Interest Rate Swaps: *Reconciliation of Models*

PETER KLEIN

PETER KLEIN
is an associate professor of finance in the Faculty of Business Administration at Simon Fraser University in Vancouver, Canada.
pklein@sfu.ca

Interest rate swaps have typically been modeled either as an exchange of non-callable bonds or as a portfolio of interest rate forwards. The first model implies that swap rates should be related to the yield on corporate bonds, and the second that swap rates should be related to forward LIBOR. This study reconciles the two models and their conflicting empirical implications by taking into account the effect on swap rates when financial institutions make markets in swaps in a competitive environment.

It shows that swap rates should be closely related to the yield on fixed-rate bank liabilities less the floating-rate bank credit spread, and should not be directly influenced by the yield on corporate bonds. Data on two-, three-, and five-year swap rates provide empirical support for these hypotheses. The general level of interest rates is also found to be significant in explaining divergence of swap rates and bank paper yields.

Interest rate swaps have typically been modeled in one of two ways. The first considers a swap as an exchange of non-callable bonds between counterparties. The second models a swap as a series of forward agreements. Which view is more appropriate has not been resolved at a theoretical level, and empirical support has been mixed.

Minton [1997] finds some evidence of a relation between swap rates and yields on fixed-rate corporate bonds, but she also finds evidence of a relation between swap rates and Euro currency futures rates. As a result, she concludes that neither model is completely consistent with swap rates observed in the market. More recently, Gupta and Subrahmanyam [2000] report large differences between swap rates and futures-implied LIBOR, but attribute these differences to the convexity bias caused by the daily marking to market of futures contracts.

My intention is to reconcile these two models of interest rate swaps and their conflicting empirical implications. This reconciliation is based on a closer examination of the swap market-making activities of large financial institutions, and the resulting impact on swap rates. For both the traditional models of swaps, profitable opportunities are shown to be available to market makers unless swap rates are consistent with the yields on their fixed- and floating-rate funding opportunities. Since most swap market makers are banks, this implies that swap rates should be closely related to the yield on fixed-rate bank liabilities less the floating-rate bank credit spread, and should not be directly influenced by the yield on corporate bonds.

These hypotheses are tested with data on two-, three-, and five-year swap rates. Empirical evidence is found that swap rates are closely related to yields on similar-term bank liabilities, and not directly related to corporate bond yields. The empirical tests also find that the general level of interest rates can explain differences between swaps rates and the yield on bank liabilities.

I. SWAPS AS AN EXCHANGE OF NON-CALLABLE BONDS—REVISITED

The analogy between swaps and an exchange of non-callable bonds has been made by many authors, such as Bicksler and Chen [1986], Turnbull [1987], Arak, Goodman, and Silver [1988], Smith, Smithson, and Wakeman [1988], Hull [1989], Cooper and Mello [1991], Litzenberger [1992], Sun, Sundaresan, and Wang [1993], Minton [1997], and Lang, Litzenberger, and Liu [1998]. The model is typically motivated by considering two corporate counterparties with a menu of funding opportunities who discover that they can reduce their funding costs by engaging in an interest rate swap with each other. To do so, they each issue liabilities to the market on a basis that the other counterparty wants, and agree to exchange payments on these liabilities in order to obtain net financing on the basis that they initially desire.

Exhibit 1 provides a typical example of two corporate borrowers who can reduce their combined funding costs by engaging in an interest rate swap. In this example, the total amount available to be saved by the two counterparties is 0.50%, which is the difference in their lending spreads in the fixed- and floating-rate markets. This saving is shared by the counterparties.

The sharing is implemented by increasing the fixed rate on the swap by a portion of the 0.50%, shown as α_1 in Exhibit 1. The amount of the comparative advantage that B is able to retain is α_1; A's share is $0.50\% - \alpha_1$.

There are two main difficulties with modeling swaps as an exchange of non-callable bonds. The first is that the model does not provide much guidance on how swap rates are determined. This is because the counterparties' fixed-rate bond yields do not, on their own, determine the swap rate.[1] Two additional important determinants are the floating-rate borrowing spread of the counterparties and the way the combined benefit of engaging in the swap is shared.

For example, A's floating-rate borrowing spread of 0.45% implies that A will be willing to enter into a swap provided the fixed rate it pays is *less* than 0.45% below its fixed-rate funding cost, that is, less than 10.30%. Similarly, B's floating-rate borrowing spread of 0.20% implies that B will benefit from entering into a swap provided the fixed rate it receives is more than 0.20% below its fixed-rate funding cost, that is, more than 9.80%. The traditional model provides no guidance on whether the swap rate will be closer to A's maximum acceptable rate or to B's minimum acceptable rate.

The second difficulty with the model of interest rate swaps as an exchange of non-callable bonds is that it ignores the activities of swaps market makers. Virtually all swaps are between a corporate counterparty and a swaps market maker, and not between two corporate counterparties. Although corporate counterparties still decide to enter into a swap by comparing swap rates with their fixed- and floating-rate financing costs, swap rates are not determined by negotiation among corporate counterparties as the traditional model would imply. Rather, swaps market makers post swap rates much as they quote the other financial products they offer their clients. As in the case of other mature financial products, competition among market makers means products are offered at marginal cost plus normal profits, which usually leaves very little room for negotiation by the corporate counterparties.

