

Putable common stock

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Abstract

The underpricing of initial public offerings is a well-documented phenomenon in the financial literature. The purpose of this paper is to show how this empirical regularity could be solved by an appropriate choice of financing instruments, namely, by an intelligent mix of common stocks and put options. The latter additional instrument, modeled in this paper as a lump sum paid by insiders of the firm to outsiders, helps alleviate the asymmetry of information existing between insiders and outsiders of the corporation, allowing good firms to sell the package they offer at the full information value. © 2003 Elsevier B.V. All rights reserved.

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The underpricing of initial public offerings (IPOs) is a well-documented phenomenon in the financial literature. Several studies have reported that underwriters of IPOs achieve abnormal returns in the very short term, suggesting that IPOs may be underpriced. For the U.S. market, [Ibbotson et al. \(1988\)](#) find an average return of about 16%.¹

On the theoretical side, scholars in finance have tried to justify the underpricing of IPOs as an equilibrium phenomenon and not as a consequence of the inefficiency of the capital market in general, and of the IPOs market in particular. In [Baron \(1982\)](#), underpricing emerges as an equilibrium in a model of agency. In his model, underwriters have better information about the right price for the IPOs because they have better knowledge of the tastes and risk characteristics of the investors. Underwriters, however, have incentives to set a lower price for the equity because this will attract more investors, will facilitate the

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¹ The same phenomenon has been detected in other countries. For a comprehensive review of the findings, see [Loughran et al. \(1994\)](#).

marketing of the shares of the company and, finally, will lower the probability of ending up with an unsuccessful offer. Rock (1986) presents a model based on asymmetric information across investors, and presents a theory that relies on a well-established result in the theory of auctions: the winner's curse. In his model, underpricing results as compensation to attract uninformed investors. Grinblatt and Hwang (1989) present a generalization of Leland and Pyle (1977) in which risk-averse issuers use the proportion of equity retained and the price of the issue to signal the mean and the risk of their projects. Other signaling models include work by Allen and Faulhaber (1989) and Welch (1989). In Chemmanur (1993), underpricing arises in equilibrium by the willingness to induce information production about the firm. An auction theoretical model is developed by Benveniste and Spindt (1989).

The purpose of this paper is to show how the underpricing occurring in IPOs could be mitigated or solved by giving firms the possibility of attaching an option-like claim to the portion of the equity they sell. The put option, modeled in this paper as a lump sum paid by the insiders of the firm to the new stockholders in some selected states, helps reduce the asymmetry of information existing between insiders and outsiders, allowing *good* firms, as specified later on in the paper, to sell the package they offer at the full information value.²

This paper is related to the work of Leland and Pyle (1977) and to that part of the literature that considers the underpricing phenomenon in IPOs as a way to communicate quality between informed firms and investors (Allen and Faulhaber, 1989; Grinblatt and Hwang, 1989; Welch, 1989). In our set-up, borrowed from Chemmanur and Fulghieri (1997), risk-averse entrepreneurs have to finance a positive net present value project by raising the funds from outsiders. There are for simplicity just two types of projects characterized by different values of means and risk parameters. In a model with asymmetric information, these differences make it problematic to price equity. If no instruments were available to signal the quality of the project, the only price of the equity to which the insider of the firm would be able to sell his project would be a *pooling* price.

In this paper, we give insiders the possibility of signaling their type to outsiders by the choice of the amount of equity retained as in Leland and Pyle (1977), by the choice of the price of the package they offer as in Grinblatt and Hwang (1989) and by the choice of attaching to the equity offering a lump sum (put-like claim) to be paid to the outsiders in the case of the bad state. This paper looks, in some sense, at the other side of the coin with respect to the work of Chemmanur and Fulghieri (1997). In the latter paper, the authors show how high-risk firms issue unit IPOs, a package of equity and warrants, to signal quality, and how firms optimally choose underpricing as a part of the efficient signaling mix. In this paper, we use basically the same framework, and we extend their model to tackle a different issue. The main result of the present paper is that when insiders of the corporation can use freely the three instruments described above, for some values of the parameter space, namely, when the risk parameter of the signaling firm is relatively low, underpricing is not part of the efficient signaling mix. That is, packaging equity and put option-like claims help good firms set the price of the package at the full information value. Through the put, the risk stemming from the asymmetry of information would shift from the outsider shareholders to the entrepreneur, allowing, for some parameter space, the latter to

² Chen and Kensinger (1988) tackle this issue. Their paper, however, only illustrates the idea and does not consider the conditions that determine the optimality of the package.

raise the capital necessary to finance the project at the full information value. The intuition behind the result is that the lump sum turns out to be cheaper for those firms whose risk parameters are lower. This comparative advantage helps good firms to signal their quality more easily, relaxing the additional requirements that were otherwise necessary to obtain separation. The implications stemming from this paper are in line with the recent empirical findings by [Prabhala and Puri \(1998\)](#) who examine a sample of firms whose underwriters supported the price around the IPO. They claim that price support can be seen as a put option written by the underwriters to the new shareholders, and they find that consistent with the main predictions of the present paper, (a) price support significantly reduces underpricing³ and (b) underwriters stabilize the price only for those firms whose ex ante measures of uncertainty were lower.

Finally, this paper is also closely related to the work of [Brennan and Kraus \(1987\)](#). They show in fact that despite the asymmetry of information, the investment opportunities may be efficiently financed by an appropriate choice of financing instruments that reveals private proprietary information to outsiders. This implies that when the adverse selection is severe, plain financial strategies may not constitute the efficient financing choice. Although the initial offering stage seems to be a moment that, due to the asymmetry of information, is potentially harmful for the entrepreneur's wealth, little effort has been done to determine which choice of financing is optimal at the IPO stage.⁴ In this sense, we find that common stocks may not be the optimal financing choice of the corporation and that a less standard choice of securities could help the firm to credibly signal its quality to outsiders.

The plan of the paper is as follows. Section 1 presents the model and describes the economy. Section 2 tackles the problem faced by entrepreneurs under the hypothesis of perfect information. Section 3 introduces the equilibrium concept used and describes the additional problems faced by the entrepreneur under asymmetric information. We will begin by assuming that the insider of the *good* firm (type G firm) may use as signals only the percentage of equity retained, or the percentage of equity retained and the value of the package he offers. In Section 4, we solve the more general problem faced by the insider when he can freely use not only the two instruments introduced above, but also a put-option-like claim. Additionally, other potential applications such as SEOs and privatizations are discussed. Section 5 presents some comparative statics. Section 6 concludes the paper.

1. The model

Consider two points in time. At time $t=0$, risk-averse entrepreneurs take their private firms public by selling equity to risk-neutral outsiders. The total number of shares is

³ The put option, as modeled in this paper, is a more explicit and, therefore, a more credible commitment device. Remember in fact that the law requires underwriters to disclose the possibility of price support in the IPO prospect. Ex post, only a fraction of those IPOs will be actually supported. This can explain why, even with price support, there may be a difference between the IPO offer and the after-market price.

⁴ Notable exceptions to this approach are the papers by [Biais et al. \(in press\)](#) and [Biais and Faugeron-Crouzet \(in press\)](#). These papers, keeping as given the securities issued by the entrepreneur, consider different offering methods and derive the optimal mechanism. Note that, however, even under the optimal mechanism, underpricing emerges in equilibrium.

normalized to 1 and entrepreneurs offer a fraction $(1 - \alpha)$ to outsiders. Assume for simplicity that the interest rate is zero and that the firms are either *good* (G) or *bad* (B). Each project has a cost k and a positive NPV. The cash flows are characterized by a three-point distribution such as: $\mu_\tau - \sigma_\tau/p_L$, μ_τ , $\mu_\tau + \sigma_\tau/p_H$, where $\tau \in \{G, B\}$, $\mu_G > \mu_B$ and $\mu_\tau - \sigma_\tau/p_L > 0$. The probabilities attached to each state (low, medium and high) are, respectively, p_L , p_M and p_H . At $t=1$, the cash flows of the projects are realized. Insiders know their own type, namely, mean μ_τ and risk parameter σ_τ , but not which state will be realized at $t=1$. Outsiders, however, do not know which firm is of what type.

