Valuing Euro Rating-Triggered Step-Up Telecom Bonds

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The authors value rating-triggered step-up bonds using three methods: a risk-neutral valuation framework; a similar framework based on historical probabilities; and as plain vanilla bonds. The market seems to value single step-up bonds according to the first model, while it values multiple step-up bonds as plain vanilla bonds. Further, step-up feature market premiums are more volatile than risk-neutral or historical premiums, and the risk-neutral model always approximates market premiums better than the historical method. Finally, most step-up bonds offer a cushion against rating migrations via dampened price movements.

European telecom companies have issued rating-triggered step-up coupon bonds in order to compensate bond investors for losses in the event of rating downgrades. McAdie, Martin, and Okane [2001], Fumagalli and Taurén [2001], and Sirinathsingh [2001] have analyzed step-up bonds. These authors largely use historical and subjective rating transition probabilities in their valuation, and show the results of their model for only one day.

Our approach is to apply the risk-neutral valuation framework of Jarrow, Lando, and Turnbull [1997] (JLT) to value step-up bonds, and we demonstrate the model's results over a long time period. For comparison, we also value step-up bonds using historical transition probabilities and as equivalent plain vanilla bonds, i.e., similar bonds except for the step-up feature.

We analyze the protection the step-up feature offers to investors in two new ways. First, we compare the volatility of a step-up bond to the equivalent plain vanilla bond. Second, we determine whether the step-up bond offers better returns than the equivalent plain vanilla bond in the case of rating downgrades and negative outlooks.

Our results indicate that the market seems to use the JLT model to value step-up bonds that make a single step-up after the rating trigger. Step-up bonds that make multiple step-ups seem to be treated as plain vanilla bonds. Also, the JLT model always approximates the step-up feature premium better than the historical method. We further find that the step-up feature reduces bond price volatility for most of the step-up bonds considered. Finally, for all bonds in our sample, the step-up feature does not offer investors positive excess returns in case of a rating downgrade or a trend toward a negative outlook.

Step-up bonds are a relatively new phenomenon in the euro-denominated corporate bond market. The coupon of a step-up bond depends on its issuer's rating or the rating of the issuer's long-term debt. If the rating deteriorates and hits a predefined level, the step-up feature is triggered, and the coupon rises a predefined amount. Depending on the exact specification, the coupon can rise even more if the rating deteriorates further. For most step-up bonds, the reverse also applies; the coupon is reduced if the rating is raised.

Both issuers and investors benefit from the step-up feature, as McAdie, Martin, and O'Kane [2000] indicate. Issuers have placed more debt at lower yields than would otherwise have occurred. Investors have profited too, because they will be compensated in the case of rating downgrades, and because issuers should be more committed to preserve their ratings, as a downgrade will penalize them directly in the form of higher coupons.

We analyze five step-up bonds issued by three companies: Deutsche Telecom, France Telecom, and KPN. These are the only three companies in our data set that have issued both step-up bonds and enough euro-denominated plain vanilla bonds that we can reliably estimate issuer-specific interest rate curves. Despite this limited number of step-up bonds, our study is worthwhile because it provides one of the first empirical tests of the JLT model on rating-sensitive instruments and because the behavior of step-up bonds has not yet been documented in the academic literature.

Various investment banks have published research on the characteristics and valuation of step-up bonds. J.P. Morgan uses historical transition and default probabilities in its valuation; see Sirinathsingh [2001]. Lehman Brothers estimates the probabilities subjectively according to analysts' opinions; see McAdie, Martin, and O'Kane [2000]. Schroder Salomon Smith Barney applies both subjective and historical transition probabilities; see Fumagalli and Taurén [2001]. Société General implements a JLT model form and uses risk-neutral probabilities; see Turc [2001].

These studies all show the results of their analysis for only one day. We instead price step-up bonds for a longer period (March 2001–February 2002), and implement three pricing methods.

Despite considerable interest from practitioners, only one article has appeared about the valuation of euro step-up bonds, as far as we know (Conroy [2000]). Conroy values step-up bonds using historical rating transition probabilities. Risk-neutral valuation models that can be used to price rating-triggered instruments have appeared elsewhere in the academic literature. The basis for most of these models is the JLT Markov chain model, which uses a firm's credit ratings as an indicator of the likelihood of default. Kijima and Komoribayashi [1998] adjust the JLT model to make it numerically more stable by replacing default probabilities with survival probabilities in the calculation of risk premiums.

We would expect the JLT model to generate step-up bond valuations that are more in line with corresponding market prices than the historical valuation method, because the JLT model takes the market risk premium into account, while the historical method does not.

Das and Tufano [1996] generalize the JLT model to incorporate stochastic recovery rates. Lando [1998] and Arvanitis, Gregory, and Laurent [1999] extend it to make transition and default intensities stochastic and possibly dependent on state variables. Schönbucher [1999], Bielecki and Rutkowski [2000], and Acharya, Das, and Sundaram [2002] embed the Markov chain in a Heath, Jarrow, and Morton [1992] framework with stochastic forward rates. Acharya, Das, and Sundaram [2002] also illustrate their model for a sample step-up bond.

In independent work, Lando and Mortensen [2003] also analyze step-up bonds in the JLT framework. Although their work is similar to ours, they focus on refining the JLT model, while we compare three different valuation methods. Further, we analyze the protection the step-up feature offers to investors.

I. STEP-UP BONDS

The coupon of a step-up bond depends on the issuer's rating or the rating of its long-term debt. If the rating deteriorates and hits a predefined level, the step-up feature is triggered, and the coupon rises by a predefined number of basis points. Depending on the type of step-up coupon, the coupon can rise even more if the rating deteriorates further. For most step-up bonds, the reverse also applies; the coupon is reduced if the rating improves—this is called the step-down feature. The coupon can never go below the original level at issuance, though.

Step-up conditions can differ among bonds. The most important discriminating conditions of the step-up bonds are the following: whether the coupon can step up and down or only step up; whether both Moody's and S&P or only one of them must downgrade an issuer before the step-up trigger is hit; the timing of the coupon adjustment; the rating-trigger level; and the number of basis points of the step-up.

Exhibit 1 defines three types of step-up bonds.

II. MODEL

We first describe the Jarrow, Lando, and Turnbull [1997] model, and then explain the risk-neutral valuation of step-up bonds.

EXHIBIT 1
Step-Up Bond Types

Type	Step-up[a]	Step-down[b]	One-off[c]	And/or[d]	Accrual[e]
A	✓	✓		or	next
B	✓	✓	✓	and	next
C	✓		✓	or	immediately

[a]*Coupon increases if the rating is reduced and hits the rating trigger.*

[b]*Coupon is reduced if the rating increases again.*

[c]*Coupon increases only once, even if the rating falls farther below the rating trigger; for bonds that are not one-off, each further decline in the rating causes a further increase in the coupon.*

[d]*Determines whether the coupon is adjusted if Moody's* and *S&P adjust their ratings, or if Moody's* or *S&P adjusts its rating.*

[e]*Determines the date at which the adjusted coupon starts accruing:* immediately *following a rating action, or on the* next *coupon date after the rating action.*

Rating Transitions

The value of a bond equals the sum of the discounted expected cash flows. Unlike a plain vanilla bond, a step-up bond's coupons are a function of the issuer's rating. We thus have to model the issuer's rating transition process under the equivalent martingale measure. The JLT model provides a suitable framework for this purpose, since it uses a company's credit rating as an indicator of creditworthiness.

Following the JLT framework, we assume a unique equivalent martingale measure $\tilde{\mathbb{Q}}$ exists that makes all default-free and defaultable bond prices martingales, after normalization by the default-free money market account. Finally, the recovery rate δ is constant; we follow Jarrow and Turnbull [2000] and Schönbucher [2000] by assuming that δ applies to the principal only, not to the coupons.

The company's credit rating R_t at day t is modeled as a Markov chain in a finite state space $S = \{1, \ldots, K\}$ under the historical probability measure. Under $\tilde{\mathbb{Q}}$, default-free interest rates and ratings are assumed to be independent. The state space S includes all possible ratings, including the intermediate ratings. State 1 represents the Aaa rating, state 2 Aa1, state 3 Aa2, ..., state $K-1$ Caa, and the last state, K, default. It is assumed that default is an absorbing state.