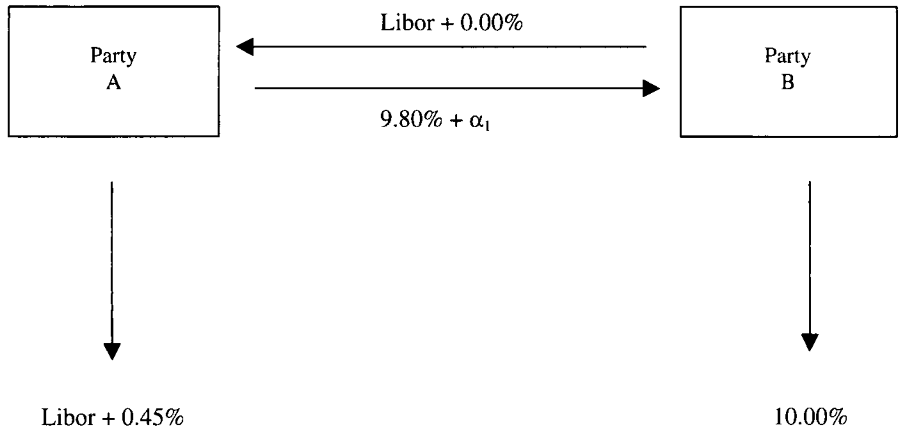
Note also that market makers typically enter into swaps with a single corporate counterparty *without* an immediate offsetting transaction. In other words, the market maker is not merely inserted into the middle of a transaction between two corporate counterparties in order to bear their respective credit risks. Instead, the market maker hedges a swap based on its effect on its overall swap portfolio without identifying an exact offsetting swap with another corporate counterparty. At some later time, a similar offsetting swap is identified, and the hedge is unwound.

The typical series of transactions for a market maker in the middle of swaps with two corporate counterparties is illustrated in Exhibits 1 and 2. A bank is generally willing to receive a fixed rate on a swap, provided that rate is in excess of its own fixed-rate financing cost less its floating-rate borrowing spread. This case is illustrated in Exhibit 1, assuming B is the bank.

Banks are also willing to enter into swaps in the reverse direction when that is advantageous to them. Exhibit 2 illustrates the reverse swap where the bank (Party B) is willing to pay a fixed rate, provided this rate is below the rate on its fixed-rate liabilities less its floating-rate spread. In this case, the bank will borrow at a floating rate and enter into the swap instead of borrowing on a fixed-rate basis.

Viewed together, the swaps in Exhibits 1 and 2 leave the bank in the middle of two swaps with A and C, with a positive spread of $\alpha_1 + \alpha_2$. After both the swaps have been transacted, this net position is identical to the position of a financial institution in a swap that is negotiated between two corporate counterparties. It is important to recognize, however, that the rates on these swaps are not negotiated between A and C, but are set by the market maker (B), one swap at a time.

Exhibit 1
Party B Pays Floating Rate



	Party A	Party B
Cost of funding via swap		
Interest on financing	LIBOR + 0.45%	10.00%
Receipt on swap	LIBOR + 0.00%	9.80% + α_1
Payment on swap	9.80% + α_1	LIBOR+ 0.00%
Net	10.25% + α_1	LIBOR+ 0.20% $-\ \alpha_1$
Cost of funding directly	10.75%	LIBOR + 0.20%
Savings due to swap	0.50% $-\ \alpha_1$	α_1

There are a number of reasons corporate counterparties prefer to deal at the rates posted by swaps market makers instead of searching for other corporate counterparties. One reason is that financial institutions are usually less credit-risky than other corporate counterparties. Dealing with a well-known bank also avoids the costly search associated with finding another corporate counterparty that has exactly offsetting swap needs at the same time. Moreover, corporate counterparties can reap greater comparative advantage if they are able to choose the best swap rate from a number of financial institutions.

Competition among market makers implies that the share of the comparative advantage that financial institutions could retain for themselves should be small. Thus swap rates should fall within a range determined by the yield on the fixed-rate liabilities less the floating-rate credit spread of the marginal swaps market maker. This also implies that swaps rates are not linked to the funding opportunities or yield on fixed-rate bonds of corporations.[2]

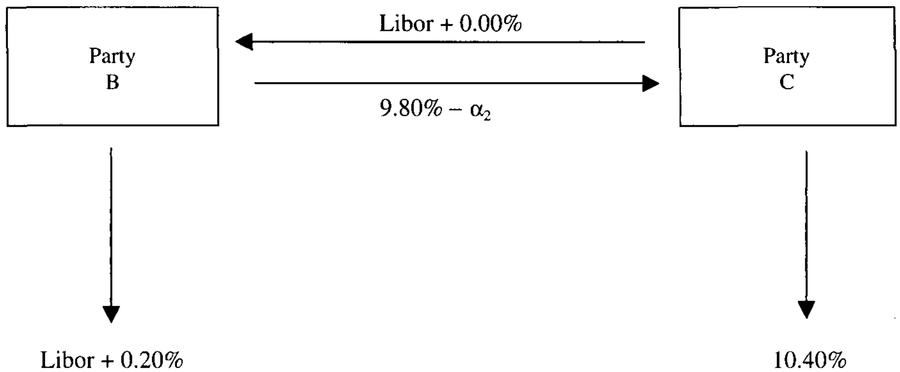
Lang, Litzenberger, and Liu [1998] recognize that the allocation of the comparative advantage is an important determinant of swap rates. They model swaps as negotiations between lower-grade corporations and higher-grade government agencies, where the financial intermediary is present only to assist in arranging the transaction or to absorb the credit risk of the counterparties.