The put option will be modeled in the paper as a lump sum, denoted Δ_τ , that will be paid by the insiders of the firms to the outsiders should the bad state occur. Thus, for a given retained proportion of the equity α , the payoff accruing to the outsiders of the firm will be either $(1 - \alpha)(\mu_\tau - \sigma_\tau/p_L + \Delta_\tau)$, $(1 - \alpha)\mu_\tau$ or $(1 - \alpha)(\mu_\tau + \sigma_\tau/p_H)$ for state low, medium or high, respectively.

We further assume that risk-averse insiders have a negative exponential utility function with risk parameter C . The insider has wealth at the beginning of the period equal to w_0 that is invested in some other asset and cannot be used to finance the current investment (in other words, the entrepreneur is financially constrained and has to raise in the market whatever funds the firm needs).

In such an environment, pricing the equity can be problematic. Statements by the insider regarding the quality of the firm are cheap and can be passed by. The problem of the insider of the good firm is, therefore, to choose some instruments to credibly signal his firm's quality. We tackle the problem in different steps in order to highlight the trade-offs involved in the analysis.

2. The insider's problem under perfect information

Denote by $V^G(\alpha, \Delta_G)$, the full information value of the proceeds of the IPO for the type G firm. These are equal to the expected value of the stream of the payoffs accruing to the buyer of the IPO under full information. Since risk-neutral outsiders price claims at their expected value, if the insider sells $(1 - \alpha)$ of the firm, the proceeds of the IPO are given by:

$$V^G(\alpha, \Delta_G) = (1 - \alpha)(\mu_G + p_L \Delta_G) \quad (1)$$

Define w_τ^s as the time 1 wealth of the insider of the type τ firm in state s ($s = L, M, H$). In the different states, this is given by the following equations:

$$\tilde{w}_\tau^L = w_0 + \alpha \left(\mu_\tau - \frac{\sigma_\tau}{p_L} \right) + V - (1 - \alpha)\Delta_\tau - k$$

$$\tilde{w}_\tau^M = w_0 + \alpha \mu_\tau + V - k$$

$$\tilde{w}_\tau^H = w_0 + \alpha \left(\mu_\tau + \frac{\sigma_\tau}{p_H} \right) + V - k$$

The type G insider's problem under full information is as follows:

$$\max_{\alpha, \Delta_G, V} E[U(\tilde{w}_1)] = -e^{-C\tilde{w}_1} = -p_L e^{-C\tilde{w}_L^G} - p_M e^{-C\tilde{w}_M^G} - p_H e^{-C\tilde{w}_H^G}$$

subject to

$$V \leq V^G(\alpha, \Delta_G)$$

$$(1 - \alpha)\Delta_G \leq \alpha(\mu_G - \sigma_G/p_L)$$

$$V - k \geq 0$$

with $0 \leq \alpha \leq 1$, $V \geq 0$, $\Delta_G \geq 0$.

Insiders want to maximize their end of period wealth, subject to some constraints. The first constraint is a rational individual constraint: no investor would be willing to pay more for the package than it is worth under full information. The second constraint is a basic credibility constraint: the entrepreneur must have the appropriate amount of funds to finance the payment of the put option in the bad state. The logic behind limiting the wealth of the entrepreneur to the stream of payoffs coming from the investment relies on limited liability and on the relative inability of the new shareholders to attack the entrepreneur's wealth. The third constraint states that the funds raised in the IPO stage must be such as to fully finance the project. The other constraints are self-evident and are not discussed.

Proposition 1 (Chemmanur–Fulghieri). *With symmetric information, $\alpha_\tau^* = 0$ and $\Delta_\tau^* = 0$, maximize the objective of both firms, and the prices of the equity for the good and the bad firm are, respectively, μ_G and μ_B .*

The proof is left to the reader. The intuition is easy to grasp. The entrepreneur in this framework does not have any informational advantage and is risk-averse. Being risk-averse, he, thus, finds it optimal to sell all the projects to the outsiders who pay a fair price for the claim they buy.

3. Equilibrium under asymmetric information

Under asymmetric information, insiders of the type G firm may find it optimal to distinguish their firm from the type B firm. We restrict our attention to separating equilibria where the type G firm structures the IPO so as to impose on the type B firm trying to mimic such a high cost that the type B firm prefers to be revealed as it is. Thus, equilibrium strategies and beliefs are defined as those that constitute a separating sequential equilibrium as defined by [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1986\)](#). Henceforth, denote $U^\tau(\alpha, \Delta_\tau, V)$ as the expected utility for type τ insider. The equilibrium emerges as the solution to the following problem faced by type G firms:

$$\max_{\alpha, \Delta_G, V} U^G(\alpha, \Delta_G, V) \tag{2}$$

subject to

$$U^B(\alpha, \Delta_G, V) \leq U(\mu_B) \quad (3)$$

$$V \leq V^G(\alpha, \Delta_G) \quad (4)$$

$$(1 - \alpha)\Delta_G \leq \alpha(\mu_G - \sigma_G/p_L) \quad (5)$$

$$\Delta_G \leq \sigma_G/p_L \quad (6)$$

$$V - k \geq 0 \quad (7)$$

and $0 \leq \alpha \leq 1$, $V \geq 0$, $\Delta_G \geq 0$.

Eq. (3) is an incentive compatibility constraint: it states that the expected utility to the type B firm under mimicking, denoted as $U^B(\alpha, \Delta_G, V)$, must be at most equal to the expected utility that the type B firm would get under revelation of his type, defined as $U(\mu_B)$. Eq. (6) puts an upper bound on the promised payment: no extra premium is paid to the outsiders should the bad state occur. All the other constraints are as in the program under perfect information and do not need further explanation.

The problem of the type G firm insiders is, thus, that of revealing their type to the outsiders without incurring excessive costs. Insiders will, thus, find the optimal combination of the three instruments mentioned above to signal their type. Therefore, the solution to this program gives the percentage of the equity retained, the price of the equity and the value of the put option that ensure that the market will perceive the type G firm as a good firm (separation).

As mentioned above, we will solve the problem step by step, assuming that the type G firm can use only the percentage of equity retained, the percentage of equity retained and the price for the equity and, finally, assuming that the firm can mix the three instruments.

Definition. Define $\alpha_F = 1 - k/\mu_G$ as the maximum amount of equity that the entrepreneur of the type G firm has to retain at the full information value μ_G in order to finance the cost of the project k .

Lemma 1 (Signaling with α). *If the firm can use only the percentage of equity retained α , then the maximization problem of the type G firm does not have any solution $\alpha^*(\alpha)$ if $\alpha^*(\alpha) > \alpha_F$.*

As shown in Leland and Pyle (1977), retaining part of the equity is costly for the entrepreneur. We know that under full information, the optimal amount of equity retained is zero. Under asymmetric information, however, with $V = V^G(\alpha, \Delta_G)$ and $\Delta_G = 0$, setting α equal to zero is not incentive compatible. Given that signaling is more costly for a type B firm than for a type G firm, there is a level of α such that the bad firm will prefer to reveal itself as it is rather than mimic the type G firm.

Fig. 1 makes the intuition clear. Because underpricing is not a choice variable in this simplified version of the problem, the solution, if any, must lie on the downward sloping line $V^G(\alpha)$. The type B firm, by directly (and credibly) revealing itself as a bad firm, can

Assume that the type G firm can use only the percentage of the equity retained and the price of the equity V . Thus, for the remaining of this section, assume that $\Delta_G = 0$.

