Recovery Assumptions in the Valuation of Credit Derivatives

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Reduced-form models are now widely applied in the pricing of credit derivatives. These models infer a risk-neutral stochastic process governing default events from the market prices of risky debt (i.e., credit spread term structures). Compared to structural models, the reduced-form approach obviates the collection and interpretation of large volumes of firm-specific data pertaining to capital structure, and avoids reliance on unobservable parameters such as firm asset value and volatility.

When a firm defaults, a debtholder (lender, bondholder, swap counterparty) typically receives a recovery amount that is only a fraction of the legal claim on remaining assets. Recovery assumptions will affect the pricing of credit derivatives for two reasons:

- The recovery amount for the reference asset may be a direct determinant of the credit derivative payoff.
- An expected recovery amount is embedded in the price of risky debt and the implied risk-neutral default process.

Recovery of debt is clearly an uncertain outcome. Models typically decompose this uncertainty into two factors: 1) claim amount, and 2) recovery rate representing the fraction of the claim that is actually recovered.

Recovery rates may be estimated from historical data under some generally accepted reporting convention. Empirical evidence for bonds and bank loans indicates that recovery rates have highly dispersed and skewed distributions that depend strongly on seniority (see, for example, "Bankrupt Bank Loan Recoveries" [1998]). A common simplification is to use historically estimated mean recovery rates (assumed to represent both the real-world and risk-neutral expectations) and ignore debt seniority in implementing a reduced-form model.

A variety of assumptions have been proposed for recovery amount (recovery rate times claim amount) in pricing risky debt. Longstaff and Schwartz [1995] assume that the recovery amount is a fixed fraction of an equivalent-maturity risk-free bond within a structural model. Jarrow, Lando, and Turnbull [1997] assume that debtholders receive a fixed fraction of the face value at maturity. Duffie and Singleton [1999] obtain simplified expressions for the value of risky debt using the recovery of market value, just prior to default, where default is driven by a hazard rate model. Another alternative is to take recovery as a fixed fraction of face value at the time of default (prior to maturity).[1]

We examine the impact of recovery assumptions on the implied risk-neutral default process and the pricing of credit default swaps. Using a reduced-form approach, risk-free and risky discount bond prices are specified exogenously and used to infer risk-neutral default probabilities. The price of a credit default swap is then obtained in a discrete-time form.[2]

Three recovery assumptions are considered: recovery of market value (RMV), recovery of treasury (RT), and recovery of face value (RFV). As with all reduced-form models, the risk-neutral default process will reproduce the prices of the given risky and risk-free bonds exactly, regardless of the recovery assumption. The price of a credit-contingent claim such as a default swap, however, is not expected to be independent of the model for recovery.

We investigate the differences that result from the recovery assumption, and we show that the price of a default swap can exhibit varying degrees of sensitivity with respect to recovery rate, depending on the particular choice of model.

I. FORMULATION OF RISK-NEUTRAL DEFAULT PROBABILITIES

We consider a simplified model of risky debt where the borrower is in one of two states, default or non-default.[3] The present value of a risky bond with maturity T, unit face value, and credit rating i is denoted as $v_i(0, T)$. Default can occur at a set number of time points $0 < t_1 < t_2 < \ldots < t_N = T$.[4]

We assume that the price of a risk-free zero-coupon bond $v(0, t_j)$ maturing on any of these dates is given. If a risky bond defaults at some time, it is assumed that all bonds issued by the firm will default. We also make the common simplifying assumptions that the risk-free interest rate, the recovery rate, and the claim amount are independent of the default mechanism.

The value of a risky bond is given then by the risk-neutral expected value of future payments discounted at the risk-free rate:

$$v_i(0,T) = \sum_{j=1}^{N}\left[v(0,t_j)\delta C[v(0,t_j), v_i(0,t_j)]\prod_{k=1}^{j-1}[1-q(t_{k-1},t_k)]q(t_{j-1},t_j)\right] + v(0,T)[1-q(0,T)]$$

where $q(t_{k-1}, t_k)$ is the conditional (risk-neutral) probability of default in the period $(t_{k-1}, t_k]$ if no default has occurred prior to t_{k-1}. Note that these default probabilities are associated with the initial rating that is common to all bonds defining the risky term structure. We do not show the explicit dependence to simplify the notation.

The expected cash recovery amount in case of default is $\delta C(v, v^i)$, where δ is the expected recovery rate, and the expected claim amount C may, in general, depend on the forward risky and risk-free bond prices. A term structure of conditional default probabilities (corresponding to a particular initial credit rating) can be determined iteratively using the basic valuation formula and given term structures for the risky and risk-free zero-coupon bond prices.