Under the Markov property, it holds for all $i, j \in S$, $t \geq s \geq 0$ and $r_u \in S$, $0 \leq u < s$ that:

$$q_{ij}(s,t) = \mathbb{P}(R_t = j | R_s = i, R_u = r_u, 0 \leq u < s)$$

$$= \mathbb{P}(R_t = j | R_s = i)$$

That is, the probability of going from rating i to rating j in the period from s to t depends only on the rating R_s at time s and not on the history R_u, $0 \leq u < s$, of reaching that rating. We are aware of the limitations of this assumption, as past rating movements do seem to affect future rating transitions, as shown by Nagpal and Bahar [2000], but this is an assumption common to most theoretical models.

To value step-up bonds, we need the rating transition process under the risk-neutral measure $\tilde{\mathbb{Q}}$. JLT start with the observed historical transition probabilities, such as from a rating transition matrix of Moody's or S&P, and apply risk premiums to transform these into risk-neutral probabilities. We choose to adjust the $(T-t)$-year transition matrix $Q(t, T)$ to get the risk-neutral transition matrix $\tilde{Q}(t, T)$ as follows:[1]

$$\tilde{q}_{ij}(t,T) = \begin{cases} \pi(t,T)q_{ij}(t,T) & \text{for } j \neq K \\ 1 - \pi(t,T)(1 - q_{iK}(t,T)) & \text{for } j = K \end{cases} \quad (1)$$

for some risk premium $\pi(t, T)$. It follows that the risk premium can be calculated as

$$\pi(t,T) = \frac{1 - \tilde{q}_{R_t,K}(t,T)}{1 - q_{R_t,K}(t,T)} \quad (2)$$

The risk premium for time T is the ratio of the risk-neutral survival probability to the historical survival probability, so that we retain the numerical stability of Kijima and Komoribayashi [1998] (hereafter KK). Note that we use cumulative probabilities $\tilde{q}_{ij}(t, T)$ instead of JLT's and KK's forward probabilities $\tilde{q}_{ij}(t, t+1)$:

$$\tilde{q}_{ij}(t, t+1) = \pi_i(t, t+1) q_{ij}(t, t+1)$$

The advantage of our risk premium approach is that the calculation is easier, because risk premiums do not require matrix inversion; see JLT's Equation (16) and KK's Equation (19). Both approaches, forward and cumulative, though, generate the same results. Note also that we use only one risk premium for all rating categories. Ideally, we would like to use a separate risk premium for each rating, but each euro-denominated telecom issuer does not cover the full rating spectrum, so we derive one risk premium from the issuer's current rating, and apply that to all ratings.

For each day, we estimate the issuer-specific survival probability curve of each telecom company. Following Houweling and Vorst [2002, Section 4], we specify a linear hazard function, assume a recovery rate of 50%, and use the euro zero-coupon swap curve as a proxy for the default-free term structure. The parameters of the hazard function are estimated from the market prices of the issuer's plain vanilla bonds using non-linear least squares.

Given the estimated survival probability curve for a company on a particular day, we calculate its risk premium (2) and risk-neutral transition matrices (1) for all required maturities. These risk-neutral matrices are used to calculate theoretical values for step-up bonds. We also calculate 95% confidence bounds for the survival probability curve, and repeat this series of calculations for the upper and lower bound, hence obtaining upper and lower bounds for the step-up bond values as well.

Step-Up Valuation

To determine the theoretical value of a defaultable step-up and step-down bond, we add another assumption to the set of JLT assumptions: Both Moody's and S&P alter their ratings of an issuer at the same time.[2] In our analysis, we apply the Moody's rating actions.

We use three methods to value step-up bonds of types A and B in Exhibit 1, starting with the JLT model. The path-dependent step-up bonds, type C, are not covered by these methods.

Consider a step-up and step-down bond with n remaining coupon payments and a face value of 1. The bond issuer makes the j-th coupon payment at day t_j, $j = 1, 2, \ldots, n$, but only if the firm has not gone into default before t_j. If the rating at t_{j-1} is equal to r, the coupon payment at t_j is equal to c_r, $r = 1, \ldots, K$. The coupon payment at t_1 depends on the rating at t_0, which we define as the previous coupon date, or, if there is no previous coupon, the issue date.

The step-up bond's principal amount is paid at maturity t_n, again only if the issuer has not defaulted before t_n. If the issuer does default before the bond matures, the constant recovery rate δ of the notional is paid at the default time.

Applying the risk-neutral valuation principle to these coupon, principal, and recovery cash flows yields:

$$\begin{aligned} B^{JLT}(t, \mathbf{t}, \mathbf{c}) = \sum_{j=1}^{n} p(t, t_j) \tilde{\mathbb{E}}_t \left[\mathbf{1}_{\{\tau > t_j\}} c_{R_{t_{j-1}}} \right] + \\ p(t, t_n) \tilde{\mathbb{E}}_t \left[\mathbf{1}_{\{\tau > t_n\}} \right] + \tilde{\mathbb{E}}_t \left[p(t, \tau) \mathbf{1}_{\{\tau \leq t_n\}} \delta \right] \quad (3) \end{aligned}$$

where $\mathbf{t}$ is an n-vector with the coupon payment dates; $\mathbf{c}$ is a K-vector with the coupon percentages per rating category; $p(t, T)$ denotes the time t default-free discount factor for time T; $\tilde{\mathbb{E}}_t [X]$ denotes the $\tilde{\mathbb{Q}}$-expected value of X, given the information at day t; and $\mathbf{1}_{\{A\}}$ is the indicator function of event A. τ denotes the first date on which a default occurs, and $\tau > t_n$ indicates no default before maturity. The first line of Equation (3) expresses the coupon payments, the second line the principal payment and the potential recovery payment.

For $j > 1$, the first risk-neutral expectation in Equation (3) can be evaluated as:[3]

$$\begin{aligned} \tilde{\mathbb{E}}_t \left[\mathbf{1}_{\{\tau > t_j\}} c_{R_{t_{j-1}}} \right] &= \sum_{k=1}^{K} \tilde{\mathbb{P}}_t (\tau > t_j \wedge R_{t_{j-1}} = k) c_k \\ &= \sum_{k=1}^{K} \tilde{\mathbb{P}}_t (\tau > t_j | R_{t_{j-1}} = k) \tilde{\mathbb{P}}_t (R_{t_{j-1}} = k) c_k \\ &= \sum_{k=1}^{K} \tilde{q}_{R_t, k}(t, t_{j-1}) (1 - \tilde{q}_{k,K}(t_{j-1}, t_j)) c_k \end{aligned}$$

where $\wedge$ is the logical *and* operator, and $\tilde{\mathbb{P}}_t(A)$ is the risk-neutral probability of event A, given the information at day t; for $j = 1$, the coupon amount is already known (because R_{t_0} is already known), so:

$$\tilde{\mathbb{E}}_t\left[\mathbf{1}_{\{\tau>t_1\}}c_{R_{t_0}}\right] = \tilde{\mathbb{P}}_t(\tau > t_1)c_{R_{t_0}}$$

$$= (1 - \tilde{q}_{R_t,K}(t,t_1))c_{R_{t_0}}$$

After evaluating the second and third risk-neutral expectations in Equation (3), the JLT value of our rating-triggered step-up and step-down bond equals

$$B^{JLT}(t,\mathbf{t},\mathbf{c}) = p(t,t_1)(1-\tilde{q}_{R_t,K}(t,t_1))c_{R_{t_0}} +$$
$$\sum_{j=2}^{n}\left[p(t,t_j)\sum_{k=1}^{K}\tilde{q}_{R_t,k}(t,t_{j-1})(1-\tilde{q}_{k,K}(t_{j-1},t_j))c_k\right] +$$
$$p(t,t_n)(1-\tilde{q}_{R_t,K}(t,t_n)) +$$
$$\sum_{j=1}^{n}p(t,t_j)(\tilde{q}_{R_t,K}(t,t_j)-\tilde{q}_{R_t,K}(t,t_{j-1}))\delta \qquad (4)$$

where we follow Houweling and Vorst [2002, Section 4] by replacing the integral that results from the recovery payment with a numerical approximation with potential default dates equal to the coupon payment dates.