Lemma 2 (Signaling with α and V). *If the firm can use both the percentage of the equity retained and the price of the package V , when $\sigma_G < \sigma_G^*$, where σ_G^* is defined in the proof, the equilibrium strategies of the type G firm involve:*

- (a) *Retaining the minimum amount of equity compatible with separation, that is $\alpha^*(\alpha, V) = \alpha_V$.*
- (b) *Underpricing the equity.*

Unlike the case in which the only signal available is the percentage of the equity retained, now the entrepreneur can choose underpricing as part of the efficient signaling mix. Thus, as shown in Fig. 1, the solution does not have to lie on the downward straight line $V^G(\alpha)$, but rather in the shaded area. The optimal solution will be the locus of points in (μ_B, α_V) . When $\sigma_G \geq \sigma_G^*$, the solution, the tangency point of the indifference curve of the type B firm calculated at $U(\mu_B)$ and the indifference curve of the type G will be in (μ_B, α_V) (point P_1 in Fig. 1). As the risk parameter of the type G firm increases, the relative cost of signaling with the percentage of equity retained increases for a risk-averse entrepreneur. The reason is that by having a higher stake in the company, the entrepreneur will bear more of the risk coming from the variability of the payoffs. Thus, he will voluntarily decide to increase the underpricing while retaining a lower stake in the company. On the other hand, as long as the risk parameter of the type G firm decreases, the type G firm will be willing to ride the indifference curve of the type B firm from the left to the right, trading off underpricing with the stake retained in the firm. This process will stop at α_V defined in the proof as the minimum amount of α such that: (a) the financial constraint is satisfied and (b) separation is still possible (point P_V in Fig. 1).⁵ In fact, any point to the right of α_V , such as point P_2 in Fig. 1, will be such as to ensure separation, but the total proceeds of the IPO will not be enough to finance the cost of the investment k , and any other point P_3 , even if financially feasible, will not be compatible with separation. When a financial constraint is introduced, firms with a relative lower level of risk will also find it necessary to underprice to credibly convey information to the outsiders.

4. An equilibrium with put options

This section tackles the more general problem faced by the entrepreneur who can use as signalling devices not only the percentage of equity retained and the price of the equity, but also a put option-like claim attached to the percentage of equity sold. The option, modeled here as a lump sum, will insure uninformed outsiders against the occurrence of a bad state.

⁵ Notice that since $\alpha_F < \alpha^*(\alpha)$, then α_V exists. Graphically, α_V is given by the intersection of the financial constraint with the downward sloping indifference curve of the type B firm valued at $U(\mu_B)$.

Forming the Lagrangian for the maximization problem (Eqs. (2)–(7)), we obtain:

$$\begin{aligned}\mathcal{L} = & U^G(\alpha, \Delta_G, V) + \lambda_1(U(\mu_B) - U^B(\alpha, \Delta_G, V)) + \lambda_2(V^G(\alpha, \Delta_G) - V) \\ & + \lambda_3(\alpha(\mu_G - \sigma_G/p_L) - (1 - \alpha)\Delta_G) + \lambda_4(\sigma_G/p_L - \Delta_G) + \lambda_5(V - k) \\ & + \lambda_6(1 - \alpha)\end{aligned}$$

with $\alpha \geq 0$, $\Delta_G \geq 0$ and $V \geq 0$.

The Kuhn–Tucker conditions for the optimum are given below, where subscripts for $\mathcal{L}$, U and V denote partial derivatives (arguments are disregarded for simplicity):

$$\mathcal{L}_\alpha = U_\alpha^G - \lambda_1 U_\alpha^B + \lambda_2 V_\alpha^G + \lambda_3(\mu_G - \sigma_G/p_L + \Delta_G) - \lambda_6 \leq 0$$

$$\mathcal{L}_{\Delta_G} = U_{\Delta_G}^G - \lambda_1 U_{\Delta_G}^B + \lambda_2 V_{\Delta_G}^G - \lambda_3(1 - \alpha) - \lambda_4 \leq 0$$

$$\mathcal{L}_V = U_V^G - \lambda_1 U_V^B - \lambda_2 + \lambda_5 \leq 0$$

$$\alpha \mathcal{L}_\alpha + \Delta_G \mathcal{L}_{\Delta_G} + V \mathcal{L}_V = 0$$

$$\mathcal{L}_{\lambda_1} = U(\mu_B) - U^B \geq 0$$

$$\mathcal{L}_{\lambda_2} = V^G - V \geq 0$$

$$\mathcal{L}_{\lambda_3} = \alpha(\mu_G - \sigma_G/p_L) - (1 - \alpha)\Delta_G \geq 0$$

$$\mathcal{L}_{\lambda_4} = \sigma_G/p_L - \Delta_G \geq 0$$

$$\mathcal{L}_{\lambda_5} = V - k \geq 0$$

$$\mathcal{L}_{\lambda_6} = 1 - \alpha \geq 0$$

$$\lambda_1 \mathcal{L}_{\lambda_1} + \lambda_2 \mathcal{L}_{\lambda_2} + \lambda_3 \mathcal{L}_{\lambda_3} + \lambda_4 \mathcal{L}_{\lambda_4} + \lambda_5 \mathcal{L}_{\lambda_5} + \lambda_6 \mathcal{L}_{\lambda_6} = 0$$

$$V \geq 0, \alpha \geq 0, \Delta \geq 0, \lambda_i \geq 0, i = 1, 2, 3, 4, 5, 6.$$

The solution to the maximization problem can be characterized in the following proposition.

Proposition 2 (An equilibrium with put options). *If Eq. (5) is not binding at the optimum, there exists a separating equilibrium for the parameter of the problem. In such a separating equilibrium, if the risk parameter of the type G firm σ_G is such that $\sigma_G \in (\sigma_G^Z, \sigma_G^V)$, defined in the proof, then the equilibrium strategies of the type G firm involve the following:*

- The firm issues a package of equity with put options ($\Delta_G^* > 0$).*
- The package of equity and put option is priced at the full information value ($V^* = V^G$).*
- The insider retains a positive fraction of the equity α^* such that $0 < \alpha^* < \alpha_F$.*

Proposition 2 shows the conditions that have to be satisfied for a firm to efficiently package equity plus a put option-like claim in an IPO. The importance of the result relies on the fact that attaching a lump sum to the percentage of the equity sold may help the entrepreneur solve the asymmetric information problem while setting the price of the package at the full information value. The main intuition of the result is as follows. As long as $\alpha_F < \alpha^*(\alpha)$, optimal separating behavior is not dictated exclusively by the firm's risk parameter. In fact, when the signals available to the entrepreneur are only the percentage of equity retained and the price of the package, if $\sigma_G < \sigma_G^*$, then all firms, no matter the value of the risk parameter, will choose point P_V in Fig. 1. We know, in fact, that any α to the right of α_V is not compatible with separation. When, however, Δ_G is part of the signals available, as long as the variability of the returns is not *high*, setting the price of the package at the full information value is optimal for a risk-averse entrepreneur. That is, under this condition, he prefers to sell a relatively low fraction of the firm at $V = V^G$, rather than to sell a higher stake of the firm at a discounted price. If the risk parameter is very low (or lower than σ_G^*), risk-averse entrepreneurs will find it advantageous to fix the price of the equity at the full information value, to sell as much of the equity as needed to make the investment viable (that is, to retain α_F), and to issue a put option-like claim. In this case, the only relevant motivation pushing the entrepreneur to go public is the need for funds so that he will sell only that part of the firm that will make it possible to finance the cost of the investment k . If the risk coming from the project is higher, but lower than σ_B^V , then the type G firm insider will find it convenient to reduce his stake in the firm, that is setting $\alpha^* < \alpha_F$, but still maintaining the value of the equity at the full information value: as σ_G gets higher, the project will bring additional risk, and the entrepreneur will be better off reducing the actual stake retained in the firm. When $\sigma_G > \sigma_G^V$, the put option will not be part of the efficient signalling mix anymore. In fact, a higher stake in the firm means bearing considerable risk coming from the project and, as long as the variability of returns is higher than σ_G^V , the entrepreneur will prefer trading off V for α .

