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Determinants of hedging and its effects on investment and debt

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Abstract

Froot et al. [J. Finance 48 (1993) 1629] develop a framework in which a firm trades derivatives on the financial markets to coordinate its investing and financing decisions. This work specifies this framework by assuming that the firm faces a risk of going bankrupt. By deriving an approximated analytical solution, some properties of the optimal hedging strategy and the effects of hedging on a firm's investing and financing behaviour are developed and discussed. Numerical simulations of the nonclosed-form optimal solution are also obtained to validate the approximation.

JEL classification: G19; G31; G32
Keywords: Hedging; Investment; Debt; Bankruptcy costs; Internal funds

1. Introduction

Trading derivatives on the financial markets can be helpful to any firm wishing to modify the exposure of its cash flow to some marketable risk, such as currency, interest rate, or commodity price risks. When the same risk exposure also affects the investment opportunity and the risk of default, risk management becomes a crucial strategic tool to coordinate investment and financing decisions. By building on a framework of Froot et al. (1993), this work contributes to discussion of how risk management can affect investment and debt decisions of a firm facing costly bankruptcy.

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Different models on risk management share the common consideration that hedging against some marketable risk can affect the payoff of a risk-neutral firm as long as some market imperfections make the firm's payoff a concave function of some state contingent variable. Smith and Stulz (1985) argue that hedging is value-enhancing when the effective corporate tax code is progressive, because it reduces expected tax liabilities by lowering the variability of pre-tax corporate income around its expected value. They also argue that, when bankruptcy is costly, hedging can mitigate the firm's financial distress by reducing the probability of the firm being unable to repay the debt. Stulz (1996), Ross (1997), and Leland (1998) integrate tax motivations and risk of default motivations for hedging by showing that reducing risk exposure has two positive effects on firm value, as it simultaneously reduces expected default rates and allows higher tax benefits. Other studies (Stulz, 1984, 1990; Smith and Stulz, 1985; Rebello, 1995; De Marzo and Duffie, 1995) emphasise that risk averse managers may have an incentive to hedge at the corporate level to the extent that their remuneration is partially held in the company's shares. While most of the models on corporate hedging focus on the optimal capital structure given the investment decision, a few approaches argue that hedging may also determine the optimal investing decision of a firm (Bessembinder, 1991; Stulz, 1990; Froot et al., 1993). Froot et al. (1993), in particular, address the link between capital structure and investment through hedging by modelling a firm facing incentives to underinvest due to increasing costs of external finance. Costly external finance is a general assumption, which may in turn arise from several types of capital market imperfections.

This paper works through the Froot et al. (1993) setting where the expected return to investment is correlated to a hedgeable source of internal fund fluctuations. The paper contributes to the analysis of corporate hedging in two aspects. First, it specifies the costly external finance function of Froot et al. (1993) in terms of expected costs of bankruptcy, thereby integrating default risk motivations with underinvestment motivations to hedge. Second, by using an approximated analytical solution, it sheds new light on some properties of hedging and on its effects on investment and capital structure, which are not fully captured by the general nonclosed form of the original framework.

As Branch's (2001) survey points out, bankruptcy may be associated to several real costs for the firm, such as the cost of losing stable relationships with suppliers (Shapiro and Titman, 1998) or costs of reorganisation (Greenwald and Stiglitz, 1993). By hedging against shortfalls in cash flow availability, a firm can affect the probability of going bankrupt and substitute virtually costless internal funds to an unexpected rise in costly debt. As in this paper, the risk exposure of the cost structure is connected to the risk exposure of the investment decision, hedging mitigates the incentive to underinvest by lowering the probability of going bankrupt. This paper derives sufficient conditions to make the general solution for hedging applicable to the cost structure specified in terms of bankruptcy risk.

The implicit nonclosed-form solution derived by Froot et al. (1993) for the optimal hedging strategy shows that hedging depends on the correlation between the return to investment and a shock to internal funds, on the level of the internal funds available when the hedging decision is taken, and on the concavity structure of its payoff function, the latter capturing the effect of the hedgeable shock. It also clearly implies that the riskiness of the hedgeable shock, being associated with the riskiness of the investment and the capital structure decisions, is a crucial determinant of the optimal hedging strategy.