Recovery of Treasury (RT)

The recovery of treasury assumption implies that, when default occurs, the debtholders recover a fraction of the value of a risk-free bond with similar maturity at the time of default. The expected claim amount is given by

$$C[v(t,T), v_i(t,T)] = v(t,T)$$

where $v(t, T)$ is to be interpreted as the *forward* price of the risk-free bond when it appears in subsequent valuation formulas.

In the absence of arbitrage opportunities, this forward price obeys the rule

$$v(0,T) = v(0,t)v(t,T)$$

We solve for conditional default probabilities beginning with the shortest-maturity bond and proceeding to the longer maturities. The value of the risky zero-coupon bond with shortest maturity is

$$v_i(0,t_1) = v(0,t_1)\left[\delta q(0,t_1) + 1 - q(0,t_1)\right]$$

where we use the fact that the recovery amount is $\delta v(t_1, t_1) = \delta$.

Solving for $q(0, t_1)$ yields

$$q(0,t_1) = \frac{1 - \dfrac{v_i(0,t_1)}{v(0,t_1)}}{1-\delta}$$

The value of the risky bond with the second-shortest maturity is

$$
\begin{aligned}
v_i(0,t_2) &= v(0,t_1)\delta\, v(t_1,t_2) q(0,t_1) + \\
&\quad v(0,t_2)\left[1-q(0,t_1)\right]\left[\delta\, q(t_1,t_2)+1-q(t_1,t_2)\right] \\
&= v(0,t_2)\begin{Bmatrix} \delta\, q(0,t_1) + \left[1-q(0,t_1)\right] + \\ (\delta-1)\left[1-q(0,t_1)\right] q(t_1,t_2) \end{Bmatrix} \\
&= v(0,t_2)\begin{Bmatrix} \left[v_i(0,t_1)/v(0,t_1)\right] + \\ (\delta-1)\left[1-q(0,t_1)\right] q(t_1,t_2) \end{Bmatrix}
\end{aligned}
$$

Solving for $q(t_1, t_2)$ yields

$$
q(t_1,t_2) = \frac{\dfrac{v_i(0,t_1)}{v(0,t_1)} - \dfrac{v_i(0,t_2)}{v(0,t_2)}}{(1-\delta)[1-q(0,t_1)]}
$$

Similarly, the value of the risky bond with the third-shortest maturity is

$$
\begin{aligned}
v_i(0,t_3) &= v(0,t_1)\delta v(t_1,t_3) q(0,t_1) + \\
&\quad v(0,t_2)\delta v(t_2,t_3)[1-q(0,t_1)] q(t_1,t_2) + \\
&\quad v(0,t_3)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right] \times \\
&\quad \left[\delta\, q(t_2,t_3) + 1 - q(t_2,t_3)\right] \\
&= v(0,t_3)\{\delta\, q(0,t_1) + 1 - q(0,t_1) + \\
&\quad (\delta-1)\left[1-q(0,t_1)\right] q(t_1,t_2) + \\
&\quad (\delta-1)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right] q(t_2,t_3)\} \\
&= v(0,t_3)\begin{Bmatrix} \left[v_i(0,t_2)/v(0,t_2)\right] + \\ (\delta-1)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right] q(t_2,t_3) \end{Bmatrix}
\end{aligned}
$$

Solving for $q(t_2, t_3)$ yields

$$
q(t_2,t_3) = \frac{\dfrac{v_i(0,t_2)}{v(0,t_2)} - \dfrac{v_i(0,t_3)}{v(0,t_3)}}{(1-\delta)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right]}
$$

By induction, the recovery of treasury conditional default probabilities for future periods will be given by the recurrence relation:

$$
q(t_{n-1},t_n) = \frac{\dfrac{v_i(0,t_{n-1})}{v(0,t_{n-1})} - \dfrac{v_i(0,t_n)}{v(0,t_n)}}{(1-\delta)\prod_{j=1}^{n-1}\left[1-q(t_{j-1},t_j)\right]} \qquad (1)
$$

Recovery of Market Value (RMV)

The recovery of market value assumption implies that the recovery amount is a fraction of a non-defaulted risky bond of similar credit quality and maturity. The expected claim amount is given by

$$
C[v(t,T), v_i(t,T)] = v_i(t,T)
$$

As in the RT model, the value of the risky zero-coupon bond with shortest maturity is

$$
v_i(0,t_1) = v(0,t_1)\left[\delta\, q(0,t_1) + 1 - q(0,t_1)\right]
$$

Solving for $q(0, t_1)$ yields

$$
q(0,t_1) = \frac{1 - \dfrac{v_i(0,t_1)}{v(0,t_1)}}{1-\delta}
$$

The value of the risky bond with the second-shortest maturity is

$$
\begin{aligned}
v_i(0,t_2) &= v(0,t_1)\delta\, v_i(t_1,t_2) q(0,t_1) + \\
&\quad v(0,t_2)\left[1-q(0,t_1)\right]\left[\delta\, q(t_1,t_2) + 1 - q(t_1,t_2)\right]
\end{aligned}
$$