The assumption we made in the proposition for deriving the results was that the constraint attached to the Lagrangian multiplier λ_3 is not binding at the optimum, which in turns implies that the fraction of the cash flows of the company in the bad state to the insiders will be such as to fully pay the lump sum promised by the entrepreneur. From Eq. (5), we know that the smaller the value of α , the lower the admissible values that Δ_G can take. For the problem to be interesting (we want to find ranges of the variance of the type G firm in which it would change α for Δ), we should assume that the equilibrium level Δ_G^* is such that constraint (5) is not binding. This may seem a quite strong assumption, but it is necessary for the problem to be meaningful. Constraint (5) expresses a basic credibility problem faced by outsiders. It has been motivated earlier in the paper by limited liability and by the impossibility or the objective difficulty of attacking the entrepreneur's wealth in the case of no payment. There are, however, several ways in which the problem can be solved successfully. The entrepreneur can write a contract with outsiders that in case of default, will give outsiders the possibility of recouping the losses on the entrepreneur's wealth (i.e. a collateral). Another option is to let the investment bank that takes the

firm public issue the put on its own behalf, ensuring the position of the investment bank with some contractual arrangement that allows the investment bank to attack the entrepreneur's wealth. This contract should be easier to write than one between insiders and outsiders. Although the model explicitly deals with new equity offerings, the package described in this paper can also be used in seasoned equity offerings when insiders of the corporation believe that stocks are undervalued by the market.⁶ Another possible application of the device introduced in the paper is in privatization plans. Perotti and Gunev (1993), using data from 12 countries, document that privatizations tend to be highly discounted fixed price offerings.⁷ While we present an IPO framework, the additional instrument introduced in the paper can be used in privatizations. One example of the described package can be found in the offer that ENI, an Italian at-that-time state owned firm, made when it decided to go public (Fig. 2). The deal that the company proposed to the investors was a package of common stocks and standard put options that allowed investors to sell back their stocks to the firm 1 year later (European puts). The issue price of the stock was fixed at Italian liras 5250. ENI started trading on November 28, 1994 on the Milan Stock Exchange, and the first day closing price was Italian liras 5250. ENI's closing prices do not resemble the average price reaction that most stocks experience in the first days of trading.

Instead of realizing the usual high jump, the ENI stock price held steady for 2 days and then started fluctuating with the market. Finally, when the option expired, the stock price was much higher than the strike price, so that investors did not exercise their options. We are aware that this single observation does not constitute a solid statistical basis to conclude that the instrument proposed in this paper can actually resolve underpricing, but it does provide a starting point for further discussion and experimentation. Other state-owned companies have recently decided to go public in a similar fashion.⁸ Through the use of the put, the risk coming from the project shifts from outsider shareholders to the entrepreneur. Given that the credit risk is a real problem in these deals (even though we claimed above that the option could be written by the investment bank whose reputation should be known by the market), it is clear why this instrument has been used by state-governed enterprises: issues such as government reputation, the possibility of jeopardizing future privatizations,

⁶ In 1985, Gearhart Industries needed to raise a fairly large amount of money during a period when the market price of its stocks was perceived as low by corporate management (the price was about US\$ 10.75 per share). Gearhart's investment bank advised the management to issue a package of common stock and put options. The offering price was set at US\$ 15 per share, therefore, allowing the firm to find better financing conditions.

⁷ Jankinson and Mayer (1988) report that the average underpricing for U.K. privatizations has been even higher than that for other new equity offerings.

⁸ The instrument described also resembles the deal offered by Argentaria (Spain) in 1994 when this state-owned company went public. We believe, however, that the put option-like claim (lump sum), as modeled in the paper, presents a substantial advantage over the standard put option, especially in privatization processes. Consider what would happen if the bad state occurred (that is, if the put option expired in-the-money). If the firm issued a standard put option, outsiders would exercise their option and would put back their stocks. This is an event that has a cost, not only at the corporate level (the firm would become state-owned again), but also at the social level because it would hurt the government's reputation and, consequently, jeopardize future privatization plans. This unfortunate situation would not occur with the lump sum proposed in the paper.

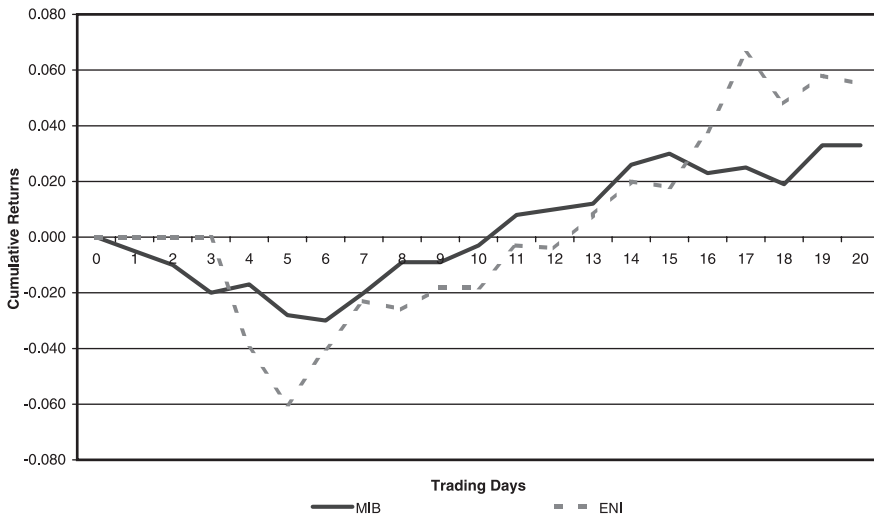


Fig. 2. The case of ENI.

as well as political costs, would be so high that any government would be willing to honor the put if necessary.

The use of put options in privatizations can also be justified in a completely different setup than the one outlined in this paper. [Perotti \(1994\)](#) presents a theory of privatizations based on government impossibility to commit to a future policy. The idea of the paper is that the structure of the offering, that is partial sales (and underpricing), as documented in the aforementioned literature, can be seen as signals coming from the government to reassure investors. In fact, by retaining a significant stake in the firm, the government, being a residual claimant as any other shareholder, will convey credibly its willingness to bear all the risks and the consequences of its actions. In such a framework, the stake retained in the firm and the put option are completely interchangeable. Both instruments constrain the government's actions and can be used as signals. Additionally, the put options would have the advantage of reducing underpricing and could help resolve the critique raised by [Gale and Stiglitz \(1989\)](#) regarding the congruence of using the retained part of the equity in a model without commitment.⁹

5. Comparative statics

In this section, we develop some comparative analysis for the general case where the entrepreneur is free to use the three instruments.

⁹ [Gale and Stiglitz \(1989\)](#) show that underdiversification, as pointed out by [Leland and Pyle \(1977\)](#), may not convince investors in a model with asymmetric information if the entrepreneur can sell his stake in the secondary market.

Proposition 3 (Comparative statics on σ_G^V). *Keeping everything else constant, the cut-off point of the variance of the type G firm σ_G^V gets higher:*

- (a) *as the risk parameter of the type B firm gets higher;*
- (b) *as the difference $(\mu_G - \mu_B)$ gets higher;*
- (c) *as the lump sum Δ_G gets higher in the admissible range $0 \leq \Delta_G \leq \sigma_G/p_L$.*

Proof. By implicit differentiation of the LHS of Eq. (13). □

The results are intuitive and deserve little comment. As the risk parameter of the type B firm gets higher, the stake in the firm that the type G entrepreneur has to maintain in order to obtain separation gets lower. This in turn implies that the entrepreneur of the type G firm is less subject to the risk coming from the project, so he will need a higher risk parameter level to choose underpricing as part of the signalling mix. The second result of the proposition is along the same lines. As μ_G gets higher (or conversely μ_B gets lower), the advantages of underpricing vanish for the type G firm. Thus, the type G insider will be willing to underprice his package only for higher levels of σ_G .

