Dividend Protection at a Price

CHRISTINE BROWN AND KEVIN DAVIS

CHRISTINE BROWN is an associate professor of finance at the University of Melbourne in Australia.
christine.brown@unimelb.edu.au

KEVIN DAVIS is a professor of finance at the University of Melbourne in Australia.

The development of long-term equity warrant markets has been hampered by the difficulty of providing dividend protection to investors in an easily understood form. A recent Australian innovation, the endowment warrant, is a long-term call option offering a form of dividend protection. Its simple structure has made it popular with retail investors, although valuation of the option is complex, since it is a long-term option with a stochastic strike price.

This article reviews what is meant by dividend protection, and demonstrates that endowment warrants provide only a form of partial dividend protection. Examination of the cost of incomplete dividend protection to investors indicates that the warrants issued have traded at prices significantly in excess of their fair value. Some possible causes of this divergence and lessons for the construction of new securities are discussed.

The adjustment of contract terms required to provide dividend protection to investors in option contracts on dividend-paying stocks has been established for three decades (see Merton [1973]). Despite that, and surprisingly, given the explosive growth of financially engineered products, there has been very little development of longer-term equity option or warrant contracts providing dividend protection. One explanation lies in the difficulty of packaging a dividend protection feature into a warrant product in a form that can be easily understood by and marketed to the general investment community.

Long-dated call options, known as *endowment warrants*, which were introduced to the Australian stock market in February 1996, appear to have overcome the problem of providing a simple form of dividend protection, and have proven popular with retail investors partly for this reason. For a price generally set between one-third and two-thirds of the share price, the investor purchases an endowment warrant whose outstanding amount (the amount owed on the optional delayed share purchase) is set by the issuer. This outstanding amount grows at the risk-free interest rate and is reduced periodically by application of each dividend paid on the stock to the outstanding amount.

In effect, the endowment warrant is a long-dated call option with a varying exercise price given by the terminal date outstanding amount. We discuss the exact nature of the variation of the exercise price, and present a valuation model for the case of known dividends.

Holders of unprotected call options have a claim only on the capital gains portion of the total stock return. Endowment warrant holders, however, have a claim on both the capital gains and the dividends, because the dividends that are paid on the stock are used to reduce the ultimate exercise price. Yet the form of the adjustment for dividends that is made to the endowment warrant contract does not fully protect the holder of the endowment warrant, as we demonstrate. We show that the endowment warrant price declines with

EXHIBIT 1
Notation

C_t	Price of warrant at time t.
S_t	Price of underlying stock at time t.
σ	Volatility of underlying stock.
K_t	Outstanding amount at time t.
T	Expected maturity date of warrant.
Z_t	Date t value of initial outstanding amount compounded at the interest rate.
d_t	Dividend at date t.
D_t	Date t value of accumulated dividends (reinvested at the interest rate).
D*	Present value of dividends over the expected life of the warrant.
D	Future value (at date T) of dividends.
r	Risk-free interest rate.

increasing dividends, as a result of the fact that the dividend protection is incomplete.

Because endowment warrants provide incomplete dividend protection, the valuation model proposed by Hoang, Powell, and Shi (HPS) [1999] is incorrect. We highlight the pricing differences between our model and that of HPS, and the implication for their conclusion that endowment warrants are fairly priced on issue. We develop a simulation model to value endowment warrants when interest rates and dividends are stochastic, and discuss some of the explanations for why endowment warrants may trade at prices in excess of fair value.

I. ENDOWMENT WARRANT VALUATION WITH KNOWN DIVIDENDS

Endowment warrants are exchange-traded, third-party issued, long-dated European call options with a stochastic exercise price, written on listed stock of major Australian companies.[1] They were issued at prices between around 30% to 65% of the current share price of the underlying stock, and the initial outstanding amount (sometimes loosely referred to as the initial exercise price) has varied from between 95% to 60% of the current share price.

