Evidence on Theta and Convexity in Treasury Returns

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In the absence of yield changes, a time passage (theta) effect fully accounts for Treasury returns. For example, in the case of coupon-paying Treasuries, the coupon and its reinvestment are the only sources of investment income, for which the elapse of time is the sole determinant. In the presence of yield changes, a duration effect accounts for most of the Treasury returns as it captures the capital gains and losses due to yield changes.[1]

A finer approximation of Treasury returns, especially for long-maturity Treasuries and large yield changes, requires consideration of the convexity effect that adjusts the non-negligible downward bias in the duration-based approximation of the capital gains and losses.[2]

A finer approximation of Treasury returns using the three measures, duration, convexity, and theta, can serve as a practical alternative to the total return analysis of option-free Treasuries. This alternative analysis becomes equally accurate when the effect of the interactions between yield changes and time passage is negligible, which is often the case for most interest rate environments in developed economies. It becomes even simpler if the relative importance of the theta effect is known.

From mathematical expressions of the convexity and theta effects on Treasury returns, it can be shown that, all else equal, the theta effect for Treasuries will be considerable at high levels of yield, while the convexity effect will be slight (great) at high levels of yield (coupon rate). The relative importance of the theta (or convexity) effect can be determined if the magnitude of yield changes and the length of time passage are given, although the extent of yield changes can be estimated from historical yield changes. Hence, determination of relative importance becomes an empirical issue.

This empirical issue was first studied by Chance and Jordan [1996]. Using data on selected U.S. Treasuries traded between 1977 and 1993, they find, that even for relatively large yield changes and very short time periods, the theta effect dominates the convexity effect in explaining the variability in Treasury returns. Their evidence on the theta effect, however, is not conclusive. This is because their sample includes selected maturity sectors traded in a period of relatively high yields; hence, their analysis is biased for the theta effect (against the convexity effect). Therefore, we cannot draw strong implications from this finding unless it also holds for different Treasury markets, various interest rate environments, and diverse maturity sectors.

We examine the relative importance of the theta and convexity effects in the major international Treasury markets that are most suitable for testing for additional evidence of the theta effect. We use monthly data on option-free Treasuries of eleven relatively developed economies including the U.S. and eight major Organization for Economic Cooperation and Development countries, plus Hong Kong and Singapore. The sample period for the U.S. Treasury market is from 1982 through 2000; the sample periods for the non-U.S. Treasury markets vary but are a subset of the period running from 1975 through 2000.

We closely follow the methodology employed in Chance and Jordan [1996], which approximates Treasury returns in terms of duration, theta, convexity, interaction, and second-order time passage effects. For the purpose of international comparison, we construct a summary measure for the relative importance of the theta and convexity effects. The main finding is that, in all major international Treasury markets, the theta effect is much more important than the convexity effect, even for a one-day period and for quite large yield changes.

I. RESEARCH METHODOLOGY

Decomposition of Treasury Returns

To understand the Chance-Jordan decomposition, let us define the Treasury return R_t as a percentage change in the Treasury price P_t between dates t and $t + q$ and define it as:

$$R_t \equiv \frac{P_{t+1} - P_t}{P_t} \equiv \frac{\Delta P_t}{P_t} \tag{1}$$

In the case of a coupon-paying Treasury, the price is a full price inclusive of accrued coupons. In the case of T-bills, the price is the discounted price. The price change in a one-day period, for example, would reflect the time passage or pull-to-par effect, and, if there are yield changes in the same period, the effect of the yield changes. The time passage and yield changes between dates t and $t + q$ are denoted Δt and Δy_t, respectively.

A Taylor series expansion of the Treasury price change (ΔP_t) around an arbitrary pair of values for its two arguments (Δy_t and Δt) yields the approximate Treasury price change as:

$$\Delta P_t \approx \frac{\partial P_t}{\partial y}\Delta y_t + \frac{\partial P_t}{\partial t}\Delta t + \frac{\partial^2 \mathrm{P_t}}{\partial \mathrm{y} \partial \mathrm{t}}\Delta y_t \Delta t + \frac{1}{2}\frac{\partial^2 P_t}{\partial y^2}\Delta y_t^2 + \frac{1}{2}\frac{\partial^2 P_t}{\partial t^2}\Delta t^2 \tag{2}$$

When yield changes are large, most changes in the Treasury price as the result of yield changes are explained by the dollar duration effect (the first term), and the remainder are explained by the dollar convexity effect (the fourth term). When the time passage is long, most of the time passage effect is explained by the theta effect (the second term), and the remainder by the second-order time passage effect (the fifth term). Since yield changes can covary with the length of time passage, there is an interaction effect (the third term), which can be negligible for either very small yield changes or very short time periods.