While parts (a) and (b) of the proposition apply also when the type G firm is constrained to choose as signals only α and V , the third result is the most important in this section and is peculiar to the case in which Δ_G is a possible signalling instrument. The result in the proposition states that as Δ_G grows in the admissible region, the cut-off point of the risk parameter σ_G^* gets higher. The importance of the result relies on the fact that when a put option-like claim can be attached to the percentage of equity retained α , then the range of values for which underpricing is *not* an optimal choice gets unambiguously higher. That is, there are some values of the risk parameter of the type G firm for which attaching Δ_G to the equity helps the entrepreneur to set the price of the equity at the full information value.

Proposition 4 (Comparative statics on Δ_G^*). *Keeping all the other variables constant, the magnitude of the lump sum Δ_G^* that has to be attached to the equity retained to allow separation gets lower as σ_B gets higher.*

Proof. The lump sum Δ_G , as a mean of being a signal, is costly. This implies that the type G firm will choose the minimum level of Δ_G such that separation occurs. That is, in equilibrium, the incentive compatibility constraint will be satisfied with equality, that is:

$$U^B(\alpha, \Delta_G, V) - U(\mu_B) = 0$$

Implicit differentiation of the above equation with respect to σ_B will give the result. □

The intuition of this result goes as follows: as long as the risk parameter of the type B firm increases, mimicking the behavior of the type G firm by attaching a put option to the equity sold becomes more and more expensive for the type B firm. Therefore, the insider of the type G firm may decrease the lump sum Δ_G and still obtain separation. This translates

also to the statement that fully insuring the outsiders might not be an optimal strategy for the insiders of the type G firm.

6. Conclusions

The optimality of common stocks (and bonds) has been and is still an important issue in the finance literature. Many authors (e.g., Zender, 1991; Aghion and Bolton, 1992; Dewatripoint and Tirole, 1994; Koskinen, 1997) address the issue of optimal security design, and show that the aforementioned instruments are indeed optimal financial instruments under different assumptions and at different moments in the life of the firm. Determining the optimal set of instruments at the IPO stage, however, does not seem to have been in the research agenda for scholars in finance, probably because firms *do* go public by issuing stocks. Along these lines, another interpretation of the paper could be that given some asymmetry of information between insiders and outsiders, stand-alone stocks may not be the optimal instrument. Allen and Gale (1992) show that in an economy characterized by asymmetry of information on securities' characteristics and payoffs (what the authors call *product uncertainty*), too much standardization is possible. If this is the case, and both the magnitude and the pervasiveness of underpricing lead us toward this conclusion, the results of this paper could be seen as a first attempt to tackle this issue. In fact, the main contribution of the paper is to show that packaging put option-like claims along with equity may help the entrepreneur to avoid or to reduce the discount that has been documented in the IPO market. For this to happen, the risk parameter of the type G firm should be low, as specified in the paper. The force driving the results is that the marginal cost of the lump sum Δ_G decreases, the lower the variability of the returns of the firm, so that firms with relatively lower variances may find it optimal to include the put as part of the efficient signalling mix.

In this paper, we take the security type as given, and we demonstrate the existence of a separating equilibrium. Deriving the optimal set of instruments at the IPO stage would constitute an interesting extension. Additionally, we modeled an economy with risk-neutral shareholders. In a model, like the one presented here in which two parameters are unknown (namely, mean and variance), and *risk-averse* investors, the derivative security could be validly used as a way of communicating the risk parameter to the market. An analysis with this perspective would enrich our understanding of the multisignaling literature and of the possible uses of the derivative securities in corporate finance.

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Appendix A

Proof of Lemma 1. In this simplified version of the model, the entrepreneur can use only the percentage of equity retained as signaling instrument. That is, set $\Delta_G = 0$ and $V = V^G(\alpha)$. A necessary condition for an equilibrium to exist is that the incentive compatibility constraint must be satisfied with equality, i.e. will be binding. In this way, the bad firm will be left indifferent between mimicking and revealing itself. That is:

$$U^B = (\alpha) = U(\mu_B) \quad (8)$$

where

$$U(\mu_B) = -\exp(-C(w_0 + \mu_B - k))$$

In what follows, we will look for the smallest value of α such that Eq. (8) will be satisfied with equality. The expected utility for the type B firm under mimicking, denoted $U^B(\alpha)$, can be written as

$$\begin{aligned} U^B(\alpha) = & -p_L \exp(-C(w_0 + \alpha(\mu_B - \sigma_B/p_L) + (1 - \alpha)\mu_G - k)) \\ & - p_M \exp(-C(w_0 + \alpha(\mu_B) + (1 - \alpha)\mu_G - k)) \\ & - p_H \exp(-C(w_0 + \alpha(\mu_B + \sigma_B/p_H) + (1 - \alpha)\mu_G - k)) \end{aligned}$$

Evaluating $U^B(\alpha)$ at $\alpha = 0$ and 1, we obtain that:

- (a) $U^B(\alpha = 0) = -\exp(-C(w_0 + \mu_G - k)) > U(\mu_B)$ because $\mu_G > \mu_B$;¹⁰
- (b) $U(\mu_B) > U^B(\alpha = 1)$ because of risk aversion,

where

$$\begin{aligned} U^B(\alpha = 1) = & -p_L \exp(-C(w_0 + \mu_B - \sigma_B/p_L - k)) - p_M \exp(-C(w_0 + \mu_B - k)) \\ & - p_H \exp(-C(w_0 + \mu_B + \sigma_B/p_H - k)) \end{aligned}$$

Because of continuity, there is at least one solution to the problem with α bounded away from 0 and 1. The uniqueness will be proven by showing that $U^B(\alpha)$ is strictly decreasing

¹⁰ Recall also that with $V = V^G(\alpha)$, $\alpha = 0$ is not incentive compatible.

and convex in α . In fact, taking the first and the second derivatives of $U^B(\alpha)$ with respect to α , we obtain:

$$\begin{aligned}\frac{\partial U^B(\alpha)}{\partial \alpha} &= p_L C(\mu_B - \sigma_B/p_L - \mu_G) e^{(-C(w_0 + \alpha(\mu_B - \sigma_B/p_L) + (1-\alpha)\mu_G - k))} \\ &\quad + p_M C(\mu_B - \mu_G) e^{(-C(w_0 + \alpha\mu_B + (1-\alpha)\mu_G - k))} \\ &\quad + p_H C(\mu_B + \sigma_B/p_H - \mu_G) e^{(-C(w_0 + \alpha(\mu_B + \sigma_B/p_H) + (1-\alpha)\mu_G - k))} < 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 U^B(\alpha)}{\partial \alpha^2} &= p_L C^2(\mu_B - \sigma_B/p_L - \mu_G)^2 e^{(-C(w_0 + \alpha(\mu_B - \sigma_B/p_L) + (1-\alpha)\mu_G - k))} \\ &\quad + p_M C^2(\mu_B - \mu_G)^2 e^{(-C(w_0 + \alpha\mu_B + (1-\alpha)\mu_G - k))} \\ &\quad + p_H C^2(\mu_B + \sigma_B/p_H - \mu_G)^2 e^{(-C(w_0 + \alpha(\mu_B + \sigma_B/p_H) + (1-\alpha)\mu_G - k))} > 0\end{aligned}$$

We have, thus, shown that Eq. (8) has a solution (because of parts (a) and (b) above and because of continuity) and that the solution is unique (because $U^B(\alpha)$ is strictly decreasing and convex in α). Because of the definition of α_F , when $\alpha^*(\alpha)$ is such that $\alpha^*(\alpha) \geq \alpha_F$, then the project is not feasible. Therefore, there is no solution to this problem that is at the same time incentive compatible and satisfies the financial constraint. $\square$

The following notation is used extensively throughout the paper, define ρ_L^τ and ρ_M^τ as:

$$\begin{aligned}\rho_L^\tau &= \frac{\exp(-C(\tilde{w}_\tau^L))}{\exp(-C(\tilde{w}_\tau^H))} = \exp(C(\alpha(\sigma_\tau/p_H + \sigma_\tau/p_L - \Delta_G) + \Delta_G)) \geq 1 \\ \rho_M^\tau &= \frac{\exp(-C(\tilde{w}_\tau^M))}{\exp(-C(\tilde{w}_\tau^H))} = \exp(C(\alpha\sigma_\tau/p_H)) \geq 1\end{aligned}$$

with $\tau \in \{G, B\}$.