Now, the forward risky bond price can be expressed as a risk-neutral expected value of future cash flows discounted at the risk-free rate. This can be written as

$$
v_i(t_1,t_2) = v(t_1,t_2)\left[\delta\, q(t_1,t_2) + 1 - q(t_1,t_2)\right]
$$

Substituting this expression into the formula for $v_i(0, t_2)$, we obtain

$$
\begin{aligned}
v_i(0,t_2) &= v(0,t_2)\left[\delta\, q(0,t_1) + 1 - q(0,t_1)\right] \times \\
&\quad \left[1 + (\delta-1)\, q(t_1,t_2)\right]
\end{aligned}
$$

Solving for $q(t_1, t_2)$ yields

$$q(t_1,t_2)=\frac{1-\frac{v_i(0,t_2)}{v(0,t_2)}\frac{1}{1+(\delta-1)q(0,t_1)}}{1-\delta}$$

By induction, the recovery of market value conditional default probabilities for future periods will be given by the recurrence relation:

$$q(t_{n-1},t_n)=\frac{1-\frac{v_i(0,t_n)}{v(0,t_n)}\prod_{j=1}^{n-1}\left(\frac{1}{1+(\delta-1)q(t_{j-1},t_j)}\right)}{1-\delta}$$

$$=\frac{1-\frac{v_i(0,t_n)}{v(0,t_n)}\prod_{j=1}^{n-1}\left(\frac{v(t_{j-1},t_j)}{v_i(t_{j-1},t_j)}\right)}{1-\delta} \qquad (2)$$

From Equation (2), we see that if the credit spread term structure is flat, the conditional default probabilities will be constant.

Recovery of Face Value (RFV)

The recovery of face value assumption implies that the recovery amount is a fraction of the par or face of the claim. The expected claim amount is given by

$$C[v(t,T),v_i(t,T)]=1$$

Since the face value and market value of the shortest-maturity bond are identical at maturity, $q(0,\ t_1)$ is given the same expression derived for the RT and RMV models:

$$q(0,t_1)=\frac{1-\frac{v_i(0,t_1)}{v(0,t_1)}}{1-\delta}$$

The value of the risky bond with the second-shortest maturity is

$$v_i(0,t_2)=v(0,t_1)\delta q(0,t_1)+ v(0,t_2)\left[1-q(0,t_1)\right]\left[\delta q(t_1,t_2)+1-q(t_1,t_2)\right]$$

Solving for $q(t_1,\ t_2)$ yields

$$q(t_1,t_2)=\frac{v_i(0,t_2)-v(0,t_1)\delta q(0,t_1)}{v(0,t_2)(\delta-1)\left[1-q(0,t_1)\right]}+\frac{1}{1-\delta}$$

Similarly, the value of the risky bond with the third-shortest maturity is

$$\begin{aligned}v_i(0,t_3)=\ &v(0,t_1)\delta q(0,t_1)+\\ &v(0,t_2)\delta[1-q(0,t_1)]q(t_1,t_2)+\\ &v(0,t_3)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right]\times\\ &\left[\delta q(t_2,t_3)+1-q(t_2,t_3)\right]\end{aligned}$$

Solving for $q(t_2,\ t_3)$yields

$$\begin{aligned}q(t_2,t_3)=&\left[\frac{v_i(0,t_3)-\delta v(0,t_1)q(0,t_1)}{v(0,t_3)(\delta-1)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right]}\right]-\\ &\left[\frac{\delta v(0,t_2)q(t_1,t_2)[1-q(0,t_1)]}{v(0,t_3)(\delta-1)\left[1-q(0,t_1)\right]\left[1-q(t_1,t_2)\right]}\right]+\\ &\frac{1}{1-\delta}\end{aligned}$$

By induction, the recovery of face value conditional default probabilities for future periods will be given by the recurrence relation:

$$\begin{aligned}q(t_{n-1},t_n)=&\left[\frac{v_i(0,t_n)}{v(0,t_n)(\delta-1)\prod_{j=1}^{n-1}\left[1-q(t_{j-1},t_j)\right]}\right]-\\ &\left[\frac{\delta\sum_{j=1}^{n-1}v(0,t_j)q(t_{j-1},t_j)\prod_{k=1}^{j-1}\left[1-q(t_{k-1},t_k)\right]}{v(0,t_n)(\delta-1)\prod_{j=1}^{n-1}\left[1-q(t_{j-1},t_j)\right]}\right]+\\ &\frac{1}{1-\delta}\end{aligned} \qquad (3)$$