Note that the only difference between Equation (4) and the value of a plain vanilla (PV) bond:

$$B^{PV}(t,\mathbf{t},c) = \sum_{j=1}^{n}p(t,t_j)(1-\tilde{q}_{R_t,K}(t,t_j))c +$$
$$p(t,t_n)(1-\tilde{q}_{R_t,K}(t,t_n)) +$$
$$\sum_{j=1}^{n}p(t,t_j)(\tilde{q}_{R_t,K}(t,t_j)-\tilde{q}_{R_t,K}(t,t_{j-1}))\delta \qquad (5)$$

is the part relating to the specific coupon structure of the step-up bond. If we set $c_k = c$ for all k, the expected value of the j-th coupon payment reduces to that of a plain vanilla bond:

$$\sum_{k=1}^{K}\tilde{q}_{R_t,k}(t,t_{j-1})(1-\tilde{q}_{k,K}(t_{j-1},t_j))c =$$
$$c\left(\sum_{k=1}^{K}\tilde{q}_{R_t,k}(t,t_{j-1}) - \sum_{k=1}^{K}\tilde{q}_{R_t,k}(t,t_{j-1})\tilde{q}_{k,K}(t_{j-1},t_j)\right)$$
$$= c(1-\tilde{\mathbb{P}}_t(R_{t_j}=K))$$
$$= c(1-\tilde{q}_{R_t,K}(t,t_j))$$

In a second valuation method, we treat the step-up bond as a bond that is identical except for the step-up feature. We refer to this bond as the *equivalent plain vanilla bond* (EPV). As in the step-up bond, the first coupon of this bond also depends on the rating at t_0, but the remaining coupons are assumed to follow from the current rating; i.e., they are all equal to c_{R_t}. The value of the equivalent plain vanilla bond thus equals:

$$B^{EPV}(t,\mathbf{t},\mathbf{c}) = p(t,t_1)(1-\tilde{q}_{R_t,K}(t,t_1))c_{R_{t_0}} +$$
$$\sum_{j=2}^{n}p(t,t_j)(1-\tilde{q}_{R_t,K}(t,t_j))c_{R_t} +$$
$$p(t,t_n)(1-\tilde{q}_{R_t,K}(t,t_n)) +$$
$$\sum_{j=1}^{n}p(t,t_j)(\tilde{q}_{R_t,K}(t,t_j)-\tilde{q}_{R_t,K}(t,t_{j-1}))\delta$$

The difference from the JLT formula (4) is in the second line, which no longer includes a summation over possible future ratings, but instead assumes the current rating R_t will prevail. The EPV formula also strongly resembles the PV Equation (5), except that the first coupon may differ from the other coupons.

The third method we consider to value step-up bonds, the *historical valuation method*, is based on the methods that investment banks often apply, e.g., McAdie, Martin, and O'Kane [2000] and Fumagalli and Taurén [2001]. This method uses the telecom company's zero-coupon curve to discount expected coupons, where the expectation is calculated using historical transition probabilities rather than risk-neutral probabilities; again, the first coupon is known.

This gives:

$$B^{H}(t,\mathbf{t},\mathbf{c}) = v(t,t_1)c_{R_{t_0}} +$$
$$\sum_{j=2}^{n}\left[v(t,t_j)\sum_{k=1}^{K}q_{R_t,k}(t,t_{j-1})c_k\right] + v(t,t_n)$$

where $v(t, T)$ denotes the issuer's time t discount factor for time T.

EXHIBIT 2
Numbers of Bonds for All Telecom Companies

	All bonds	Quoted, plain vanilla bonds	Step-up bonds
British Telecom Group	5	0	3
Deutsche Telecom	29	14	2
France Telecom	37	7	2
KPN	10	5	2
Olivetti SPA/Tecnost	12	0	4
Telecom Italia	7	2	2

III. DATA

Most euro-denominated step-up bonds have been issued by telecom companies. Analyzing these rating-triggered step-up corporate bonds thus automatically means we focus on the telecom sector. Step-up bonds are an important source of financing for telecom companies. As of the end of March 2001, step-up bonds accounted for 42% of the market capitalization of the telecom bond market; see Fumagalli and Taurén [2001].

We analyze the euro-denominated step-up coupon telecom bonds as listed by both Lehman Brothers and J.P. Morgan (see McAdie, Martin, and O'Kane [2000] and Marchakitus, Soderberg, and Bramley [2001]). For these European telecom companies, we download their main characteristics and the price time series for all their bonds from Bloomberg on a daily basis for the period from January 4, 1999, through February 13, 2002. We use the Bloomberg generic (BGN) price. The BGN price is an average of prices quoted by a long list of banks and brokers, and reflects the bid side of London closing. BGN prices are also used to price the Bloomberg/EFFAS government bond indexes; see Brown [1994].

We make sure the bonds used in the curve estimation are plain vanilla. We classify a bond as a plain vanilla bond if the bond has no step-up language and no embedded options, and if it is not floating or convertible. From the downloaded prices, we remove quotes that equal the quote of the preceding days and quote spikes. We seldom have to remove quotes, though.

Exhibit 2 shows the number of all euro-redenominated bonds, plain vanilla bonds, and euro-denominated step-up coupon bonds of the telecom companies that have issued step-up bonds. Three issuers, Deutsche Telecom, France Telecom, and KPN, have more quoted plain vanilla bonds than the other telecom companies, British Telecom, Olivetti/Tecnost, and Telecom Italia.

EXHIBIT 3
Characteristics of Step-Up Bonds

	Type[a]	Step-up[b]	Trigger[c]
DEUTSCHE TELECOM			
2005	B	50	Baa1/BBB+
2010	B	50	Baa1/BBB+
FRANCE TELECOM			
2004	A	25	Baa1/BBB+
2008	A	25	Baa1/BBB+
KPN			
2006	A	37.5	Baa3/BBB−

[a] *Type of step-up bond; see Exhibit 1.*
[b] *Number of basis points the coupon steps up.*
[c] *Rating-trigger level (Moody's/S&P).*

Our analysis focuses on these step-up bonds of the first three issuers only, because we need to estimate issuer-specific survival probability curves, which requires a certain number of plain vanilla bonds. These three companies are large corporate bond issuers as together they represent 6.5% of the Lehman Brothers Euro-Aggregate Corporate Bond Index on May 31, 2002.

Exhibit 3 displays the characteristics of the step-up bonds we use in our analysis: the step-up type, as defined in Exhibit 1; the number of basis points step-up; and the rating-trigger level. We restrict ourselves to step-up bonds with step-up *and* step-down coupons, because among our three telecom issuers just one KPN bond has step-up-only language.

A step-up-*only* bond, type C in Exhibit 1, differs from a bond with step-up and step-down features, types A and B, because the coupon of a step-up-only bond is path-dependent. This means that if the rating is below the trigger-level at any time before the coupon date, the coupon payment includes the step-up, even after the issuer is upgraded to or even above the pre-step-up rating again; this path-dependence characteristic necessitates a different valuation procedure from that in Equation (4). In short, once the step-up is triggered, a step-up-only bond becomes a plain vanilla bond.

This is exactly what happened to the KPN step-up only bond seven months after its issuance. Therefore, from the first coupon date after this rating event, we treat it as a plain vanilla bond and use it in our estimation of the KPN curve.

We download the Moody's and S&P rating and outlook history of Deutsche Telecom, France Telecom, and KPN from Bloomberg as well. From Moody's, we use the issuer's rating and from S&P the rating of long-term debt in local currency.

Exhibit 4 shows the rating and outlook migrations of Deutsche Telecom, France Telecom, and KPN by both Moody's and S&P for the sample period. A positive (negative) outlook is denoted by a +(-) sign. The plots show that the rating dynamics of both agencies are very similar. At the end of our period, the two agencies assign identical ratings to each telecom company.

For a historical transition matrix, we use Moody's average one-year senior rating transition matrix for corporate bond issuers, estimated from 1983 through 2001; see Cantor, Hamilton, and Ou [2002].

Finally, euro swap rates are downloaded from Bloomberg. We apply a standard bootstrapping procedure to extract zero-coupon rates and interpolate linearly between the available maturities to get a curve for all required maturities.

IV. RESULTS

We present general results and results for each company and elements of the analysis. We also examine the protection that step-up procedures offer bondholders.

Step-Up Bond Values and Step-Up Premiums

We define the *pricing error* of a step-up bond as its market price minus its theoretical value. Specifically, PE_{bt}^{i} is the pricing error of bond b at day t, if method i is used to calculate the theoretical value, $i \in \{EPV, JLT, H\}$ (EPV is the equivalent plain vanilla bond; JLT is the Jarrow, Lando, and Turnbull model; and H is the historical model).

APE_{bt}^{i} denotes the *absolute pricing error* for the three methods. The sample means of these six statistics are denoted by MPE_{b}^{i} and $MAPE_{b}^{i}$. A negative (positive) sign of an MPE statistic indicates that theoretical values are, on average, too high (too low).

