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THE NASDAQ VOLATILITY INDEX DURING AND AFTER THE BUBBLE 9

DAVID P. SIMON

The VIX index of implied volatilities for OEX index options was introduced in the early 1990s and is widely followed as an investor "fear index." This article examines the VXN, a similar volatility index for Nasdaq 100 options, over a period that includes both the inflation of the Internet "bubble" and its bursting. If the VXN is the market's best estimate of the future volatility of the Nasdaq 100 index, it should be an unbiased forecast of subsequent realized volatility. But if the VXN represents a "fear index," it will reflect variations in investors' emotions, for example rising after a sharp market drop to reflect heightened concern and an increase in investors' demand to buy put options. It may also be influenced by technical indicators of market direction. Simon finds that even after correcting for the effect of a little-known built-in bias in the way it is constructed, the VXN averages about 7-1/2 percentage points higher than subsequent realized volatility. It also shows a strong asymmetrical response to positive and negative index returns, as has been found in other implied volatility studies. A GARCH model fitted to the returns on the actual index also reveals an asymmetrical response of volatility to returns, but much smaller than for the VXN. The evidence suggests that implied volatilities from options on the Nasdaq 100 index reflect the stochastic properties of the index itself, but they also show behavior that appears to be more closely related to investor sentiment.

VALUATION OF STOCK OPTION GRANTS UNDER MULTIPLE SEVERANCE RISKS 25

GURUPDESH S. PANDHER

Grants of stock options as part of total employee compensation are now commonplace, but how such options should be valued and reported in firm financial statements is still an unsettled issue. The Black-Scholes (BS) model is the best-known and most widely accepted approach to option valuation as a benchmark for accounting and legal purposes, so it is seems like a natural place to begin. However, employee stock options (ESOs) are subject to several additional contingencies that are not in the BS equation, but significantly alter the possible payoffs. Specifically, severance with or without cause, including by death of the option holder, typically alters the payoff on an ESO. For example, termination with cause may also entail forfeiture of the employee's options; termination without cause may simply advance the option's maturity date and require the departing employee to exercise immediately, or not at all. In this article, Pandher presents a risk-neutral valuation approach for pricing ESOs under a realistic set of severance possibilities and examines the impact on valuation and on expected exercise dates. He shows that proper treatment of these additional features can make a substantial difference in the ESO value relative to Black-Scholes.

HOW VALUABLE IS CREDIT CARD LENDING? 39

ARKADEV CHATTERJEA, ROBERT JARROW, ROBERT NEAL, AND YILDIRAY YILDIRIM

Standard methodology for valuing interest-dependent assets is by now well established, with the Heath-Jarrow-Morton (HJM) model being one of the most widely-used approaches. However, despite representing a significant portion of many banks' portfolios, credit card loans as an asset class have not been brought into the standard valuation framework. Like a bond position, valuing a bank's credit card loan portfolio involves projecting the levels of future interest rates. But a portfolio of credit card loans also has several special features not shared by bonds. One is that the total loan face value shows long-run growth, but also short-term fluctuation, reflecting both seasonal effects and interest rate changes. A second is that the profitability, and hence the capitalized value, of the loan portfolio depends on the difference between the rate earned on the loans and the rate the bank must pay to fund those loans, so both rates need to be modeled. In this article, Chatterjea, Jarrow, Neal, and Yildirim offer a general modeling approach for valuing a bank's credit card loan portfolio within an extended HJM paradigm, and then illustrate how it works by fitting it to the credit card portfolios of five different banks.

The Nasdaq Volatility Index During and After the Bubble

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The Nasdaq 100 Volatility Index (VXN) was constructed by the Chicago Board Options Exchange to reflect the implied volatility of near-term at-the-money options on the Nasdaq 100 index (NDX). An examination of its behavior indicates that the response of the VXN to NDX changes is remarkably stable across the technology stock bubble in the late 1990s and its aftermath, falling in response to large positive NDX returns, and rising in response to large negative NDX returns. This opposite response to positive and negative returns is consistent with the idea that the VXN is a "fear index." The results also demonstrate that the VXN rises in response to technical indicators suggesting stronger positive or negative NDX trends, such as greater deviations of the NDX from moving averages.

Asymmetric GARCH models, however, indicate that the conditional volatility of the underlying NDX returns shows no tendency to fall in response to large positive NDX returns. The results also indicate that, although the VXN has predictive power for actual volatility, it is biased, averaging 7.5 percentage points higher than actual volatility. The elevated level of implied volatility substantially raises the extent of the NDX change that causes gamma to offset time decay for delta-hedged at-the-money NDX options.

Fleming, Ostdiek, and Whaley [1995] and Whaley [2000] examine the behavior of the Volatility Index (VIX), which is a measure of implied volatility calculated by the Chicago Board Options Exchange (CBOE) from near-term close-to-the-money options on the S&P 100 index. The VIX often is referred to as a *fear index*, because its level indicates how much market participants are willing to pay in terms of implied volatility to hedge stock portfolios with S&P 100 index put options or to be long (with limited downside risk) by buying S&P 100 index call options.

Evidence is consistent with this view, as Fleming, Ostdiek, and Whaley find that the VIX tends to rise following large sell-offs and to fall following large rallies. Equity market strategists often interpret extreme levels of the VIX as contrary trading signals. High VIX levels indicate extreme pessimism, which causes equity prices to overshoot on the downside and leads to subsequent rallies.[1]

Extremely low VIX levels reflect complacency among market participants, which sets up the market for disappointment and raises the odds of a downward market correction. McMillan [1996, p. 32), for example, states that "when options are too cheap, too many people are selling options at lower and lower prices. Contrary thinking tells us that such a market is about to explode."

Another issue concerning the VIX (and implied volatility more generally) is whether implied volatility predicts actual volatility. While market participants commonly view implied volatility as a consensus forecast of actual volatility, evidence suggests that the predictive power of the implied volatility of S&P 100 index options is mixed. Canina and Figlewski [1993] examine the forecasting power of the implied volatility of S&P 100 index options for the actual volatility of S&P 100 index returns through option expiration over 1983–1987, and find that these forecasts are biased and, more strikingly, typically do not have statistically significant forecasting power.[2]

Christensen and Prabhala [1998] examine the implied volatility of S&P 100 index options one month before expiration for a longer time span over 1983-1995, and find significant and unbiased predictive power, while Fleming [1998] finds significant but biased predictive power in the implied volatility of S&P 100 index options over 1985–1992.