II. VALUATION OF CREDIT DEFAULT SWAPS

We consider the valuation of a credit default swap using risk-neutral default probabilities inferred from a reduced-form model. A default swap provides a payoff to a counterparty (*buyer*) in the event of default by the issuer of a bond (*reference security*) during the life of the swap. For a plain vanilla default swap, the payoff is the difference between the face value of the bond and the recovery amount.[5]

The buyer must make periodic premium payments to the seller during the life of the swap until final maturity or a default event, whichever comes first. If the default event occurs between the payment dates, the buyer may be required to a make a partial payment (or equivalently accept a reduced payoff), depending upon the contractual terms. We consider only the case where the payoff triggered by a default event between payment dates is reduced by the straight-line accrual of the premium payment.

Let $\mathbf{T_P} = \{T_1, \ldots, T_M: 0 < T_1 < \ldots < T_M\}$ be the dates on which the premium payments are made. These payments are assumed to be a fixed percentage x of the face value of the reference asset. We also assume that the default swap ends on the final payment date T_M.

The value of the default swap is the risk-neutral expectation of the discounted future cash flows.[6] To evaluate the expected value in a discrete-time setting, we consider a set of dates $\mathbf{T_D} = \{t_1, \ldots, t_N: 0 < t_1 < \ldots < t_N\}$ on which default is considered.[7] It is desirable for accurate pricing that this set of default dates include all premium payment dates, i.e., $\mathbf{T_P} \subset \mathbf{T_D}$.

The default swap value (relative to the face value of the reference security) is then given by

$$\begin{aligned} V_{DS}(0) = \sum_{i=1}^{N} \nu(0,t_i)\left[1-\hat{\delta}-xf_a(t_i)\right]\prod_{j=1}^{i-1}\times \\ \left[1-q(t_{j-1},t_j)\right]q(t_{i-1},t_i) - \\ \sum_{i=1}^{N} \nu(0,t_i)x1_{\mathbf{T_P}}(t_i)\prod_{j=1}^{i}\times \\ \left[1-q(t_{j-1},t_j)\right] \end{aligned} \tag{4}$$

where $\hat{\delta}$ is the expected recovery rate for the reference security, $1_{\mathrm{T_P}}(t_i)$ is an indicator function that equals one if $t_i \in \mathbf{T_P}$ and zero otherwise, and f_a is an accrual function given by:[8]

$$f_a(t) = \frac{t - \max\{T \in \mathbf{T_P}: T < t\}}{\min\{T \in \mathbf{T_P}: T \geq t\} - \max\{T \in \mathbf{T_P}: T < t\}}$$

The two sums on the right-hand side of Equation (4) are, respectively, the value of the *contingent side*, accounting for the potential payoff from the seller, and the value of the *premium side*, accounting for the periodic payments made by the buyer. The value of the premium rate x that makes the default swap value equal to zero is called the *default swap spread*.

III. NUMERICAL EXAMPLES

We examine some numerical examples to show the effects of the three recovery assumptions. The input parameters of the reduced-form model are the risky and risk-free term structures along with the recovery rate δ. We assume a flat risk-free term structure with zero-coupon bond prices generated by

$$\nu(0,t) = e^{-r_f t}$$

where r_f is a constant, continuously compounded, interest rate.[9] The price of a risky bond can be represented as a product of a risk-free bond price and a credit spread factor. The spread rate is assumed to be a constant plus a component that increases linearly with bond maturity.

