian prices.

Equations (3) and (4) express the Laplace transforms of the delta and gamma of a down-and-in call option. We can obtain numerical inversions for these Greeks using the approximation method we have developed.

DIC deltas are represented along D and S_0 in Exhibit 7, while DIC gammas are given in Exhibit 8. Exhibit 9 plots the numerical results.

As expected from the monotonically declining relation of DIC prices with respect to S_0, DIC deltas are negative. This sensitivity is high in absolute value when S_0 is close to the barrier, which is logical because deltas are higher for narrow values D of the window (when D is close to zero, the activation and thus the call price become highly sensitive to small changes in S_0). Gammas are systematically positive for Parisian DICs. Consistent with the results for deltas, their values rise when D is narrow and S_0 is close to the barrier.

IV. SUMMARY

Parisian options can be priced by inverse Laplace transforms followed by quadratures. We provide for this some new formulas, including one for the gamma of a Parisian down-and-in call and one for the valuation of Parisian digital options.

Our main purpose has been to introduce a new simple numerical method allowing for the performance of inverse Laplace transforms and thus the computation of Parisian prices and Greeks. This method, based on approximation of the function to be inverted by a function whose inverse is known, proves extremely quick in yielding results for DIC and DOP prices and their Greeks.

Parisian options are interesting not only for their practical use as derivatives, but also because they are involved in many theoretical financial problems, ranging from real options to deposit guarantee valuation. In these contexts, the formula given for Parisian digital options should prove particularly useful.

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