The problem with this approach is that it does not take into account the role of the swaps market maker. It also ignores the fact that there is no opportunity for negotiations to occur between the corporate and government agency counterparties because each negotiates with the swaps market maker instead.

II. SWAPS AS A PORTFOLIO OF INTEREST RATE FORWARDS—REVISITED

The model of interest rate swaps as a portfolio of interest rate forward contracts implies that the swap rate should be set with reference to the rate at which a series of forward contracts can be hedged. An example of the transactions required to hedge one of the exchanges of payments on an interest rate swap, which is analogous to

EXHIBIT 2
Party B Pays Fixed Rate



	Party B	Party C
Cost of funding via swap		
Interest on financing	LIBOR + 0.20%	10.40%
Receipt on swap	LIBOR + 0.00%	9.80% – α_2
Payment on swap	9.80% – α_2	Libor + 0.00%
Net	10.00% – α_2	Libor + 0.60% – α_2
Cost of funding directly	10.00%	Libor + 0.90%
Savings due to swap	α_2	0.30% – α_2

a single forward contract, is depicted in Exhibit 3. We assume for simplicity that the yield curve is flat, and that the market maker is paying a fixed rate and receiving a floating rate for five periods.[3]

Exhibit 3 illustrates that the hedging required for the fixed-rate payment in the third period consists of lending at a fixed rate for three periods and borrowing at a fixed rate for two periods. Since the yield curve is flat, the fixed interest rates net for the first two periods, leaving a single fixed-rate receipt for the third period that offsets the fixed-rate payment on the swap. The repayment of the two-period borrowing is financed by borrowing at the start of the third period at interest rates prevailing at that time. This payment hedges the floating-rate receipt in the third period on the interest rate swap.[4]

The model of swaps as a portfolio of interest rate forwards suffers from two difficulties concerning the way the market maker is assumed to be able to hedge or replicate the swap. The first is the choice of the spot rates that are used to determine the forward interest rates constituting the *fixed* leg of the interest rate swap.

Typically it is assumed that the market maker can borrow and lend at the riskless rate, or at the repo rate. This assumption is relatively sound for short-dated forward contracts, but becomes inaccurate when the delivery date on the forward contract increases beyond a few months, because the market maker is unable to hedge at riskless or repo rates for longer-dated maturities. This point is particularly important because most swaps have relatively long terms to maturity of between two and five years.

The second difficulty relates to the interest rate used on the hedge for the *floating* leg of the swap. Typically the floating rate is a publicly available benchmark rate such as LIBOR, which is determined as the average of LIBOR posted on a publicly available financial news service by a number of reference banks. The identity of the banks providing the LIBOR used to compute the average is determined by industry convention, and can change from time to time. As depicted in Exhibit 3, the swaps market maker hedges this floating rate by borrowing at interest rates prevailing at the start of each swap period.

Basis risk arises, however, since the rate at which the market maker borrows may not exactly equal the precise benchmark rate used for the floating rate on the interest rate swap. This difference is likely negligible when hedging the earlier floating-rate payments in the interest

EXHIBIT 3
Hedging a Single Exchange of Payments

Time	Swap Payment	Swap Receipt	Hedge for Third Fixed Swap Payment *Borrow for 2 Periods*	*Invest for 3 Periods*	*Net Fixed Hedge*	Hedge for Third Floating Swap Receipt *Borrow for 1 Period at Start of Period 3*
0			100	-100		
1			$-R_{fixed}$	R_{fixed}		
2			$-(100 + R_{fixed})$	R_{fixed}	-100	100
3	$-S_{fixed}$	$S_{floating,3}$		$100 + R_{fixed}$	$100 + R_{fixed}$	$-(100 + R_{short,3})$
4						
5						

rate swap, but becomes substantial when hedging floating-rate payments several years into the life of the swap.[5]

It is also important to recognize that when it hedges a swap, the market maker is simultaneously hedging a series of fixed- and floating-rate exchanges that occur over the life of the swap. Since the maturity of one forward contract corresponds to the start of the next, the principal amount of the *borrowing* required to hedge a forward rate at one delivery date is offset by the principal amount of the *investment* required to hedge the forward rate at the next delivery date.

Exhibit 4 shows that when the borrowing and the lending for each of the five periods are considered altogether, only the initial and the final principal amounts and a series of interest payments remain. The last column of Exhibit 4 indicates these net cash flows are identical to the cash flows associated with repurchasing a fixed-rate liability with a term to maturity equal to the tenor of the interest rate swap. This implies that the market maker can hedge the fixed swap payments with a single transaction involving its own fixed-rate liabilities.

