Contingent Claims Analysis Applied in Credit Risk Modeling

ANNE M. ANDERSON, WILLIAM MAXWELL, AND THEODORE M. BARNHILL, JR.

ANNE M. ANDERSON is a Ph.D. candidate in finance at the Eller College of Business and Public Administration at the University of Arizona in Tucson, AZ.
annea@mail.bpa.arizona.edu

WILLIAM MAXWELL is an assistant professor of finance at the Eller College at the University of Arizona in Tucson, AZ.
maxwell@eller.arizona.edu

THEODORE M. BARNHILL, JR., is a finance professor at the School of Business and Public Management at George Washington University in Washington, DC.
barnhill@gwu.edu

Credit risk analysis assesses the probability and effect of stochastic changes in credit quality (including default) on the value of a fixed-income security or a portfolio of fixed-income securities. Methodologies that assess credit risk are described in, for example, "CreditMetrics"™, "CreditRisk+"™, and Barnhill and Maxwell [2002].[1]

One of the key issues for an accurate credit risk system is a methodology to estimate the probabilities associated with a change in the credit quality of individual assets in a portfolio. "CreditRisk+" relies on historical transition probabilities to assess the probability of a change in credit quality. "CreditMetrics," KMV, and *ValueCalc* (see Barnhill [1997]) use a contingent claims framework to simulate credit transition probabilities.

Banks, insurance companies, and other institutions use credit risk analysis to set acceptable risk levels and capital requirements over predetermined time horizons. Credit risk analysis allows institutions to actively manage and mitigate the level of risk in their portfolios by adjusting their operating strategies or by using financial derivatives.

Credit risk is commonly thought of as the probability of default. This perspective views a fixed-income security in only two states, however, defaulted or not defaulted, and is appropriate only if the security is held to maturity. In the more complex environment necessary to price fixed-income securities before maturity in a mark-to-market setting, credit risk is a continuum, with multiple states of nature, each representing a different associated probability of default. Thus, temporal credit risk is a function of the probability of change in the value of the bond associated with a transition in the probability of default over time.

A positive credit change diminishes the likelihood of default and thus any reduction in the required yield. A negative credit change increases the likelihood of default, which reduces the value of the bond by increasing the required yield. Given variations in required yields within bond rating categories, this may be an overly restrictive definition of credit quality changes. Nonetheless, given the importance placed on credit ratings for assessing the risk associated with financial institutions, mutual funds, and pension funds, credit ratings by credible third parties serve an important function in credit risk analysis.

Credit risk analysis typically estimates the probability that financial assets will move to different risk categories, normally associated with bond ratings, over a predetermined horizon. The values of the financial assets are then estimated at some future time for each possible risk category using forward rates for the appropriate risk class.

The use of historical transition matrices, as in "CreditRisk+", to determine the probability of transition is appealing, but the practice is fraught with the danger that risk may be estimated incorrectly. Most historical transition matrices do not calculate the correlation of changes, which implies that credit risk on individual issues is independent. In fact, credit risk is clearly correlated with macroeconomic conditions (Fridson, Garman, and Wu [1997]), and hence correlated across issues. Thus credit transition probabilities likely differ depending on macroeconomic factors (e.g., economic expansion or contraction).

Furthermore, some historical rating transition data sets treat all firms the same. Clearly, industry- and firm-specific conditions of companies can influence the probability of credit changes. Finally, historical transitions are compiled over a number of years that may not necessarily reflect conditions prevailing in the market today.

A second dynamic approach, as in "CreditMetrics", KMV, and *ValueCalc*, simulates probability distributions in an option-theoretic or a contingent claims model as developed by Black and Scholes [1973] and Merton [1974]. In this framework, the firm's stockholders hold a call option on the firm, and the debt ratio is a measure of how far the call option is in the money. In Merton's model, default risk is a function of the firm's leverage and standard deviation of return.[2]

Risk management systems use a contingent claims framework to determine default risk as follows. First, the value of a firm's assets or equity is simulated. A distribution of possible future asset values and debt ratios is estimated. The simulated debt ratios are then mapped into bond ratings.

