Contingent Claims Valuation When Capital Structure Includes Options Liability

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Derivatives are often ignored in firm valuation because of their small transaction size. The importance of the leverage they introduce is also unclear, perhaps because of their complexity. In light of the growing use of derivatives by financial and non-financial firms, however, one cannot ignore the combined effect of several derivatives on other claims of equal or lower seniority.

This article shows how to adjust the traditional credit model based on contingent claims analysis when a capital structure includes a short position in a call or put option. The stochastic features of the asset underlying the option introduce added risk elements to the analysis of corporate claims valuation, complicating the pricing of credit risk for firms with significant off-balance sheet liabilities. Typical examples are financial or commodities firms.

The vulnerable options methodology shows how much options influence corporate claims. It is found that the values of debt, option, and equity are lower than in otherwise comparable firms. Numerical illustrations show that senior and subordinated debt values are lower and spreads are wider than without the options. The volatility of the asset underlying the call or put option and the volatility of the assets of the firm also appear to have a negative influence on debt value.

The Merton [1974] contingent claims approach to corporate debt valuation is an integral component of modern corporate finance. Although the basic model is theoretically sound, there have been several problems with fitting it empirically to observed bond prices. Some authors find the model tends to underestimate observed bond spreads (see Jones, Mason, and Rosenfeld [1984]). The model's unsatisfactory performance can be attributed to bond call features, bond covenants, sinking fund provisions, deviations from the absolute priority of claims, or bankruptcy costs.

The literature tends to focus on departures from the main assumptions of the Merton [1974] model. Recently, however, Anderson and Sundaresan [2000] have estimated an adjusted contingent claims model that nests alternative firm value-based models, to find that leverage and asset volatility explain much of the variation in observed corporate bond yields.

Another strand of literature has developed common ties with this research. Johnson and Stulz [1987] study the pricing of credit-risky options for the firm writing them. Holders of these "vulnerable" options are entitled to receive either the standard option payoff upon expiration, or the assets of the firm if that payoff is not fully attainable. The firm's assets enter the option valuation problem along with the underlying asset.

Thus, vulnerable options are more comprehensive than Black-Scholes [1973] options, which constitute the limiting case when credit risk is absent. Johnson and Stulz [1987] have shown that, by construction of the payoff, vulnerable options are worth less than otherwise identical standard options.

Hull and White [1995], Jarrow and Turnbull [1995], Klein [1996], and Klein and Inglis [1999, 2001] have extended Johnson and Stulz [1987] to include the valuation of options in the presence of stochastic interest rates or fixed liabilities of equal seniority to the written option. Their focus is mainly on the option and not the debt, even though debt is priced in some cases such as Klein [1996].

Cao and Wei [2001] directly address the effect of a derivative in the liability structure of the firm on debt spreads. They find that adding an option can lead to a positive short-term credit spread and a variety of possible credit spread term structures. One of their conclusions is that vulnerable options can sometimes be worth *more* than standard options. This counter-intuitive result may be attributable to their different assumptions about option payoffs.

A natural criticism of capital structure models attempting to include derivative liabilities would be the relatively small importance of short option positions for most firms. Although the economic value of a single option transaction may not be significant in altering a firm's leverage, multiple options are. Thus, the verdict on these models depends on the extent of the firm's derivatives activities.

Some real-world examples should indicate the nature of the problem. Financial firms take short positions in derivatives as part of their trading operations. Commodities firms also write large amounts of derivatives in an effort to hedge their output. In addition, many off-balance sheet (OBS) activities have option characteristics (e.g., an investment bank's option position in a takeover or an insurance company's position in a policy claim), or can be reduced to options for pricing purposes (e.g., a long forward contract may be decomposed into a long call and a short put).

The aggregate size of derivatives positions by financial firms has been growing dramatically in the last two decades. Even non-financial firms, especially large ones, tend to use derivatives, as shown by Howton and Perfect [1998] as well as Bodnar, Hyat, and Marston [1998] in studies of derivatives usage by U.S. corporations.

OBS activities can introduce huge leverage effects. Breuer [2002] attempts to measure derivatives leverage in banking by decomposing each claim into the equity and debt components of the portfolio replicating the derivative. He finds that OBS leverage of U.S. commercial banks exceeds on-balance sheet leverage several times over, especially for the top 25 banks. The failures of Barings Bank and the natural resources firm Enron should serve as reminders of the importance of OBS activities for leverage and the risk of corporate claims.

We use the Klein and Inglis [2001] model to price senior debt and a call option written by a firm. It is shown that the computed bond spreads are wider than in the Merton [1974] model. This result is further evidence of a bond spread underestimation problem, particularly for financial firms or any firm with significant OBS activities. Adding another liability in the capital structure obviously diminishes the value of other claims of equal or lower seniority than the option.

Because derivative liabilities do not show on the balance sheet, it is often unclear how they influence leverage, values, and spreads. We address this problem by refocusing the research started by Johnson and Stulz [1987] from the vulnerable option to other important claims in the capital structure such as debt and equity. This ties into the contingent claims research that associates the complexity of capital structure with the underperformance of the Merton [1974] credit model.