However, it does not show how exactly the riskiness of the internal fund fluctuations affects investment, capital structure and hedging, nor does it show how the sensitivity of the hedging strategy on the correlation parameter is affected by the internal funds volatility and by the current cash flow available at the date when the hedging decision is taken. Moreover, while the original model clearly implies that hedging ultimately affects the decisions on capital structure and investment, its general solution does not allow us to derive the explicit investment and external finance choices as a function of the shock to the internal funds before and after hedging.

The approximated analytical solution proposed in this paper allows one to overcome these above mentioned limits and to shed light on precisely how the sensitivity of the optimal hedging strategy to the correlation parameter depends on the riskiness of the variable to be hedged, as well as on the level of the internal funds available when the hedging decision is taken. A nonintuitive implication, which cannot be easily inferred from the nonclosed-form general solution, is that the firm is motivated to perform less hedging when the riskiness of internal fund fluctuations grows larger. The approximated solution also allows one to derive sufficient conditions under which the hedging strategy results in debt stabilisation rather than investment stabilisation. The approximated analytical solution is evaluated and validated in the light of numerical simulations of the nonclosed-form solution. Both approximation and simulations are used to illustrate, firstly, the effects of hedging availability on the firm's investment and debt decisions and, secondly, the effects of some changes in the elasticity parameters. It is shown numerically that the firm, under conditions of high profitability of the investment, may decide to adopt a speculative strategy with the sole purpose of stabilising its debt.

The remainder of the paper is organised as follows. In Section 2, the model of risk management is presented in the general formulation with costly external finance specified in terms of bankruptcy costs. Section 3 derives the approximated analytical solution for hedging, investment and debt. Section 4 demonstrates and comments on the properties of the optimal hedging strategy and its effects on investment and debt. In Section 5, the approximated analytical solution for optimal debt, investment and risk management is compared to the numerically simulated nonclosed-form solution. Section 6 concludes.

2. The model of risk management

This section works through the model of Froot et al. (1993) using a specification of the cost of external finance function in terms of expected bankruptcy costs.

Consider the example of a European company, 75% of whose product is sold in US dollars to customers located all over the world. From the ordinary activity of selling the product, it has receivables in US dollars in 6 months' time that are exposed to changes in the rate of exchange for the Euro against the US dollar. As this transactional risk is peripheral to the core business, the firm may want to decide fully to hedge against it. Suppose now that the core business of the company is also exposed to the foreign exchange risk. For example, the firm is investing in developing plant and machinery to make its exports more competitive abroad. The demand for its product (thus, the revenue to its investment) is now exposed to the exchange rate risk, as exports rise when the Euro

Table 1
Time structure

Time 0	Time 1	Time 2
Hedging strategy h^*	Investment and Debt I^*, D^*; ε is realised	Output is sold, debt is repaid π; μ is realised

Time structure of the hedging model in which both the return on investment and the probability of default are exposed to the same risk as the internal funds (cash flow). ε and μ represent, respectively, a hedgeable shock to the internal funds and a nonhedgeable shock to the return on investment. π is the profit of the firm after both shocks are realised. h^* is the optimal hedging decision against the realisation of ε. I^* and D^* are the optimal investment and debt decision taken when ε is known with certainty.

falls against the US dollar. Both the return to investment (economic exposure) and the cash flow (transactional exposure) are exposed to the same hedgeable source of shock (currency risk); thus, they are partially correlated. Moreover, if the firm partially finances its investment by borrowing money from European lenders, it also runs a risk of default. This third risk has exchange rate, transactional and economic exposure, to the extent that the probability of going bankrupt is affected by the amount of internal funds available to finance the investment. Finally, hedging against the transactional exposure of the cash flow plays a central part in reducing the core business risk and the risk of default.