Proof of Lemma 2. Set $\Delta_G = 0$. Forming the Lagrangian of the problem (Eqs. (2)–(7)), we obtain:

$$\begin{aligned}\mathcal{L} &= U^G(\alpha, V) + \lambda_1(U(\mu_B) - U^B(\alpha, V)) + \lambda_2(V^G(\alpha) - V) + \lambda_3(V - k) \\ &\quad + \lambda_4(1 - \alpha)\end{aligned}$$

with $\alpha \geq 0$, $V \geq 0$, $\lambda_i \geq 0$ and $i = 1, 2, 3, 4$.

The Kuhn–Tucker conditions for the problem are as follows, where subscripts for $\mathcal{L}$, U and V^G denote partial derivatives:

$$\mathcal{L}_\alpha = U_\alpha^G - \lambda_1 U_\alpha^B + \lambda_2 V_\alpha^B - \lambda_4 \leq 0$$

$$\mathcal{L}_V = U_V^G - \lambda_1 U_V^B - \lambda_2 + \lambda_3 \leq 0$$

$$\alpha \mathcal{L}_\alpha + V \mathcal{L}_V = 0$$

$$\mathcal{L}_{\lambda_1} = U(\mu_B) - U^B(\alpha, V) \geq 0$$

$$\mathcal{L}_{\lambda_2} = V^G - V \geq 0$$

$$\mathcal{L}_{\lambda_3} = V - k \geq 0$$

$$\mathcal{L}_{\lambda_4} = 1 - \alpha \geq 0$$

$$\lambda_1 \mathcal{L}_{\lambda_1} + \lambda_2 \mathcal{L}_{\lambda_2} + \lambda_3 \mathcal{L}_{\lambda_3} + \lambda_4 \mathcal{L}_{\lambda_4} = 0$$

with $V \geq 0$, $\alpha \geq 0$, $\lambda_i \geq 0$, $i = 1, 2, 3, 4$.

Since $k > 0$, then it follows that $\alpha_F < 1$ (and $\lambda_4 = 0$) and $V^* > 0$. Thus, from the Kuhn–Tucker conditions, we have that $\mathcal{L}_V = 0$. Solving $\mathcal{L}_V = 0$ for λ_1 , we obtain:

$$\lambda_1 = \frac{U_V^G - \lambda_2 + \lambda_3}{U_V^B}$$

Define α_V as the minimum amount of α such that: (a) the financial constraint is satisfied and (b) separation is still possible. This cut-off point is given by solving the following implicit expression:

$$U^B(\alpha_V, k) = U(\mu_B)$$

With $\alpha^*(\alpha, V) = \alpha_V$, we have that $\mathcal{L}_\alpha = 0$. Plugging the expression of λ_1 in $\mathcal{L}_\alpha = 0$, we obtain, after some manipulation:

$$\frac{U_\alpha^G}{U_V^G} - \frac{U_\alpha^B}{U_V^B} = -\frac{\lambda_2}{U_V^B U_V^G} (U_\alpha^B + V_\alpha^G U_V^B) + \lambda_3 \frac{U_\alpha^B}{U_V^B U_V^G} \quad (9)$$

The RHS of Eq. (9) is greater than or equal to zero because the term in the brackets is negative. Now, consider the LHS of Eq. (9). While U_α^B/U_V^B does not depend on the risk parameter of the type G firm, the term U_α^G/U_V^G (that is the absolute value of the marginal rate of substitution between V and α for the type G firm) does depend on it. The term U_α^G/U_V^G can be written as follows:

$$U_\alpha^G/U_V^G = \mu_G + \frac{\sigma_G(1 - \rho_L^G)}{p_L \rho_L^G + p_M \rho_M^G + p_H} \quad (10)$$

Taking the derivative of Eq. (10) with respect to the risk parameter of the type G firm, we obtain:

$$\begin{aligned} \frac{\partial (U_x^G/U_V^G)}{\partial \sigma_G} &= \frac{\left(1 - \rho_L^G - \sigma_G \frac{\partial \rho_L^G}{\partial \sigma_G}\right)(p_L \rho_L^G + p_M \rho_M^G + p_H)}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} \\ &\quad - \frac{\left(p_L \frac{\partial \rho_L^G}{\partial \sigma_G} + p_M \frac{\partial \rho_M^G}{\partial \sigma_G}\right)(\sigma_G(1 - \rho_L^G))}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} \\ &\leq \frac{-\sigma_G \frac{\partial \rho_L^G}{\partial \sigma_G}(p_L \rho_L^G + p_M \rho_M^G + p_H) - \left(p_L \frac{\partial \rho_L^G}{\partial \sigma_G} + p_M \frac{\partial \rho_M^G}{\partial \sigma_G}\right)(\sigma_G(1 - \rho_L^G))}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} \\ &= \frac{-\frac{\partial \rho_L^G}{\partial \sigma_G}(p_H + p_L) - \frac{\partial \rho_M^G}{\partial \sigma_G} p_M - p_M \left(\frac{\partial \rho_L^G}{\partial \sigma_G} \rho_M^G - \frac{\partial \rho_M^G}{\partial \sigma_G} \rho_L^G\right)}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} < 0 \end{aligned}$$

where the first inequality follows from $\rho_L^G \geq 1$, the second inequality from the fact that the term in brackets is always positive and the two following expressions:

$$\frac{\partial \rho_L^G}{\partial \sigma_G} = \frac{C\alpha}{p_H} \rho_L^B + \frac{C\alpha}{p_L} \rho_L^B > 0$$

and

$$\frac{\partial \rho_M^G}{\partial \sigma_G} = \frac{C\alpha}{p_H} \rho_M^B > 0.$$

We have shown that the derivative of U_x^G/U_V^G with respect to σ_G is negative. Thus, there exists a value of σ_G , denoted σ_G^* , such that the LHS of Eq. (9) is equal to zero. Thus, $\forall \sigma_G \geq \sigma_G^*$, Eq. (9) is violated. Then, with $\sigma_G \geq \sigma_G^*$, $\alpha^*(\alpha, V) < \alpha_V$. Also notice that $\alpha^*(\alpha, V)$ cannot be 0 because at that value no separation occurs. For the same token, with $\sigma_G \leq \sigma_G^*$, then $\alpha^*(\alpha, V) = \alpha_V$. Since $\alpha^*(\alpha) > \alpha_F$, $V^* < V^G$. $\square$