To illustrate, the initial issue price of an endowment warrant on a stock trading at \$10.00 might be set at \$5.00 with an outstanding amount of \$6.50. This outstanding amount is adjusted over time as follows. It increases daily by application of a benchmark interest rate (reset at regular intervals) to the outstanding amount, and declines stepwise (semiannually) by the amount of dividends (and other distributions) paid on the stock.[2]

Over time, therefore, the outstanding amount is expected to decline in a sawtoothed pattern (provided dividends on the stock exceed interest accruing on the outstanding amount), and is initially set so that its expected termination date value is zero, provided that the dividend growth rate and interest rate forecasts of the issuer are realized. The termination date is set at around 10.5 years from the issue date.

An endowment warrant is therefore a call option on the underlying stock with a stochastic exercise price (due to stochastic dividends and interest rates) and stochastic exercise date (because the outstanding amount may reach zero before the termination date). The exercise date of the warrant occurs 30 days after the earlier of 1) the termination date or 2) the date a dividend payment is sufficient to reduce the outstanding amount to zero.[3]

We first provide a simple derivation of the appropriate pricing model in the special case where dividends are known and the interest rate is constant, before relaxing these assumptions later. Exhibit 1 provides a summary of the notation.

The endowment warrant is a European option, so the terminal date (T) value of the outstanding amount process (K_T) is the relevant option exercise price. That value is the outcome of the process K_t, which commences at K_0 and grows continuously at rate r, with six monthly discrete drops equal to the dividend amount paid at those dates. K_T can also be related to the outcome of an alternative process Z_t that commences at $Z_0 = K_0$. The process Z_t grows continuously at rate r such that $Z_T = K_0e^{rT}$.

The relationship between Z_T and K_T is given by:

$$K_T = Z_T - D$$

where D is the future (date T) value of all dividends paid between 0 and T and reinvested at the interest rate r. In the case of a fixed interest rate r, Z_T is known at date 0, and the endowment warrant exercise price is thus equal to this known exercise price reduced by the future value of dividends paid on the stock. This deduction is also known at date 0 in the case of known dividends, and thus, with fixed r and known dividends, the exercise price is non-stochastic.

How should we value this simplified, non-stochastic exercise price, endowment warrant that is equivalent to a standard call option on a dividend-paying stock with exercise price of $Z_T - D$? Let the date 0 present value of dividends to be paid between 0 and T be represented by D^*. It is well known that the Black and Scholes [1973] and Merton [1973] European option pricing formula denoted by $C(S, X, \sigma, r, T)$ is modified to $C(S - D^*, X, \sigma, r, T)$ for an option with exercise price X on a stock paying known dividends with present value D^*.

In the case of an endowment warrant, the exercise price is given by $K_T = Z_T - D = K_0e^{rT} - D$, and the relationship between the future and the present value of dividends is given by $D^* = De^{-rT}$. Hence, the appropriate pricing formula can be represented by $C_0\ (S_0 - D^*, Z_T - D, \sigma, r, T)$.

Using the standard modified Black-Scholes formula and substituting for Z_T and D gives

$$C_0 = (S_0 - D^*)\ N(d_1) - e^{-rT}(Z_T - D)N(d_2) \tag{1}$$
$$= (S_0 - D^*)\ N(d_1) - (K_0 - D^*)N(d_2) \tag{2}$$

where:

$$d_1 = [\ln[(S_0 - D^*)/(Z_T - D)] + (r + \sigma^2/2)T]/(\sigma\sqrt{T})$$
$$= [\ln[(S_0 - D^*)/(K_0 - D^*)] + \sigma^2 /2T]/(\sigma\sqrt{T}) \tag{3}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

We refer to Equation (2) as the *EW model*, which would apply to endowment warrants in the special case of known dividends and fixed interest rates. Hoang, Powell, and Shi (HPS) [1999] published a different model for the case of constant interest rates. Their model assumes that the dividend protection is complete, resulting in valuation as follows:

$$C_0 = S_0\ N(\delta_1) - K_0\ N(\delta_2) \tag{5}$$

where:

$$\delta_1 = [\ln(S_0/K_0) + (\sigma^2/2)T]/(\sigma\sqrt{T}) \tag{6}$$
$$\delta_2 = \delta_1 - \sigma\sqrt{T} \tag{7}$$

HPS argue that the endowment warrant can be thought of as equivalent to a dividend-protected call option with an exponentially increasing strike price. It is true that the form of dividend protection given in Equation (4) of their article results in the same payoff for an endowment warrant and a "dividend-protected" call option. Yet the form of dividend protection created by subtracting the dividend from the outstanding amount, or equivalently subtracting the future value of dividends from the terminal date outstanding amount, is incomplete.