This methodology assumes a deterministic relationship between the firm's debt ratio and its bond rating. Jones, Mason, and Rosenfeld [1984] find weak support for contingent claims models as a method for determining bond prices. Ogden [1987], however, finds that a firm's standard deviation of return and leverage ratio can be used to explain 78% of the variation in firms' bond ratings. The strongest argument for this approach can be found in Campbell and Taksler [2002], who find idiosyncratic risk is the most significant factor (including bond rating) in describing cross-sectional bond returns.

We examine the appropriateness of mapping debt ratios into bond ratings. To do this, we first examine the distributions of debt ratios in each bond rating category to determine if there are statistically significant differences. First, we analyze the distributions for all firms. Second, we refine the analysis for broad industry categories (manufacturing, service, utility, wholesale/retail, oil and gas, financial, real estate, and business service firms). Third, we analyze whether the relationships between debt ratio and bond ratings are stable over time. Finally, we draw conclusions about the use of debt ratios in simulating bond ratings.

I. DEBT RATIOS AND BOND RATINGS

To examine the relationship of debt ratios and bond ratings, a cross-sectional time series of debt ratios is calculated for all firms with bond ratings. Annual data are collected for the 1985-2001 time period from Compustat's combined active and research databases for firms with bond ratings (20,966 firm-year observations). We include defaulted bonds, but obviously these ratios are problematic. Debt ratios are calculated as total debt (long-term debt plus debt in current liabilities) divided by total debt plus market value of equity.[3] Compustat is also used to identify Standard & Poor's debt ratings and primary SIC codes for applicable firms.

We first analyze the distributional properties of debt ratios by bond rating category and find they are not normally distributed. Hence, a mean-variance framework does not adequately describe the underlying distribution of debt ratios. Thus, we rely on a non-parametric approach, and report median and quartile statistics instead of means and standard deviations.

For debt ratios to be useful in the mapping of bond ratings there must be some statistically significant difference in the distributions of debt ratios across rating categories. To test whether such differences are statistically significant, we use a Scheffe test, the equivalent of the Wilcoxon rank sum test for multiple categories. The Scheffe test is a non-parametric test that ranks observations in two samples from highest to lowest. The relative proportions of the rankings are then tested for statistical significance.

The Scheffe test is run for all rating categories. For example, AAA bonds are tested to see whether their debt ratios are statistically different from the debt ratios for AA bonds and so forth.

The results of the analysis of the relation between debt ratios and bond ratings are found in Exhibit 1. The results show a statistically significant difference (99% confidence level) between debt ratios and bond ratings in all categories. We also note a monotonic relation between bond rating and debt ratios for investment-grade firms. This suggests that as an overall methodology the use of simulated debt ratios to estimate bond rating probability distributions has promise.

EXHIBIT 1
Debt Ratios and Bond Ratings for All Firms: 1985-2001

S&P Rating	Observations	Median	Third Quartile	First Quartile
AAA	395	0.076*	0.174	0.028
AA	1921	0.161*	0.326	0.071
A	5271	0.232*	0.360	0.131
BBB	5379	0.329*	0.468	0.202
BB	4038	0.440*	0.608	0.280
B	3409	0.561*	0.758	0.348
CCC/C	425	0.778*	0.920	0.533
Defaulted	128	0.865	0.969	0.555

**Represents significance at the 99% confidence level.*

II. DEBT RATIOS AND INDUSTRY-SPECIFIC CATEGORIES

A basic tenet of corporate finance is that business and financial risk are inversely related. We therefore wish to see if there are significant differences in the debt ratios for firms with similar bond ratings across different industries, where the industry classification serves as the proxy for business risk.

By differentiating firms by industry, we expect to find more refined estimates of the difference between debt ratios and bond ratings. To test this hypothesis, we segment the firms into broad industry categories: Manufacturing (SIC 2000-3999), Service (SIC 7000-8999), Utilities (SIC 4000-4999), Wholesale/Retail (SIC 5000-5499), Oil and Gas Exploration (SIC 1300s), Financial (SIC 6000-6399), and Real Estate (SIC 6700s).