The partial derivatives used in the proof of Proposition 2 are given by the following expressions:

$$\begin{aligned} \frac{\partial U^G(\alpha, \Delta_G, V)}{\partial \alpha} &= p_L C(\mu_G - \sigma_G/p_L + \Delta_G) \exp(-C(\tilde{w}_G^L)) + p_M C\mu_G \exp(-C(\tilde{w}_G^M)) \\ &\quad + p_H C(\mu_G + \sigma_G/p_H) \exp(-C(\tilde{w}_G^H)) > 0 \end{aligned}$$

$$\frac{\partial U^G(\alpha, \Delta_G, V)}{\partial \Delta_G} = -p_L C(1 - \alpha) \exp(-C(\tilde{w}_G^L)) < 0$$

$$\frac{\partial U^G(\alpha, \Delta_G, V)}{\partial V} = p_L C \exp(-C(\tilde{w}_G^L)) + p_M C \exp(-C(\tilde{w}_G^M)) \\ + p_H C \exp(-C(\tilde{w}_G^H)) > 0$$

$$\frac{\partial U^B(\alpha, \Delta_G, V)}{\partial \alpha} = p_L C(\mu_B - \sigma_B/p_L + \Delta_G) \exp(-C(\tilde{w}_B^L)) + p_M C(\mu_B \exp(-C(\tilde{w}_B^M))) \\ + p_H C(\mu_B + \sigma_B/p_H) \exp(-C(\tilde{w}_B^H)) > 0$$

$$\frac{\partial U^B(\alpha, \Delta_G, V)}{\partial \Delta_G} = -p_L C(1 - \alpha) \exp(-C(\tilde{w}_B^L)) < 0$$

$$\frac{\partial U^B(\alpha, \Delta_G, V)}{\partial V} = p_L C \exp(-C(\tilde{w}_B^L)) + p_M C \exp(-C(\tilde{w}_B^M)) \\ + p_H C \exp(-C(\tilde{w}_B^H)) > 0$$

Also, $V_\alpha^G = -(\mu_G + p_L \Delta_G) \leq 0$ and $V_{\Delta_G}^G = p_L(1 - \alpha) \geq 0$.

Proof of Proposition 2. We will prove our statements in four steps. First, we show that $\alpha^* > 0$. Second, we will state which are the conditions that have to be satisfied so that $V^* = V^G$ and $\Delta_G^* > 0$. Third, we will find conditions such that $\alpha^* < \alpha_F$. Finally, it will be shown that there exists a unique Δ_G^* such that for $\Delta_G^* = \bar{\Delta}_G > 0$ separation occurs.

Step 1: $\alpha^* > 0$.

We know that $\alpha^* = 1$ is not in the solution set as long as $k > 0$. For the same reason, $V^* > 0$. It follows that $\lambda_6 = 0$. Thus, from the Kuhn–Tucker conditions, we have that $\mathcal{L}_V = 0$. Solving $\mathcal{L}_V = 0$ for λ_1 , we obtain:

$$\lambda_1 = \frac{U_V^G - \lambda_2 + \lambda_5}{U_V^B}$$

Plug this expression in $\mathcal{L}_\alpha$ and we have:

$$U_\alpha^G - U_\alpha^B \frac{U_V^G}{U_V^B} + \lambda_2 \frac{U_\alpha^B}{U_V^B} + \lambda_2 V_\alpha^G - \lambda_5 \frac{U_\alpha^B}{U_V^B} + \lambda_3 (\mu_G - \sigma_G/p_L + \Delta_G) \leq 0.$$

Dividing each member for U_V^G , after some manipulation, we get:

$$\frac{U_\alpha^G}{U_V^G} - \frac{U_\alpha^B}{U_V^B} \leq -\frac{\lambda_2}{U_V^B U_V^G} (U_\alpha^B + V_\alpha^G U_V^B) - \lambda_3 \frac{\mu_G - \sigma_G/p_L + \Delta_G}{U_V^G} + \lambda_5 \frac{U_\alpha^B}{U_V^B U_V^G} \quad (11)$$

Suppose that $\alpha = 0$. This in turn implies that $\Delta_G = 0$ (because of the limited liability constraint) and, therefore, $\lambda_2 = 0$ because the only variable to signal is V and, thus, under separation, underpricing must occur. In addition, note that at $\alpha = 0$, $V - k > 0$, since the type

G firm is a positive NPV project. Thus, at $\alpha=0$, $\lambda_5=0$. If this is the case, Eq. (11) simplifies to

$$\frac{U_\alpha^G}{U_V^G} - \frac{U_\alpha^B}{U_V^B} \leq -\lambda_3 \frac{\mu_G - \sigma_G/p_L}{U_V^G} \quad (12)$$

Valued at $\alpha=0$, the LHS of Eq. (12) is strictly positive:

$$\frac{U_\alpha^G}{U_V^G} - \frac{U_\alpha^B}{U_V^B} = \mu_G - \mu_B > 0.$$

Since $\mu_G - \sigma_G/p_L$ is strictly positive by assumption and $U_V^G \geq 0$, then Eq. (12) is violated. Thus, $\alpha^* > 0$ and $\mathcal{L}_\alpha = 0$.

Step 2: $V^* = V^G$ and $\Delta_G^* > 0$.

Again, $V^* > 0$ and $\mathcal{L}_V = 0$. Solving $\mathcal{L}_V = 0$ for λ_1 , we obtain:

$$\lambda_1 = \frac{U_V^G - \lambda_2 + \lambda_5}{U_V^B}$$

Plugging this equation into $\mathcal{L}_\alpha = 0$, we get:

$$\frac{U_\alpha^G}{U_V^G} - \frac{U_\alpha^B}{U_V^B} = -\frac{\lambda_2}{U_V^B U_V^G} (U_\alpha^B + V_\alpha^G U_V^B) - \lambda_3 \frac{\mu_G - \sigma_G/p_L + \Delta_G}{U_V^G} + \lambda_5 \frac{U_\alpha^B}{U_V^B U_V^G} \quad (13)$$

Consider the LHS of Eq. (13). While U_α^B/U_V^B is independent of σ_G , U_α^G/U_V^G (that is equal to the absolute value of the marginal rate of substitution between α and V for the type G firm) does depend on it. This figure can be written as:

$$\frac{U_\alpha^G}{U_V^G} = \mu_G + \frac{\sigma_G(1 - \rho_L^G) + p_L \Delta_G \rho_L^G}{p_L \rho_L^G + p_M \rho_M^G + p_H}. \quad (14)$$

Taking the derivative of Eq. (14) with respect to σ_G , we find that:

$$\begin{aligned} \frac{\partial (U_\alpha^G/U_V^G)}{\partial \sigma_G} &\leq \frac{\left(-\sigma_G \frac{\partial \rho_L^G}{\partial \sigma_G} + p_L \Delta_G \frac{\partial \rho_L^G}{\partial \sigma_G} \right) (p_L \rho_L^G + p_M \rho_M^G + p_H)}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} \\ &\quad - \frac{\left(p_L \frac{\partial \rho_L^G}{\partial \sigma_G} + p_M \frac{\partial \rho_M^G}{\partial \sigma_G} \right) (\sigma_G(1 - \rho_L^G) + p_L \Delta_G \rho_L^G)}{(p_L \rho_L^G + p_M \rho_M^G + p_H)^2} < 0 \end{aligned}$$

where the first inequality follows from $\rho_L^G \geq 1$, the second inequality follows from $0 \leq p_L \Delta_G \leq \sigma_G$ and

$$\frac{\partial \rho_L^G}{\partial \sigma_G} = C \alpha \rho_L^G \left(\frac{1}{p_H} + \frac{1}{p_L} \right) > 0$$

$$\frac{\partial \rho_M^G}{\partial \sigma_G} = \frac{C \alpha \rho_L^G}{p_H} > 0.$$

Then, there exists a value of the risk parameter for the type G , defined as σ_G^V , such that:

$$\frac{U_\alpha^G}{U_V^G} = \frac{U_\alpha^B}{U_V^B}.$$

Thus, $\forall \sigma_G < \sigma_G^V$, the LHS of Eq. (13) is positive. Noting that $U_\alpha^B + V_\alpha^G U_V^B < 0$, then the only way the RHS of Eq. (13) could be positive is if $\lambda_2 > 0$, which in turn implies that $V^G - V^* = 0$ or $V^* = V^G$. Since by assumption, $\alpha^*(\alpha) > \alpha_V$, separation is attainable only if $\Delta_G^* > 0$.