Consider a yield change of Δy_t and a Treasury pricing model:

$$P_t = \sum_{i=1}^{T} X_i \left(1 + \frac{y_t}{n}\right)^{-(i-t)}$$

with cash payments of X_i at time t_i for $i = 1, 2, ..., T$ and the remaining maturity of T/n years, in which n is the fixed number of cash payments per year.[3]

The approximate Treasury return can then be decomposed as:

$$R_t \approx D + T_1 + I + C + T_2 \tag{3}$$

where D is the first-order approximate percentage change in the Treasury price as the result of yield changes. It is measured as

$$-\left[\left(1 + \frac{y_t}{n}\right)^{-1} D_t\right]\Delta y_t$$

where D_t is the Macaulay duration, and the term in brackets is the modified duration. T_1 is the first-order approximation of the percentage increase in the Treasury price as the result of time passage. T_1 is computed as $\theta_t \Delta t$, where the theta (θ_t), measured as $\ln(1 + y_t/n)$, reflects the approximate percentage increase in the Treasury price as the result of only the passage in one payment-unit time.

$$C \equiv \frac{1}{2}\left[\left(1 + \frac{y_t}{n}\right)^{-2} C_t\right](\Delta y_t)^2$$

is the second-order approximation of the percentage change in the Treasury price as the result of only the yield changes, where C_t is a non-modified convexity measure, and the bracketed term is the convexity measure.

$$I \equiv -\left(1+\frac{y_t}{n}\right)^{-1}\left(\theta_t D_t - \frac{1}{n}\right)\Delta y_t \Delta t$$

is the approximate percentage change in the Treasury price as the result of simultaneous changes in yield and time.

$$T_2 = \frac{1}{2}\theta_t^2\left(\Delta t\right)^2$$

is the second-order approximate percentage increase in the Treasury price as the result of only the time passage.

The approximate Treasury return in Equation (3) is decomposed in terms of duration (the D term); convexity (the C term); theta or the first-order time passage (the T_1 term); interaction (the I term); and the second-order time passage effects (the T_2 term). If yield changes (Δy) are negligible, the duration, convexity, and interaction effects do not contribute much to Treasury returns. If the length of time passage (Δt) is very short, then the theta, interaction, and second-order time passage effects do not contribute much to Treasury returns.[4]

Given the decomposition of Treasury returns, the percentage error in the approximation of Treasury returns can be given by:

$$\begin{aligned} e_t &= \frac{R_t - \left(D + T_1 + I + C + T_2\right)}{R_t} \\ &= 1 - \left[\frac{D}{R_t} + \frac{T_1}{R_t} + \frac{I}{R_t} + \frac{C}{R_t} + \frac{T_2}{R_t}\right] \\ &= \left[1 - \frac{D}{R_t}\right] - \left(\frac{T_1}{R_t} + \frac{I}{R_t} + \frac{C}{R_t} + \frac{T_2}{R_t}\right) \end{aligned} \tag{4}$$

Each fraction in the first set of brackets, when multiplied by 100, is the percentage of Treasury returns that is explained by the particular term. The $[1 - D/R_t]$ term reflects the portion of Treasury returns that is not explained by the duration effect.

The formulas for other effects suggest that the theta effect (T_1/R_t) and the convexity effect (C/R_t) will explain most of the unexplained variability in Treasury returns. Hence, the relative importance of the theta effect, compared to the convexity effect, is determined by the relative magnitude of T_1 and C. For the U.S. market, we compute the Chance-Jordan decomposition and compare the results with those in Chance and Jordan [1996].

Theta-to-Convexity Ratio

International comparisons of the Chance-Jordan decomposition for 10 maturity sectors across 11 markets are too complicated. To focus on comparisons of the relative importance of the theta, we compute the theta-to-convexity ratio as the summary measure for the relative importance of the theta effect. We construct, for each country, an equal-weighted Treasury portfolio from different-maturity Treasuries, and compare the theta-to-convexity ratios of the various country portfolios.

The theta-to-convexity ratio is given by:

$$\begin{aligned} \frac{T_1}{C} &= \frac{\theta_t \Delta t}{\frac{1}{2}\left(1+\frac{y_t}{n}\right)^{-2} C_t\left(\Delta y_t\right)^2} \\ &= \frac{ln(1+y_t/n)\Delta t}{0.5(1+y_t/n)^{-2}\left[\frac{1+y_t}{y_t} + \frac{T(y_t - c) - (1+y_t)}{c[(1+y_t)^T - 1] + y_t}\right]\left(\Delta y_t\right)^2} \end{aligned} \tag{5}$$

where every notation is as defined.