To test whether this over- or underestimation of step-up bond prices is significant for method i, we apply a *one-sample Z-test* (see, e.g., Arnold [1990, Chapter 11]):

$$Z_b^i = \sqrt{N_b}\frac{\text{MPE}_b^i}{S_b^i}$$

where N_b is the sample size for bond b, and S_b^i is the sample standard deviation of the PE_b^i series. Asymptotically, Z_b^i has a standard normal distribution. Similarly, in order to determine if there are significant performance differences among the three methods, we also use a *paired Z-test*; see Arnold [1990, Chapter 11]. This test tells us whether two time series have the same mean, while allowing for non-zero correlation and unequal variances. The test statistic to compare measures i and j is defined as:

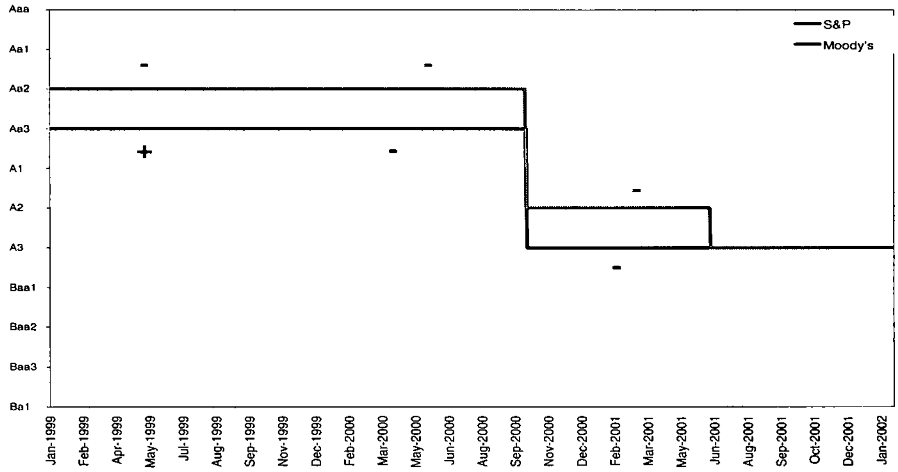
$$Z_b^{ij} = \sqrt{N_b}\frac{M_b^{ij}}{S_b^{ij}}$$

where M_b^{ij} and S_b^{ij} are the sample mean and sample standard deviation, respectively, of D_{bt}^{ij}: $= APE_{bt}^{i} - APE_{bt}^{j}$, $i, j \in \{EPV, JLT, H\}$, $i \neq j$, $t = 1, \ldots, N_b$. Asymptotically, Z_b^{ij} also has a standard normal distribution.

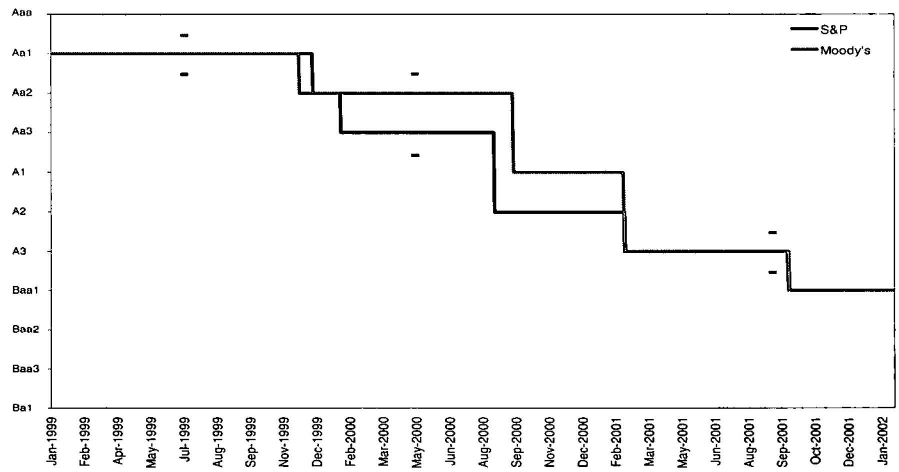
Exhibit 5 shows the MPE and MAPE statistics for the telecom step-up bond values generated by the three valuation methods and, for comparison, the MAPE values for the plain vanilla bonds used in estimation of the issuer-specific probability curves.[4] The paired Z-tests are presented in Exhibit 6 for all combinations of the three valuation methods. In an alternative test of the performance of the three methods, we calculate in Exhibit 7 the percentage of days that the market price of a step-up bond lies between the 95% confidence bounds of the calculated theoretical values. This tells us how uncertainty in the estimated survival probability curves translates into uncertainty in the calculated theoretical step-up bond values.

We also zoom in on the value of the step-up fea-

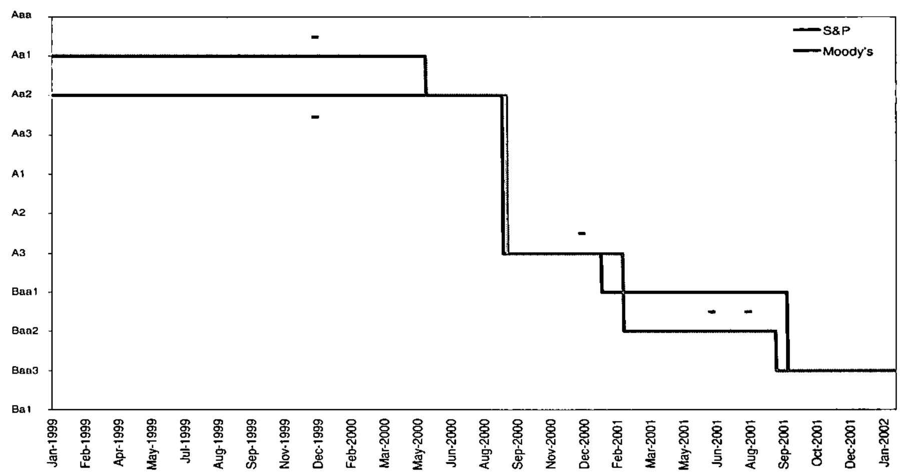
Exhibit 4
Credit Ratings History



Deutsche Telecom



France Telecom



KPN

EXHIBIT 5
Pricing Errors†

	Plain vanillas	Step-ups					
		EPV		JLT		Historical	
	MAPE	MPE	MAPE	MPE	MAPE	MPE	MAPE
DEUTSCHE TELECOM							
plain vanilla	0.29						
2005		0.15*	0.24*	-0.09*	0.25*	-0.29*	0.36*
2010		1.05*	1.15*	0.20*	0.82*	-0.34*	0.89*
FRANCE TELECOM							
plain vanilla	0.64						
2004		-0.26*	0.33*	-0.41*	0.46*	-0.56*	0.57*
2008		-0.87*	0.94*	-1.99*	1.99*	-2.57*	2.57*
KPN							
plain vanilla	1.09						
2006		-0.44*	1.13*	-0.95*	1.42*	-2.28*	2.29*

† Mean absolute pricing errors (MAPE) of plain vanilla bonds and both mean pricing errors (MPE) and MAPEs of step-up bonds for equivalent plain vanilla (EPV), Jarrow, Lando, and Turnbull [1997, JLT], and historical valuation methods.

**Indicates significance of the one-sample Z-test at a 95% confidence level.*

ture. We define the *market premium* of the step-up feature as the step-up bond's market price minus the value of the equivalent plain vanilla bond. Similarly, we calculate the *JLT premium* (or the *historical premium)* of the step-up feature as the JLT (or the historical) value of the step-up bond minus the value of the equivalent plain vanilla bond.

By subtracting the value of the equivalent plain vanilla bond, we "correct" the market, JLT, and historical values for all bond characteristics except for the step-up feature. We thus assume that the step-up feature fully determines the remainder of performance, and no other factors are of importance. We calculate the 95% confidence interval around the premium using the confidence bounds of the equivalent plain vanilla bond.

As in the pricing errors introduced for the step-up bond prices, we look at the differences between the market step-up premium and the JLT and historical premiums. As all three figures incorporate the value of the equivalent plain vanilla bond as a correction term, the last four columns in Exhibit 5 and the last column in Exhibit 6 also apply to the step-up premium. Exhibit 7 does not pertain to the step-up premium; the coverage percentages for step-up premiums are very similar to those for the step-up bond prices, and are therefore omitted.

Deutsche Telecom. The MPE values in Exhibit 5 tell that the EPV method typically undervalues and the JLT and historical methods usually overvalue the 2005 bond. The JLT model has the smallest bias to the market price, since both MPE^{EPV} and MPE^{H} differ more from zero than MPE^{JLT}, even though all three MPEs are statistically different from zero. The percentages in Exhibit 7 also show that the JLT model better approximates the step-up market price than the EPV and historical methods. Although the bond values vary across methods, the paired Z-test in Exhibit 6 shows that these differences are small, and in the case of the difference between the EPV and JLT methods not even statistically significant.