Step 3: $\alpha^* < \alpha_F$.

$\Delta_G^* > 0$ implies that $\mathcal{L}_{\Delta_G} = 0$. Solving this expression for λ_1 we get:

$$\lambda_1 = \frac{U_{\Delta_G}^G + \lambda_2 V_{\Delta_G}^G - \lambda_3(1 - \alpha) - \lambda_4}{U_{\Delta_G}^B}. \quad (15)$$

At $\alpha = \alpha_F$, $\mathcal{L}_\alpha = 0$. Plugging Eq. (15) into $\mathcal{L}_\alpha = 0$, we obtain, after some manipulation:

$$\begin{aligned} \frac{U_\alpha^G}{U_{\Delta_G}^G} - \frac{U_\alpha^B}{U_{\Delta_G}^B} = & -\frac{\lambda_3}{U_{\Delta_G}^G} ((1 - \alpha) U_\alpha^B / U_{\Delta_G}^B + (\mu_G - \sigma_G / p_L + \Delta_G)) \\ & - \frac{\lambda_2}{U_{\Delta_G}^G} \left(\frac{V_{\Delta_G}^G U_\alpha^B}{U_{\Delta_G}^B} - V_\alpha^G \right) - \lambda_4 \frac{U_\alpha^B}{U_{\Delta_G}^B U_{\Delta_G}^B}. \end{aligned} \quad (16)$$

Assume that Δ_G is such that the constraint attached to the Lagrangian multiplier λ_3 is not so large that the constraint is binding. Then $\lambda_3 = 0$, and the RHS of the last equation is nonpositive. Now, consider the LHS of the last equation. Again, while the absolute value of the marginal rate of substitution (MRS) between Δ_G and α for the type B firm does not depend on σ_G , the absolute value of the $MRS^G(\Delta_G, \alpha)$ does depend on it. This figure can be written as:

$$\frac{U_\alpha^G}{U_{\Delta_G}^G} = -\frac{\mu_G(p_L \rho_L^G + p_M \rho_M^G + p_H) + \sigma_G(1 - \rho_L^G) + p_L \Delta_G \rho_L^G}{p_L(1 - \alpha) \rho_L^G}. \quad (17)$$

Taking the derivative of expression (17) with respect to the risk parameter of the type G firm σ_G , we find, after some tedious calculations, that

$$\frac{\partial(U_\alpha^G / U_{\Delta_G}^G)}{\partial \sigma_G} > 0.$$

Then, there exists a value for the risk parameter of the type G firm, defined as σ_G^α , such that the difference

$$\frac{U_\alpha^G}{U_{\Delta_G}^G} - \frac{U_\alpha^B}{U_{\Delta_G}^B} = 0.$$

Thus, for all $\sigma_G > \sigma_G^\alpha$, Eq. (16) is violated. Thus, $\alpha^* < \alpha_F$.

Step 4: $\Delta_G^* = \bar{\Delta}_G > 0$.

The uniqueness of the solution will be proven by showing that the indifference curves for the type B firm in the space (V^G, Δ_G) are strictly concave. Recall that the marginal rate of substitution between Δ_G and V for the type B firm can be written as follows:

$$\text{MRS}^B(V, \Delta_G) = \frac{p_L(1-\alpha)\rho_L^B}{p_L\rho_L^B + p_M\rho_M^B + p_H}. \quad (18)$$

Differentiating Eq. (18) with respect to Δ_G , we find that:

$$\begin{aligned} \frac{\partial \text{MRS}^B(\Delta_G, V)}{\partial \Delta_G} &= \frac{C p_L(1-\alpha)^2 \rho_L^B (p_L \rho_L^B + p_M \rho_M^B + p_H) - p_L^2 C(1-\alpha)^2 \rho_L^{B2}}{(p_L \rho_L^B + p_M \rho_M^B + p_H)^2} \\ &= \frac{C p_L(1-\alpha)(\rho_L^B(p_M \rho_M^B + p_H))}{(p_L \rho_L^B + p_M \rho_M^B + p_H)^2} > 0. \end{aligned}$$

This and linearity of V^G implies that the intersection is unique. Defining this point as $\bar{\Delta}_G$ ends the proof. $\square$

References

- Aghion, P., Bolton, P., 1992. An incomplete contracts approach to financial contracting. *Review of Economic Studies* 59, 473–494.
- Allen, F., Faulhaber, G.R., 1989. Signalling by underpricing in the IPO market. *Journal of Financial Economics* 23, 303–323.
- Allen, F., Gale, D., 1992. Standard securities. *Review of Financial Studies* 5, 731–755.
- Baron, D., 1982. A model of demand for investment banking advising and distribution services for new issues. *Journal of Finance* 37, 955–976.
- Benveniste, L., Spindt, P., 1989. How investment bankers determine the offer price and allocation of new issues? *Journal of Financial Economics* 28, 173–207.
- Biais, B., Faugeron-Crouzet, A.M., 2002. IPO auctions. *Journal of Financial Intermediation* 11, 9–36.
- Biais, B., Bossaert, P., Rochet, J.C., 2002. An optimal IPO mechanism. *Review of Economic Studies* 69, 117–146.
- Brennan, M., Kraus, A., 1987. Efficient financing under asymmetric information. *Journal of Finance* 42, 1225–1243.
- Chemmanur, T., 1993. The pricing of initial public offerings: a dynamic model with information production. *Journal of Finance* 48, 285–304.
- Chemmanur, T., Fulghieri, P., 1997. Why include warrants in new equity issues? A theory of unit IPOs. *Journal of Financial and Quantitative Analysis* 32, 1–24.
- Chen, A., Kensinger, J., 1988. Putable stock: a new innovation in equity financing. *Financial Management*, 27–37 (Spring).
- Dewatripoint, M., Tirole, J., 1994. A theory of debt and equity: diversity of securities and manager shareholders congruence. *Quarterly Journal of Economics* 109, 709–737.
- Gale, I., Stiglitz, J., 1989. The informational content of initial public offerings. *Journal of Finance* 44, 469–477.
- Grinblatt, M., Hwang, C.Y., 1989. Signalling and the pricing of unseasoned new issues. *Journal of Finance* 44, 393–420.
- Ibbotson, R., Sindelar, J.L., Ritter, J., 1988. Initial public offerings. *Journal of Applied Corporate Finance*, 37–45 (Summer).
- Jankinson, T., Mayer, C., 1988. The privatization process in France and in the U.K. *European Economic Review* 32, 482–490.

- Koskinen, Y., 1997. A theory of debt and equity based on monitoring incentives. Working paper, INSEAD.
- Kreps, D.M., Wilson, R., 1982. Sequential equilibrium. *Econometrica* 50, 863–894.
- Leland, H.E., Pyle, D.H., 1977. Information asymmetries, financial structure and financial intermediation. *Journal of Finance* 32, 371–387.
- Loughran, T., Ritter, J., Rydquist, K., 1994. Initial public offerings: international insights. *Pacific Basin Finance Journal* 2, 165–199.
- Milgrom, P., Roberts, J., 1986. Price and advertising signals of product quality. *Journal of Political Economy* 94, 797–824.
- Perotti, E., 1994. Credible privatizations. *American Economic Review* 85, 847–859.
- Perotti, E., Guney, S., 1993. Successful privatization plans. *Financial Management* 22 (1), 84–98.
- Prabhala, N.R., Puri, M., 1998. How does underwriter price support affect IPOs? Empirical evidence. Working paper, Yale University.
- Rock, K., 1986. Why new issues are underpriced? *Journal of Financial Economics* 15, 187–212.
- Welch, I., 1989. Seasoned offerings, imitation costs and the underpricing of initial public offerings. *Journal of Finance* 44, 421–449.
- Zender, J., 1991. Optimal financial instruments. *Journal of Finance* 46, 1645–1664.