A special feature of the research is inclusion of subordinated debt (SD) as an additional component in the liability structure, following Black and Cox [1976]. Theoretical support for inclusion of SD is found in its role in solving incentive problems in banking. The market discipline it introduces may be useful in curbing excess risk-taking by bank owners. The significance of SD is highlighted in the Financial Services Modernization Act of 1999, which explicitly provides for a study on the use of such debt (see Lang and Robertson [2002] for a survey of the relevant proposals).

In our contribution to the discussion about the behavior of SD prices and spreads, we show that the Black-Cox [1976] model of SD as a bull spread on the assets of the firm can be adjusted when the financial firm has an over-the-counter call or put option liability. SD values in the presence of an option are lower than otherwise comparable SD values, implying, of course, wider spreads.

I. MODEL WITH A CALL OPTION AND NO SUBORDINATED DEBT

Suppose a financial firm with asset value W has issued zero-coupon non-callable debt with a face value of D maturing at time T. The firm has also written a European OTC call option on a non-dividend-paying asset S with an exercise price X expiring at the same time as the debt. Asset values follow logarithmic diffusions. The volatility of each asset is denoted by σ_W and σ_s, and the correlation coefficient between the two asset returns is $\rho_{S,W}$. The debt and option liabilities have equal seniority as claims on the final assets of the firm in the event of bankruptcy.

Suppose also that the value of the firm is independent of its capital structure, and financial distress does not affect the joint distribution of asset returns. We assume that markets are complete in order to create replicating portfolios for assets that are not traded.

Under these assumptions, the value of claims can be found by solving the usual partial differential equations for derivatives, subject to appropriate boundary constraints. The values can also be found by discounting the risk-neutral expected payoffs at the risk-free rate, r. Note that the firm can be in default even if its assets are sufficient to meet the debt obligation. Debt or option holders receive either their full claim or a prorated amount of the firm's assets. Thus, the values of debt, G, option, c, and equity, H, are given by:

$$G = e^{-rT} E_{S,W}\left\{\min\left(aW_T, D\right)\right\} \tag{1}$$

$$c = e^{-rT} E_{S,W}\left\{\min\left(bW_T, \max\left(S_T - X, 0\right)\right)\right\} \tag{2}$$

$$H = W - G - c \tag{3}$$

where $E_{S,W}$ denotes the expectations operator under the risk-neutral probability measure over the two assets. The proportions of the two claims on the final assets of the firm are given by:

$$a = \frac{D}{\max\left(S_T - X, 0\right) + D} \quad \text{and} \quad b = \frac{\max\left(S_T - X,0\right)}{\max\left(S_T - X,0\right) + D} \tag{4}$$

The min function in Equation (1) gives the final payoff to bondholders, which is D if the firm is solvent and aW_T if the firm's assets are insufficient to meet both claims outstanding. Similarly, the min function in Equation (2) shows that the payoff to the option holder is either the standard Black-Scholes payoff or a proportion of the firm's assets.

We have ignored for simplicity any collateral posted by the option writer as well as a write-down factor in the value of the options as in Klein and Inglis [2001]. The possibility of early default is precluded for tractability. The assumption is not as restrictive as it seems. As explained in Klein and Inglis [1999], OTC options are governed by the standardized contract recommended by the International Swaps and Derivatives Association, which does not require payment acceleration when financial distress occurs.

Because of the non-linear nature of Equation (2), Klein and Inglis [2001] solve for the option price numerically using three-dimensional binomial trees, but the problem structure accepts a straightforward calculation by integration. If lower-case symbols denote asset values at time T, the values of the debt and option are:

$$G = \frac{e^{-rT}}{\sigma_S \sigma_W T}\left[\int_0^\infty \int_0^\infty \frac{1}{sw} \min\left(aw, D\right) f_2 \, ds\, dw\right] \tag{5}$$

$$c = \frac{e^{-rT}}{\sigma_S \sigma_W T}\left[\int_0^\infty \int_0^\infty \frac{1}{sw} \min\left(bw, \max(s - X, 0)\right) f_2 \, ds\, dw\right] \tag{6}$$

where:

$$f_2 = f_2(x) = (2\pi)^{-1} |J|^{-1/2} \exp\left[-\frac{1}{2} x' J^{-1} x\right] \tag{6-A}$$

is the density function of the standard bivariate normal distribution with mean (0, 0) and correlation matrix:

$$J = \begin{bmatrix} 1 & \rho_{S,W} \\ \rho_{S,W} & 1 \end{bmatrix} \tag{6-B}$$

The elements of vector $x = (x_1, x_2)$ are given by:

$$x_1 = \frac{\ln\left(s/S\right) - \left(r - \sigma_S^2 T/2\right)}{\sigma_S \sqrt{T}} \tag{6-C}$$

$$x_2 = \frac{\ln\left(w/W\right) - \left(r - \sigma_W^2 T/2\right)}{\sigma_W \sqrt{T}} \tag{6-D}$$

All the integrals are computed using the *Mathematica* software, and can be easily generalized to multiple options.*

In the original Merton [1974] model, creditors hold a portfolio of risk-free debt and a short position in a put option on the assets of the firm where the terminal value of debt plays the role of exercise price. Creditors write this put to stockholders, who exercise it when the assets of the firm are exceeded by the firm's debt obligation at debt maturity.