If the theta-to-convexity ratio equals one, the convexity and theta effects contribute equally to Treasury returns. If the ratio is less than (more than) one, the theta effect contributes to Treasury returns less than (more than) the convexity effect. Contrary to appearance, the relative contribution ratio does not actually depend on the length of time passage.[5]

Given a particular passage of time, the theta effect is mainly determined by the current level of yields. All else equal, the theta effect will be greater in countries with higher levels of yields. If the prevailing level of yields is higher (lower) than the prevailing level of coupon rates, Treasuries will trade at a discount (at a premium). Hence, all else equal, the theta effect for Treasuries trading at a discount will be greater than for Treasuries trading at a premium.

Given yield changes, the convexity effect is determined mainly by the current levels of both yields and coupon rates. It can easily be shown that, all else equal, the convexity effect is slighter (greater) at higher levels of yields (at higher levels of coupon rates). All else equal, the convexity effect will be greater in countries with lower levels of yields (and higher levels of coupon rates). The convexity effect and its dependence on the yields and coupon rates suggest that, all else equal, the convexity effect for Treasuries trading at a discount (at a premium) will be greater (slighter) than for Treasuries trading at par.

These major determinants of the theta and convexity effects indicate that, all else equal, the theta effect will be more important in countries with higher levels of yields (and lower levels of coupon rates). A ranking of the theta-to-convexity ratios between par and non-par Treasuries cannot be determined a priori.[6]

Computation Details

The contributions of the five components in Equation (4) and the theta-to-convexity ratio in Equation (5) depend mainly on the magnitude of yield changes and the length of time passage. As in Chance and Jordan, we consider two lengths of time, one day and thirty days. These lengths of time are short, and hence they are not biased against presence of the convexity effect.

For each of the two time periods, two possible yield changes are considered. They are 1) the mean of the historical yield changes plus/minus one standard deviation, and 2) the mean of the historical yield changes plus/minus two standard deviations. These yield changes are relatively large, and hence they are not biased against presence of the convexity effect.

II. DATA

The main source of data is Datastream, which provides monthly data on different-maturity Treasuries of OECD and non-OECD countries. In selection of the major international Treasury markets, we consider the availability of data, the variety of available maturities, and the period of available data. These considerations prompted the choice of eleven Treasury markets comprising nine OECD countries and two non-OECD countries. The OECD countries are the U.S., Japan, the U.K., Germany, Canada, Sweden, Belgium, Australia, and New Zealand. The two non-OECD countries are Hong Kong and Singapore.

The data include constant-maturity yields, coupon rates, and other Treasury-specific characteristics. Because the coupon rate information for Singapore Treasuries is not available on Datastream, the relevant data are obtained from various issues of the *Stock Exchange of Singapore Journal* (now replaced by PULSES).

The sample period runs from February 1975 through August 2000. The sample period for U.S. Treasuries is from February 1982 through August 2000. The periods for non-U.S. Treasury markets are subsets of the period from February 1975 through August 2000.

Exhibit 1 reports average historical yield changes and associated p-values by Treasury market and maturity sector. For the U.S., Japan, the U.K., Germany, Canada, and Hong Kong, the data were available for seven or more maturity sectors. For other countries, the data were available for five or fewer maturity sectors.

Historical yield change averages are relatively great in New Zealand, Sweden, Belgium, and Canada, but modest in Singapore, Japan, and Hong Kong. The p-values indicate, however, that nearly all the averages of the historical yield changes are not statistically different from zero.

In the 70 maturity-country observations, there are only three exceptions. They are the 30-year U.K. Treasury and the one-year Treasuries of Belgium and New Zealand. Given this finding, one standard deviation and two standard deviations of the historical yield changes are treated as the two possible yield changes.

Exhibit 2 reports statistics on coupon rates. U.S. Treasury coupon rates exhibit the widest range, 3.54% to 15.75%, as the result of relatively diverse maturity sectors. Similar diversity in sector maturities is observed also in the Treasury markets of the U.K., Canada, and Sweden.

The two highest median coupon rates, 9.38% and 8.0%, are observed in Canada and the U.K. The two lowest median coupon rates, 4.20% and 4.56%, are observed in Japan and Singapore.