As in Exhibit 1, we examine the statistical significance of differences between adjacent rating categories. The differences in the debt ratios within categories but across industry classifications are also tested. For example, we test whether the debt ratio for a BBB bond is statistically different across manufacturing, service, and utility firms.

The results are presented in Exhibit 2. Overall debt ratios serve as a good proxy for bond ratings. Generally, we find statistically significant differences between adjacent rating categories. The only time this becomes problematical is for small sample sizes. In fact, in some industries, we find no AAA bonds and very few AA bonds. Even in these instances, we find significant differences between the most likely bond rating categories for the particular industry.

Next, we examine the differences in debt ratios for similarly rated bonds across industry classification. We find clear and statistically significant differences for ratings across industry classifications.

For example, in the A rating category, the median debt ratio for all firms is 0.232 (Exhibit 1), but 0.177 for manufacturing firms, 0.153 for service firms, 0.371 for utilities, 0.217 for wholesale/retail, 0.188 for oil and gas exploration, 0.235 for financial firms, and 0.287 for real estate firms. As expected, utilities are associated with the highest debt ratios, holding rating constant. Utilities, with lower levels of business risk, can sustain a higher degree of financial risk and maintain a bond rating similar to service or manufacturing firms. Service firms and oil and gas firms have the lowest debt ratios.

III. SEGMENTING INDUSTRIES

We find significant differences among manufacturing, service, utilities, wholesale/retail, oil and gas, financial, and real estate firms, which is consistent with the industries' different levels of business risk. As there still can be greater variance in business risk within these categories, we analyze the relationship of debt ratios and bond rating in more narrowly defined industry categories based on two-digit SIC codes. Given the limited number of observations and diversity of subcategories, service, wholesale, oil and gas, financial, and real estate firms are not segmented further.

Note that even a two-digit classification can be problematical. For example, the 2800s (Chemicals and Allied Products) includes traditional chemical producers, with high levels of business risk, and pharmaceutical companies, with low levels of business risk, which suggests very different levels of financial risk.

Exhibit 2

Debt Ratios and Bond Rating for Various Industries: 1985-2001

S&P Rating	Manufacturing Firms (M) (SIC 2000-3999) Obs	Median	Test for Diff	Service Firms (S) (SIC 7000-8999) Obs	Median	Test for Diff	Utilities (U) (SIC 4000s) Obs	Median	Test for Diff	Wholesale/Retail (W) (SIC 5000-5499)	Median	Test for Diff
AAA	229	0.048	U^a,W^a,F^a	19	0.028	W^b,F^a	54	0.173^a	M^a	112	0.137	M^a,S^b
AA	736	0.085^a	U^a,W^a,F^a	26	0.064	U^a,W^a,F^a	455	0.326^a	M^a,S^a,W^a,R^a	690	0.176	M^a,S^a,U^a,F^a,R^c
A	1934	0.177^a	U^a,W^a,F^a	170	0.153^a	U^a,W^a,F^a	1251	0.371^a	M^a,S^a,W^a,O^a,F^a,R^a	1909	0.217^a	M^a,S^a,U^a,O^a,F^a
BBB	1905	0.281^a	U^a,W^a,F^a,R^a	346	0.272^a	U^a,R^a	1156	0.448^a	M^a,S^a,W^a,O^a,F^a	1950	0.319^a	M^a,S^b,R^a
BB	1575	0.420^a	U^a,R^a	520	0.428^a	U^a,R^a	590	0.517	M^a,S^a,W^a	1476	0.432^a	U^a,R^a
B	1273	0.555^a	O^b	588	0.555^a	O^b	572	0.543^a	O^b	1222	0.600^a	O^a
CCC/C	99	0.735		67	0.769		96	0.725		159	0.827	
Defaulted	38	0.710		18	0.869		18	0.916		53	0.917	