This is a special case of Equation (1) if we set a equal to unity. Because of the joint normality of the variables, the underlying asset price is "integrated out" and eliminated. Thus, when there is no option claim, equity is reduced to a standard call option on the value of the firm where debt plays the role of the exercise price (Black and Scholes [1973]). We may call this value of equity, H_{BS}.

Similarly, in Johnson and Stulz [1987], the financial firm has no other liabilities except the option, so their model is a special case of the above if we set $D = 0$ in Equation (2). The problems of a firm issuing warrants on its own stock or distributing employee stock options are related to this model, but the complexity of warrant valuation, especially with respect to equity dilution, makes these problems beyond the scope of this research.

II. NUMERICAL ILLUSTRATIONS: CALL OPTION

Some numerical examples will illustrate the difference between standard models and our models above. We must emphasize that the nominal amounts involving the option are set artificially large to make the differences in values easier to see, although reducing the size of the derivative claim leaves the qualitative results intact.

Exhibit 1, Panel A, shows the results of Equations (1)-(3).

Lower Values Due to the Call

A general observation is that the values of all three claims are lower than their standard counterparts, as predicted by the model. For example, the value of debt in the presence of the call option, G, is lower than the value of the Merton [1974] debt, G_M. The value of the vulnerable call, c, is lower than the value of the Black-Scholes [1973] call, c_{BS}. This result is contrary to that in Cao and Wei [2001], who find that sometimes their vulnerable options can be worth more than standard options. The value of equity, H, is lower than the value of equity as a call option on the assets of the firm, H_{BS}.

These results are to be expected, because there is an added liability, namely, the option that takes a share of the final assets of the firm, which remain constant. In this example, the sum of the vulnerable debt and the option creates a combined contingent liability that is greater than standard debt alone. Actually, equity is negatively affected by the presence of the option, particularly for lower levels of the value of the firm, just where credit risk becomes binding.

These results have significant implications for credit risk measurement. Exhibit 2 graphs the value of risk-free debt, Merton [1974] debt, and debt in the presence of the option as a function of firm value. We can clearly see that the shareholders' put option component of the debt value is much higher in this case. This makes G smaller than G_M. Furthermore, the difference between risk-free debt and risky debt becomes pronounced at much lower asset values than the Merton [1974] debt does. Since debt values translate to spreads for traded bonds, the model predicts wider spreads than the Merton [1974] model, especially for poorer credit quality bonds.

Exhibit 3 displays the values of vulnerable and Black-Scholes options as a function of the underlying asset price, S. As the option is more in the money, the option value curve becomes concave with respect to S. The reason is that the option commands a greater share of the aggregate claim, eventually approaching the value of the firm's assets, W, which stays fixed. This extreme case is likely when the firm is close to financial distress due to excessive derivative liabilities.

Changes in Parameters and the Effect on Contingent Claims

It would be useful to see how changes in one parameter affect each contingent claim if the other parameters are held constant. Relevant predictions of the model are not always clear a priori, and an interpretation of the results is necessary.

Effect of volatility of the asset underlying the option. Increases in volatility, σ_S, of the asset underlying the option reduce the value of debt, as the increased option value takes a greater share of the final assets. The Merton [1974] debt is unaffected by changes in this volatility, as expected. At the same time, the values of both the Black-Scholes and the vulnerable option rise as expected.

The effect of increased σ_S on equity is unclear, depending on the relative size of the drop in debt and the increase in option value. In this particular example, as shown in the last column of Panel A in Exhibit 1, equity value increases. The usual Black-Scholes equity remains unchanged because it does not depend on the volatility of the optioned asset.

Effect of volatility of the assets of the firm. The volatility of the assets of the firm writing the option, σ_W, has a negative effect on the option value (compare the three figures \$8.66, \$8.76, and \$8.49 in the vulnerable option, c, column). This interesting result can be explained intuitively by the fact that the value of the firm is like collateral to option holders. If that collateral cannot be