III. EVIDENCE IN THE U.S. TREASURY MARKET

Exhibit 3 provides the details of the Chance-Jordan decomposition in the U.S. Treasury market. To save space and to provide a stronger test against the theta effect, it reports only the results obtained from assuming a one-day time passage and the yield change equivalent to two standard deviations of the historical yield changes.

As par yields are assumed for coupon-paying Treasuries in the U.S., our Exhibit 3 is comparable to Exhibit 5 in Chance and Jordan [1996]. Our Exhibit 3, however, reports results for ten maturity sectors over the 1982-2000 period, while Exhibit 5 in Chance and Jordan reports results for five maturity sectors (0.5, 1, 5, 15, and 30 years) over the 1975-1993 period.

The yield change, the length of one day in decimals, and initial and new prices are reported in Panel A. Treasury returns, Macaulay duration, the convexity measure, and theta are reported in Panel B. The Chance-Jordan decomposition and the theta-to-convexity ratio, which are computed using the inputs reported in Panels A and B, are reported in Panels C and D.

EXHIBIT 1

Average Historical Yield Changes by Country and Maturity Sector

Constant	U.S.			Japan			U.K.		
Maturity	#Obs	Mean	p-value	#Obs	Mean	p-value	#Obs	Mean	p-value
3 mo	210	-0.0472	0.11	175	-0.0355	0.08	277	-0.0200	0.65
6 mo	210	-0.0482	0.09	61	-0.0364	0.21	na	na	na
1 yr	157	-0.0079	0.74	61	-0.0374	0.26	na	na	na
2 yr	157	-0.0073	0.78	43	-0.0074	0.81	43	-0.0129	0.76
3 yr	157	-0.0079	0.77	43	-0.0108	0.76	42	-0.0189	0.67
5 yr	157	-0.0088	0.74	43	-0.0251	0.52	42	-0.0278	0.51
7 yr	157	-0.0076	0.76	43	-0.0294	0.43	42	-0.0361	0.41
10 yr	157	-0.0092	0.70	43	-0.0312	0.39	42	-0.0494	0.26
20 yr	70	0.0040	0.89	43	-0.0233	0.51	42	-0.0672	0.08
30 yr	157	-0.0069	0.74	na	na	na	42	-0.0768	0.05*

Constant	Germany			Canada			Sweden		
Maturity	#Obs	Mean	p-value	#Obs	Mean	p-value	#Obs	Mean	p-value
3 mo	na	na	na	114	-0.0668	0.20	123	-0.0706	0.30
6 mo	na	na	na	114	-0.0642	0.23	123	-0.0693	0.23
1 yr	na	na	na	234	-0.0336	0.49	123	-0.0637	0.19
2 yr	43	-0.0079	0.78	205	-0.0540	0.15	na	na	na
3 yr	43	-0.0060	0.86	205	-0.0525	0.14	na	na	na
5 yr	43	-0.0144	0.69	205	-0.0510	0.11	na	na	na
7 yr	43	-0.0237	0.49	174	-0.0302	0.32	na	na	na
10 yr	43	-0.0254	0.47	205	-0.0525	0.06	na	na	na
20 yr	43	-0.0290	0.39	na	na	na	na	na	na
30 yr	43	-0.0264	0.43	na	na	na	na	na	na

Constant	Belgium			Australia			New Zealand		
Maturity	#Obs	Mean	p-value	#Obs	Mean	p-value	#Obs	Mean	p-value
3 mo	174	-0.0468	0.14	223	-0.0335	0.58	na	na	na
6 mo	174	-0.0463	0.13	na	na	na	na	na	na
1 yr	99	-0.0651	0.03*	na	na	na	145	-0.0948	0.04*
2 yr	na	na	na	na	na	na	172	-0.0796	0.13
3 yr	na	na	na	na	na	na	na	na	na
5 yr	na	na	na	282	-0.0126	0.69	172	-0.0711	0.10
7 yr	na	na	na	na	na	na	na	na	na
10 yr	na	na	na	282	-0.0132	0.56	172	-0.0622	0.08

Constant	Hong Kong			Singapore		
Maturity	#Obs	Mean	p-value	#Obs	Mean	p-value
3 mo	97	-0.0118	0.90	138	-0.0108	0.85
6 mo	97	-0.0101	0.90	na	na	na
1 yr	97	-0.0095	0.90	na	na	na
2 yr	92	0.0078	0.91	24	-0.0192	0.81
3 yr	69	0.0339	0.66	na	na	na
5 yr	58	-0.0174	0.83	138	-0.0188	0.52
7 yr	44	0.0073	0.94	24	0.03	0.72
10 yr	33	0.0042	0.97	12	-0.0683	0.53