Exhibit 8, Panel A, shows the market, JLT, and historical step-up premiums and the 95% confidence bounds around the market premium for the 2005 Deutsche Telecom bond.[5] Both the JLT and historical premiums are positive and more or less constant, except for a small increase caused by Moody's downgrade of the company on June 13, 2001; the historical premiums are greater than the JLT premiums. The market premiums by contrast are much more volatile and sometimes even negative.[6]

A negative market premium for the step-up feature is counterintuitive; as long as the step-up has not been trig-

EXHIBIT 6
Paired Z-Tests†

	EPV – JLT	EPV – Historical	JLT – Historical
DEUTSCHE TELECOM			
2005	0.00	-0.12*	-0.12*
2010	0.33*	0.26*	-0.07*
FRANCE TELECOM			
2004	-0.13*	-0.24*	-0.11*
2008	-1.04*	-1.63*	-0.58*
KPN			
2006	-0.30*	-1.16*	-0.86*

†Pairwise differences between mean absolute pricing errors of the equivalent plain vanilla (EPV), Jarrow, Lando, and Turnbull [1997, JLT], and historical values for the step-up bonds.

**Indicates significance of the paired-sample Z-test at a 95% confidence level.*

EXHIBIT 7
Confidence Interval Coverage Percentages

	EPV	JLT	Historical
DEUTSCHE TELECOM			
2005	73%	85%	62%
2010	33%	68%	50%
FRANCE TELECOM			
2004	90%	85%	72%
2008	48%	0%	6%
KPN			
2006	82%	76%	33%

gered yet, the coupon cannot be reduced, and hence the step-up feature must have a positive value. Even though the negative market premiums may suggest inefficient market pricing, in practice it is very hard or even impossible to exploit this imperfection, as there are no bonds available that are comparable to the step-up bonds except for the step-up feature. Even if there were such comparable bonds, it would be either impossible or very expensive to take a short position in these bonds.

From the confidence bounds of the market premiums, however, it follows that the negative market premiums are not statistically significant. The market premium fluctuates mostly between 0 and 1, except in August 2001, when it is above 1, and in November and December 2001, when it is below 0.

For the 2010 bond, the MPE values in Exhibit 5 for the JLT and historical methods are relatively low, while for the EPV method the MPE value is much higher. Further, all three valuation methods give statistically significant different values at a confidence level of 95%, according to Exhibit 6. Exhibit 7 also shows that the JLT model generates the highest coverage percentages.

The market step-up premium of the 2010 bond displays an even more volatile pattern than the premium of the 2005 bond; see Exhibit 8, Panel B. This makes sense, because with its longer maturity, more coupons are affected by the step-up language. During the first half of the period, until October 2001, the market premium is higher than the JLT and historical premiums; later this pattern is reversed. The market step-up premium becomes negative in December 2001 and January 2002, but again the value is not significant. Both JLT and historical premiums remain roughly constant over the entire period.

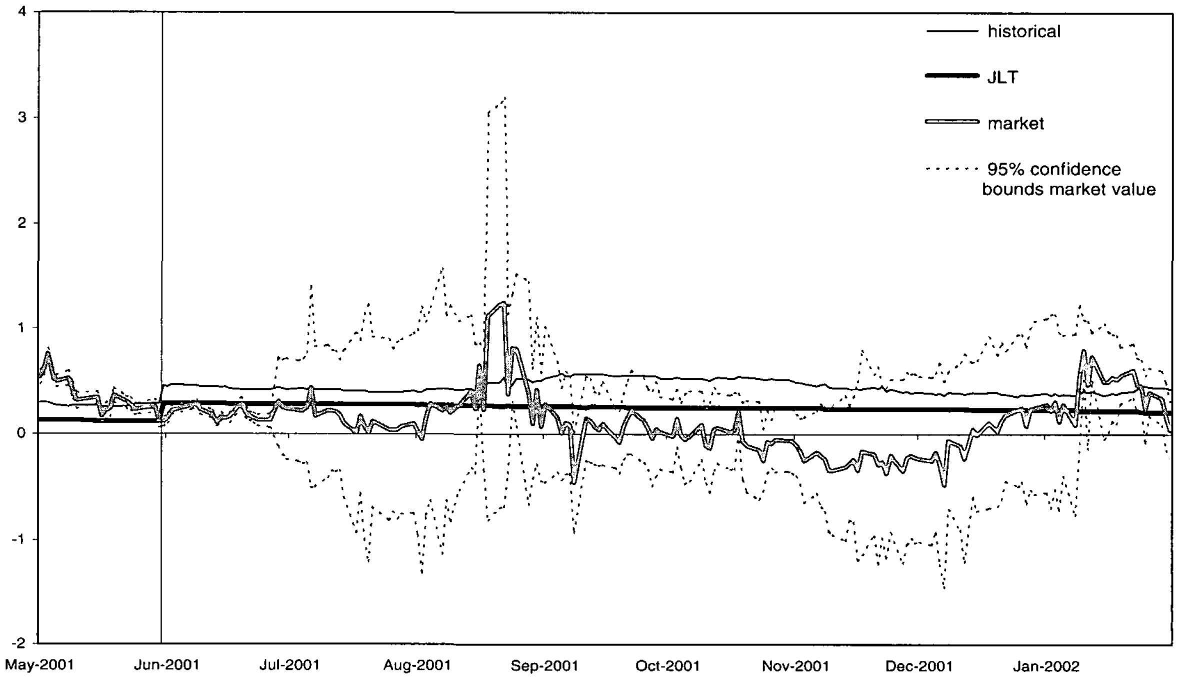
France Telecom. For the 2004 France Telecom step-up bond, the MPE and MAPE values for the EPV method in Exhibit 5 approximate the market prices better than the JLT and historical methods. Apparently, the market values this bond as a plain vanilla bond. In Exhibit 7, we observe that the 95% confidence bounds of the EPV values enclose the market prices on 90% of the days, but for the JLT and historical methods only 85% and 72%, respectively. Again, Exhibit 6 shows that the approximations produced by the three valuation methods differ significantly.

Exhibit 9, Panel A, displays the market step-up premium with its 95% confidence bounds and the JLT and historical premiums. The same pattern as for Deutsche Telecom emerges, with volatile although insignificant market premiums and steady JLT and historical premiums. Also, there is a large drop in both theoretical premiums on September 26, 2001, after a downgrade by Moody's from A3 to Baa1. This downgrade triggered the step-up feature, so that there are fewer remaining step-ups than the number of step-ups at issuance, and the coupon can also step down as the rating improves again. The consequence is a decline in the theoretical step-up premiums, as observed in Exhibit 9, Panel A.

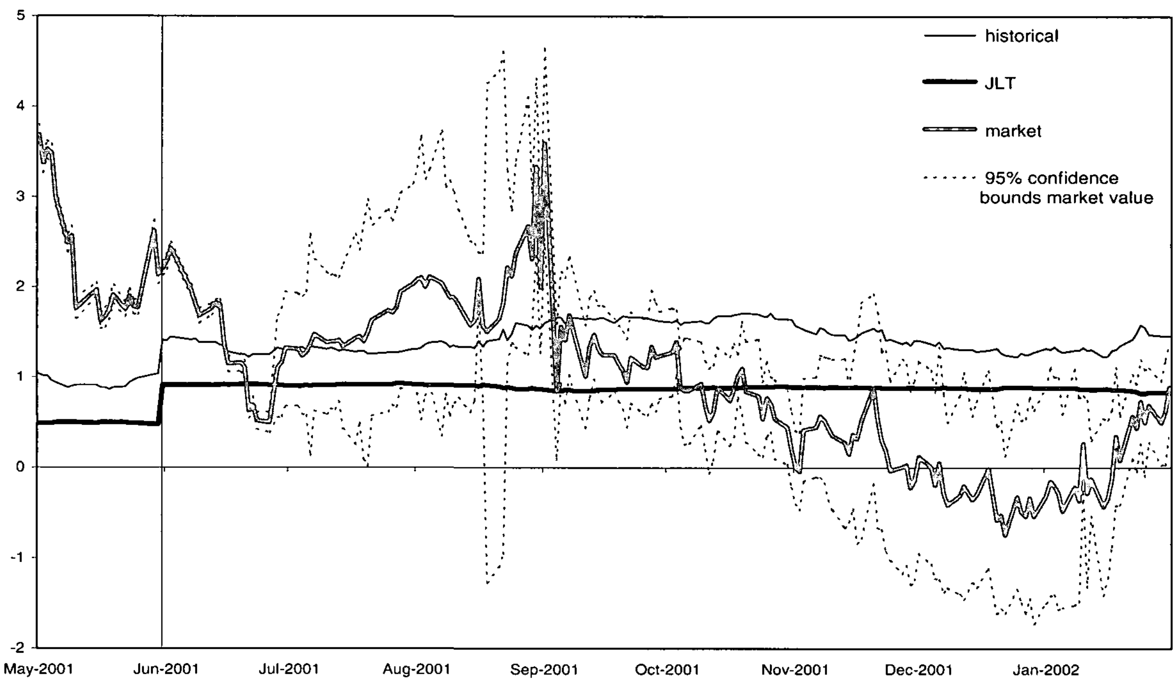
For the 2008 France Telecom bond, the MPE value is (in absolute value) equal to the MAPE value for both the JLT and historical methods (*Exhibit 5*). This means that both methods always overestimate the market price. The MPE value for the EPV method is the lowest, but there is still a large bias. The confidence bound statistics in Exhibit 7 reveal that on 48% of the days the market price and EPV value are statistically indistinguishable, while the confidence bounds of the JLT and historical methods almost never include the market price. The paired Z-tests also indicate that the JLT and historical values do not vary