EXHIBIT 1

Numerical Values of Contingent Claims: Call Option

								PANEL A							PANEL B				
Parameters								Senior Debt			Call Option		Equity		Subordinated Debt			Equity	
σ_S	σ_W	$\rho_{S,W}$	r	T	S	W	k	G_{RF}	G_M	G	c_{BS}	c	H_{BS}	H	g_{RF}	g_{BC}	g	H_{BC}	H_g
0.1								32	31.53	28.18	10.31	7.92	8.47	3.90	2.90	2.49	1.22	5.98	2.68
0.2	**0.2**	**0.0**	**0.10**	**1.0**	**100**	**40**	**0.8**	**32**	**31.53**	**27.16**	**13.27**	**8.66**	**8.47**	**4.18**	**2.90**	**2.49**	**1.31**	**5.98**	**2.87**
0.3								32	31.53	26.43	16.73	9.09	8.47	4.48	2.90	2.49	1.37	5.98	3.11
	0.1							32	31.98	27.66	13.27	8.76	8.02	3.58	2.90	3.04	1.47	4.98	2.11
	0.3							32	30.59	26.36	13.27	8.49	9.41	5.14	2.90	2.13	1.23	7.28	3.92
		-0.9						32	31.53	26.27	13.27	7.37	8.47	6.36	2.90	2.49	1.51	5.98	4.85
		0.9						32	31.53	28.54	13.27	10.25	8.47	1.21	2.90	2.49	0.81	5.98	0.40
			0.05					32	31.53	28.09	10.45	6.97	8.47	4.93	3.04	2.49	1.53	5.98	3.41
			0.15					32	31.53	26.14	16.36	10.41	8.47	3.44	2.75	2.49	1.09	5.98	2.35
				0.5				32	31.88	29.22	8.28	6.31	8.12	4.47	3.04	2.80	1.66	5.33	2.80
				1.5				32	31.14	25.60	17.67	10.33	8.86	4.07	2.75	2.31	1.12	6.55	2.95
					90			32	31.53	29.25	6.95	4.92	8.47	5.83	2.90	2.49	1.78	5.98	4.05
					110			32	31.53	24.64	21.25	12.66	8.47	2.70	2.90	2.49	0.87	5.98	1.83
						30		24	23.64	19.44	13.27	7.64	6.36	2.91	2.17	1.87	0.90	4.49	2.01
						50		40	39.41	35.02	13.27	9.41	10.59	5.57	3.62	3.11	1.74	7.48	3.83
							0.7	28	27.90	24.60	13.27	9.17	12.10	6.23	2.53	2.59	1.48	9.51	4.75
							0.9	36	34.56	29.25	13.27	8.16	5.44	2.59	3.26	2.06	1.02	3.37	1.57

σ_S = volatility of underlying asset, σ_W = volatility of firm's assets, $\rho_{S,W}$ = correlation coefficient between assets, r = risk-free rate, T = time to expiration and debt maturity, S = price of asset underlying the call option, W = value of firm's assets, k = quasi-debt ratio $e^{-rT}D/W$ (D = amount due to senior creditors at time T. Amount due to SD creditors = 10% of D).

G_{RF} = risk-free debt, G_M = Merton [1974] debt ignoring option, G = debt in the presence of option, c_{BS} = Black-Scholes [1973] call, c = vulnerable call, H_{BS} = Black-Scholes [1973] equity ignoring option, H = equity in the presence of option and debt, g_{RF} = risk-free subordinated debt (SD), g_{BC} = Black-Cox [1976] SD, g = SD in the presence of option and debt, H_{BC} = Black-Cox [1976] equity ignoring option, H_g = equity in the presence of SD and option.

Except when otherwise indicated, parameter values are: W = \$40, S = \$100, k = 0.80, $\sigma_W = \sigma_S = 0.2$, $\rho_{S,W} = 0$, r = 0.1. The base case appears in boldface.

EXHIBIT 2
Value of Debt

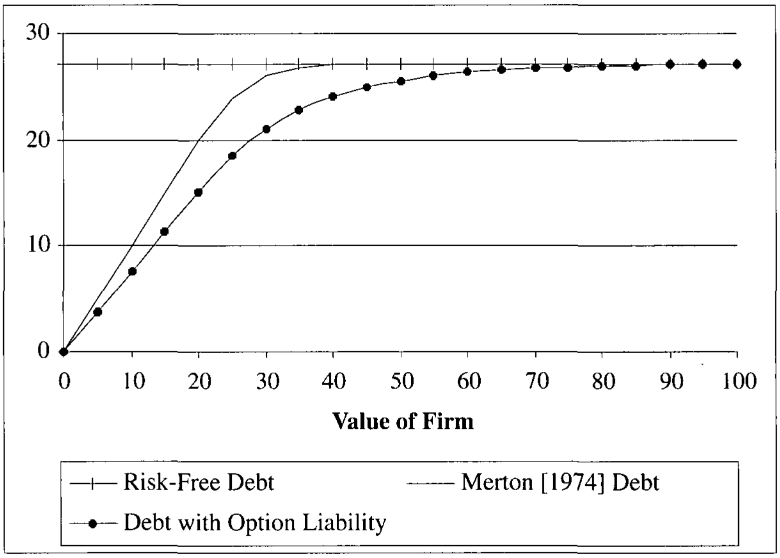
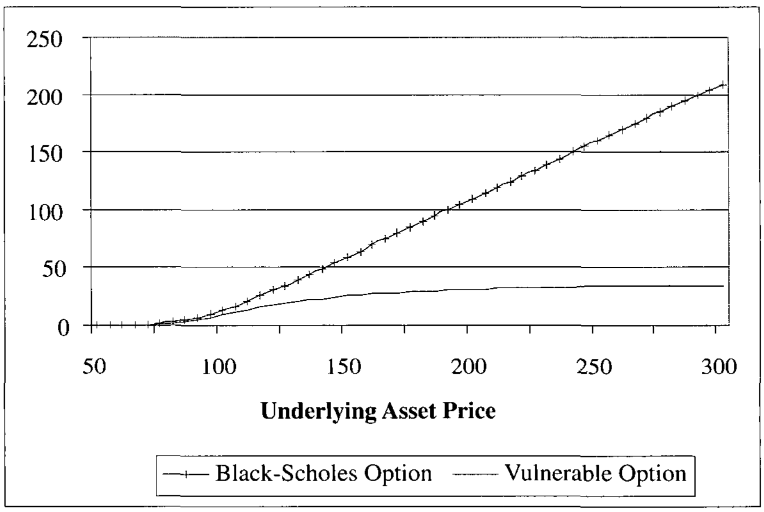