EXHIBIT 2
Average Treasury Coupons by Country

Country	Mean	Median	Minimum	Maximum	Range	Std Dev	Skewness	Kurtosis
U.S.	7.2425%	6.3750%	3.5404%	15.7500%	12.2096%	2.3867%	0.0129	0.0128
Japan	4.1763%	4.2000%	0.9000%	7.9000%	7.0000%	1.6609%	0.0005	-0.0094
U.K.	7.9172%	8.0000%	3.5000%	13.5000%	10.0000%	2.3290%	0.0022	-0.0046
Germany	5.0712%	5.0000%	2.2500%	9.0600%	0.0681%	1.9738%	0.3412	-1.1458
Canada	9.2278%	9.3750%	4.0000%	15.7500%	11.7500%	2.9037%	0.0013	-0.0061
Sweden	6.5617%	5.7500%	3.5000%	13.0000%	9.5000%	2.9627%	0.7486	-0.6489
Belgium	6.1380%	6.2500%	2.4850%	10.0000%	7.5150%	1.8313%	0.001	-0.0111
Australia	8.4185%	7.5000%	5.2500%	13.0000%	7.7500%	2.6551%	0.0053	-0.009
New Zealand	7.1550%	7.5000%	4.5500%	10.0000%	5.4500%	1.5728%	0.0005	0.0005
Hong Kong	7.4025%	7.0000%	5.9700%	10.0000%	4.0300%	1.1280%	0.0100	-0.0005
Singapore	4.3899%	4.5625%	1.2500%	6.2500%	5.0000%	1.5155%	-0.3207	-1.0822

Notes:

1. *Coupon-paying Treasuries in many countries pay coupons semiannually; in Germany, Sweden, and Belgium, they pay annually.*
2. *Practices in counting number of days in a year differ by country. In the U.S., Australia, and New Zealand, it is the actual number of days. In Japan, the U.K., Canada, Singapore, and Hong Kong, it is 365 days. In Germany, Sweden, and Belgium, it is 360 days.*
3. *The three large Treasury markets are the U.S., Japan, and Germany, while the three small markets are New Zealand, Hong Kong, and Singapore.*

EXHIBIT 3
Chance-Jordan Decomposition of Treasury Returns and Theta-to-Convexity Ratios: U.S. Treasury Market

Maturity	3M	6M	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr	Portfolio
Panel A. Inputs											
Change in Yield (Δy_t)	0.157%	0.151%	0.110%	0.122%	0.125%	0.1208%	0.1161%	0.1093%	0.0880%	0.0949%	0.1193%
Change in Time (Δt)	0.0056	0.006	0.0056	0.0056	0.006	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056
Initial Price (P_t)	$98.431	$96.91	$100	$100	$100	$100	$100	$100	$100	$100	$995.3422
Price after Yield Change (P_{t+a})	$98.402	$96.86	$99.91	$99.792	$99.68	$99.509	$99.373	$99.2226	$99.0376	$98.7680	$989.8528
Panel B. Treasury Returns and Three Major Components											
Treasury Returns (R_t)	-0.0003	-0.0006	-0.0009	-0.0021	-0.003	-0.0049	-0.006	-0.0078	-0.0096	-0.0123	-0.0055
Macaulay Duration (D_t)	0.2500	0.5000	0.9846	1.9092	2.778	4.3592	5.754	7.5445	11.5724	13.7230	4.9586
Convexity Measure (C_t)	0.1250	0.5000	1.4691	4.6982	9.468	22.8492	40.233	71.5159	190.0750	296.4475	64.0102
Theta (θ_t)	0.0158	0.0314	0.0314	0.0314	0.031	0.0314	0.0314	0.0314	0.0314	0.0314	0.0314
Panel C. Chance-Jordan Decomposition of Treasury Returns											
Duration Effect (D)	130.5%	132.3%	120.5%	108.7%	105.8%	103.9%	103.2%	102.78%	102.54%	102.44%	103.98%
Convexity Effect (C)	-0.05%	-0.097%	-0.096%	-0.159%	-0.218%	-0.319%	-0.406%	-0.5159%	-0.7180%	-1.0175%	-0.7761%
Theta Effect (T_1)	-29.8%	-31.47%	-20.07%	-8.369%	-5.503%	-3.551%	-2.780%	-2.2422%	-1.8114%	-1.4150%	-3.1608%
Interaction Effect (I)	-0.71%	-0.712%	-0.319%	-0.139%	-0.087%	-0.048%	-0.032%	-0.0199%	-0.0067%	-0.0029%	-0.0401%
2nd-Order Time Effect T_2)	-0.001%	-0.003%	-0.002%	-0.0007%	-0.0005%	-0.0003%	-0.0002%	-0.0002%	-0.0002%	-0.0001%	-0.0003%
Panel D. Theta-to-Convexity Ratio											
T_1 / C	591.887	324.64	210.028	52.8010	25.295	11.14	6.8500	4.3461	2.5228	1.3907	4.0726