The setup in which the firm operates is divided into three periods (see also Table 1): at time 0, when both return to investment and internal funds available are uncertain, it chooses its hedging strategy against its internal fund fluctuations. At time 1, when internal funds are realised, it chooses the amount of investment and debt. At time 2, the return on investment is realised and sold and the debt is repaid. The return on investment is correlated to the same random variable affecting the value of the internal funds.

The net present value of investment expenditure is given by

$$f(I) = \theta \frac{\omega I^{1-\beta}}{1-\beta} - I, \qquad 0 < \beta < 1 \tag{1}$$

where I is the investment, $f(I)=(\omega I^{1-\beta})/(1-\beta)$ is the expected revenue of the output, specified in terms of an exponential function (equivalent to a Cobb–Douglas normalised with respect to the labour factor). The first and second derivatives are given, respectively, by $f_{\mathrm{I}} = \omega I^{-\beta} > 0$ and $f_{\mathrm{II}} = -\omega\beta I^{-1-\beta} < 0$. θ is a variable representing a shock to the expected outcome of the investment decision. The discount rate is assumed to be equal to 0 for simplicity.

The budget constraint at time 1 is given by $I = V + D$, where V is the value of the internal funds available and D the level of debt to be raised at time 1. As debt is costly, the firm prefers to raise the debt level only when it cannot provide enough internal funds to its project of investment. Other sources of external finance are not considered.[1] Without any hedging

[1] In Froot et al. (1993), costly external finance may also include new equities (pp. 1633–1634). Even though such a more general assumption would not change the structure and the results of this work, in this paper the emphasis is on the debt as a more important source of external finance than new equity issues. This simplification is also carried on in the work of Whited (1992), on the ground of several empirical contributions showing that "share issues typically account for less than 5% of total new external finance" (p. 1426). It can also be justified by an assumption of equity rationing, as in Greenwald and Stiglitz (1993).

policy, the value of the internal funds is given by $V = V_0\varepsilon$, where V_0 is the initial value of the assets and ε is the hedgeable source of uncertainty occurring at time 1, distributed as a Normal with mean 1 and variance σ^2. The budget constraint of a nonhedging firm would be given by

$$I = D + V_0\varepsilon \tag{2}$$

By trading derivatives at time 0, the firm can modify the distribution of its cash flow across possible values of the shock ε. The choice to hedge is modelled under the following simplifying assumptions: (i) the fluctuations of the cash flow, V, are completely hedgeable; (ii) hedging does not alter the expected value of the cash flow; (iii) hedging is linear, i.e. the sensitivity of the cash to the changes of the random variable is constant. The latter assumption concretely means that the usage of derivatives described in this model is limited only to forward and futures contracts, therefore, options contracts are ruled out. This restrictive assumption is made to keep the analysis as simple as possible, as the purpose of this model is not to analyse what kinds of hedging instruments are optimal, but rather to what extent the firm should hedge against its risk exposure.[2] The internal funds after hedging are given by $V = V_0[h + (1-h)\varepsilon]$, and the budget constraint becomes

$$I = D + V_0[h + (1-h)\varepsilon], \tag{3}$$

where the value of h is determined at time 0, as a solution of the maximization problem. In the special case of full hedging, where $h = 1$, the distribution collapses to the mean, and the value of the existing assets becomes nonstochastic: $V = V_0$.

The shock to the outcome of the investment is given by

$$\theta = \alpha(\varepsilon - 1) + \mu, \qquad \text{with } E(\mu) = 1, \tag{4}$$

where α is a measure of the correlation between return to investment and the hedgeable shock to the internal funds, ε, and μ is a nonhedgeable shock to revenues, arising between time 1 and time 2 and uncorrelated to the shock to internal funds. The expected value of θ given the information available at time 1 is, thus

$$\theta_\varepsilon = E(\theta \mid \varepsilon) = \alpha(\varepsilon - 1) + 1. \tag{5}$$

The cost function is specified in terms of bankruptcy costs. Let r be the interest rate on one-period debt and let the value of the debt be positive (i.e. $I > V$). The expected cost of debt is given by the interest on borrowing plus the expected bankruptcy costs. The firm