EXHIBIT 3
Value of Call Options



counted on to be available at expiration of the option, the option becomes less valuable. Of course, the Black-Scholes call value is unaffected, because it does not depend on σ_W.

Both types of debt, G_M and G, are reduced by increases in σ_W, while both types of equity, H_{BS} and H, are increased with it, since equity is a residual claim. We can conclude that the two sources of uncertainty, that is, from the asset underlying the option and from the assets of the firm, appear to have a positive effect on equity value.

Effect of correlations among assets. The correlation between the underlying asset and the assets of the firm needs special mention. As the correlation coefficient, $\rho_{S,W}$, increases from −0.9 to 0.0 and to 0.9, this increases the value of the vulnerable call dramatically, while it leaves the standard call value intact. Perhaps a positive correlation means that, when the underlying asset price increases, the value of the assets of the firm also tends to increase. In this example, the reverse does not lead to a large loss in collateral value, so the total effect is positive.

The case is similar for debt, G. If both debt and the option have a higher value due to the increased correlation, however, this takes a toll on equity. As expected, the usual debt value, G_M, is unaffected by the correlation parameter.

This result has important implications for banking in general because correlations among assets tend to be higher in bad times. It follows that the equity of financial firms can be severely affected by increased correlations among the assets underlying options and the assets of those firms, other things held constant.

Effect of underlying asset price and value of the assets of the firm. A rise in the underlying asset price has a positive effect on the value of both types of call options and a negative effect on the value of debt and equity, while the Merton [1974] debt and standard equity are unaffected by such changes. Similarly, the value of the firm is positively related to the values of all claims except the Black-Scholes option, as expected.

Effect of interest rate, time to maturity, and debt leverage. A rise in the interest rate has a positive effect on both options. In fact, the effect on the vulnerable call is more pronounced. The reason that the risk-free debt and the Merton [1974] debt are not affected is rather technical. In order to maintain a constant quasi-debt ratio, $k = e^{-rT}D/W$, we have set D as the solution to this equation. A constant quasi-debt ratio (as in Merton [1974]) serves as a fixed point of reference in the analysis.

We see that G is reduced by interest rate increases, apparently because the option is receiving a greater share of the final payoff. Time to expiration T has a similar effect. Finally, increasing the quasi-debt ratio leaves fewer assets for the option holders and reduces the value of the vulnerable call, while the standard option is unchanged. Debt value increases, because debt now represents a greater share of the firm's assets, and both types of equity are negatively affected.

These results have implications for hedging. As an example, let us take the vulnerable option. The usual option delta does not fully capture the effect of changes in the underlying asset.

It is instructive to look at the delta of c_{BS} and the delta of c. Exhibit 4 shows that the vulnerable option delta is lower than that of the Black-Scholes option and eventually declines with option moneyness (see also Exhibit 3).

III. SUBORDINATED DEBT INCLUDED

The value of zero-coupon subordinated debt has the unique property of being the difference between the values of two call options on the value of the firm, as shown in Black and Cox [1976]. Actually, investors holding SD hold a bull spread with exercise prices D and $D + B$ where B is the nominal amount of junior debt due at time T.

In banking theory, SD is considered a useful market discipline tool. SD spreads over risk-free debt have been proposed as a mechanism triggering prompt corrective action by bank regulators. Therefore, it should be useful to see the behavior of SD as parameters change, with and without an option claim.

First, let us set up the model. If there is no option liability, the standard Black-Cox [1976] value of SD, g_{BC}, can be found by discounting the expected payoff to the SD holders:

$$g_{BC}=e^{-rT}E_W\left\{\min\left(B, \max\left(W_T - D, 0\right)\right)\right\} \tag{7}$$

According to Equation (7), SD holders receive the smaller of either the amount B that is due to them or the difference between final asset value and the payoff to senior creditors. In that case, the value of equity, H_{BC}, is just the residual:

$$H_{BC} = W - G_M - g_{BC} \tag{8}$$

If there is an option liability, its value has to be subtracted from the payoff to SD holders, while equity is again given by the residual payoff:

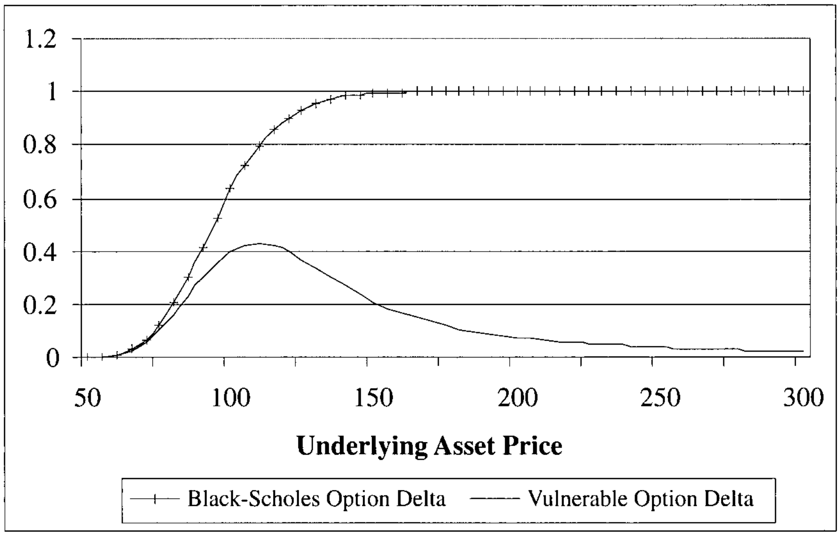
$$g = e^{-rT}E_{S,W}\left\{\min\left(B, \max\left(\begin{matrix} W_T - D - \\ \max(S_T - X, 0),0\end{matrix}\right)\right)\right\} \tag{9}$$

$$H_g = W - G - c - g \tag{10}$$

Lower SD Values Due to the Option

Equations (7) and (9) are computed using numerical integration. Panel B of Exhibit 1 displays the value of standard SD, g_{BC}, and SD with the option claim, g, as a function of firm value. Note the difference in value from the risk-free SD, as predicted by the model. Differences are translated into wider SD spreads, which become obvious much earlier as the value of the assets falls. Apparently, without the option written, we have the Black-Cox [1976] SD as a special case.

Exhibit 4
Call Option Delta



Effect of Changes in Parameters on SD and Equity

When SD is added, the values of debt and the option do not change, because these claims are senior to SD. Thus, equity is the only claim that is affected.

Effect of volatility of asset underlying the option. The first three rows in Panel B of Exhibit 1 display the value of standard SD without the option liability, g_{BC}, and the value of SD with the option liability for different volatilities of the underlying asset. We observe that both are lower than the risk-free debt level, but the standard SD and equity are naturally unaffected by σ_S (compare the equity columns in Panels A and B).

Effect of volatility of the assets of the firm. It is interesting to see the effect of asset volatility on junior debt. Because the value of standard SD is the difference between two calls, increases in σ_W increase both calls, leading to a less pronounced SD graph in Exhibit 5. This is a strong point in favor of using SD as a tool for market discipline. SD holders would be very sensitive to drops in asset value *and* to increases in volatility. Of course, at asset values lower than the first exercise price, D, increases in volatility are welcome.

Suppose now we add the call option to the firm's liability structure. The value of SD appears to be less sensitive to increases in volatility than without the option (according to a comparison of the proportionate changes in g and those of g_{BC}). Equity appears to be negatively affected by this parameter.

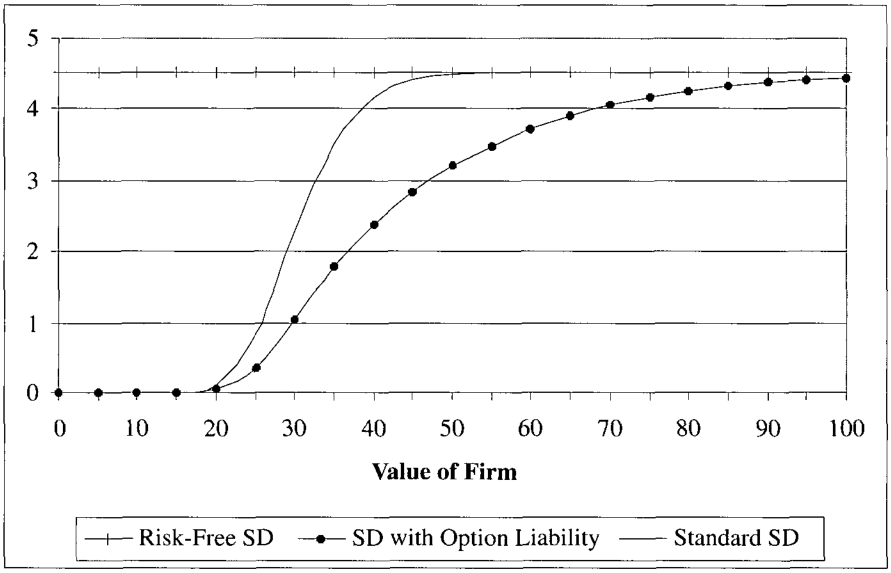
Effect of correlation between assets. Standard junior debt is unaffected by the correlation of the asset underlying the option and the assets of the firm, but g is negatively affected in this case. This is because correlation increases the value of the senior debt and the option, leaving less to claim for the SD and equityholders.