EXHIBIT 8
Deutsche Telecom Step-Up Premiums

A. 2005 Step-Up Bond



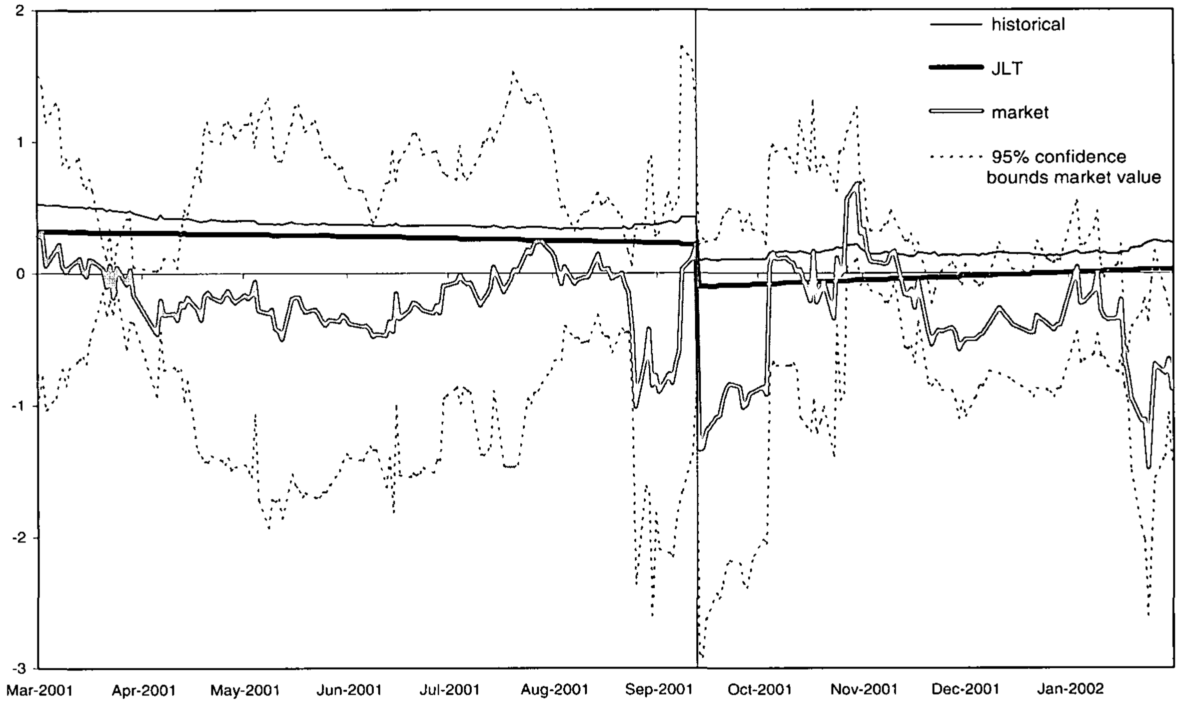
B. 2010 Step-Up Bond



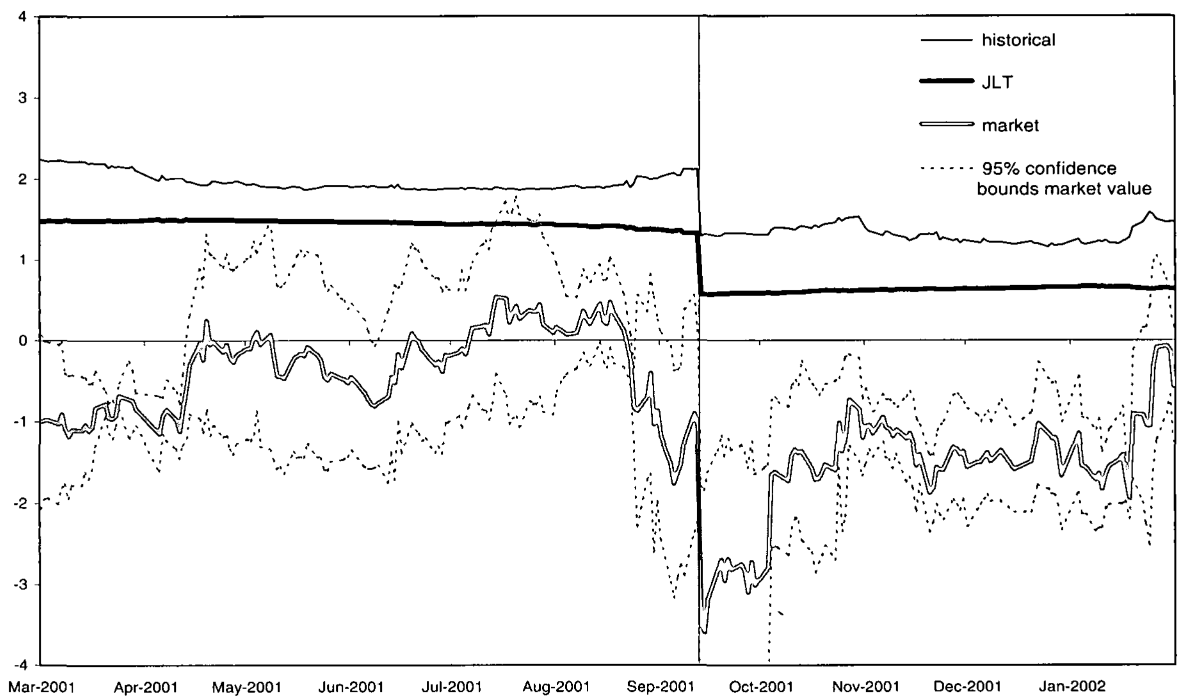
Vertical line indicates a rating downgrade by Moody's.

Exhibit 9
France Telecom Step-Up Premiums

A. 2004 Step-Up Bond

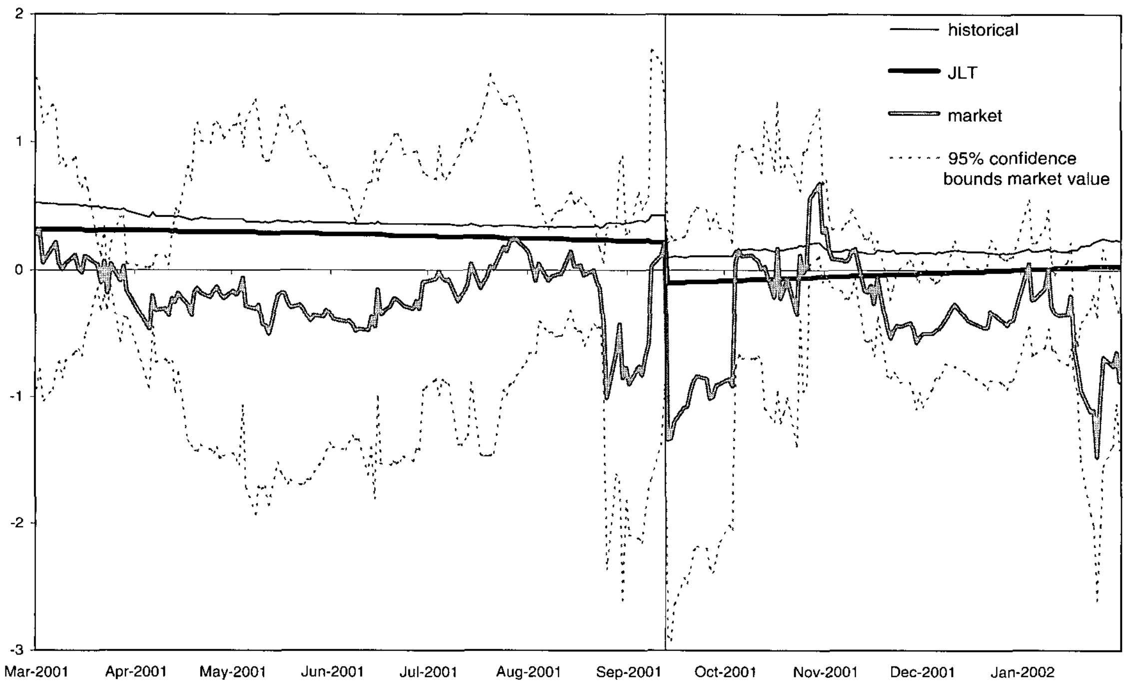


B. 2008 Step-Up Bond



Vertical line indicates a rating downgrade by Moody's.