Exhibit 5 depicts the hedging required for the entire floating leg of the swap. Note in the last column that the net cash flows on the entire series of floating-rate hedges are identical to the cash flows from a floating-rate liability issued by the market maker with a term to maturity equal to the tenor of the interest rate swap. This hedging strategy can expose the market maker to basis risk because it may find at any given reset date during the life of the swap that its short-term borrowing cost does not equal the LIBOR receipt on the swap. In order to eliminate this basis risk, the market maker must fix in advance the amount by which its short-term funding rates can differ from LIBOR over the life of the swap.[6]

A simple way to do so is to issue a floating-rate note instead of a series of short-term notes. The rate for such floating-rate funding is typically not equal to LIBOR + 0.00%, but instead will be at some premium above LIBOR depending on the floating-rate credit spread of the market maker.

This analysis implies that profitable opportunities are available to financial institutions making markets in swaps unless swap rates are equal to the fixed-rate yield, less the floating-rate borrowing spread, of their own liabilities. This result is identical to the implications of the model of swaps as an exchange of non-callable bonds as discussed at the outset.[7]

III. EFFECT OF CREDIT RISK

Hull [1989], Usmen [1994], Duffie and Huang [1996], Cossin and Pirotte [1997], and Duffie and Singleton [1997] consider the effect of credit risk on swap rates. Duffie and Huang [1996] show that the appropriate adjustment depends on a switching rate that is based on the credit-riskiness of both counterparties. They estimate that for a typical counterparty that issues bonds at 2% to 3% above riskless rates, the appropriate credit adjustment for a five-year interest rate swap is in the range of 0.03%. This analysis does not imply that swap rates posted by market makers should not be based on their own fixed- and floating-rate funding costs. Instead, it merely implies that a market maker would need to receive a few basis points more from its counterparty, or pay a few basis points less, in order to compensate it for the expected credit loss on these swaps.

Credit risk can also affect the rates at which corporate counterparties are willing to enter into swaps with the market maker. For example, Party A in Exhibit 1 will enter into a swap only if the rate provided by the market maker after A's own adjustment for the credit risk in the swap is attractive enough compared to A's direct financing alternative. Recall, however, that competition among market makers allows A to retain for itself most of the comparative advantage of entering into the swap with B. If the amount is insufficient to compensate the counterparty for the credit risk associated with the swap, no swap will occur. Typically, however, a corporate counterparty gains sufficient benefit from the swap to more than cover the adjustment for credit risk.

Sun, Sundaresan, and Wang [1993] observe that swap rates of AAA-rated market makers bracket the rates of A-rated market makers. This observation is consistent with the effect of credit risk on the corporate counterparty that I have noted. The credit risk of a lower-rated market maker is higher, so a corporate counterparty needs to obtain a better swap rate in order to deal with such a market maker. This implies the swap rates of A-rated market makers are unacceptable to corporate counterparties unless they are closer to midmarket rates than are the rates of AAA-rated market makers.

IV. TESTING RECONCILIATION OF THE MODELS

The model of swaps as an exchange of non-callable bonds and the model of swaps as a portfolio of interest rate forwards have the same empirical implications, once the effect of financial institutions making markets in swaps in a competitive environment is properly taken into account. Despite the credit risk in swaps, corporate bond yields should have a negligible direct influence on swap rates. Instead, swap rates are related to the yield on similar-term fixed-rate liabilities less the similar-term floating-rate credit spread of the marginal swaps market maker.

My principal empirical hypothesis is that swap rates should be related to the yield on similar-term fixed-rate liabilities less the floating-rate borrowing spread of the marginal swaps market maker. If swap rates are regressed on both variables, the coefficients on the fixed-rate yield and the floating-rate borrowing spread should equal 1.0 and be positive and negative, respectively. If swap rates are regressed on the fixed-rate yield by itself, the coefficient should still be positive, but its value should be less than 1.0 if fixed-rate yields and floating-rate credit spreads are positively correlated.

Longstaff and Schwartz [1995] show that the general level of interest rates is a major determinant of corporate credit spreads. Since a corporate swap counterparty often views a floating-rate liability and an interest rate swap as an alternative to a fixed-rate bond, it is possible that the general level of interest rates will also affect swap rates within the arbitrage bounds. Thus a proxy for the general level of interest rates will also be included in the regressions.

It is also important to test the traditional view that swap rates are related to corporate bond yields. Accordingly, the yield on similar-term corporate bonds is added to the regressions, even though my swaps pricing theory

EXHIBIT 4
Hedging the Fixed Rate on a Swap

Time	Swap Payment	Net Hedges for Each Fixed Rate Swap Payment: *Payment 1*	*Payment 2*	*Payment 3*	*Payment 4*	*Payment 5*	Net Hedge for All Fixed Payments
0		-100					-100
1	$-S_{fixed}$	$100 + R_{fixed}$	-100				R_{fixed}
2	$-S_{fixed}$		$100 + R_{fixed}$	-100			R_{fixed}
3	$-S_{fixed}$			$100 + R_{fixed}$	-100		R_{fixed}
4	$-S_{fixed}$				$100 + R_{fixed}$	-100	R_{fixed}
5	$-S_{fixed}$					$100 + R_{fixed}$	$100 + R_{fixed}$

Exhibit 5
Hedging the Floating Rate on a Swap

Time	Swap Receipt	Hedges for Each Floating Rate Swap Receipt *Receipt 1*	*Receipt 2*	*Receipt 3*	*Receipt 4*	*Receipt 5*	Net Hedge for All Floating Payments
0		100					100
1	$S_{floating,1}$	$-(100 + R_{short,1})$	100				$-R_{short,1}$
2	$S_{floating,2}$		$-(100 + R_{short,2})$	100			$-R_{short,2}$
3	$S_{floating,3}$			$-(100 + R_{short,3})$	100		$-R_{short,3}$
4	$S_{floating,4}$				$-(100 + R_{short,4})$	100	$-R_{short,4}$
5	$S_{floating,5}$					$-(100 + R_{short,5})$	$-(100 + R_{short,5})$

implies no significant relationship between swap rates and corporate bond yields.