S&P Rating	Oil and Gas Exploration (O) (SIC 1300s) Obs	Median	Test for Diff	Financial Firms (F) (SIC 6000-6399) Obs	Median	Test for Diff	Real Estate (R) (SIC 6700s) Obs	Median	Test for Diff
AAA	-	-		73	0.138	M^a,S^b	-		
AA	12	0.151		519	0.231	M^a,S^a,W^a,R^b	6	0.006	U^a,W^c,F^b
A	105	0.188^a	F^a	1236	0.235^a	M^a,S^a,U^a,W^a,O^a	64	0.287^b	U^a
BBB	221	0.304^b	U^a,R^a	728	0.287^a	M^a,U^a,R^a	376	0.419^a	M^a,S^a,W^a,O^a,F^a
BB	190	0.360	R^a	256	0.436^a	R^c	74	0.609^a	M^a,S^a,W^a,O^a,F^c
B	186	0.444^a	M^b,S^b,U^b,W^a,F^b	101	0.669	O^b	26	0.634	
CCC/C	49	0.848		32	0.917		15	0.848	
Defaulted	8	0.809		10	0.918		3	0.961	

a, b, c denote statistical significance at the 99%, 95%, and 90% confidence levels, respectively.

EXHIBIT 3
Debt Ratios for Manufacturing Firms: 1985-2001

	Panel 1 SIC 2000 *Food and Kindred Products*			Panel 2 SIC 3600s *Electronics*			Panel 3 SIC 3700s *Transportation*		
S&P Rating	Obs	Median	Diff Between 2-Digit SIC	Obs	Median	Diff Between 2-Digit SIC	Obs	Median	Diff Between 2-Digit SIC
AAA	41	0.076	3[a]	13	0.066	3[a]	3	0.209	1[a],2[b]
AA	105	0.107	2[b],3[a]	91	0.082	1[b],3[a]	56	0.246	1[a],2[a]
A	219	0.148	3[a]	156	0.149	3[a]	200	0.229	1[a],2[a]
BBB	136	0.234	2[a],3[a]	119	0.213	1[a],3[a]	164	0.351	1[a],2[a]
BB	55	0.507	2[a]	214	0.248	1[a],3[a]	152	0.530	2[a]
B	65	0.477	2[a]	193	0.351	1[a],3[a]	52	0.617	2[a]
CCC/C	7	0.622		12	0.588		2	0.750	
Defaulted	4	0.586	3[c]	2	0.746		3	0.806	1[c]

[a, b, c] *denote statistical significance at the 99%, 95%, and 90% confidence levels, respectively.*

EXHIBIT 4
Debt Ratios for Utilities: 1985-2001

	Panel 1 SIC 4800s *Communications*			Panel 2 SIC 4900s *Electric, Gas, and Sanitation*		
S&P Rating	Obs	Median	Diff Between 2-Digit SIC	Obs	Median	Diff Between 2-Digit SIC
AAA	41	0.186		10	0.159	
AA	96	0.174	2[a]	337	0.343	1[a]
A	232	0.231	2[a]	899	0.399	1[a]
BBB	163	0.259	2[a]	733	0.486	1[a]
BB	211	0.432	2[a]	190	0.506	1[a]
B	393	0.471	2[a]	63	0.767	1[a]
CCC/C	57	0.617	2[b]	7	0.938	1[b]
Defaulted	11	0.880		3	0.888	

[a, b, c] *denote statistical significance at the 99%, 95%, and 90% confidence levels, respectively.*

In Exhibit 3, we divide manufacturing firms into categories with at least 600 total observations. These include Food and Kindred Products (SIC 2000s), Electronics (SIC 3600s), and Transportation (SIC 3700s). Overall, we see a consistent pattern between debt ratios and bond ratings. There are significant differences between categories, however; transportation firms have the highest debt ratios and electronics firms the lowest; food firms are somewhere in the middle. For example, the debt ratios for BBB-rated bonds are 0.351 for transportation, 0.234 for food, and 0.213 for electronics.

We perform a similar analysis for the utility industry by segmenting utilities into Communications (SIC 4800s) and Electric, Gas, and Sanitation (SIC 4900s). The results are presented in Exhibit 4.