The implication is that the presence of off-balance sheet activities may make holders of subordinated debt highly sensitive to firms' asset correlation with the optioned assets. The same is true for equityholders.

Effect of underlying asset price and value of the assets of the firm. The underlying asset price is positively related to the option value, which translates into a lower SD and equity value. The firm's asset value has a positive effect on SD and equity that is more pronounced than in the standard case.

Effect of interest rate, time to maturity, and debt leverage. Increases in the interest rate do not affect g_{st} and H_{st}, because B and D are stated as discounted proportions of the firm value to maintain the quasi-debt ratio constant. Time to maturity of the option and debt have a negative effect on SD and equity. Finally, increases in the debt ratio reduce the value of SD and equity, as debt takes a greater share of firm value.

EXHIBIT 5
Value of Subordinated Debt (SD)



IV. MODEL WITH A PUT OPTION

Exhibit 6 describes values in the case of the vulnerable put option. Only the most important results are noted here.

The qualitative results are similar to those in the call option case. In particular, all contingent claim values are lower than their standard counterparts. Furthermore, the direction of the effect of parameter changes is also the same, with two notable exceptions: The effect of the underlying asset price and the correlation between assets are negatively related to the value of the vulnerable put option.

The first relation is to be expected because the option becomes more out of the money. The second result is not obvious, but it is opposite to that in the call case.

V. VARYING SIZE OF OPTION LIABILITY

Investors must sometimes compare debt spreads of two firms with almost identical equity and risk characteristics. Thus, it would be useful to see how the presence of a call or put affects debt values. This can be achieved by using a factor, q, indicating the size of the firm's liability on the option. For instance, a factor of 0.5 would mean the firm is taking only half of the given option payoff. Then the firm size can be varied, along with increases in q from 0 to 3, so that equity value stays constant.

The new claims are given by the modified Equations (1)-(4) for a given q:

$$G = e^{-rT} E_{S,W}\left\{\min\left(aW_T, D\right)\right\} \tag{1'}$$

$$c = e^{-rT} E_{S,W}\left\{\min\left(bW_T, q\max\left(S_T - X, 0\right)\right)\right\} \tag{2'}$$

$$H = W - G - c \tag{3'}$$

The new proportions now are:

$$a = \frac{D}{q\max\left(S_T - X, 0\right) + D} \quad \text{and} \quad b = \frac{q\max\left(S_T - X, 0\right)}{q\max\left(S_T - X, 0\right) + D} \tag{4'}$$

The prediction of the model is that, as we add more of the option, the option liability will dominate the total senior claims of the firm [proportion b will rise—see also Equation (2′)], while debt's share of the firm value, a, will decline. Thus, debt spreads will widen.

The first half of Exhibit 7 displays the numerical results of computing Equations (1′)-(4′) for the call option. The put results are displayed in the second half. Interestingly, the Merton [1974] debt spreads actually narrow. This is natural because, as firm size grows and the quasi-debt ratio declines, the value of the Merton [1974] debt

EXHIBIT 6
Numerical Values of Contingent Claims: Put Option

								PANEL A							PANEL B				
Parameters								Senior Debt			Put Option		Equity		Subordinated Debt			Equity	
σ_S	σ_W	$\rho_{S,W}$	r	T	S	W	k	G_{RF}	G_M	G	p_{BS}	p	H_{BS}	H	g_{RF}	g_{BC}	g	H_{BC}	H_g
0.1								32	31.53	31.33	0.79	0.73	8.47	7.94	2.90	2.49	2.37	5.98	5.57
0.2	**0.2**	**0.0**	**0.10**	**1.0**	**100**	**40**	**0.8**	**32**	**31.53**	**30.33**	**3.75**	**3.03**	**8.47**	**6.64**	**2.90**	**2.49**	**2.02**	**5.98**	**4.63**
0.3								32	31.53	29.09	7.22	5.17	8.47	5.75	2.90	2.49	1.75	5.98	4.00
	0.1							32	31.98	30.90	3.75	3.10	8.02	6.01	2.90	3.04	2.37	4.98	3.63
	0.3							32	30.59	29.37	3.75	2.93	9.41	7.70	2.90	2.13	1.79	7.28	5.92
		-0.9						32	31.53	31.41	3.75	3.68	8.47	4.90	2.90	2.49	2.21	5.98	2.70
		0.9						32	31.53	29.55	3.75	2.44	8.47	8.02	2.90	2.49	2.19	5.98	5.83
			0.05					32	31.53	29.70	5.57	4.35	8.47	5.95	3.04	2.49	1.82	5.98	4.13
			0.15					32	31.53	30.78	2.43	2.01	8.47	7.21	2.75	2.49	2.17	5.98	5.04
				0.5				32	31.88	30.93	3.40	2.93	8.12	6.14	3.04	2.80	2.22	5.33	3.93
				1.5				32	31.14	29.92	3.74	2.91	8.86	7.18	2.75	2.31	1.92	6.55	5.26
					90			32	31.53	29.07	7.43	5.74	8.47	5.20	2.90	2.49	1.62	5.98	3.58
					110			32	31.53	30.99	1.73	1.44	8.47	7.56	2.90	2.49	2.26	5.98	5.30
						30		24	23.64	22.40	3.75	2.79	6.36	4.81	2.17	1.87	1.46	4.49	3.36
						50		40	39.41	38.27	3.75	3.18	10.59	8.54	3.62	3.11	2.60	7.48	5.95
							0.7	28	27.90	27.12	3.75	3.20	12.10	9.68	2.53	2.59	2.18	9.51	7.50
							0.9	36	34.56	32.96	3.75	2.84	5.44	4.19	3.26	2.06	1.62	3.37	2.57