[2] However, some survey evidence reports that nonfinancial hedging firms use a substantial percentage of linear hedging instruments, equivalent in UK to 56% of foreign exchange derivatives and 38% of all types of derivatives (Mallin et al., 2001).

goes bankrupt when the net revenue from the sale of its product is not sufficient to pay the interest on debt, i.e. when

$$\theta f(I) - I < rD, \tag{6}$$

thus, the critical level of θ under which the firm goes bankrupt (bankruptcy threshold) is given by

$$\bar{\theta} = \frac{I + rD}{f(I)}. \tag{7}$$

Given the realisation of ε, let the shock to the investment function, $\theta(\mu)$, from Eq. (4), be distributed as $G(\theta)$, with density $g(\theta)$ and the probability of bankruptcy be given by $G(\bar{\theta})$. For simplicity, let the bankruptcy costs be proportional to the debt level, i.e. $B = bD$ with $b>0$. Then the expected cost of debt at time 1 is given by

$$C(D) = rD + bDG(\bar{\theta}). \tag{8}$$

Froot et al. (1993) assume that both the first and second derivatives of the cost function with respect to the debt level are positive, i.e., respectively, $C_D>0$ and $C_{DD}>0$. Such an assumption is indispensable to obtain a solution for the optimal hedging strategy. With bankruptcy cost specification, the requirement of positive first and second derivatives is respected under nonrestrictive sufficient conditions, as stated in the next proposition.

Proposition 1. *If (i) the bankruptcy threshold,* $\bar{\theta}$, grows with the debt level and (ii) the probability of $\theta = \bar{\theta}$ *does not decrease with* $\bar{\theta}$*, then both the first and the second derivatives of the expected cost function with respect to debt are positive (*$C_D > 0$*, and* $C_{DD} > 0$*).*

Proof. The first derivative of Eq. (8) with respect to debt is given by $C_D = r + b(G(\bar{\theta}) + Dg(\bar{\theta})\bar{\theta}_D)$, where $\bar{\theta}_D = (d\bar{\theta})/(dD)$. It is straightforward to verify that it is always positive if $\bar{\theta}_D > 0$ (condition (i)). The second derivative of Eq. (8) is given by $C_{DD} = 2bg(\bar{\theta})\bar{\theta}_D + bD(dg)/(d\bar{\theta})\bar{\theta}_D^2 + bDg(\bar{\theta})\bar{\theta}_{DD}$, where $\bar{\theta}_{DD} = (d^2\bar{\theta})/(dD^2)$. C_{DD} is greater than 0 if $-(dg/d\bar{\theta}) < (g(\bar{\theta})(2b\bar{\theta}_D + bD\bar{\theta}_{DD})/(bD\bar{\theta}_D^2))$. If the derivative of the density function with respect to the bankruptcy threshold is positive, i.e. $dg/d\bar{\theta} > 0$ (condition (ii)), then a positive RHS is sufficient to verify the inequality above. The denominator on the RHS is always positive for positive values of the debt level; $g(\bar{\theta}) > 0$ by definition of probability density; $(2b\bar{\theta}_D + bD\bar{\theta}_{DD}) > 0$ when $\bar{\theta}_D > -1/2D\bar{\theta}_{DD}$. The latter inequality is always verified if $\bar{\theta}_D > 0$ (condition (i)), which implies that $\bar{\theta}_{DD} = -(2f_I)/(f(I))\bar{\theta}_D + (f_{II})/(f(I))\bar{\theta} < 0$ because, by assumption, $f_{II} < 0$, $f_I > 0$, $f(I) > 0$ and $\bar{\theta} > 0$ (Eqs. (1) and (7)). □

Condition (i) implies that bankruptcy is more likely when the debt is higher with respect to the internal funds available. Condition (ii) implies that the marginal increment in the probability of bankruptcy following a rise in the bankruptcy threshold must be nonnegative. For example, if the source of shock to revenues to be realised at time 2 (i.e. μ from Eq. (4)) is normally distributed, condition (ii) requires that $\bar{\theta}$ must be lower than the expected value of θ, therefore bankruptcy must correspond to a reasonably low realisation of θ.