A major difference in these studies is that Canina and Figlewski [1993] examine daily overlapping forecasts, and the two other studies examine monthly non-overlapping implied volatility forecasts.

We examine the empirical properties of the Nasdaq Volatility Index from January 1995 through May 2002. The Nasdaq Volatility Index, referred to as the VXN, is calculated by the CBOE in a manner similar to the VIX. It is based on the implied volatilities of at-the-money near-term options on the Nasdaq 100 index (NDX).

We first model the time series properties of the VXN, focusing on how much implied volatility responds differently to NDX rallies and declines. Because the time series includes both the technology stock bubble in the late 1990s and its aftermath, we can see whether at-the-money implied volatility responds differently to positive and negative NDX returns across these periods of extreme bullishness and bearishness.

We also explore whether the VXN responds to technical indicators suggesting that the market is trending, such as deviations of the NDX from moving averages. We then examine whether the conditional volatility of the underlying NDX index responds in a manner consistent with the response of the VXN to NDX rallies and sell-offs.

We examine the predictive power of the VXN, asymmetric GARCH out-of-sample forecasts, and exponentially weighted moving-average forecasts for actual volatility, and whether the VXN encompasses these forecasts. Finally, we look at the implications of the finding that implied volatility substantially exceeds actual volatility for the break-even change in the NDX that causes the effect of gamma to offset time decay for delta-hedged at-the-money NDX options.

The results indicate that the VXN responds to the same extent but in opposite directions to positive and negative NDX returns. Larger NDX sell-offs lead to greater VXN increases, while larger NDX rallies lead to greater VXN declines. This phenomenon is extremely stable across the technology stock bubble and the post-bubble periods. In addition, the VXN increases in response to technical indicators that suggest the NDX is trending in either direction.

The results also demonstrate that, although the VXN has predictive power, it is a biased forecast of actual volatility and overforecasts actual volatility on average by about 7.5 percentage points. The higher implied volatility substantially raises the daily break-even NDX price change, where gamma just offsets time decay for delta-hedged at-the-money NDX options.

I. NASDAQ VOLATILITY INDEX

The instrument underlying the Nasdaq Volatility Index (VXN) is the Nasdaq 100 index (NDX). The NDX is a capitalization-weighted index comprising 100 of the largest non-financial equities listed on the Nasdaq Stock Market. The NDX index was created in 1985 by the Chicago Board Options Exchange (CBOE); it is calculated and reported to market participants every 15 seconds during the trading day.

Options traded on the NDX are European and may be exercised only on the last business day before the expiration date, which is the Saturday after the third Friday of the expiration month. NDX options are cash-settled on the basis of the first five minutes of trading on the final trading day before expiration. NDX option maturities include the three near-term months and up to three additional months from the March quarterly schedule.

The CBOE began calculating and disseminating the VXN on an intraday basis in January 1998, and has made the series available from the beginning of 1995. The closing VXN is based on quotes at the 4 p.m. EST close of regular hours trading on the Nasdaq.

The VXN provides market participants with real-time information about the volatility assumptions embedded in the pricing of NDX options, using the Black-Scholes [1973] model with an adjustment for dividends. Like the more widely followed Volatility Index (VIX) based on S&P 100 index options, the VXN is constructed from the implied volatilities of eight options—the two closest to-the-money calls and puts for the two closest-to-expiration option contracts that have at least eight calendar days until expiration.

The VXN is calculated as follows. At-the-money implied volatilities are obtained for calls (puts) from each of the two closest-to-expiration months by interpolating between the implied volatilities of the two closest-to-the-money calls (puts) for the two front contracts. These eight implied volatilities are calculated using calendar days to expiration assuming a 365-day year from the midpoints of the bid-ask spreads of the options rather than from the most recent transactions to avoid both bid-ask spread bounce and the possibility of stale option prices. The implied volatilities are then adjusted to a trading day basis, and the VXN is interpolated from these implied volatilities to reflect the implied volatility of at-the-money options with 22 business days until expiration.

The reason business days rather than calendar days are used to calculate the VXN is that, as shown by French and Roll [1986], stock return volatilities from Friday through Monday closes are similar to what they are from the close to close of consecutive trading days during the same week.[3]

Note that the procedure for converting calendar day implied volatilities to trading day implied volatilities biases the VXN upward compared to actual volatility. To adjust implied volatilities from a calendar day basis to a trading day basis, the CBOE sets the number of trading days until expiration:

$$N_t = N_c - 2\text{int}(N_c/7) \tag{1}$$

where N_t and N_c are the number of trading and calendar days until expiration, respectively.

The CBOE then multiplies the calendar day implied volatilities by the ratio of the square root of the number of calendar days to the square root of the number of trading days to obtain the implied volatilities on a trading day basis:

$$\sigma_t = \sigma_c\left(\frac{\sqrt{N_c}}{\sqrt{N_t}}\right) \tag{2}$$

which are then interpolated to form the VXN.

Bilson [2003] points out that this adjustment causes the VXN to be higher than actual volatility, which we can see in an example. Suppose that both actual and implied volatilities on a daily calendar day basis are 1%, or equivalently 1.2% on a daily trading day basis. Multiplying 1% by $\sqrt{365}$ or multiplying 1.2% by $\sqrt{252}$ results in annualized implied and actual volatilities of 19.1%. Equation (2), however, results in a VXN of 22.3%, despite starting with equal daily implied and actual volatilities.

Because the VXN overstates volatility by 16.77%, it is important to use comparable volatilities in comparison of the VXN and actual volatility. We adjust the VXN to a basis comparable to actual volatility by undoing the effect of Equation (2) on the reported VXN. Henceforth, the term VXN refers to the *adjusted VXN* as described, rather than the reported VXN.

Exhibit 1 shows the daily history of the VXN and the NDX. There was a massive and largely uninterrupted NDX rally from a level of 400 in January 1995 to a peak of 4700 in March 2000 preceding the sell-off that subsequently took the NDX down to 1160 in May 2002. From January 1995 through March 2000, the VXN moved mostly in a relatively narrow range between 15% and 40%, with one major departure to the upside in late 1998 associated with the Southeast Asian crisis, the Russian default, and the problems at Long-Term Capital Management. Following the peak of the NDX in March 2000, the VXN moved in a range with a lower bound of about 35% and an upper bound of around 75% reached during sharp NDX sell-offs.