Assuming complete dividend protection applies (which they do) implies that the warrant can be valued *as though* the stock does not pay dividends, which results in Equation (5), a simple model with no dividend terms in it. This assumption is not correct for the strike price adjustment of the endowment warrant. Indeed, Merton [1973] shows that only when dividends are reinvested in the stock is full dividend protection afforded.

Merton [1973] and Geske, Roll, and Shastri [1983] demonstrate that the form of adjustment in which the strike price is reduced by the size of the cash dividend at the ex-dividend date (called OTC protection by Geske, Roll, and Shastri) results in imperfect dividend protection. Geske, Roll, and Shastri show that the loss to a call option holder as a result of imperfect dividend protection is small for short-maturity options, and the loss is an increasing function of the amount of the dividends, the interest rate, and the maturity date. They compare the ex-dividend date value of an over-the-counter protected call option on a stock with cum-dividend price of S, dividend payment of d, and preadjustment strike price of X with the corresponding value of a call option on an otherwise equivalent zero-dividend stock. To a first approximation, the value of the OTC protected call is d/S% lower.

Similarly, the endowment warrant adjustment of reducing the exercise price by the future value of the dividend (or equivalently reducing the outstanding amount by the actual value of the dividend) does not provide full dividend protection. This is illustrated by differentiating Equation (2) with respect to D^* and simplifying, resulting in:

$$\frac{\partial C}{\partial D^*} = N(d_2) - N(d_1) < 0 \tag{8}$$

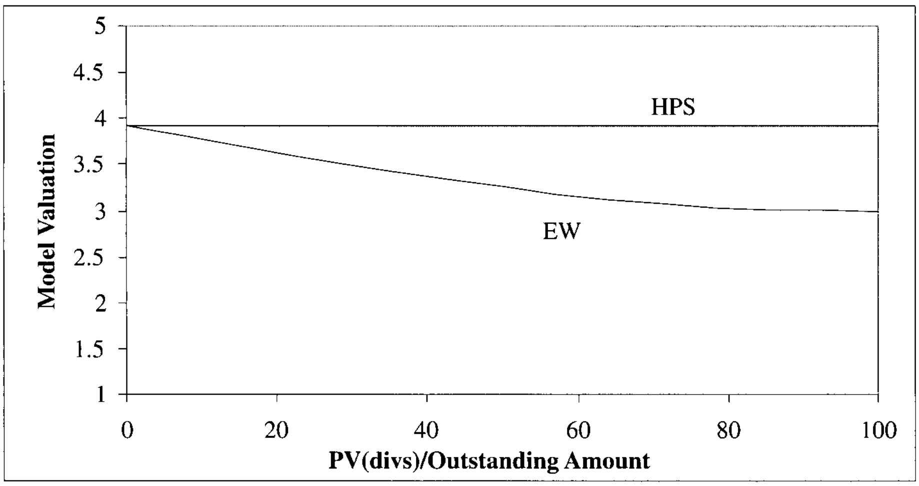
which indicates that dividend protection is less than complete. Thus the HPS [1999] formula will overvalue endowment warrants, except in the trivial case of zero dividends.

EXHIBIT 2
Illustration of Cost of Incomplete Dividend Protection

Volatility (%)	Outstanding Amount as Percentage of Stock Price			
	50%	60%	70%	80%
10	0.22	1.36	5.68	19.87
20	5.99	14.10	30.71	68.30
30	17.06	32.27	59.95	118.30

Percentage of overpricing of the HPS model for endowment warrant with maturity of ten years and constant interest rate of 7.5%, assuming that present value of known future dividends equals the outstanding amount, so that the terminal date exercise price is zero and exercise is certain.

EXHIBIT 3
Endowment Warrant Valuation—HPS and EW Models



S_0 = 10, outstanding amount = 7, σ = 0.2, T = 10.