Notes:

1. *Par yields are assumed for coupon-paying Treasuries.*
2. *Portfolio refers to an equal-weighted portfolio, which is constructed from Treasuries of diverse maturity sectors.*
3. *One-day time increment and a yield change equivalent to two standard deviations of the historical yield changes are assumed for the computation.*

The Chance-Jordan decompositions indicate that, for all maturity sectors, the duration effect overestimate Treasury returns, a finding that is consistent with the fact that dollar duration underestimates Treasury price change and the findings in Chance and Jordan [1996]. For example, the duration contribution in the case of the five-year Treasury is 103.92%, which is greater than that in Chance and Jordan by only a negligible fraction (24 basis points).

The theta contribution in the case of the five-year Treasury is about –3.55% compared to –3.20% in Chance and Jordan. The convexity contribution is –0.32% compared to –0.44% in Chance and Jordan. As in Chance and Jordan, the contributions of interaction and second-order time effects are negligible.[7]

In all maturity sectors, the theta effect dominates the convexity effect as a component of Treasury returns. The theta effect is particularly strong in three-year or shorter maturity sectors. But, its dominance declines monotonically with sector maturity. Such a pattern in the theta effect is not surprising, because the shorter the remaining maturity, the smaller the duration and convexity effects. In the one-year or shorter maturity sectors, even the interaction effect dominates the convexity effect. The theta-to-convexity ratio in the three-month U.S. Treasury sector, 591.9, is the highest, while the ratio in the 30-year U.S. Treasury sector, 1.39, is the lowest. It is 4.07 for the equal-weighted Treasury portfolio, which has a weighted average maturity of about 7.85 years. This finding is not surprising, given the strong positive association between duration and maturity.

All the findings in Exhibit 3 are by and large consistent with the findings in Chance and Jordan's [1996] Exhibit 5.

IV. EVIDENCE IN INTERNATIONAL TREASURY MARKETS

To facilitate the comparison of the theta-to-convexity ratios across international Treasury markets and their non-comparable maturity sectors, we construct equal-weighted Treasury portfolios for Treasury markets. Then, for each country, we compute the theta-to-convexity ratios for par, discount, and premium portfolios. The par (non-par) portfolios consist of the coupon-paying Treasuries (not) trading at par and Treasury bills.

Exhibit 4 reports, for 11 Treasury markets, the theta-to-convexity ratios for par and non-par equal-weighted Treasury portfolios. Although various magnitudes of premium and discount are considered, we report only the results for non-par Treasuries trading at a 3% premium and a 3% discount.

The mean (median) is the mean (median) of the theta-to-convexity ratios for country Treasury portfolios

EXHIBIT 4

Theta-to-Convexity Ratios of Par and Non-Par Equal-Weighted Portfolio: by Country

Country	Mean	Med	Min	Max	Par Portfolio		Discount (3%) Portfolio		Premium (3%) Portfolio	
					1SD1D	2SD1D	1SD1D	2SD1D	1SD1D	2SD1D
U.S.	9.81	8.80	3.30	17.62	16.29	4.07	17.62	4.41	13.18	3.30
Japan	14.92	12.46	4.52	27.44	27.44	6.86	26.10	6.52	18.06	4.52
U.K.	15.36	14.02	5.15	29.71	20.61	5.15	29.71	7.43	23.38	5.85
Germany	6.74	6.15	2.30	12.45	10.71	2.68	12.45	3.11	9.20	2.30
Canada	20.12	17.64	6.51	36.90	36.90	9.22	33.63	8.41	26.06	6.51
Sweden	9.23	5.54	0.70	25.23	19.05	4.76	25.23	6.32	2.80	0.70
Belgium	64.66	40.59	11.40	142.30	122.5	30.62	142.30	35.57	45.62	11.40
Australia	8.47	8.01	3.03	15.53	12.14	3.03	15.53	3.88	12.99	3.25
New Zealand	10.95	9.96	3.71	20.32	17.40	4.35	20.32	5.08	14.83	3.71
Hong Kong	7.87	6.83	2.55	13.81	13.81	3.45	13.75	3.46	10.21	2.55
Singapore	9.13	8.49	3.17	17.14	13.98	3.49	17.14	4.28	12.69	3.17

Notes:

1. The par (non-par) portfolio consists of the coupon-paying Treasuries (not) trading at par and Treasury bills. For non-par portfolios, only the results for 3% premiums and 3% discounts are reported.