Time Series Model of the VXN

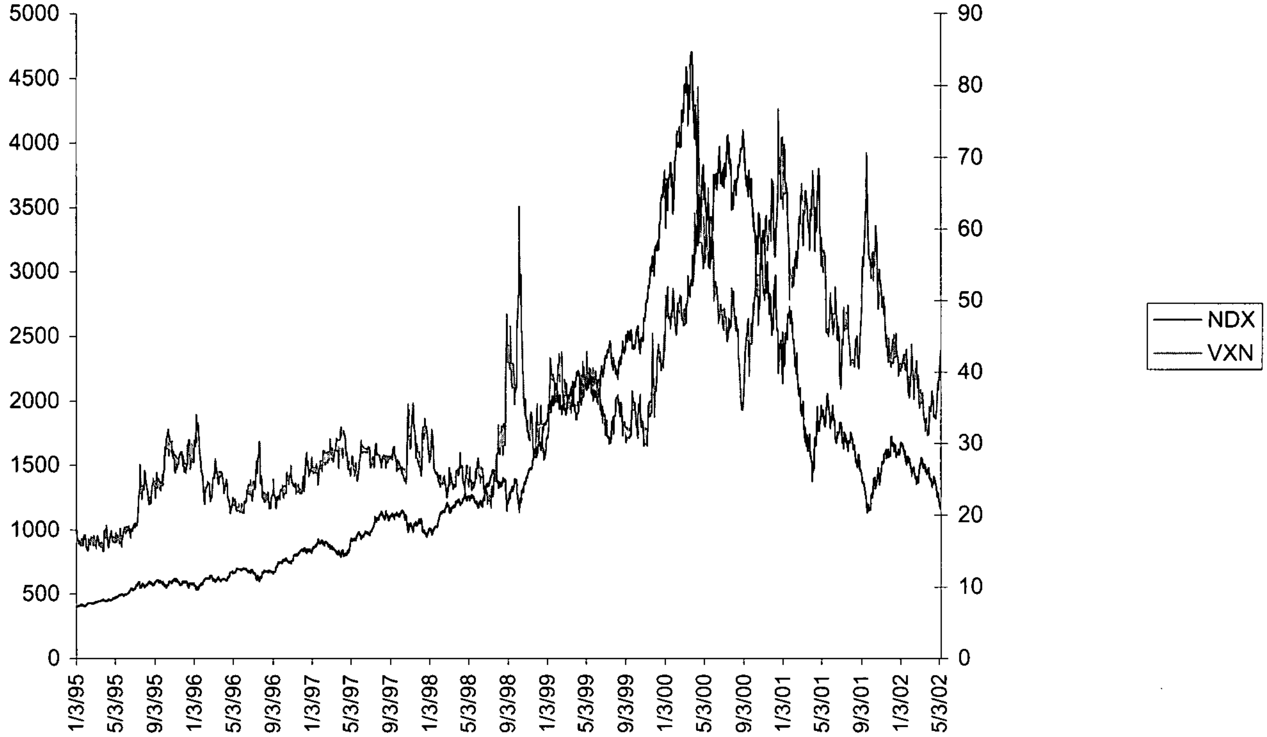
The model specifies the daily change of the VXN as a function of a constant, the lagged level of the VXN, separate variables for contemporaneous positive and negative NDX returns, and technical indicators. Inclusion of the lagged level of the VXN allows for the possibility that the VXN reverts toward a typical level.[4]

The specification also allows equal-magnitude positive and negative NDX returns to have different effects on the VXN. Fleming, Ostdiek, and Whaley [1995] find that the VIX moves in opposite directions in response to positive and negative S&P 100 returns; negative returns are associated with implied volatility increases, and positive returns are associated with implied volatility declines.[5]

This phenomenon has been attributed to the tendency of actual volatility to increase more following large negative returns than large positive returns of equal magnitude, owing to financial leverage (see Black [1976] and Christie [1982]). French, Schwert, and Stambaugh [1987] and Schwert [1989, 1990], however, demonstrate that this asymmetry is too strong to be explained by leverage alone.

The opposite reaction of the VIX to positive and negative returns may stem from the ways market participants use options to manage risk. One possible explanation is that market participants have more of an inclination to buy put options to hedge portfolios following large market declines than following large rallies. Similarly,

EXHIBIT 1
NDX and VXN



market declines may also give rise to a greater demand for the limited downside risk associated with buying call options rather than stocks, while rallies lead to a diminished demand for the limited risk associated with buying call options rather than stocks. Either of these mechanisms—an increased demand for calls or puts—would have the same effect on implied volatility because put-call parity links the implied volatilities of calls and puts that have the same strike price and expiration date.[6]

Another possible explanation for the opposite response of the VIX to positive and negative S&P 100 returns is the volatility skew, which in equity index options is represented in a pattern of higher implied volatilities at lower strike prices for both calls and puts. An implication of the skew is that if the underlying index rallies, the at-the-money options from which the VIX is calculated have higher strike prices and lower implied volatilities than the options that were previously at the money and used to calculate the VIX. Thus, the volatility skew could cause the VIX to fall (rise) in response to an increase (a decline) in the S&P 100 index without necessarily a drop (rise) in the implied volatilities of specific strike options.[7]

An important implication of this finding is that it could be misleading to interpret VIX changes as changes in the implied volatilities of specific strike price options. At the same time, VIX changes for whatever reason affect the ex ante trade-offs for those entering delta-hedged at-the-money option positions.[8]

Although the opposite response of at-the-money implied volatility to positive and negative returns has been documented in S&P 100 index options, such a response has not been documented in Nasdaq index options. We examine how the VXN responds to positive and negative NDX returns by regressing the change of the VXN on separate variables for positive and negative NDX returns. This basic specification is augmented by variables that technical analysts use to gauge the strength of trends, which could affect the demand for options and hence implied volatility.

If, for example, the NDX is expected to trend strongly upward, call option prices and their implied volatilities may be bid up by investors who prefer to be long calls rather than long delta-equivalent stock positions. One explanation is that the presence of trends suggests a greater probability that the underlying NDX will be considerably higher or lower in the future, which would increase the potential payout of options and cause implied volatility to be bid higher.[9]

EXHIBIT 2

Summary Statistics of VXN, NDX Returns, and Technical Indicators (January 3, 1995–May 7, 2002)

Variable	Mean	Std Dev	Top Decile	Bottom Decile	ρ_1	ρ_2	ρ_3	ρ_4
VXN	0.3494	0.1283	0.5463	0.2195	0.99	0.98	0.98	0.97
ΔVXN	0.0001	0.0176	0.0191	-0.0183	-0.04	-0.10	0.01	-0.06
NDXRET	0.0009	0.0240	0.0270	-0.0282	-0.04	-0.08	0.01	0.02
DEVMA5	0.0013	0.0248	0.0285	-0.0290	0.62	0.30	0.08	-0.04

VXN is the Nasdaq volatility index, calculated from at-the-money near-term options on the Nasdaq 100 index. NDXRET are returns on the NDX index, and DEVMA5 is the percentage deviation of the NDX from its five-day moving average.