EXHIBIT 5

Debt Ratios and Bond Ratings for Three Segments Across Time

Time Period	Panel 1 1985-198			Panel 2 1990-1991			Panel 3 1992-2000			Panel 4 2001		
Panel A: Manufacturing Firms												
S&P Rating	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods
AAA	74	0.074		29	0.057	3^{c}	118	0.037	2^{c}	8	0.031	
AA	234	0.098	3^{b}	97	0.102		379	0.075	1^{b}	26	0.086	
A	508	0.198	$2^{a},3^{a},4^{c}$	198	0.235	$1^{a},3^{a},4^{a}$	1126	0.160	$1^{a},2^{a}$	102	0.152	$1^{c},2^{a}$
BBB	346	0.302	$2^{b},3^{a}$	156	0.357	$1^{b},3^{a},4^{b}$	1245	0.266	$1^{a},2^{a}$	158	0.291	4^{b}
BB	356	0.412		128	0.449		971	0.422		120	0.427	
B	427	0.519		84	0.555		670	0.566		92	0.657	
CCC/C	40	0.602		17	0.738		35	0.813		7	0.865	
Defaulted	12	0.547		3	0.453		16	0.746		7	0.806	
Panel B: Service Firms												
S&P Rating	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods
AAA	5	0.058		2	0.215		10	0.021		2	0.009	
AA	11	0.127	$3^{a},4^{c}$	2	0.119	3^{b}	12	0.034	$1^{a},2^{b}$	1	0.033	1^{c}
A	43	0.247		13	0.234		106	0.128		8	0.079	
BBB	45	0.413	$3^{b},4^{a}$	19	0.288		247	0.269	1^{b}	35	0.088	1^{a}
BB	110	0.433		34	0.451		335	0.431		41	0.195	
B	158	0.603		43	0.666	3^{c}	342	0.537	2^{c}	45	0.305	
CCC/C	18	0.660		13	0.836		30	0.839		6	0.421	
Defaulted	7	0.881		1	0.217		7	0.839		3	0.857	
Panel C: Utilities												
S&P Rating	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods	Obs	Median	Diff Between Periods
AAA	17	0.207	3^{c}	7	0.228		29	0.135	1^{a}	1	0.078	
AA	186	0.361	3^{a}	66	0.336	3^{b}	197	0.287	$1^{a},2^{b}$	6	0.296	
A	318	0.416	$3^{a},4^{c}$	127	0.392	3^{b}	734	0.344	$1^{a},2^{b}$	72	0.377	1^{c}
BBB	284	0.515	$3^{a},4^{a}$	124	0.497	$3^{a},4^{b}$	667	0.402	$1^{a},2^{a}$	81	0.409	$1^{a},2^{b}$
BB	95	0.550		35	0.567		415	0.499		45	0.561	
B	106	0.657	3^{a}	21	0.763	3^{b}	394	0.487	$1^{a},2^{b}$	51	0.636	
CCC/C	16	0.780		5	0.710		60	0.621		15	0.913	
Defaulted	2	0.684		2	0.482		7	0.945		7	0.963	

a, b, c denote statistical significance at the 99%, 95%, and 90% confidence levels, respectively.

Communications utilities, with greater business risk, are associated with lower debt ratios within bond rating categories. These differences are considerable. The debt ratios for the traditional utilities are close to double the communications utilities in investment rating categories.

Overall, we find that bond ratings and debt ratios differ significantly among manufacturing, service, utilities, wholesale/retail, oil and gas, financial, and real estate firms, although there still can be great and significant variation within these industry classifications.

IV. DIFFERENCES IN DEBT RATIOS OVER TIME

A major concern in the use of historical transition matrices in a credit risk management system is the use of a long-term historical average to represent current market conditions. To see if this problem is present in the use of debt ratios as a methodology of mapping out bond rating changes, we separate the data into four time periods representing different economic conditions. Low economic growth periods are defined as the recessionary time periods of 1990-1991 and 2001. Growth periods are defined as 1985-1989 and 1992-2000.