σ_S = volatility of underlying asset, σ_W = volatility of firm's assets, $\rho_{S,W}$ = correlation coefficient between assets, r = risk-free rate, T = time to expiration and debt maturity, S = price of asset underlying the put option, W = value of firm's assets, k = quasi-debt ratio $e^{-rT}D/W$ (D = amount due to senior creditors at time T. Amount due to SD creditors = 10% of D).

G_{RF} = risk-free debt, G_M = Merton [1974] debt ignoring option, G = debt in the presence of option, p_{BS} = Black-Scholes [1973] put, p = vulnerable put, H_{BS} = Black-Scholes [1973] equity ignoring option, H = equity in the presence of option and debt, g_{RF} = risk-free subordinated debt (SD), g_{BC} = Black-Cox [1976] SD, g = SD in the presence of option and debt, H_{BC} = Black-Cox [1976] equity ignoring option, H_g = equity in the presence of SD and option.

Except when otherwise indicated, parameter values are: W = \$40, S = \$100, k = 0.80, $\sigma_W = \sigma_S = 0.2$, $\rho_{S,W}$ = 0, r = 0.1. The base case appears in boldface.

EXHIBIT 7
Changing Option Size While Holding Equity Constant

			Senior Debt			Call	Put	Equity
W	k	q	G_{RF}	G_M	G	c	p	H
40.00	0.80	0	32	31.53	31.53	0.00		8.47
44.75	0.72	0.5	32	31.85	30.49	5.79		8.47
47.38	0.68	1	32	31.93	28.99	9.92		8.47
49.04	0.65	1.5	32	31.95	27.61	12.95		8.47
50.18	0.64	2	32	31.97	26.43	15.27		8.47
51.01	0.63	2.5	32	31.97	25.42	17.12		8.47
51.65	0.62	3	32	31.98	24.55	18.63		8.47
40.00	0.80	0	32	31.53	31.53		0.00	8.47
41.48	0.77	0.5	32	31.67	31.26		1.74	8.47
42.37	0.76	1	32	31.73	30.75		3.15	8.47
42.91	0.75	1.5	32	31.77	30.18		4.25	8.47
43.26	0.74	2	32	31.79	29.65		5.14	8.47
43.50	0.74	2.5	32	31.80	29.17		5.86	8.47
43.68	0.73	3	32	31.81	28.74		6.46	8.47

W = value of firm's assets, k = quasi-debt ratio e-rTD/W (D = amount due to senior creditors), q = factor showing the size of the option liability [see Equations (1')-(4')], G_{RF} = risk-free debt, G_M = Merton [1974] debt ignoring option, G = debt in the presence of option, c = vulnerable call, p = vulnerable put, H = equity in the presence of option and debt.

Parameter values are: W = \$40, S = \$100, $k = 0.80$, $D = 35.37$, $\sigma_W = \sigma_S = 0.2$, $\rho_{S,W} = 0$, $r = 0.1$, $T = 1$.

approaches the value of risk-free debt.

This exercise reaffirms our main claim that ignoring option liabilities leads to an underestimation of spreads. In fact, the more active a firm is with respect to derivative liabilities, the more severe the spread underestimation problem becomes.

VI. CONCLUSION

We have demonstrated how the traditional credit model based on contingent claims analysis can be adjusted when the capital structure includes short positions in options. Some characteristic results from the numerical illustrations are that senior debt and vulnerable option, equity, and subordinated debt claims all have lower values than their standard counterparts. Debt values are lower than in the standard case without the option liability, particularly for risky firms, and spreads are wider. This result may be useful in explaining the credit underestimation problem of standard credit models based on contingent claims.

The numerical illustrations also show that debt value is reduced by increases in volatility of both the assets of the firm and the price of the asset underlying the option. Additional results regarding the effect of other parameters on the value of contingent claims can be useful in the valuation and hedging of all claims of financial firms as well as firms with considerable OBS activities.

ENDNOTES

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*We can efficiently approximate multiple integrals in higher dimensions using a quasi-Monte Carlo (QMC) technique. The typical estimation error in QMC increases with the integral dimension but it is mainly dependent on the sample size (see Evans and Swartz [2000]).

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