Leland [1996] demonstrates that investors who believe market returns are less mean-reverting than the average investor will be inclined to buy options, while investors who believe that market returns are more mean-reverting than the average investor will be inclined to sell options. Also, because the delta of an option increases in absolute terms as its moneyness rises, a trending market would tend to cause options to outperform originally delta-equivalent stock positions, all else equal.

A common indicator of trends is the deviation of the current level of the price of an instrument from a moving average of its recent levels. We use separate variables for positive and negative percentage deviations of the NDX index from its five-day moving average—similar results are obtained using positive and negative deviations from three- and ten-day moving averages. If an increased demand for options causes implied volatility to rise as trends become stronger, the coefficient on positive and negative percentage deviations from moving averages should be significantly positive and negative, respectively. By including both the contemporaneous NDX return and the contemporaneous deviation of the NDX from its five-day moving average, the model specification allows the impact of NDX returns on implied volatility to vary depending on the extent to which they lead to deviations of the NDX from its recent central tendency.

The model estimated is:

$$\begin{aligned} \Delta VXN_t = \beta_0 + \beta_1 VXN_{t-1} + \beta_2\, NDXRET_t^+ + \\ \beta_3\, NDXRET_t^- + \\ \beta_4\, DEVMA5_t^+ + \beta_5\, DEVMA5_t^- + \varepsilon_t \end{aligned} \quad (3)$$

where the daily VXN change from the close at time t − 1 to the close at time t is regressed on its lagged level, separate variables for positive and negative same day NDX returns, and separate variables for positive and negative percentage deviations of the closing NDX from its five-day moving average, respectively.

Data

Information on the variables used in the model is shown in Exhibit 2, including the means, standard deviations, top and bottom deciles, and first four autocorrelations of the levels and daily first differences of the VXN, the daily NDX returns, and the deviations of the NDX from its five-day moving average.

The VXN over the sample period averages 34.94%, with highest and lowest decile cutoffs of 54.6% and 22.0%, respectively. The daily first difference of the VXN averages 0.01 percentage points, with highest and lowest decile cutoffs of 1.9% and −1.8%, respectively. Daily NDX returns over the sample period average 0.09% with highest and lowest decile cutoffs of 2.7% and −2.8%, respectively. Deviations of the NDX from its five-day moving average have a mean of 0.13% with highest and lowest decile cutoffs of 2.9% and −2.9%, respectively.

Not surprisingly, autocorrelation is evident for the VXN and for the deviation of the NDX from its five-

EXHIBIT 3
Model of Nasdaq 100 Volatility Index (VXN)

Variable	Whole Sample Period (1/3/95-5/7/02)	Bubble Period (1/3/95-3/27/00)	Post-Bubble Period (3/28/00-5/7/02)
β_0	0.0047**	0.0020	0.0105**
	(0.0008)	(0.0013)	(0.0035)
β_1	-0.0227**	-0.0119*	-0.0367**
	(0.0036)	(0.0053)	(0.0091)
β_2	-0.4853**	-0.5849**	-0.4045**
	(0.0414)	(0.0488)	(0.0587)
β_3	-0.5184**	-0.6907**	-0.3995**
	(0.0473)	(0.0564)	(0.0704)
β_4	0.1842**	0.2281**	0.1582*
	(0.0419)	(0.0367)	(0.0756)
β_5	-0.1885**	-0.0151	-0.3150**
	(0.0461)	(0.0644)	(0.0649)
$\bar{R}^2$	0.515	0.534	0.520

*Daily change of the VXN regressed on a constant, the lagged level of the VXN, separate variables for contemporaneous positive and negative returns on the underlying Nasdaq 100 index (NDX), and separate variables for positive and negative percentage deviations of the contemporaneous closing level of the NDX from its five-day moving average. Models are estimated using the Newey-West correction for autocorrelation at the first lag and heteroscedasticity. *Statistically significant at the 5% level. **Statistically significant at the 1% level.*

day moving average, but less so for the first difference of the VXN and the daily NDX returns.

Model Estimation Results

The model estimates are shown in Exhibit 3. The model is estimated with the Newey-West correction for heteroscedasticity and autocorrelation at the first lag, as the autocorrelations of the residuals from the model show a spike at the first lag, but not at higher lags. The model is estimated over the whole sample period from January 3, 1995, through May 7, 2002, and for subperiods corresponding to what is referred to as the bubble period from January 3, 1995, through the peak of the NDX on a closing basis on March 27, 2000, and what is referred to as the post-bubble period from March 28, 2000, through May 7, 2002.

The results for the whole sample period indicate that the VXN is mean-reverting. A higher (lower) level of the VXN is associated with a subsequent decline (rise) in the VXN that is statistically significant at the 1% level. The coefficient estimate of –0.0227 on the lagged level of the VXN indicates that implied volatility shocks have half-lives of roughly 30 business days. The results also indicate that the impact of positive and negative NDX returns on the VXN is highly directional—higher positive returns are associated with greater VXN declines, while more negative returns are associated with greater VXN increases.[10] Both coefficients are statistically significant at 1% confidence levels.

The coefficient estimates indicate that a 1% greater decline in NDX returns is associated with a 0.49 percentage point greater VXN increase, while a 1% greater increase in NDX returns is associated with a 0.52 percentage point greater VXN decrease. T-tests not shown indicate that the absolute values of these coefficients are not significantly different from each other. These results imply that the VXN responds in equal and in opposite directions to positive and negative NDX returns.[11]

The results also indicate that both greater positive and negative deviations of the NDX from its five-day moving average lead to statistically significant (at the 1% level) greater VXN increases. These results are robust and are little changed when other specifications are used, such as deviations of the NDX from three- and ten-day moving averages.[12]

A 1% increase in both positive and negative deviations of the NDX from its five-day moving average is associated with a roughly one-fifth percentage point greater increase in the VXN. Thus, despite the finding that positive NDX returns are associated with greater VXN declines, stronger positive NDX trends are associated with greater VXN increases.