V. DESCRIPTION OF DATA AND SUMMARY STATISTICS

Tests of swap pricing models have generally used U.S. data because of the liquidity and history of swap markets in that currency. These characteristics also make it much more difficult to identify the funding costs of the marginal swaps market maker because many U.S. dollar swaps market makers are not banks, but rather subsidiaries of other financial institutions. For example, neither of the two market makers providing swap quotes for the study in Sun, Sundaresan, and Wang [1993] are banks; one is an insurance company and the other a major investment bank.

Although my swaps pricing theory would still apply to whatever institution is pricing swaps at the margin, it is not possible to identify a priori the sector of the U.S. financial services industry in which the marginal swaps market maker operates. Thus testing the relation between bank paper yields and swap rates in a market that is influenced by non-bank market makers may provide inconclusive results.

Further, even if one assumes that the marginal swaps market maker in the U.S. market is a bank, the illiquidity of the U.S. interbank market causes significant problems in testing swaps pricing theory, as noted by Sun, Sundaresan, and Wang [1993]. Most researchers have attempted to overcome this second problem by substituting yields implied by a strip of Eurodollar futures contracts. As Gupta and Subrahmanyam [2000] show, however, estimation error in the convexity bias between futures and forward prices limits the comparability of these rates.

The theory I have outlined should apply to swap rates in any currency, but its principal empirical implication is not easily tested for U.S. swaps because we cannot clearly identify the marginal swaps market maker. The theory is, however, able to say that even U.S. swap rates should not be *directly* linked to corporate bond yields, as the marginal swaps market maker is not likely to be a non-financial corporation. Since the yields on bonds of all issuers are related to some extent, however, an indirect link between swap rates and corporate bonds has been reported by other researchers because their empirical models are underspecified.

The market for Canadian dollar swaps and related products does allow a direct test of the theory I propose. In Canada, there are only six banks of significant size, all making markets in Canadian dollar swaps. Each of these banks also actively issues paper in the form of debentures and term certificates of deposit denominated in Canadian dollars, and each is of a roughly similar credit rating. These characteristics allow easy identification of the appropriate yield on the paper of the marginal swaps market maker, and thus allow testing of my revised swap pricing model.

Data for this study came from the research department of a major Canadian bank. Swap rates as well as yields on similar-term fixed-rate bank paper, corporate bonds, and government of Canada bonds for terms of two, three, and five years were provided, based on weekly closing rates, for the six-year period from November 8, 1990, through January 31, 1997.

Swap rates are the average of bid-ask quotes on fixed-for-floating Canadian dollar swaps. Bank paper yields are the average of yields on AA-rated fixed-rate debentures issued by Canadian banks. Data on banks' floating-rate credit spreads were unavailable. Corporate bond yields are the average for single-A rated bonds issued by corporate issuers.

The average of bid-ask yields on similar-term current-coupon government of Canada Treasury bonds (GB) is used as a proxy for the general level of interest rates. All swaps rates as well as yields on bank paper and corporate bonds are restated as spreads over the similar-term Canada bond. These items are referred to as swap spreads (SS), bank paper spread (BPS), and corporate bond spread (CBS) throughout.

Exhibit 6 presents summary statistics for these variables for terms of two, three, and five years. The average values of BPS and CBS are similar at each maturity, and the term structure of each is upward-sloping. The average values of SS are noticeably lower, approximating only 60% of the average values of BPS and CBS. This finding is consistent with the empirical implications of the reconciled swap pricing model, since swap spreads should be related to the yield on similar-term fixed-rate bank liabilities *less* the similar-term floating-rate bank credit spread. It is also consistent with the finding of Sun, Sundaresan, and Wang [1993] that interbank rates are generally higher than swap rates.

VI. EMPIRICAL RESULTS

Exhibit 7 presents results of regressions of two-, three-, and five-year swap spreads on various combinations of bank paper spread (BPS), corporate bond spread (CBS), and Canadian government bond (GB) yield. The first three sets of results in Panels A, B, and C show that each of the independent variables is significant when included individually with an intercept in the regressions. The coefficients on these variables generally increase with the term of the swap, which is consistent with a generally increasing term structure of credit spreads.

These univariate results provide no evidence on which of the independent variables is most directly responsible for the variation in swap spreads. This finding is not surprising because of the correlation among these variables.

Further insight into which of the independent variables is most closely linked to swap spreads can be gained by studying the results for various combinations of the regressors. The strongest results relate to the bank paper spread. For all maturities, and no matter which additional explanatory variables are included, the coefficients on BPS are positive, significant, and somewhat lower than 1.0 in all cases. These results are robust to the addition of the other independent variables, including CBS.

These findings are precisely what is implied by the reconciled model of swap pricing. They indicate a strong relationship between swap spreads and bank paper spreads, which supports my central hypothesis.