EXHIBIT 10
KPN Step-Up Premiums



Vertical line indicates a rating downgrade by Moody's.

much, but that they both differ to a great extent from the EPV values.

As Exhibit 9, Panel B, shows, the market premium of the step-up feature of the 2008 bond is again volatile and even significantly negative during the first few months and also after the downgrade. Both the JLT and historical premiums remain positive during the full period, including after the downgrade, when their values drop as well. As above, we observe that the JLT and historical premiums are typically higher than the market premium and this difference is quite large, because they virtually never lie within the market premium's confidence bounds.

KPN. For the 2006 KPN step-up bond, again the MPE value of the EPV method is closest to zero, although it is significantly below zero; see Exhibit 5. The JLT model also typically overvalues the market price, and the historical method even more, as evidenced by their negative MPE values. The same ranking of the three methods emerges from Exhibit 7, as in 82% of the cases the market price lies within the EPV confidence bounds; for the JLT and historical methods the figures are 76% and 33%. Therefore, the market seems to value this bond as a plain vanilla bond.

Exhibit 10 plots the market, JLT, and historical step-up premiums. As in the two France Telecom bonds, the JLT and historical premiums are less volatile and higher than the market premium, but the premiums of the JLT model often lie within the 95% confidence bounds.

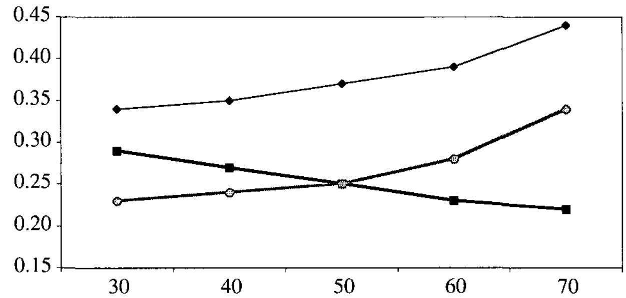
The drops in all premiums on September 6, 2001, follow Moody's rating downgrade from Baa2 to Baa3. The market premium first shows a dramatic fall but then an impressive recovery in the subsequent months to become significantly positive, before it then falls for the second time and becomes negative. The JLT premium also drops after the downgrade and becomes virtually equal to zero. A similar pattern can be observed for the historical method, since it drops following the downgrade too; unlike the JLT premium, however, it stays positive. Again, the historical premium is always higher than the market premium, something we also noticed for the Deutsche Telecom and France Telecom step-up bonds.

Implications. What is the value of the step-up features at bond issuance? Exhibit 9 shows that the values of both France Telecom bonds do not differ significantly from zero at their issue date, while Exhibit 10 shows that the same applies to the 2006 KPN step-up bond.[7]

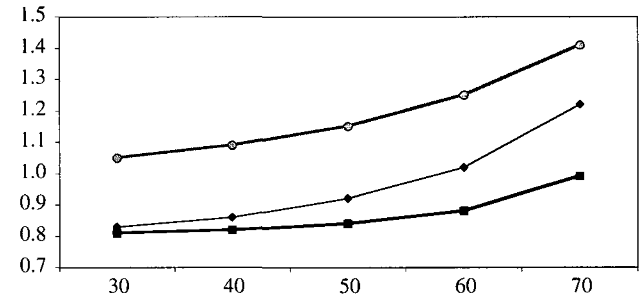
Exhibit 11

Recovery Rate Sensitivity Analysis

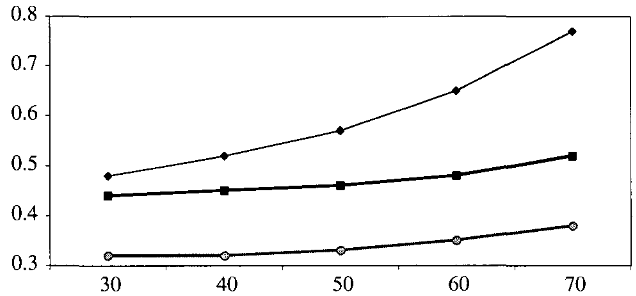
A. 2005 Deutsche Telecom Step-Up



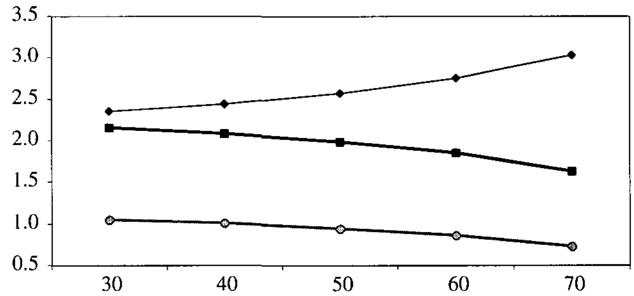
B. 2010 Deutsche Telecom Step-Up



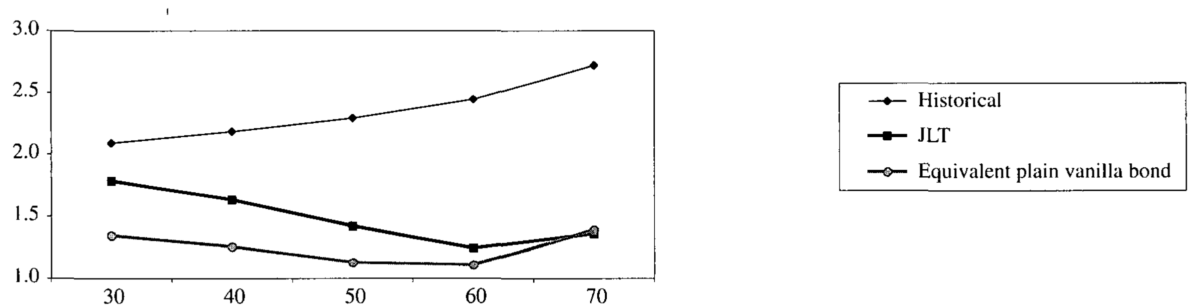
C. 2004 France Telecom Step-Up



D. 2008 France Telecom Step-Up



E. 2006 KPN Step-Up



In theory, the step-up feature should have a positive value. Thus, we conclude these three step-up bonds are undervalued at their issue date. Consequently, the issuer could have issued bonds with a lower (expected) coupon, thereby saving interest payments.

This implies that issuing step-up bonds has increased the costs of capital of France Telecom and KPN; see also Lando and Mortensen [2003].

Recovery Rate. The step-up bond values constructed by the EPV, JLT, and historical methods all use a recovery rate of 50%. We should note that the true recovery rates for these three telecom issuers are not known exactly. Of course, this is the case for all corporate issuers that have not experienced default. We thus analyze here how sensitive the mean absolute pricing errors (MAPEs) are to variations in recovery rates ranging from 30% to 70%.[8]

Exhibit 11 shows that, for the 2005 Deutsche Telecom bond, the MAPE values produced by the historical and EPV valuation methods always increase, while the MAPE of the JLT valuation method always declines as the recovery rate increases. For the 2010 Deutsche Telecom bond, both MAPEs and recovery rate always move in the same direction. The same applies for the 2004 France Telecom bond. The MAPEs generated by the historical valuation method rise as the recovery rate increases for both the 2008 France Telecom and the 2006 KPN bond. The opposite pattern can be observed for the MAPEs of the JLT and EPV valuation methods, with the exception of the MAPE of the 2006 KPN bond, which rises again at the 70% recovery rate.

We can see that for three of the five step-up bonds—the 2010 Deutsche Telecom and both France Telecom bonds—the order of the valuation methods is always the same, whatever the recovery rate. For the other two bonds, the 2005 Deutsche Telecom bond and the 2006 KPN bond, the historical method always performs most poorly, but the JLT and EPV methods cross at a recovery rate of 50% and 70%, respectively; at lower values, the EPV method is better, while for higher values the JLT method produces smaller errors.