Given the demonstrated differences among industries, the data are divided further into the three largest categories: manufacturing firms, service firms, and utilities. A similar analysis is performed for the other smaller categories (results not reported). The results are consistent, although the significance of the results is reduced, which is not surprising, given the small sample sizes.

The results of this analysis appear in Exhibit 5. As expected, we generally find an increase in debt ratios for the 1990-1991 period over the 1985-1989 period; the increase is most apparent in manufacturing firms. After the 1990-1991 time period, we find significant declines in debt ratios across the board within bond rating categories for manufacturing, service, and utility firms.

The lower debt ratios during the period of higher economic growth seem counter-intuitive. It would be expected that during periods of low or negative economic growth, which implies higher default risk (Fridson, Garman, and Wu [1997]), firms would need to have lower debt ratios (less firm-specific risk) to maintain the same debt rating. The results are just the opposite.

Part of the explanation is that debt ratios calculated on a market value basis tend to go up (down) when market equity value goes down (up).[4] In addition, these results are consistent with findings by Blume, Lim, and MacKinlay [1998] that rating agencies have become more stringent in their rating requirements over this time period.

We do not find significant changes between the 1992-2000 period and the recessionary 2001 time period. This is probably due in part to the small sample size and a possible lag effect in ratios. Any conclusions regarding service firms are limited by the small sample.

In any event, the temporal variations in the relationships between debt ratios and bond ratings point out the need to use relationships that are consistent with the time period of analysis. If long-term historical averages are used, there may be biases in simulating bond ratings.

V. CONCLUSION

We empirically examine the accuracy of using debt ratios alone to estimate bond ratings. While no one would argue that additional information not incorporated in the debt ratio is not important in assessing default probabilities, our results clearly show a statistically significant relationship between debt ratios and bond ratings, which is consistent with a contingent claims framework.

We also conclude that mapping of debt ratios into bond ratings becomes more accurate as the industry segment is defined more precisely. Care must be used in relying on the historical relations between debt ratios and bond ratings, as these relationships may change over time.

While the results clearly suggest a statistically significant relationship of debt ratios and bond ratings, one would want to exercise caution in using a mapping system where the median value for a higher-rated bond may be higher than the first-quartile observation of a lower-rated bond. This occurs mostly in the non-investment-grade categories.

We believe that the crossing of median debt ratios for higher-rated bonds and the first quartile of lower-rated bonds is consistent with known market conditions or a sampling error. Market pricing of bonds is not always consistent. There is a great deal of variation in the pricing of similarly rated bonds, and crossing of yields may occur, when lower-rated bonds will be trading at a lower yield to maturity than higher-rated bonds. These market imperfections are more prevalent in the non-investment-grade categories (as in our results), which suggests imperfections in the bond rating system.

We also rely on the primary SIC code to segment the data. No method is available to distinguish firms with substantial operations in diverse industries. This would clearly limit the comparability of firms even within primary SIC codes.

Finally, the crossing may be due to the time period analyzed. The longer the time period, the greater the expected variation within bond rating categories.

Overall, we conclude that credit risk simulation methodologies, which rely on debt ratios to map bond ratings, can produce useful estimates of bond rating migrations. Nonetheless, to produce the most accurate simulations the estimated relationships between debt ratios and bond ratings should be as industry-specific as possible, and reflect market conditions at the time the simulation is undertaken.

ENDNOTES

[1]Altman and Saunders [1998] provide an overview of credit risk measurement.

[2]Merton [1974] defines debt ratio as debt over equity. To simplify and for comparison purposes, we use the algebraically equivalent debt over total market capitalization, defined as book value of debt/(book value of debt + market value of equity), which we refer to throughout as the debt ratio.

[3]To check for the robustness of our findings, we also examine alternative definitions of indebtedness, including total debt minus secured obligations (leases and mortgages), total debt

minus cash, total debt minus notes payable, and total term debt minus secured obligations and cash and notes payable. The results are qualitatively similar using any of the measures, although the measure reported in the text produces the most consistent results across industry classification and time period.

[4]A similar analysis with book value in the debt ratio shows book value ratios to be more consistent although with the same general trends.

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