The expected profit function at time 1 is given by the difference between the expected net revenues on investment and the expected cost of debt:

$$\pi = f(I) - C(D) = \theta_{\varepsilon}\frac{\omega I^{1-\beta}}{1-\beta} - I - rD - bDG(\bar{\theta}). \tag{9}$$

As at time 1 the value of the available internal funds is known with certainty, the firm maximizes its profit function with respect to I according to the following first-order condition:

$$\theta_{\varepsilon}\omega I^{-\beta} = 1 + r + b(G(\bar{\theta}) + Dg(\bar{\theta})\bar{\theta}_{\mathrm{D}}), \tag{10}$$

where $\bar{\theta}_{\mathrm{D}}=(\mathrm{d}\bar{\theta})/(\mathrm{d}D)>0$, from Proposition 1.[3] The LHS of Eq. (10) represents the expected marginal revenue to the investment. The RHS represents the expected marginal cost of debt, which includes the repayment to lenders, $1+r$, and the expected marginal cost of bankruptcy, $b(G(\bar{\theta})+Dg(\bar{\theta})\bar{\theta}_{\mathrm{D}})$, the latter being affected by the debt level. Moving back to period 0, when the internal funds are still uncertain, the firm maximizes its expected profit with respect to the hedging ratio, h:

$$\max_{\mathrm{h}} E_0[\pi(V(\varepsilon, h)]. \tag{11}$$

Froot et al. (1993) show that the solution to Eq. (11) is given by the following formula for the optimal hedging strategy:[4]

$$h^* = 1 + \frac{\alpha}{V_0}\frac{E_0\left[\frac{-f_{\mathrm{I}}C_{\mathrm{DD}}}{\theta_{\varepsilon}f_{\mathrm{II}}-C_{\mathrm{DD}}}\right]}{E_0\left[\frac{-\theta_{\varepsilon}f_{\mathrm{II}}C_{\mathrm{DD}}}{\theta_{\varepsilon}f_{\mathrm{II}}-C_{\mathrm{DD}}}\right]}. \tag{12}$$

Expression (12) is a nonclosed-form solution for hedging: the ratio between expected values on the RHS includes, firstly, the levels of the investment and the debt, both depending on ε and h^*, secondly, a direct effect of ε on h^* through the expected investment opportunities, θ_{ε}.

The informational content of the formula for the optimal hedging solution does not allow one to derive easily evident implications on the relationship between parameters involved in the hedging decision. Expression (12) shows clearly that the hedging strategy depends on the parameter α, capturing the relation between return to investment and internal fund fluctuations. The higher α is, the lower is the hedge ratio, h, which is equal to 0 when the firm does not hedge at all and lets its internal funds fluctuate according to the movement of ε in the primary market. Clearly, hedging is also dependent on the second moment of the variable to be hedged (ε), which is

[3] At time 1, V is given, hence $(\mathrm{d}D)/(\mathrm{d}I)=1$.

[4] The proof is based on a result by Rubinstein (1976). See Froot et al. (1993), note 18 p. 1639.

included in the second derivatives f_{II} and C_{DD}. However, Eq. (12) does not show exactly how the volatility of the shock to the internal funds (σ) affects hedging decisions, nor does it show how the sensitivity of the hedging strategy to the correlation parameter (α) is affected by the internal funds volatility (σ) and by the current cash flow available at the date when the hedging decision is taken (V_0). Moreover, as hedging in this model is a tool to coordinate financing and investing policy, more information would also be desirable on how hedging depends on the parameters affecting investment and cost function (and the concavity of the payoff function), as well as on how investment and debt decisions are in turn affected by the hedging strategy. The next section makes this issue clearer.

3. The approximated analytical solution

This section derives an approximated analytical solution for optimal hedging strategy, investment and debt functions.

Eqs. (3), (10) and (12) constitute an unsolved system of three equations with three unknowns: I, D, h. Solving this system of equations would lead to expressing the three unknowns as a function of the hedgeable shock, ε. While it does not seem to be possible to find a closed-form solution, the system can be solved, instead, by using second-order Taylor expansions of the investment and the cost functions, respectively, around the expected levels of the investment, I_0, and the debt, D_0.