Hence, the risky bond price is given by

$$\nu_i(0,t) = \nu(0,t)e^{-(s_0+s_1 t)}$$

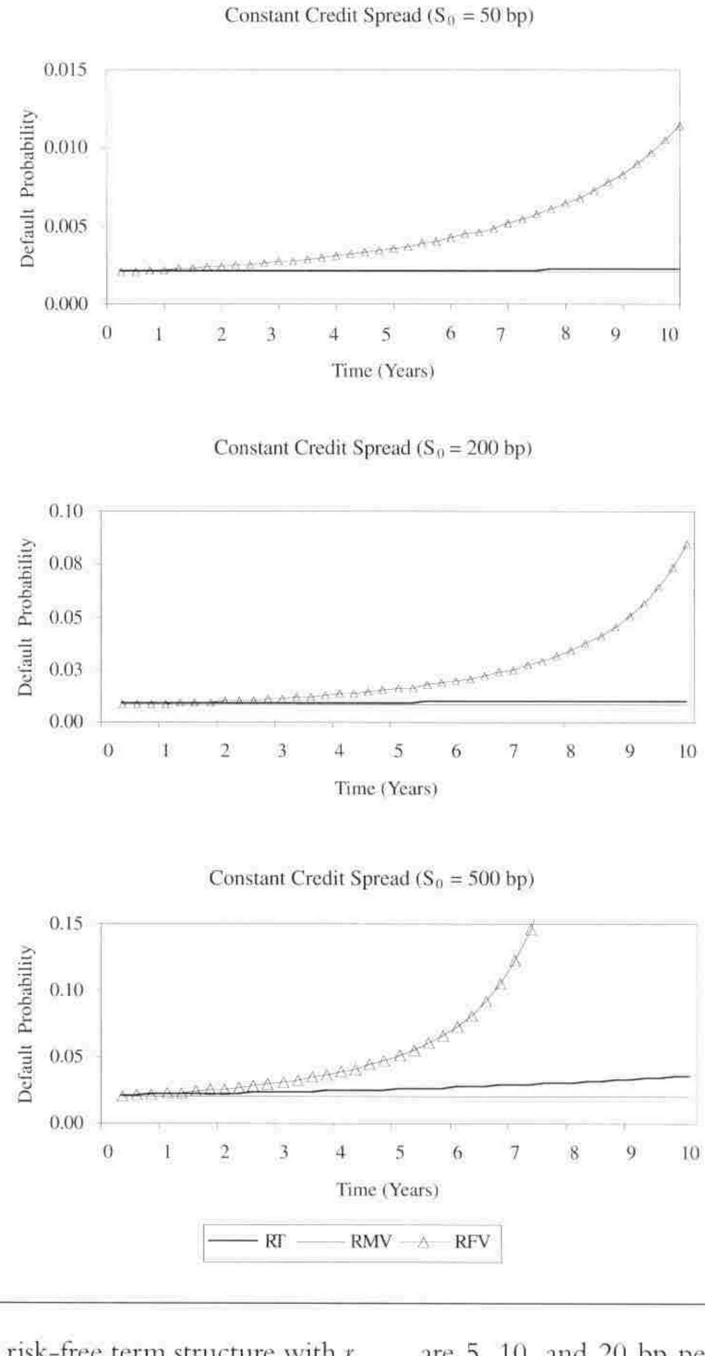
where s_0 is the base spread level, and s_1 is the slope of the credit spread term structure. This form permits construction of flat, upward-sloping, or downward-sloping credit spread term structures.

We first consider six cases where only the level and the slope of the credit spread term structure vary. The parameters for the six cases are shown in Exhibit 1.

EXHIBIT 1
Case Definitions

Case	R_f	S_0 (bp)	S_1 (bp)	δ
1	0.06	50	0	0.40
2	0.06	200	0	0.40
3	0.06	500	0	0.40
4	0.06	50	5	0.40
5	0.06	200	10	0.40
6	0.06	500	20	0.40

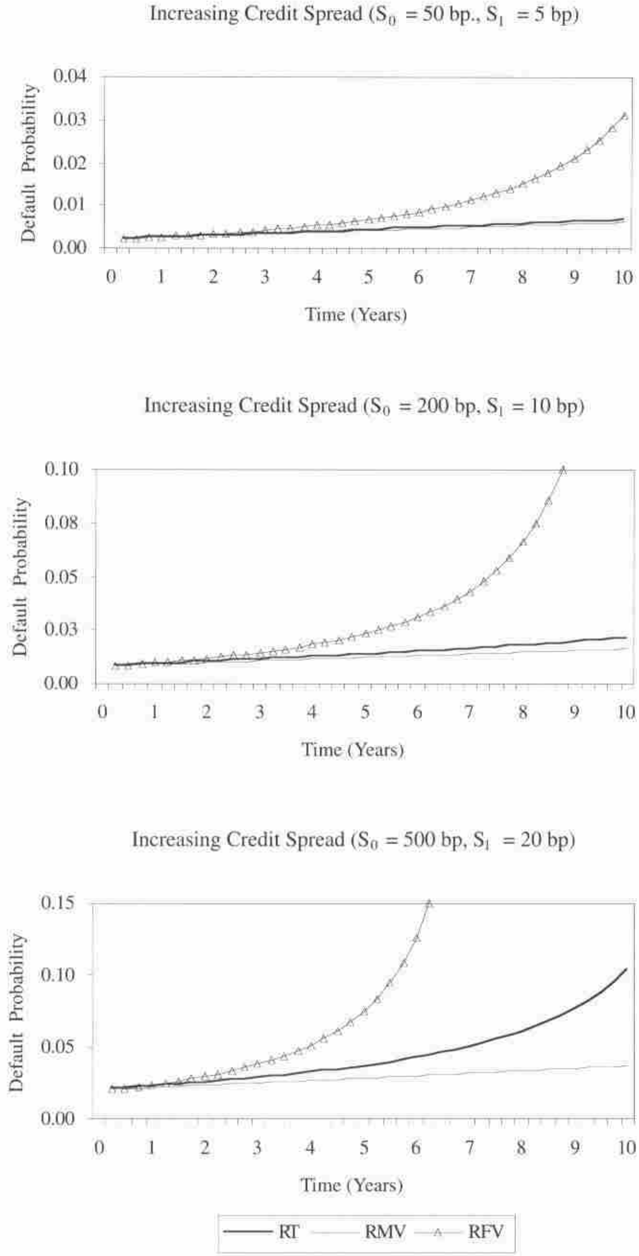
EXHIBIT 2
Implied Default Probabilities: Flat Credit Spread