Step-Up Protection

We analyze the protection of a step-up from two perspectives: volatility and excess returns.

Volatility Analysis. We hypothesize that step-up bond prices are less volatile than an equivalent plain vanilla bond price, because the step-up feature compensates for a lower rating with a higher coupon, and for a higher rating with a lower coupon; see also McAdie, Martin, and O'Kane [2000].[9] The step-up feature should work as a cushion against the impact of rating migrations on bond prices.

We compare the variance of a step-up bond's market prices, $\sigma^2_{step\text{-}up}$, with the variance of its equivalent plain vanilla bond's values, σ^2_{EPV}. The null hypothesis

$$H_0 : \frac{\sigma^2_{step-up}}{\sigma^2_{EPV}} = 1$$

is tested using the test statistic

$$V = \frac{S^2_{step-up}}{S^2_{EPV}}$$

where $S^2_{\text{step-up}}$ and S^2_{EPV} are the sample variances of the step-up bond market prices and equivalent plain vanilla bond values, respectively. V follows an F-distribution with $n_{step\text{-}up} - 1$ and $n_{EPV} - 1$ degrees of freedom, where n_i equals the number of i observations, with $i \in$ {step-up, EPV}; see, e.g., Madsen and Moeschenberger [1986].

EXHIBIT 12
Volatility Analysis

	Test statistic	p-value
DEUTSCHE TELECOM		
2005	0.72	0.025
2010	0.56	0.000
FRANCE TELECOM		
2004	1.04	0.768
2008	0.75	0.031
KPN		
2006	1.00	1.000

Exhibit 12 shows mixed outcomes. The null hypothesis is easily rejected at the 95% confidence level for both Deutsche Telecom bonds and the 2008 France Telecom bond. So, for these three bonds, the step-up bond price variance is lower than the variance of the equivalent plain vanilla bond.[10] For the 2004 France Telecom and the 2006 KPN bonds, the two variances are statistically indistinguishable.

Event Analysis. As long as the timing or extent of a rating downgrade or negative outlook change is not fully anticipated by the market, plain vanilla bonds should have a negative return on a downgrade or negative information release event date; see, e.g., Hand, Holthausen, and Leftwich [1992]. Since a step-up bond compensates investors for poorer creditworthiness via a higher coupon, we hypothesize that it has a higher return than its equivalent plain vanilla bond.

To test this hypothesis, we define the *excess step-up return* as the step-up bond market return minus the return of its equivalent plain vanilla bond, so the step-up bond return is fully corrected for all bond characteristics except for the step-up. We first calculate the excess return ER_{it} for bond i for the event day, $t = 0$, and the three succeeding trading days, $t = 1, 2, 3$. Then, we calculate the average ER_t of these excess returns for each day for the five step-up bonds. We also consider *cumulative excess returns*, defined as:

$$CER_{it} = \sum_{s=0}^{t} ER_{is}$$

and their averages CER_t. We test the significance of the average excess returns and the cumulative average excess

EXHIBIT 13
Event Analysis

	Returns				Cumulative returns			
Day	Step-up	EPV	Excess	t-value	Step-up	EPV	Excess	t-value
0	-1.89%	-0.77%	-1.12%	-1.14				
1	0.36%	-0.14%	0.50%	0.65	-1.53%	-0.90%	-0.63%	-0.97
2	-0.11%	-0.61%	0.50%	1.72	-1.64%	-1.51%	-0.13%	-0.20
3	0.16%	0.18%	-0.02%	-0.15	-1.47%	-1.32%	-0.15%	-0.23

returns using simple t-tests as described in Ritter [1991]. If both Moody's and S&P change their ratings of (or outlooks for) the same company at the same time, we treat this as a single event.

Exhibit 13 shows the results of this event analysis. On the event date, the return on the EPV bond is indeed negative, as expected. The return on the step-up bond, however, is even more negative, so that the excess step-up return is negative. Although this negative excess return is not significant, it is not consistent with our expectations. During the post-event period, the average excess return becomes positive, but stays statistically insignificant. The cumulative excess returns are always negative and insignificant for the entire post-event period.

It is counter to our expectations that the step-up feature does not offer investors positive excess returns in the case of a rating downgrade or a negative outlook, but rather statistically identical returns as the EPV bond.

V. SUMMARY

We have compared several pricing methods for rating-triggered step-up coupon bonds. European telecom companies have issued these bonds in order to compensate bond investors for losses in the event of rating downgrades. The coupon of a step-up bond depends on its issuer rating or the rating of the issuer's long term debt. If a rating deteriorates to a predefined level, a step-up coupon is triggered, and the coupon rises by a predefined number of basis points.

We apply both risk-neutral and historical transition probabilities using the Jarrow, Lando, and Turnbull [1997] (JLT) framework to value these rating-triggered step-up bonds. For comparison purposes, we also value step-up bonds as plain vanilla bonds comparable to step-up bonds except for the step-up feature. We demonstrate our results over a long time period; compare the volatility of a step-up bond and the equivalent plain vanilla bond; and perform a rating and outlook change event analysis of excess step-up bond returns.

We find that the market seems to value Deutsche Telecom step-up bonds whose coupons make a single step-up after the rating hits the trigger level according to the JLT model. For the France Telecom and KPN step-up bonds whose coupons step up every time a rating hits a trigger level, the market seems to resort to valuation as plain vanilla bonds. Market premiums for step-ups have been much more volatile than the JLT and historical premiums. The theoretical premiums are stable, except for changes caused by downgrades. Further, as expected, the JLT model always approximates the market premiums better than the historical valuation method.

Prices of the two Deutsche Telecom bonds and the 2008 France Telecom bond were significantly less volatile than prices of their equivalent plain vanilla bonds at a 95% confidence level. These three step-up bonds thus offered protection in terms of less price volatility. Price volatilities of the 2004 France Telecom and the KPN step-up bonds were statistically indistinguishable from their equivalent plain vanilla bonds. Therefore, these step-up bonds did not offer a cushion against rating migrations in the form of dampened price movements.

The results of an event analysis, another way of looking at the step-up protection, demonstrate that step-up bonds have not offered superior returns to an investor in the case of rating downgrades or negative outlooks.

ENDNOTES

The authors thank Arjan van Dijk, Martijn Krijger, and Joost Kromhout for computational assistance. The views expressed are the authors' own and do not necessarily reflect those of any of their employers.

[1]Moody's and S&P usually report (multiples of) one-year

transition matrices, but the time between the valuation day and coupon and redemption dates almost never exactly equals (multiples of) one year. We adjust the historical transition matrix and make it maturity-dependent using a generator matrix, as described by Israel, Rosenthal, and Wei [2001].

[2]Fumagalli and Taurén [2001] assume that a rating action of one agency is followed by the other agency within six months.

[3]Note that the summation may be reduced from K terms to $K-1$ terms, since the K-th term is zero: $\tilde{\mathbb{P}}_t(\tau > t_j \wedge R_{t_{j-1}} = K) = 0$.

[4]Note that per definition MPE for the plain vanilla bonds is zero, since the pricing error for a plain vanilla bond is simply its residual from the least squares estimation of the survival probability curve.

[5]Because there were no plain vanilla Deutsche Telecom bonds before May 14, 2001, DT figures start well after the step-up bond's issue date of July 6, 2000.

[6]McAdie, Martin, and O'Kane [2000] also found a negative value in their analysis of euro step-up bonds on July 13, 2000.

[7]For Deutsche Telecom, we cannot value the step-up features at the bond issue dates.

[8]All other things equal, a higher recovery rate should increase the theoretical value of a step-up bond. Here, however, the recovery rate is first used in estimation of the survival probabilities, where higher recovery rates result in lower survival probabilities, and vice versa. Next, both the recovery rate and the survival probabilities are used in the valuation of the step-up bonds. How the changing recovery rates (and thus the changing survival probabilities) affect the step-up valuation ultimately is therefore not at all straightforward.

[9]Olivetti/Tecnost's chief financial officer, after linking the coupons of its bonds to its credit rating, stated: "we think having these sort of volatility protection measures associated with our bonds should result in a lower capital cost" (*Bloomberg Equity News*, June 16, 2000, quoted in Acharya, Das, and Sundaram [2002, p. 43]).

[10]If the step-up bonds were less liquid than the plain vanilla bonds, this outcome could be explained by the occurrence of stale prices. In fact, step-up bonds are probably *more* liquid than plain vanilla bonds, because they are younger and have higher notionals, see, e.g., Houweling, Mentink, and Vorst [2003].

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