These findings can be reconciled as follows. Positive returns by themselves lead to a reduction in fear and less of a demand for protection from puts and for the limited risk associated with being long calls. To the extent that positive returns lead to positive deviations of the NDX from its five-day moving average, however, the decline in the VXN is mitigated.

One possible explanation is that the trending behavior of the NDX leads to an increased demand for options because of gamma, which causes the deltas of the options to move in favor of call buyers.[13] In the case of negative returns that are associated with negative deviations of the NDX from its five-day moving average, the downward trend of stock prices reinforces the effect of the negative return, and the VXN rises by more. Overall, the model explains about half of the variation of the daily VXN change.

The results for the subperiods covering the bubble period and the post-bubble period are generally similar to the overall results. Positive and negative returns in both subperiods continue to have roughly equal but opposite effects on the VXN, and again the VXN is mean-reverting in both subperiods.

The major difference between the subperiods is the effect of the technical indicators on the VXN. During the bubble period, greater positive but not greater negative deviations from the NDX five-day moving average are associated with higher VXN increases. A possible explanation for greater negative deviations not having incremental effects on the VXN during the bubble period is that, as the trend of the NDX was overwhelmingly up, negative deviations from the five-day moving average were viewed more skeptically as being the start of sustained downward trends. During the post-bubble period, however, both positive and negative deviations of the NDX from its five-day MA led to greater VXN increases.

Exhibit 4 provides additional information about the way the VXN responds to extreme NDX returns over the entire sample period. Panel A shows that for the ten highest daily percentage NDX losses, which average 8.3%, the average VXN increase is 5.7 percentage points, to an average level of 63.1%.

As shown in Panel B, for the ten highest daily percentage NDX gains, which average 10.6%, the VXN falls an average of 5.5 percentage points to an average level of 60.4%. These VXN declines are fairly uniform. Each of the ten highest percentage daily NDX rallies are associated with VXN declines and Exhibit 4 shows that these declines typically occur from high VXN levels. These results are consistent with seeing the VXN as a fear index.

Overall, the results indicate that the VXN moves in opposite directions in response to large positive and negative NDX returns. One possibility is that the directionality of the VXN is consistent with how the actual volatility of NDX returns responds to positive and negative returns. An alternative possibility is that this directionality is driven by option trading dynamics, particularly by fluctuations in the demand for the defined risk associated with buying options.

II. ASYMMETRIC GARCH MODELS OF NDX VOLATILITY

To examine whether the actual volatility of NDX returns exhibits asymmetries with respect to positive and negative returns similar to those found in the VXN, we estimate the Glosten, Jaganathan, and Runkle (GJR) [1993] model, which nests the GARCH model of Bollerslev [1986], and allows conditional volatility to respond differently to equal-magnitude positive and negative return innovations.

The asymmetric GARCH model is specified as:

$$R_t = \mu + \eta_t$$
$$\eta_t = N(0, h_t^2)$$
$$h_t^2 = \alpha_0 + \beta h^2_{t-1} + \delta_1 h^2_{t-1} + \delta_2 D\eta^2_{t-1} \qquad (4)$$

where R_t is the daily return expressed as the difference from t − 1 to t of the natural logs of the NDX, μ is the

EXHIBIT 4
Extreme NDX Returns and VXN Changes

Panel A. Ten Greatest One-Day NDX Percentage Losses

Date	NDX Return	VXN Change	Ending VXN
08/31/98	-9.86	6.04	48.06
04/14/00	-9.80	12.19	79.79
01/02/01	-9.09	0.26	72.75
09/17/01	-8.25	8.07	62.74
12/20/00	-7.89	9.21	76.69
01/05/01	-7.81	8.05	70.87
04/03/01	-7.75	2.48	64.33
03/28/01	-7.69	1.52	58.55
10/27/97	-7.42	8.69	35.55
05/23/00	-7.39	0.69	61.67
Average	-8.30	5.72	63.10

Panel B. Ten Greatest One-Day NDX Percentage Gains

Date	NDX Return	VXN Change	Ending VXN
01/03/01	18.77	-7.67	65.07
12/05/00	11.68	-8.19	58.40
04/05/01	10.82	-0.07	65.33
04/17/00	10.10	-9.00	70.79
05/30/00	10.08	-3.61	58.03
04/18/01	9.53	-3.32	59.70
12/22/00	9.50	-5.43	67.85
10/13/00	9.10	-5.38	51.11
10/19/00	8.40	-7.08	46.47
04/25/00	7.99	-5.15	60.88
Average	10.60	-5.49	60.36

mean daily return, and η_t is the innovation of the mean daily return. The innovation is assumed to be normally distributed with a mean equal to zero and a conditional variance equal to h_t^2.

The variable D is an indicator variable that takes the value of one if the lagged innovation is negative, and zero otherwise. This specification allows equal-magnitude positive and negative squared lagged return innovations to have different effects on the conditional variance, which is captured by the δ_2 coefficient. If this coefficient is significantly positive, negative return innovations have greater effects on conditional volatility than equal-magnitude positive returns. If the δ_1 coefficient is significantly positive, positive return innovations lead to conditional volatility increases, which would be inconsistent with the negative effect of positive NDX returns on the VXN that we have found so far.[14]

Exhibit 5 presents estimates of asymmetric GARCH models of daily NDX returns for the whole sample period from January 3, 1995, through May 7, 2002, and for the two subsamples. For the whole period, the results indicate that both positive and negative return innovations give rise to significant conditional volatility increases. Negative return innovations, however, lead to roughly four times greater conditional volatility increases than positive return innovations of the same magnitude.

The estimates also indicate that conditional volatility is mean-reverting in both subperiods. For the bubble period, the results indicate that both positive and negative return innovations give rise to higher conditional volatility, although the effect on conditional volatility from negative return innovations is about three times greater than from equal-magnitude positive return innovations. For the post-bubble period, the estimates indicate that

EXHIBIT 5

Asymmetric GARCH Model of Daily Returns of NDX (January 3, 1995–May 7, 2002)

	Average Daily Return	$\alpha_0 x10^4$	β	δ_1	δ_2	Likelihood Function
Jan. 3, 1995 -May 7, 2002	0.0011* (0.0004)	0.0750** (0.020)	0.8939** (0.0135)	0.0359** (0.0131)	0.1145** (0.0171)	6242
Jan. 3, 1995 -Mar. 27, 2000	0.00167** (0.0004)	0.1831** (0.0526)	0.8202** (0.0305)	0.0636** (0.0227)	0.1267** (0.0270)	4700
Mar. 28, 2000 -May 7, 2002	-0.0033** (0.0013)	0.1218 (0.1040)	0.9287** (0.0190)	-0.0223 (0.0142)	0.1970** (0.0378)	1553

**Statistically significant at the 5% level. **Statistically significant at the 1% level.*

positive return innovations no longer lead to statistically significant conditional volatility increases.