All cases share a flat risk-free term structure with r_f = 0.06 and a recovery rate δ = 0.4. In Cases 1-3, the credit spread term structure is flat. Cases 4-6 have an upward-sloping credit spread term structure. The base spread levels are 50 basis points in Cases 1 and 4, 200 bp in Cases 2 and 5, and 500 bp in Cases 3 and 6. These choices correspond roughly to credit qualities ranging from high investment-grade to speculative. The slopes in Cases 4-6 are 5, 10, and 20 bp per year, respectively. The three-month conditional default probabilities are generated with bonds maturing at quarterly intervals.

Exhibit 2 shows the risk-neutral conditional default probabilities for the three cases with flat credit spread term structures (Cases 1-3). The RT and RMV models generate similar default probabilities, which is consistent with the findings of Duffie and Singleton [1999] and Hull and White

EXHIBIT 3
Implied Default Probabilities: Upward-Sloping Credit Spread



[2000]. The RMV model leads to constant default probabilities, as expected, with a flat credit spread term structure. The RT default probabilities, on the other hand, increase moderately with time and are higher than the RMV values.

The difference between the RT and RMV default probabilities becomes more pronounced as the credit spread widens. This is because the loss due to default increases (the recovery amount falls) with increasing credit spread only in the case of the RMV model—implying a lower default probability for a fixed risky bond price. The RFV default probabilities increase with time at the greatest rate, leading to dramatically higher values that ultimately become invalid (i.e., exceed one).

Exhibit 3 shows the results for upward-sloping credit spread term structures (Cases 4-6). The conditional default probabilities now increase with time for all three recov-

EXHIBIT 4
Default Swap Spreads (bp)

Maturity	Model	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
1	RMV	17	66	165	18	69	171
1	RT	17	67	167	18	70	173
1	RFV	17	68	171	19	72	178
3	RMV	21	85	209	27	97	231
3	RT	21	86	219	28	99	244
3	RFV	24	97	249	31	112	277
5	RMV	23	89	220	33	110	258
5	RT	23	92	239	33	114	284
5	RFV	29	118	310	42	145	369
7	RMV	23	92	226	38	120	275
7	RT	23	96	253	39	127	318
7	RFV	34	142	391	56	189	501
10	RMV	24	93	229	44	131	294
10	RT	24	99	270	45	144	369
10	RFV	46	203		90	304	

ery models. The RT and RMV default probabilities diverge more significantly, especially for the wider spreads and longer maturities. The RFV model again generates the highest default probabilities and may fail to provide permissible values at longer maturities.

Exhibit 4 shows default swap spreads for the six credit spread cases and the three recovery models. We consider swaps with semiannual premium payments and maturities ranging from one to ten years. The possibility of default is considered at quarterly intervals. The recovery rate is 0.4, both for generating the risk-neutral default probabilities and for pricing the default swap

The RT and RMV models lead to similar default swap spreads, especially for short maturities and narrow credit spreads. Hence, default swap spreads for investment-grade debt are notably insensitive to recovery assumptions. The RT and RMV models exhibit more significant differences in default swap spread as credit spreads widen and slopes increase (Cases 3 and 6).