2. The 1SD1D (2SD1D) refer to assumptions on yield changes of one (two) standard deviation(s) and one-day investment horizon.

computed under different sets of assumptions on the magnitude of yield changes and the length of time passage. The highest median value, 40.59, is observed in Belgium. The lowest median value, 5.54 is observed in Sweden. The minimum theta-to-convexity ratios in all markets are all higher than 1.00, except in Sweden, where the minimum is 0.70. When the magnitude of yield changes drops from two standard deviations to one standard deviation, the theta-to-convexity ratio increases by about four times.

Exhibit 4 provides conclusive evidence that, in the major international Treasury markets, the theta effect is much more important than the convexity effect, and, more often than not, its relative importance is stronger in international Treasury markets.

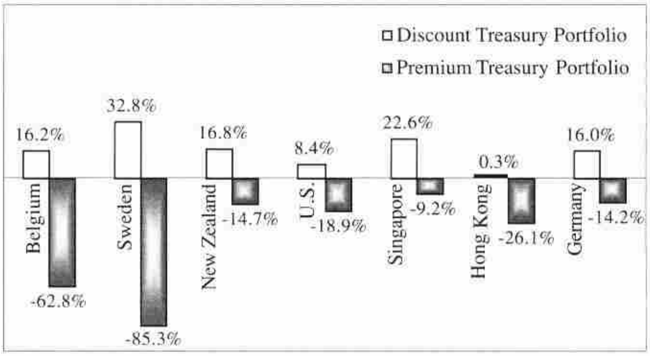
One noticeable pattern is that, in all Treasury markets, the theta-to-convexity ratios of the discount Treasury portfolios are higher than those of the premium Treasury portfolios. To summarize patterns in the ratio differences between the par and non-par portfolios, Exhibit 5 plots for each country the scaled ratio differences between the par and non-par portfolios, where the ratio of the par portfolio is used as the scaling factor.

There are two distinct patterns in the scaled ratio differences between the par and the non-par portfolios. The pattern consistent with intuition is reported in Panel A, where the ratio of the discount Treasury (premium Treasury) portfolio is higher (lower) than that of the par Treasury portfolio. This pattern is observed in seven countries, although it is relatively weak in the Hong Kong Treasury market.

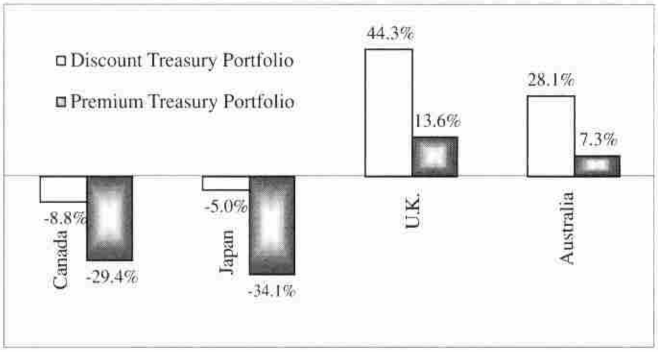
Panel B reports the counter-intuitive pattern, where the ratios of both the discount and premium Treasury portfolios are either higher or lower than that of the par Treasury portfolio. In Canada and Japan, both the ratios of non-par portfolios are lower than the ratio of the par portfolio. In the U.K. and Australia, the non-par portfolio ratios are higher.[8]

EXHIBIT 5
Scaled Ratio Differences Between Par and Non-Par Portfolios

Panel A: Countries with Different Theta-to-Convexity Ratios for Discount and Premium Portfolios



Panel B: Countries with Same Theta-to-Convexity Ratios for Discount and Premium Portfolios



Notes:

1. Scaled ratio differences are computed as the ratio difference between the par and non-par portfolios, scaled by the ratio of the par portfolio.

2. Only results for 3% premiums and 3% discounts are reported.

V. SUMMARY AND CONCLUSIONS

Chance and Jordan [1996] have documented that, even when yield changes are quite substantial, a theta and not convexity effect explains a non-negligible portion of Treasury returns. Using data on option-free Treasuries in the U.S. and ten other countries for 1975-2000, we examine whether this surprising finding holds for different Treasury markets, interest rate environments, and maturity sectors.