Overall, the results indicate that the conditional volatility of the underlying NDX index is far less directional with respect to positive and negative returns than the VXN. Contrary to the evidence that the VXN tends to drop on large NDX rallies, no tendency is found for the conditional volatility of NDX returns to drop with positive returns.

III. PREDICTIVE POWER OF VXN AND TIME SERIES FORECASTS OF ACTUAL VOLATILITY

To test the forecasting performance of the VXN, we compare it with out-of-sample asymmetric GARCH forecasts and moving-average forecasts for the volatility of NDX returns from 22 business days before option expiration through option expiration. We estimate the asymmetric GARCH model with daily NDX return data over a pre-sample period from the beginning of January 1992 through the end of December 1994 to obtain initial parameter estimates. These estimated parameters are used to obtain one-day-ahead volatility forecasts for 1995—the first year of the sample period—using the formula:

$$E(h^2_{t+1}) = \alpha_0 + \beta\, h_t^2 + \delta_1 h_t^2 + \delta_2\, D\eta_t^2 \qquad (5)$$

The conditional volatility forecasts more than one day ahead are formed using the equation:

$$E(h^2_{t+s}) = \alpha_0 + (\beta + \delta_1 + 0.5\delta_2)\, E_t\,(h^2_{t+s}) \quad \text{for } s > 1 \qquad (6)$$

Equations (5) and (6) reflect the assumption that positive and negative return innovations have asymmetric effects on conditional volatility only one day into the future. Because future positive and negative return innovations are assumed to be equally likely, more-distant conditional volatility forecasts are invariant to the sign of the most recently observed innovation.[15]

The forecast of the standard deviation of returns over the remaining life of the nearby option contract from 22 business days before expiration is equal to the mean of the annualized daily standard deviation forecasts for each day through option expiration. For each subsequent year through the end of the sample period in May 2002, GARCH volatility forecasts are obtained in the same manner by reestimating the models with the prior year of data added to the estimation period.

The forecasting power of moving averages of lagged volatility is also examined. These forecasts are obtained by constructing moving averages of lagged volatility with exponentially weighted observations, which gives more recent observations greater weight than more distant obser-

EXHIBIT 6
Volatilities and Forecast Errors (January 3, 1995–May 7, 2002)

Panel A: Volatilities

	Mean	Standard Error	Bottom Decile	Top Decile
Actual Volatility	0.2765	0.1279	0.1415	0.4317
VXN	0.3520	0.1312	0.2236	0.5557
GARCH	0.3058	0.1485	0.1786	0.5491
EWMA	0.2610	0.1183	0.1444	0.4146

Panel B: Forecast Errors

	Mean	Standard Error	Bottom Decile	Top Decile
VXN	0.0756	0.0724	-0.0101	0.1658
GARCH	0.0293	0.0893	-0.0674	0.1279
EWMA	-0.0154	0.0798	-0.1131	0.0706
Random Walk	0.0050	0.0904	-0.1030	0.1087

Panel C: Absolute Forecast Errors

	Mean	Standard Error	Bottom Decile	Top Decile
VXN	0.0867	0.0584	0.0209	0.1659
GARCH	0.0668	0.0658	0.0057	0.1474
EWMA	0.0605	0.0539	0.0106	0.1519
Random Walk	0.0651	0.0626	0.0082	0.1352

vations. These forecasts, hereafter referred to as EWMA volatility forecasts, are equal to the square root of

$$(1-\lambda)\sum_{\tau=1}^{75}\lambda^{\tau} r^2_{T+1-\tau}$$

with parameters set equal to the values used by RiskMetrics [1996]—λ is set equal to 0.94; the moving average is constructed from the most recent 75 daily NDX returns (r); and the mean return is set equal to zero. These daily volatility forecasts are converted to annualized volatility forecasts by multiplying by the square root of 252, the typical number of trading days in a year.

Data

Exhibit 6 provides background information on the predicted and actual volatilities and forecast errors over the sample period from January 1995 through May 2002. The table shows the means, standard errors, and the bottom and top deciles of the actual volatilities, the volatility forecasts, and the volatility forecast errors. Random walk forecast errors also are included for perspective.

The VXN is measured 22 business days before each of the monthly NDX option expirations, while actual volatilities and GARCH volatility forecasts are measured from 22 business days before option expiration through option expiration. The actual volatility is the sample standard deviation of NDX returns over the 22 business days before option expiration, with the mean return set equal to zero to avoid the potential bias that arises from using noisy estimates of the true mean (see Figlewski [1997] for a discussion of this issue).

Exhibit 6 indicates that the VXN averages 7.5 percentage points higher than actual volatility, while the GARCH volatility forecasts average 3.0 percentage points higher than actual, and the average EWMA volatility forecasts are 1.5 percentage points below actual volatility. The mean absolute forecast errors of the VXN are about a third greater than those of both the out-of-sample GARCH volatility forecasts and the EWMA volatility forecasts. The mean absolute forecast errors of the GARCH forecasts, the EWMA forecasts, and the random walk forecasts are about equal, which indicates that these absolute forecast errors are roughly the same magnitude as the absolute changes of actual volatility.

The VXN is more volatile than both actual volatility and the other volatility forecasts, and has an upper decile cutoff of 55.6% compared to 43.2% for actual volatility.

Predictive Power Tests

To compare predictive power and biases of the forecasts in both levels and first differences, in Equation (7) we regress the actual volatility of NDX returns on a constant and on the VXN, the out-of-sample asymmetric GARCH volatility forecasts, and the EWMA volatility forecasts separately:

$$\sigma_{t,T} = \alpha + \beta\, \sigma_t^{VXN,\, G,\, EWMA} + \varepsilon_{t,T} \tag{7}$$

where $\sigma_{t,T}$ is the actual volatility of NDX returns from 22 business days before nearby option expiration through option expiration. σ_t^{VXN}, σ_t^{G}, and σ_t^{EWMA} are the VXN, GARCH, and EWMA forecasts of actual volatility, respectively, each measured 22 business days before the nearby option contract expires.