Exhibit 2 provides an illustration of the errors involved in the HPS model when the present value of dividends equals the outstanding amount, demonstrating that the error increases as the outstanding amount relative to the stock price increases.

Equation (8) shows that the endowment warrant price increases as the present value of dividends declines. In other words, a lower dividend payout makes the warrant more valuable (contrary, we suspect, to the perceptions of most retail investors in the product). Exhibit 3 (which scales the present value of dividends by the outstanding amount) illustrates this, and also compares prices of the HPS [1999] model and the EW model [Equation (2)].

When dividends are relatively low, the cost to the warrant holder of this imperfect dividend protection becomes less significant, so prices given by the HPS model and the EW model are closer. The EW valuation line is almost horizontal when the ratio of present value of dividends to the outstanding amount is close to 1, showing that in this region the warrant valuation is relatively insensitive to changes in the present value of dividends. As expected when the ratio is 1, so that the terminal date strike price is known to be zero, the endowment warrant value equals the difference between the stock price and the outstanding amount.

More generally, as shown by Merton [1973], there is no simple dividend-related adjustment to the strike price alone that can provide full dividend protection. The endowment warrant exercise price adjustment is a simple one and easy for retail investors to understand, but it comes at the expense of incomplete dividend protection.

II. DIVIDEND UNCERTAINTY AND ENDOWMENT WARRANT VALUATION

We develop a simulation model to value endowment warrants and compare prices from the simulation model to prices from the EW model and the HPS model. For comparability reasons, we use the same data (including observed market prices) as in the HPS Exhibit 4, where

Exhibit 4

Endowment Warrant Simulation Results

Code	EW	SIM	RMSE	%OTM	HPS	Market	Difference from Traded Price EW	SIM	HPS
ANZ	4.04	4.12	0.09	0.00	4.51	4.19	-0.15	-0.07	0.32
BOR	0.33	0.16	0.02	81.00	0.49	0.55	-0.22	-0.39	-0.06
BIL	27.53	27.63	0.29	0.00	27.54	27.84	-0.31	-0.21	-0.30
CML	4.09	4.12	0.05	0.00	4.16	4.17	-0.08	-0.05	-0.01
CBA	11.19	11.36	0.10	0.00	11.32	11.43	-0.24	-0.07	-0.11
CSR	0.63	0.17	0.02	81.00	0.74	0.61	0.02	-0.44	0.13
FBG	2.08	2.04	0.04	0.00	2.12	2.16	-0.08	-0.12	-0.04
GIO	2.51	2.36	0.06	0.00	2.75	2.47	0.04	-0.11	0.28
LNN	1.76	1.60	0.04	0.00	1.98	1.89	-0.13	-0.29	0.09
MBL	7.64	7.92	0.18	0.00	8.16	7.97	-0.33	-0.05	0.19
NAB	9.79	10.07	0.12	0.00	10.17	10.10	-0.31	-0.03	0.07
STO	1.48	1.39	0.04	0.00	1.85	1.82	-0.34	-0.43	0.03
SGW	0.67	0.29	0.04	82.00	1.22	1.27	-0.60	-0.98	-0.05
TEL	2.29	1.82	0.08	1.00	2.66	1.90	0.39	-0.08	0.76
WOW	4.11	4.08	0.04	0.00	4.12	4.06	0.05	0.02	0.06
Average Difference							**-0.15**	**-0.22**	**0.07**

valuations are as of October 1, 1998.

A practical valuation model for endowment warrants must take account of the variability in the short-term interest rate applied to the outstanding amount, uncertainty in dividends that are paid semiannually, and early termination of the endowment warrant when the outstanding amount goes to zero. Clearly, the Black-Scholes-Merton style model for the case of known dividends cannot deal with these features of the endowment warrant. Complete treatment of the impact of dividend and interest rate uncertainty requires the use of numerical techniques.

We present the results of Monte Carlo simulation under the assumption of geometric Brownian motion for stock prices, using the Cox, Ingersoll, and Ross [1985] model for interest rates and introducing uncertainty into the dividend amount through an assumption of a constant dividend yield.