Sun, Sundaresan, and Wang [1993] conduct similar tests in the U.S. dollar market, although for different reasons from mine. They use swap rates provided by two counterparties: one AAA-rated and one A-rated, neither of them a bank. They seek to compare these rates with the yields on similar-term par bonds of swap counterparties, but note some difficulties in identifying the appropriate yields because of differences in credit ratings among swap counterparties. Assuming that swap counterparties are represented by participants in the interbank deposit market, they conduct their tests based on par bond yields constructed from rates in the term LIBOR deposit market.

They find that midmarket swap rates are related to the yield on interbank par bond yields, but note that the

EXHIBIT 6
Summary Statistics

	2-Year	3-Year	5-Year
Swap Spread			
Mean	0.28%	0.37%	0.41%
Standard Deviation	0.17	0.21	0.23
Minimum	0.02	0.05	0.10
Maximum	0.87	0.83	0.84
Bank Paper Spread			
Mean	0.53	0.58	0.60
Standard Deviation	0.21	0.24	0.24
Minimum	0.15	0.06	0.19
Maximum	1.11	1.26	1.12
Corporate Bond Spread			
Mean	0.54	0.60	0.65
Standard Deviation	0.19	0.21	0.22
Minimum	0.22	0.23	0.26
Maximum	1.04	1.09	1.14
Government Bond Yield			
Mean	6.90	7.23	7.61
Standard Deviation	1.55	1.45	1.24
Minimum	3.72	4.23	5.11
Maximum	10.85	10.78	10.68

illiquidity of the interbank deposit market causes significant problems in testing their model. They also find that the interbank par bond yields are generally higher than the swap rates.

Despite the empirical difficulties in the U.S. dollar market, the Sun, Sundaresan, and Wang [1993] findings are consistent with the empirical implications of my research. Swap rates are closely related to the fixed-rate yield of the marginal swaps market maker, but should be somewhat lower because the floating-rate borrowing spread must also be taken into account.

Unlike the results for BPS, the multivariate results provide little evidence that corporate bond spreads have an appreciable direct effect on swap spreads. When both CBS and BPS are included as regressors, CBS is insignificant for the two- and five-year data. These findings are consistent with my model; that is, there should be no direct relationship between swap spreads and corporate bond yields.

When all three independent variables are included in the regression, the coefficient on BPS remains significant and positive, but the coefficient on CBS is never significant and positive. For the two-year data, the coefficient on CBS is significant but *negative* when all other independent variables are included in the regression. This finding is contrary to the positive relation between swap spread and corporate bond spreads proposed by other researchers.

A possible explanation may be found in Arak, Goodman, and Silver [1988], who argue that firms issue fixed-rate obligations when their future credit prospects are good, and swap from floating-rate instruments when their future credit prospects are poor. Thus demand by corporate counterparties to pay a fixed rate on a swap will tend to be greatest when future credit prospects are poor and firms are issuing relatively few fixed-rate bonds directly. In this case, corporate credit spreads may be narrower than usual because of the relatively small supply of corporate bonds on the market.

Conversely, swaps market makers would raise the fixed rate demanded from their corporate counterparties because of the relatively high demand by firms to pay a fixed rate on an interest rate swap. Thus the relationship between corporate bond spreads and swap spreads could be negative after controlling for variation in bank paper spreads, which is consistent with my empirical results.

The regression results for GB are also interesting. When all three independent variables are included in the regressions, the coefficient on GB is positive and significant in all cases. When only BPS and GB are in the

EXHIBIT 7

Regressions of Swap Spreads on Bank Paper and Corporate Bond Spreads

Panel A: 2-Year Data

Intercept	BPS	CBS	GB	Adj. R^2
-0.05	0.63*			0.64
(-1.25)	(8.36)			
-0.02		0.55*		0.42
(-0.26)		(4.65)		
-0.12			0.06*	0.29
(-1.00)			(2.82)	
-0.03	0.74*	-0.14		0.64
(-0.72)	(14.7)	(-1.31)		
-0.19	0.55*		0.03	0.69
(-1.79)	(18.40)		(1.90)	
-0.13		0.43*	0.03	0.46
(-0.95)		(7.80)	(1.38)	
-0.22*	0.85*	-0.42*	0.04*	0.74
(-3.11)	(12.30)	(-4.26)	(5.00)	

Panel B: 3-Year Data

Intercept	BPS	CBS	GB	Adj. R^2
-0.05	0.72*			0.66
(-1.77)	(11.40)			
-0.11		0.81*		0.67
(-2.19)		(9.54)		
-0.19			0.08*	0.28
(-1.59)			(3.66)	
-0.11	0.35*	0.46*		0.70
(-2.15)	(3.40)	(2.78)		
-0.33*	0.64*		0.05*	0.75
(-3.05)	(17.20)		(2.94)	
-0.23		0.74*	0.02	0.69
(-1.63)		(14.40)	(1.16)	
-0.32*	0.57*	0.08	0.04*	0.75
(-3.12)	(7.22)	(0.80)	(3.29)	

Panel C: 5-Year Data

Intercept	BPS	CBS	GB	Adj. R^2
-0.10*	0.84*			0.85
(-4.36)	(18.40)			
-0.19*		0.92*		0.81
(-3.80)		(12.30)		
-0.24			0.08*	0.22
(-1.52)			(3.74)	
-0.14*	0.60*	0.29		0.85
(-2.94)	(8.04)	(2.17)		
-0.29*	0.79*		0.03*	0.87
(-3.05)	(27.60)		(2.55)	
-0.27		0.88*	0.01	0.81
(-2.17)		(16.90)	(0.98)	
-0.29*	0.70*	0.11	0.03*	0.87
(-2.91)	(10.10)	(1.04)	(2.70)	

T-statistics in parentheses.