Equation (8) specifies the model in first differences:

$$\sigma_{t,T} - \sigma_{t-1,T-1} = \alpha + \beta[\sigma_t^{VXN,\, G,\, EWMA} - \sigma_{t-1,T-1}] + \varepsilon_{t,T} \tag{8}$$

where the dependent variable is the change in actual volatility over the 22 business days before the previous option expiration to the 22 business days before the current option expiration, and $[\sigma_t^{VXN,\, G,\, EWMA} - \sigma_{t-1,T-1}]$ are the predicted changes of actual volatility from its level before the previous expiration according to the VXN, GARCH, and EWMA forecasts, respectively.

If the volatility forecasts in Equations (7) and (8) are unbiased, the intercept coefficients should not be significantly different from zero, and the slope coefficients should not be significantly different from one. If the volatility forecasts have predictive power, the parameter estimates of the volatility forecasts should be significantly greater than zero.

Whether the VXN encompasses information in the other forecasts is examined by regressing actual volatility on a constant and on the VXN and either the GARCH volatility forecasts or the EWMA volatility forecasts. These models are estimated both in levels and in first differences as shown in Equations (9) and (10):

$$\sigma_{t,T} = \alpha + \beta\, \sigma_t^{VXN} + \lambda\, \sigma_t^{G,\, EWMA} + \varepsilon_{t,T} \tag{9}$$

$$\sigma_{t,T} - \sigma_{t-1,\, T-1} = \alpha + \beta\, [\sigma_t^{VXN} - \sigma_{t-1,\, T-1}] + \lambda\, [\sigma_t^{G,\, EWMA} - \sigma_{t-1,\, T-1}] + \varepsilon_{t,T} \tag{10}$$

If the VXN encompasses the information in the other volatility forecasts, the coefficients on the other volatility forecasts should not be statistically significant, and the coefficient on the VXN should remain significantly positive. All these regressions are estimated for the whole sample period with White's correction for heteroscedasticity.[16]

The estimates are shown in Exhibit 7. The results for the levels regressions indicate that the VXN and the GARCH volatility forecasts have significant predictive power. They both have coefficients that are significantly less than one, indicating that they are biased forecasts of actual volatility. In addition, the intercept coefficient is significantly positive in the GARCH regression, but is insignificant in the VXN model. These coefficient estimates indicate that the VXN tends to overforecast actual volatility over its entire range, while GARCH forecasts tend to overpredict actual volatility over much of its range.[17]

The coefficient estimates on the EWMA volatility forecasts indicate that the slope coefficient is significantly greater than zero and not significantly different from one, while the intercept term is significantly positive. These coefficient estimates indicate that the EWMA volatility forecasts tend to underestimate subsequent actual volatility when EWMA volatility forecasts are at low levels, and overestimate subsequent actual volatility when EWMA volatility forecasts are at high levels.

Each of the forecasts explains roughly two-thirds of the overall variation of the level of actual volatility. When both the VXN and either the GARCH volatility forecast or the EWMA volatility forecasts are included in the models, the VXN continues to show a significantly positive coefficient, while the other forecasts are no longer statistically significant. These results indicate that the VXN encompasses the information in the other forecasts for the level of actual volatility.

When the models are estimated in first differences, both the VXN and the GARCH forecasts enter by themselves with significantly positive coefficients. The coefficient on the VXN is not significantly different from one,

EXHIBIT 7

Predictive Power of Volatility Forecasts (January 1995-May 2002)

	Constant	VXN	GARCH	EWMA	$\bar{R}^2$	D-W	Wald Test
Levels							
	-0.013	0.823**†			0.71	2.06	124
	(0.023)	(0.074)					(00)
	0.065**		0.690**†		0.64	2.13	20.12
	(0.020)		(0.075)				(00)
	0.053*			0.857**	0.62	2.21	12.55
	(0.023)			(0.099)			(00)
	-0.009	0.752**	0.067†		0.71	2.11	--
	(0.023)	(0.159)	(0.149)				
	-0.011	0.761**		0.074†	0.71	2.11	--
	(0.024)	(0.191)		(0.207)			
First Differences							
	-0.071**	0.939**			0.32	2.11	97
	(0.017)	(0.212)					(00)
	-0.012		0.458*†		0.07	2.52	13.36
	(0.009)		(0.206)				(00)
	0.021*			1.44**	0.19	2.17	5.48
	(0.009)			(0.352)			(0.06)
	-0.072**	0.970**	-0.059		0.31	2.11	--
	(0.017)	(0.220)	(0.203)				
	-0.047*	0.770**		0.807*	0.36	1.97	--
	(0.019)	(0.202)		(0.321)			

*Standard errors are estimated using the White correction for heteroscedasticity and are shown in parentheses. * and ** Significantly different from zero at the 5% and 1% level, respectively. †Significantly different from one at the 1% level. Wald tests are for the joint hypothesis that the intercept term is equal to zero and the coefficient on the volatility forecast is equal to one. P-values for the Wald tests in parentheses.*

although the constant term is significantly negative. These coefficients indicate that the VXN is a biased forecast of actual volatility changes—when the VXN is high relative to recent actual volatility and thus is predicting rising actual volatility, such predictions tend to be too high.

The coefficient on the GARCH forecasts is significantly less than one, indicating that these forecasts are biased as well. The EWMA volatility forecast enters the first difference models with a coefficient that is significantly positive and not significantly different from one, although again the constant term is significantly positive.

The VXN forecasts explain roughly 32% of the overall variation in actual volatility changes, while the GARCH and EWMA volatility forecasts explain roughly 7% and 19% of the overall variation in actual volatility changes.

When both the VXN and the GARCH forecasts are included in the first-difference models, the VXN continues to enter with a significantly positive coefficient, and again the GARCH forecast does not enter the model with a significant coefficient. When the VXN and the EWMA forecasts are included in the first-difference models, both the VXN and the EWMA forecasts enter with significantly positive coefficients. Thus, the VXN encompasses the information in GARCH forecasts, but not the information in EWMA forecasts for actual volatility changes.

Overall, these results indicate that the VXN has significant predictive power in both levels and first differences for actual volatility, but it is a biased forecast of actual volatility.

IV. TRADING IMPLICATIONS

The large 7.5 percentage point average difference between the VXN and actual volatility over the sample period has implications for traders who buy at-the-money NDX call options and delta-hedge with NDX short positions.[18] To examine the issue, we price options using implied volatilities equal to both the average level of the VXN and the average level of actual volatility over the sample period. Then we calculate hedge ratios, and determine how much the NDX must change in order for gamma to offset one day of time decay. Like the VXN, the call options are assumed to be exactly at the money and have 22 business days until expiration. The NDX is assumed to be at 1000, and the risk-free rate is assumed to be 1.5%.