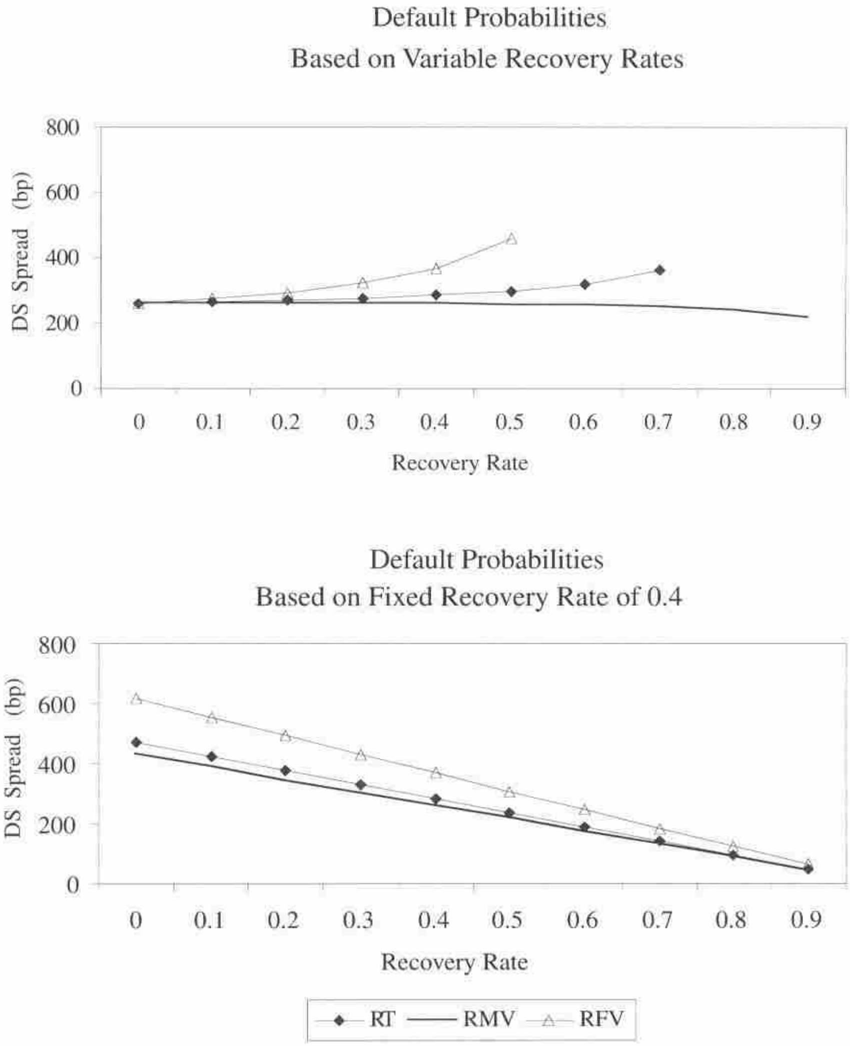
For a five-year maturity, the RT default swap spread exceeds the RMV value by 19 and 26 bp in Cases 3 and 6, respectively. For a ten-year maturity, the effect is even greater, with respective differences of 41 and 75 bp. As expected, the RFV model always produces the highest default swap spreads since it generates the highest implied default probabilities.

Another important issue is the sensitivity of default swap value to the assumed recovery rate. Hayt [2000] suggests that a plain vanilla default swap should be insensitive to the specific value of recovery rate, if the same value is used both in generating the risk-neutral default probabilities and in pricing the swap. His argument is based on a simple single-period model where assumptions about claim amount are irrelevant. We examine this issue in the context of a more realistic multiperiod model and the various claim assumptions.

We consider a five-year default swap with semiannual premium payments. For our demonstration, we use the credit spread term structure of Case 6 (widest base spread and highest slope). The default swap spread is plotted versus the recovery rate in Exhibit 5.

The first graph shows the results obtained when identical recovery rates are used in generating the risk-neutral default probabilities and in pricing the default swap. The RMV model leads to default swap spreads that are indeed insensitive to the assumed recovery rate. The RT model also exhibits little sensitivity when the recovery rate is low but ultimately diverges from the RMV results for

EXHIBIT 5
Impact of Recovery Rate on Default Swap Spread



higher recovery rates. For a recovery rate of 0.7, the RT default swap spread is about 50% higher than the RMV value. The default swap spreads are particularly sensitive to the recovery rate in the case of the RFV model.

In practice, risk-neutral default probabilities will be generated for a particular credit class using a representative average recovery rate, although assumptions about recovery rates for individual credits (and associated default swaps) within the class may vary. To examine this scenario, we first obtain a term structure of risk-neutral default probabilities based on a fixed recovery rate of 0.4. We then calculate default swap spreads using different recovery rates in the pricing formula, but using the same set of default probabilities each time.

The results are plotted in the second graph of Exhibit 5. The default swap spread is seen to decrease linearly as the recovery rate increases. The effect is significant and comparable for each of the three recovery models, with the spread falling by approximately 5 bp for each one percentage point increase in recovery rate.

Finally, we consider the effect of time resolution on the pricing of a default swap. In our discrete-form valuation formula, Equation (4), the spacing of potential default dates is arbitrary.[10] We consider the effect of this spacing in a calculation of the default swap spread for potential default frequencies ranging from semiannual to daily. We again consider a five-year swap with semiannual payments, a recovery rate of 0.4, and the credit spread conditions of Case 6.