The stock price path is simulated as geometric Brownian motion using Monte Carlo simulation with one-month time steps, assuming a dividend is paid at each sixth step. The dividend amount is calculated as a constant dividend yield multiplied by the stock price at the previous step.[4] The assumed dividend yield is derived by setting the initial dividend equal to the most recently paid dividend for each stock. We assume an ex-dividend drop-off in the share price equal to the cash dividend.

Interest rates are simulated using the Cox, Ingersoll, and Ross [1985] model with the short-term rate set to 4.88% and the long-term rate set to 5.02%, with adjustment rate of 25%.[5] A correlation of zero is assumed between movements in the stock price and short-term interest rates.

To produce prices using the analytical EW model given by Equation (2), we calculate the present value of dividends to substitute as follows. First we take the most recent dividend prior to October 1, 1998, and calculate a dividend yield using the dividend announcement date stock price. Assuming the dividend yield is constant over the life of the warrant, we then calculate the present value of dividends from that time until the maturity date of the endowment warrant.[6]

Given the valuation date of October 1998, the remaining time to maturity is 7.75 years. The proportion of simulations in which the outstanding amount reaches zero prior to 7.75 years is recorded, and the effect of earlier termination is incorporated into the warrant valuation.

The results for the analytical model (EW), the simulated warrant prices (SIM), and the HPS [1999] prices are presented in Exhibit 4, where the simulation results are based on 2,000 simulations using the antithetic variable technique. The column labeled % OTM shows the percentage of paths from the simulation in which the warrant finishes out of the money. The root mean squared error of the simulation is reported in the third column. For actual warrant prices observed on October 1, 1998, the EW and simulation models on average underprice relative to observed market prices, while the HPS model on average overprices.

It is interesting to compare the EW model prices to those produced by our simulation model. The EW model values the warrant with a fixed maturity date because the dividends are known in advance. The simulation model, however, allows for early termination of the warrant contract should the outstanding amount go to zero before the contractual maturity date. Both models have the same assumed dividend yield and present value of future dividends. Simulation model prices are close to those produced by the EW model, except in three cases.

The stocks for which there is a large difference between the EW model and the simulation model are BOR, CSR, and SGW. On October 1, 1998, each of these warrants had outstanding amounts higher than the stock price (despite initial issue with outstanding amounts around 80% of the then-stock price). These warrants were significantly out of the money, and in each case a high percentage of the simulation price paths have terminal stock prices where the warrant finishes out of the money, as column (4) of Exhibit 4 illustrates. In these cases, the simulation model produces prices that are considerably lower than those produced by the EW model.

Underlying this difference is the path-dependence of actual dividend amounts in the simulation model, where a constant dividend yield means that dividends are positively correlated with the average stock price over the path. Because dividend protection is incomplete, high dividend outcomes reduce the value of the warrant. While low dividends conversely would tend to increase the value of the warrant, this offsetting effect does not occur for those paths where the warrant finishes out of the money and thus has zero value.

The EW model, however, while assuming the same average dividend yield and dividend amount, implicitly assumes a zero correlation between the dividend amount and the stock price path. For warrants well in the money and where the simulation model has no price paths finishing out of the money, the resulting linear payoff schedule of terminal date warrant values means that the path-dependence of dividends does not affect the current warrant price, and EW and simulation results are thus relatively close.

We also observe that the simulation model produces estimates for the warrant value that are on average below market prices. This is in contrast to the HPS formula, which produces prices on average higher than market prices. The higher values resulting from the HPS formula reflect the (inappropriate) assumption of full dividend protection. These results suggest that endowment warrants have traded at prices in excess of fair value.

III. DISCUSSION AND CONCLUSIONS

The dividend protection provided to investors in the structure of the endowment warrant is incomplete. We have provided a simple analytical model for the case of known dividends, which shows that the value of the endowment warrant increases as dividends decline. This result is consistent with the findings of Merton [1973] and Geske, Roll, and Shastri [1983] and arises because investors receive higher capital gains when dividends are lower, which more than compensates for the higher strike price at the maturity date of the warrant. We show there can be a substantial pricing error induced by assuming that the dividend protection is complete.