In all major international Treasury markets, we find that the theta effect is much more important than the convexity effect, even for a one-day period and for a large yield change equivalent to two standard deviations of the historical yield changes. The results also show that, more often than not, the theta effect is even more important in major international markets than in the U.S. market but it is less (greater) for Treasuries selling at a premium (at a discount).

These findings obtain for option-free Treasuries. They may not necessarily hold for Treasuries with embedded options, because certain types of embedded options can amplify the importance of the convexity effect.

Nonetheless, the findings are useful for fixed-income practitioners and analysts. For example, given target investment horizons and expected ranges of yield changes, it is easier to approximate Treasury returns, because they can be determined using duration and theta. Furthermore, long-maturity Treasury approximations become more accurate because the often-neglected theta effect comes into play.

The dominance of the theta effect also has important implications for hedging the risks of large yield changes. For example, duration-theta hedges might replace duration-convexity hedges in hedging Treasuries of relatively short maturity because in such sectors, the dominance of the theta effect is particularly strong.

ENDNOTES

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[1]Christensen and Sorensen [1994] address the theta effect in bond price changes. For alternative duration measures of non-stochastic changes in yield, see Bierwag, Kaufman, and Latta [1988], Ho [1992], and Kishimoto [1998].

[2]Convexity is a linear approximation of the downward bias. For usage of duration and convexity, see Kritzman [1992]. Brooks and Livingston [1992] document the dominance of the duration effect (over the convexity effect) on bond price changes due to instantaneous finite yield changes. Using data on U.S. Treasuries, Ilmanen [1992] documents that in the 1980s the duration effect explained about 80% to 90% of the variability in Treasury returns. The explanatory power of the duration effect is not very great due to the stochastic nature of yield changes. For early research on stochastic duration, see Cox, Ingersoll, and Ross [1979].

[3]Replacing the subscript i with t yields an identical pricing model:

$$P_t = \sum_{t_i=1}^{T} X_{t_i}\left(1+\frac{y_t}{n}\right)^{-(t_i-t)}$$

[4]A slightly different decomposition of Treasury returns can be obtained by considering the log of relative Treasury prices as the measure for Treasury return. For details, see Barber [1995].

[5]Note that the yield changes in the developed economies of the sample can be regarded as a normal variate with zero mean. Then, the yield changes over x calendar days can be expressed in h standard deviations of the historic yield changes. To see this, consider the simple case where n is 2 and the number of days in a year is 360 days, which implies $q = x/180$. Then, given a yield change with a 67% chance of occurring, the yield changes over x days can be expressed as:

$$\Delta y_t = (\overline{\Delta y_t}) + \sigma_{x\ days} = \sigma_{x\ days} = \sigma_{30days}\sqrt{x/30}$$

where the square root of time rule on computing a period-specific volatility of yield changes is used. From this, one can easily show that the theta-to-convexity ratio does not depend on the length of time passage, q or Δt, since

$$\frac{T_1}{C} = \frac{\theta_t \Delta t}{\frac{1}{2}\left(1+\frac{y_t}{n}\right)^{-2} C_t (\Delta y_t)^2} = \frac{\theta_t\left(\frac{x}{180}\right)}{\frac{1}{2}\left(1+\frac{y_t}{n}\right)^{-2} C_t\left(\sigma_{30days}\sqrt{\frac{x}{30}}\right)^2} = \frac{[\theta_t]}{\left[3\left(1+\frac{y_t}{n}\right)^{-2} C_t \sigma_{30days}{}^2\right]}$$

[6]A numerical analysis of the theta-to-convexity ratio suggests, however, that more often than not the theta effect increase due to a given discount magnitude would be greater than the convexity effect increase, while the theta effect decrease due to a given premium magnitude would be greater in absolute amount than the convexity effect decrease. This result suggests that, all else equal, the theta effect tends to be greater (less) in discount (premium) Treasuries.

[7]Given the skewness in the contributions of the five components across maturity sectors, the median statistic summarizes the respective contributions better than the mean does. But, the median in Jordan and Chance is the five-year Treasury, while the median in Exhibit 3 is in between three- and five-year Treasuries. Hence, our comparison here is based on the contributions for five-year Treasuries.

[8]It is noteworthy that during the sample period, the median yield in Australia is quite a bit higher than the median coupon rate. In Canada, Japan, and the U.K., the median yields are lower than the median coupon rates.

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