EXHIBIT 6
Effect of Time Step on Default Swap Spread

ΔT	RT	RMV	RFV
Semiannual	260	241	328
Quarterly	233	215	301
Monthly	235	217	310
Weekly	236	217	313
Daily	236	217	314

The results are shown in Exhibit 6, where we see that the default swap spreads converge rapidly for all three recovery models. In each case, the spread obtained with quarterly steps deviates by no more than 5% from the value obtained with daily steps.

IV. SUMMARY AND CONCLUSIONS

The pricing of risky debt and credit derivatives requires some assumption about the claim made by the debtholder in the event of default. Three common models specify the claim amount as the market value of the risky debt prior to default (RMV), the market value of a risk-free bond with identical maturity (RT), or the face value of the risky debt (RFV). There are two steps in the reduced-form valuation of a credit derivative: 1) inference of risk-neutral default probabilities from risk-free and risky term structures, and 2) pricing of a credit-contingent claim as a discounted, risk-neutral expectation of future cash flows including recovery amount in case of default.

We have examined the effect of the three recovery assumptions on both steps of this process, presenting explicit formulas for the implied default probabilities generated from zero-coupon yield curves and a two-state default model. We assess the effect of these assumptions on the valuation of a credit default swap.

The RMV and RT models generate similar term structures of risk-neutral default probabilities for higher-rated debt and short maturities. Results of the two models diverge as the maturity lengthens or the credit quality deteriorates. The RFV model is distinctly different from the other two approaches, and appears to be less robust in that it may fail to generate an admissible long-dated term structure of risk-neutral default probabilities. The incorporation of risky coupon-bearing bonds in the generation of default probabilities might improve the performance of the RFV model, but we do not investigate this here.

The RMV and RT models lead to similar spreads for credit default swaps in cases with better credit quality and shorter maturities. The RT and RFV models generally produce significantly wider spreads in cases with poorer credit quality and longer maturities. The RMV model leads to default swap spreads that are relatively insensitive to recovery rate when the same rate is applied in pricing and in generation of default probabilities. This is not the case for the RT and RFV models.

Our overall conclusion is that the choice of recovery model is an important consideration for pricing long-dated credit derivatives on non-investment-grade debt. Market folklore suggesting that default swap spreads are insensitive to recovery rate and claim assumptions should be viewed with caution.

ENDNOTES

[1]Recovery rates for corporate bonds are quoted as a percentage of face value.

[2]Hull and White [2000] describe a reduced-form approach for valuing plain vanilla and binary credit default swaps in a continuous-time setting using default probabilities inferred from the coupon-bearing bonds of a specific reference issuer. In our investigation, we maintain a discrete-time approach and consider generation of default probabilities from zero-coupon term structures that would be bootstrapped, in practice, from many bond prices in a general credit class.

[3]Jarrow, Lando, and Turnbull [1997] and Kijima and Komoribayashi [1998] describe more general approaches based on Markov chains for multiple credit rating states.

[4]In our discrete model the shortest possible interval between default times is one day. Some studies express default probabilities in terms of integrals of a hazard rate function. We avoid this detail, since, in practical applications, one day is the finest period for which bond maturities are defined and default events are recorded. Furthermore, expressions involving hazard rate integrals are typically evaluated using numerical quadrature—leading back to a discrete-form model.

[5]The payoff may also include accrued interest from the reference security.

[6]This expected value should be calculated with risk-neutral default probabilities that are generated from prices of risky debt with similar credit quality (rating) as the issuer of the reference security.

[7]For maximum accuracy, the ideal spacing between the potential default dates is one day, although a wider spacing might be necessary for faster simulation of a large portfolio of default swaps.

[8]The parameter $\hat{\delta}$ represents the expected recovery amount per unit face value of the reference security. This obligor-specific recovery rate may be assigned a value that is

independent of the recovery assumptions used to generate risk-neutral default probabilities. In view of data limitations, we do not propose a more detailed model that distinguishes between recovery rate and claim amount in the case of an individual obligor.

[9]We consider only a flat risk-free term structure here because the shape will not affect the qualitative results in this investigation. We assume a reduced-form model where the risk-free interest rate is independent of the default mechanism. In the RT and RMV cases, the default probabilities depend only on ratios of risky to risk-free bond prices and are not affected by the level of the risk-free rate. In the RFV case, higher risk-free rates result in higher default probabilities.

[10]Theoretically, a firm may default on any day, but, in reality, default will be announced when a scheduled interest payment is not made. If interest payments are made quarterly, it should be necessary to consider only four potential default dates per year.

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