**Significant at the 1% level.*

EXHIBIT 8
Specification Tests

	2-Year	3-Year	5-Year	Critical Value*
Add CBS_t to (1), F =	2.2	15.6	6.7	4.7
Add BPS_t to (2), F =	72.6	10.9	35.0	4.7
Add CBS_t to (3), F =	14.3	0.3	0.7	3.9
Add BPS_t to (4), F =	82.1	19.6	33.4	3.9

*At the 1% level.

regressions, GB remains positive for the three- and five-year data, but is insignificant at two years. Thus there appears to be a direct relation between swap spreads and the general level of interest rates.

This direct relation is contrary to the inverse relation between the general level of interest rates and the corporate credit spread reported by Longstaff and Schwartz [1995]. It is consistent, however, with the explanation that future credit prospects are worst when interest rates are high.

The results of F-tests reported in Exhibit 8 confirm the overall specification of the regression. They also support the finding that swap rates are not directly determined by corporate bond spreads. These results are consistent with Minton [1997] and Lang, Litzenberger, and Liu [1998], who find that corporate bond yields are unable to explain swap rates completely.

VII. CONCLUSION

Swaps have typically been modeled either as an exchange of non-callable bonds or as a strip of forward contracts. I reconcile these two models by taking into account the presence of banks making markets in swaps in a competitive environment, to show that profitable opportunities are available to market makers unless swap rates are bounded by rates determined by the yields on similar-term bank liabilities. I also show that the yield on corporate bonds should have no significant direct influence on swap rates.

The reconciled models are tested using data on two-, three-, and five-year swap rates. These tests provide strong empirical evidence that swap rates are closely related to bank paper yields, and not directly related to corporate bond yields. The tests also find that differences between swap rates and bank paper yields can be at least partly explained by the yield on Treasury securities.

APPENDIX A

Consider a swaps market maker with \$100 nominal amount of outstanding fixed-rate bonds maturing in T years at semiannual coupon rate R_{fixed}, and \$100 nominal amount of outstanding floating-rate notes maturing at time T at coupon rate $R_{floating,i}$ paid at the end of each period $i = 1, \ldots, 2T$. Assume the market maker can also issue more bonds at these prices.

The price P_{fixed} of this fixed-rate bond can be expressed as:

$$P_{fixed} = \mathrm{E}^*\left[100e^{-\int_0^T r(s)ds} + \sum_{i=1}^{2T} R_{fixed}e^{-\int_0^i r(s)ds}\right] \tag{A-1}$$

where $\mathrm{E}^*[\cdot]$ denotes the risk-neutral expectation, and $r(s)$ is the risk-neutral discount rate at time s.

Similarly, the price $P_{floating}$ of the floating-rate note can be expressed as:

$$P_{floating} = \mathrm{E}^*\left[100e^{-\int_0^T r(s)ds} + \sum_{i=1}^{2T} R_{floating,i}e^{-\int_0^i r(s)ds}\right] \tag{A-2}$$

Assume the swaps market maker can also enter into an interest rate swap with a nominal amount of \$100 maturing at time T, where the market maker pays a fixed semiannual amount of S_{fixed} and receives a floating semiannual amount of $S_{floating,i}$ at the end of each period $i = 1, \ldots, 2T$.

The value V_{swap} of the swap to the swaps market maker can be expressed as:

$$V_{swap} = \mathrm{E}^*\left[\sum_{i=1}^{2T} S_{floating,i}e^{-\int_0^i r(s)ds} - \sum_{i=1}^{2T} S_{fixed}e^{-\int_0^i r(s)ds}\right] \tag{A-3}$$

Note that the value of a swap in the reverse direction (where the swaps market maker receives the fixed amount and pays the floating rate) is $-V_{swap}$. For ease of exposition, assume the fixed- and floating-rate bonds trade at par, no interest has yet accrued, and that the swap currently has no value, i.e., $P_{fixed} = P_{floating} = 100$ and $V_{swap} = 0$.