Exhibit 8 shows the prices and the Greeks for these two call options with implied volatility equal to both the 35.2% sample mean of the VXN and the 27.6% sample mean of actual volatility. At-the-money calls are priced at \$42.08 versus \$33.14 at the average level of the VXN versus the average level of implied volatility. The deltas of the calls are both roughly equal to 0.52, indicating that a delta-hedged position is short 52 shares of the NDX per one call option contract. Higher implied volatility translates into higher time decay—98 cents versus 78 cents per business day. Higher implied volatility also translates into a lower gamma—0.004 versus 0.005, which further works against the buyer of options.[19]

Abstracting from implied volatility changes, the delta-hedged position described above benefits from gamma and is hurt by time decay.[20] The break-even NDX change that causes gamma to offset time decay on a delta-hedged long call position is determined by solving an equation for the NDX change:

$$\text{Time Decay} = 1/2 * \text{Gamma} * (\Delta\ \text{NDX})^2 \qquad (11)$$

The break-even daily absolute NDX change rises to 2.27% from 1.78% when implied volatility is equal to the average level of the VXN over the sample period rather than equal to actual volatility. Over the sample period, absolute daily NDX returns higher than 2.27% occurred only about 28% of the time, while absolute daily NDX returns higher than 1.78% occurred roughly 38% of the time. Thus, the elevated level of implied volatility substantially raises the break-even change in the NDX for delta hedgers.

V. CONCLUSION

The evidence is clear that the Nasdaq Volatility Index (VXN) from January 1995 through May 2002 responds in opposite directions to positive and negative NDX returns, rising with NDX sell-offs and falling with NDX rallies. These parameter estimates are extremely stable across both the bubble and post-bubble periods. The tendency of the VXN to fall in response to large positive NDX returns is inconsistent with the conditional volatility of NDX returns, however, which shows no tendency to fall following large positive return innovations.

These findings suggest that the way market participants use options to manage risk explains some of the asymmetry. In particular, the asymmetry of the VXN is consistent with a conclusion that the demand for the protection associated with buying puts and for the limited downside risk associated with buying calls increases following NDX sell-offs and drops following NDX rallies. The VXN also rises when trends become stronger, as measured by greater positive and negative deviations of the NDX index from its five-day moving average.

While the VXN has statistically significant power to predict actual volatility, it is a biased forecast. In evaluating its predictive power, it is essential to distinguish between forecasting accuracy and predictive power. Biased volatility forecasts may have substantial predictive power, but may not provide reasonably accurate forecasts unless an adjustment is made for the bias.

Although the VXN has significant predictive power for actual volatility in both levels and first differences, its mean forecast error in levels is 7.5 percentage points. These results indicate that Nasdaq volatility forecasts based on the VXN should be adjusted downward substantially, even after eliminating the built-in bias in the index caused by the calendar year/trading day adjustment we have described. The elevated level of implied volatility relative to actual volatility substantially raises the magnitude of the daily NDX change that causes gamma to offset time decay for delta-hedged buyers of near-term at-the-money NDX options.

Exhibit 8
Impact of Higher Implied Volatility

	Sample Average Implied Volatility 35.2%	Sample Average Actual Volatility 27.6 %
Option Price	42.08	33.14
Delta	0.5257	0.5227
Daily Time Decay	0.9810	0.7811
Gamma	0.038	0.0049
Vega	1.178	1.176
Break-Even Price Change	2.27%	1.78%

At-the-money NDX call options priced using implied and actual volatility. Options assumed to be exactly at the money and with 22 business days until expiration. Break-even price change is the daily NDX change where gamma offsets time decay.

ENDNOTES

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[1]Simon and Wiggins [2001] find that when the VIX is in its top decile from January 1989 through June 1999, S&P futures are 2.8 percentage points higher on average over the next 30 days than they otherwise would be.

[2]In each of their subsamples, implied volatility fails the unbiasedness test, and in only 6 of 32 subsamples are the coefficients on implied volatility significantly greater than zero at the 5% level.

[3]If weekend days contribute to equity prices the same information flow (and hence volatility) as business days, the standard deviation of returns from Friday to Monday would be $\sqrt{3}$ times greater than the standard deviation of returns between consecutive trading days during the same week. The annualized standard deviations of daily returns over the sample period for consecutive days that are both trading days and for Fridays to Mondays are 27.7% and 28.3%, respectively.

[4]Whether mean reversion is different for positive and negative departures of the VXN from typical levels is also examined; asymmetries are not found.

[5]Fleming, Ostdiek, and Whaley find that the VIX falls less in response to a positive S&P 100 return than it rises in response to an equal-magnitude negative S&P 100 return, which they define as an asymmetric response.

[6]An investor who is long equities, for example, can convert this exposure to a long call position exposure by purchasing put options, or equivalently could sell the equities and buy call options.

[7]Derman [1999] documents this phenomenon for S&P 500 index options during subperiods from September 1997 through November 1998 when the S&P 500 index rallied. During these subperiods, specific strike price implied volatilities were little changed, but at-the-money implied volatilities fell as higher strike options with lower implied volatilities became the at-the-money options.

[8]For example, a trader who buys an at-the-money call against a delta-equivalent short position when the VIX is low benefits from buying the option more cheaply than otherwise. As a result, a smaller change in the underlying instrument is necessary for the effect of gamma to offset the time decay of the option.

[9]This possibility pertains more to non-professional option market participants. Options market makers who are long options on a delta-hedged basis would be inclined to rebalance their deltas frequently in order to lock in gamma profits, so the development of trends would not necessarily affect their profits.

[10]The intercept coefficient of 0.005 and the –0.48 coefficient on positive returns imply that positive NDX returns that are lower than 1.125% are associated with small VXN increases, while NDX returns higher than 1.125% are associated with VXN declines. Thus, the statement that higher positive NDX returns are associated with greater VXN declines is made for expositional ease, and should be understood to include the possibility that higher NDX returns can also lead to smaller VXN increases.

[11]The directionality found here for the VXN is greater than that found for the VIX. Fleming, Ostdiek, and Whaley [1995] find that the decline in the VIX for a positive S&P 100 index return is about half the increase in the VIX for an equal-magnitude negative S&P 100 index return using daily data for 1986-1992, excluding the stock market crash in October 1987.

[12]While it is possible that the technical indicators proxy for very near-term volatility that is not reflected in the model,