Using a simulation model that incorporates stochastic interest rates and dividends, we produce prices for endowment warrants and compare them with both market prices and prices produced using the HPS [1999] model for particular warrant prices observed on October 1, 1998. These results show that prices produced by the simulation model and the analytical model for the case of known dividends are lower on average than market prices, and lower on average than the prices produced by the HPS model.

Several possible explanations can be advanced for such apparent mispricing. Endowment warrants may provide tax arbitrage strategies for some (international) investors—although in Brown and Davis [1997] we demonstrate that the warrants are tax-disadvantaged for Australian investors. Because endowment warrants are a partial substitute for levered purchases of stock and are available in small sizes and involve wholesale market interest rates, they provide liquidity benefits and overcome borrowing market imperfections for retail investors looking for leveraged positions. In addition, legal constraints on institutional investors (such as pension funds) taking leveraged positions may be effectively circumvented by investment in endowment warrants.

Our most important contribution lies in enhancement of our understanding of the appropriate design and pricing of warrant products that provide some measure of dividend protection to investors. As we have demonstrated, the differences in fair value between longer-term warrants that provide partial and full dividend protection can be very significant, indicating the importance of full appreciation of the extent of dividend protection that innovative products provide.

ENDNOTES

This article was commenced while both authors were visiting scholars at the University of Strathclyde in Glasgow, Scotland. The authors thank Ser-Huang Poon, Pradeep Yadav, and the referees for helpful comments, and Patrick McGlenchy for research assistance.

[1]A specified number of warrants on each company were created by the issuers and sold to the public in the initial offering, as a tap issue at a price that can vary over time, through an intermediary (Challenger International). Reflecting their popularity with retail investors (for whom they are a possible alternative to buying the stock on margin) there were then several series of endowment warrants created on the same stocks.

[2]The interest rate applied on warrants issued by one of the issuers, Macquarie Bank, is the 90-day bank bill rate plus 300 basis points, while that on warrants issued by Credit Suisse Financial Products is the 180-day bank bill rate. The dividend amount credited against the outstanding amount is the cash value of the dividend in the latter case and the cash value plus the value of imputation tax credits of dividends in the former case. Dividends are typically paid semiannually in Australia and may carry tax credits for the investor. The warrants we study are those issued by Credit Suisse, where the dividends do not carry the tax credits.

[3]If the dividend payment exceeds the outstanding amount, the warrant holder is entitled to the excess.

[4]Simulations involving a dividend smoothing assumption (see Lintner [1956]) produced virtually identical results, reflecting the minor impact of smoothing on the present value of total dividends paid over such a long time horizon.

[5]These were the market rates on October 1, 1998.

[6]We use Se^{-qT} to approximate the stock net of the present value of dividends, where q is the dividend yield. Then the present value of dividends can be approximated by $S(1 - e^{-qT})$. We ran checks comparing the calculated present value of dividends against the present value of dividends as an output from the simulation, and the results were similar.

[7]Robustness checks on our simulation model indicate that the same simulation model for stocks and dividends, but holding the interest rate constant at the initial short-term rate, produces prices where the average difference in price between the two models is 2%.

REFERENCES

Black, F., and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 81 (1973), pp. 637-659.

Brown, C., and K. Davis. "Endowment Warrants: Valuing a Financial Innovation." Working Paper 97-09, University of Melbourne, 1997.

Cox, J.C., J.E. Ingersoll, and S.A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985), pp. 385-407.

Geske, R., R. Roll, and K. Shastri. "Over-the-Counter Option Market Dividend Protection and 'Biases' in the Black-Scholes Model: A Note." *Journal of Finance*, 38 (1983), pp. 1271-1277.

Hoang, P., J.G. Powell, and J. Shi. "Endowment Warrant Valuation." *The Journal of Derivatives*, 7 (1999), pp. 91-103.

Lintner, J. "Distribution of Income of Corporations among Dividends, Retained Earnings and Taxes." *American Economic Review*, 46 (1956), pp. 97-113.

Merton, R.C. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science*, 4 (1973), pp. 141-183.