This implies that a riskless profit is available to the market maker at times $i > 0$ unless:

$$
\mathrm{E}^*\left[100e^{-\int_0^T r(s)ds} + \sum_{i=1}^{2T} R_{fixed}e^{-\int_0^i r(s)ds}\right]
$$

$$
-\mathrm{E}^*\left[100e^{-\int_0^T r(s)ds} + \sum_{i=1}^{2T} R_{floating,i}e^{-\int_0^i r(s)ds}\right]
$$

$$
+\mathrm{E}^*\left[\sum_{i=1}^{2T} S_{floating,i}e^{-\int_0^i r(s)ds} - \sum_{i=1}^{2T} S_{fixed}e^{-\int_0^i r(s)ds}\right] = 0 \tag{A-4}
$$

Assume that the index used for the floating rate on the notes is the index used for the floating rate of the swap, i.e., $R_{floating,i} = S_{floating,i} + C$ for $i = 1, \ldots, 2T$, where C represents the fixed spread of the floating-rate note coupon over the floating-rate index. This implies:

$$
S_{fixed} = R_{fixed} - C \tag{A-5}
$$

APPENDIX B

Consider a swaps market maker wanting to hedge the i-th exchange of payments on an interest rate swap of \$100 nominal amount maturing at time T where it pays a semiannual fixed rate S_{fixed}, and receives a semiannual floating rate $S_{floating,i}$ at the end of period i. Assume the market maker can repurchase \$100 million of its own fixed-rate bonds bearing a semiannual coupon rate $R_{fixed,i}$ and maturing in i periods (i.e., $i/2$ years), and that it can issue \$100 nominal amount of fixed-rate bonds bearing a semiannual coupon rate $R_{fixed,i-1}$.

The price $P_{fixed,i}$ of the bond to be repurchased can be expressed as:

$$
P_{fixed,i} = \mathrm{E}^*\left[100e^{-\int_0^i r(s)ds} + \sum_{j=1}^{i} R_{fixed,i}e^{-\int_0^j r(s)ds}\right] \tag{B-1}
$$

where $\mathrm{E}^*[.]$ denotes the risk-neutral expectation, and $r(s)$ is the risk-neutral discount rate at time s.

Similarly, the price $P_{fixed,i-1}$ of the bond to be issued can be expressed as:

$$
P_{fixed,i-1} = \mathrm{E}^*\left[100e^{-\int_0^{i-1} r(s)ds} + \sum_{j=1}^{i-1} R_{fixed,i-1}e^{-\int_0^j r(s)ds}\right] \tag{B-2}
$$

At the end of the period $i - 1$, the swaps market maker will issue a six-month note with principal amount of \$100 and coupon rate $R_{short,i}$, which will be set when this note is issued. Assume for ease of exposition that the prices of both bonds are \$100, and that the yield curve is flat, i.e., $R_{fixed,i} = R_{fixed,i-1}$.

Considering on a net basis both of the fixed-rate bonds, the floating-rate note and the i-th exchange on the interest rate swap result in zero cash flow at all times except at the end of the i-th period, when the net cash flow is:

$$
S_{floating,i} - S_{fixed} + R_{fixed,i} - R_{short,i} \tag{B-3}
$$

This implies a riskless profit is available to the market maker at the end of the i-th period unless:

$$
S_{fixed} = R_{fixed,i} - C_i \tag{B-4}
$$

where C_i represents the difference between the interest rate on the short-term note and the index for the floating-rate swap payment on the i-th swap exchange, i.e., $C_i = R_{short,i} - S_{floating,i}$. If C_i is not fixed in advance, the value of (B-4) is uncertain, i.e., there is basis risk between the swap and the way it is hedged.

APPENDIX C

Consider a swaps market maker wanting to hedge each of $2T$ exchanges on an interest rate swap of \$100 nominal amount maturing at time T where it pays a semiannual fixed rate S_{fixed}, and receives a semiannual floating rate $S_{floating,i}$. As shown in Appendix B, the net cash flow on the i-th hedged exchange can be written as:

$$
S_{fixed,i} = R_{fixed,i} - C_i \tag{C-1}
$$

where $C_i = R_{short,i} - S_{floating,i}$ for each period $i = 1, \ldots, 2T$. The present value of the net cash flows can be written as:

$$
\mathrm{E}^*\left[\sum_{i=1}^{2T}\left[R_{fixed,i} - C_i - S_{fixed}\right]e^{-\int_0^i r(s)ds}\right] \tag{C-2}
$$

where $\mathrm{E}^*[.]$ denotes the risk-neutral expectation, and $r(s)$ is the risk-neutral discount rate at time s. The market maker can eliminate variability in C_i by issuing a floating- rate note at time 0 with maturity T at coupon rate $R_{floating,i} = S_{floating,i} + C$ where C is constant in all periods $i = 1, \ldots, 2T$. Assuming the yield curve is flat, i.e., $R_{fixed,i} = R_{fixed}$ for all i implies there is a riskless profit opportunity available to the market maker unless:

$$
S_{fixed} = R_{fixed} - C \tag{C-3}
$$

ENDNOTES

The author gratefully acknowledges the generosity of CIBC/Wood Gundy Financial Products in providing the data for this study.

[1]Corporations may also engage in interest rate swaps because of private information. See Titman [1992], among others.

[2]See Appendix A for a mathematical presentation of this result.

[3]See Collin-Dufresne and Solnik [2001] for an analysis of term structure effects on swap rates.

[4]See Appendix B for a mathematical presentation of this result.

[5]See Appendix B for a mathematical presentation of this result.

[6]This is analogous to the refreshed credit quality assumption in Collin-Dufresne and Solnik [2001]. Note that Sun, Sundaresan, and Wang [1993] argue that paying a fixed rate and receiving a floating rate on a swap can be replicated by a issuing a fixed-rate note and investing the notional principal amount in a LIBOR deposit maturing on the next reset date. This approach is similar to my model except that it fails to consider the floating-rate spread of the market maker.

[7]See Appendix C for a mathematical presentation of this result.

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