After the second-order Taylor expansion, the expected revenue and cost functions defined above, Eqs. (1) and (8), take the following quadratic forms:

$$f(I) \simeq \frac{a}{2}I^2 + sI + k, \tag{13}$$

with $a = f_{II}(I_0) < 0$, $s = f_I(I_0) - I_0 f_{II}(I_0) > 0$ and $k = f(I_0) - I_0 f_I(I_0) + 1/2 I_0^2 f_{II}(I_0)$, where $f_I = aI + b$, $f_{II} = a$;

$$C(D) \simeq \frac{c}{2}D^2 + wD + z, \tag{14}$$

with $c = C_{DD}(D_0) > 0$, $w = C_D(D_0) - D_0 C_{DD}(D_0) > 0$ and $z = C(D_0) - D_0 C_D(D_0) + 1/2 D_0^2 C_{DD}(D_0)$, where $C_D = r + cD$ and $C_{DD} = c$.

The concavity of the profit function is captured by a and c corresponding, respectively, to f_{II} and C_{DD} of the original functions. The interest rate on debt in the cost function specified in terms of bankruptcy costs (Eq. (8)) is captured by the parameter w (the higher r, the higher w). These approximated functions simplify the calculation of Eq. (12) as the second derivatives of both approximated functions, Eqs. (13) and (14), are constant.

The approximated expected profit function of the firm becomes

$$\pi(V(\varepsilon)) = \theta_\varepsilon\left(\frac{a}{2}I^2 + sI - k\right) - I - wD - \frac{c}{2}D^2$$

and the first-order condition of the time 1 maximization problem is now given by

$$\theta_\varepsilon(aI + s) = 1 + w + cD. \tag{15}$$

Investment and debt functions of a nonhedging firm are derived by combining the first-order condition (Eq. (15)) with the nonhedger's budget constraint (Eq. (2)):

$$I^*(\varepsilon) = \frac{\theta_\varepsilon s - (1 + w) + cV_0\varepsilon}{c - \theta_\varepsilon a}, \tag{16}$$

$$D^*(\varepsilon) = \frac{\theta_\varepsilon s - (1 + w) + \theta_\varepsilon a V_0 \varepsilon}{c - \theta_\varepsilon a}, \tag{17}$$

where θ_ε is given by Eq. (5).

By using, instead, the hedger's budget constraint, Eq. (3), the investment and the debt functions of the hedging firm are given by

$$I_{\mathrm{h}}{}^*(\varepsilon) = \frac{\theta_\varepsilon s - (1 + w) + cV_0}{c - \theta_\varepsilon a} + \frac{cV_0(1 - \varepsilon)}{c - \theta_\varepsilon a}(h^* - 1), \tag{18}$$

and

$$D_{\mathrm{h}}{}^*(\varepsilon) = \frac{\theta_\varepsilon s - (1 + w) + \theta_\varepsilon a V_0}{c - \theta_\varepsilon a} + \frac{\theta_\varepsilon a V_0(1 - \varepsilon)}{c - \theta_\varepsilon a}(h^* - 1), \tag{19}$$

In both cases of hedging and no hedging, the investment and debt choices depend on the realisation of ε at time 1. However, for the hedger they also depend on the optimal hedging strategy, h^*, which can filter the pure effects of the hedgeable shock.

From a second-order Taylor expansion of the two expected terms of Eq. (12) around $\varepsilon = 1$, after substituting for the approximated functions' derivatives, f_{I}, f_{II}, and C_{DD}, and for the optimal investment (Eq. (18)) into the expression for f_{I}, one can verify.

Proposition 2. *The optimal hedging ratio, h^*, is given by*

$$h^* \simeq 1 + \frac{\alpha}{V_0} \frac{\left(1 + w - cV_0 - \frac{sc}{a}\right)\left((a - c)^2 + 3a^2\alpha^2\sigma^2\right)}{(a - c)\left((a - c)^2 + 3ac\alpha^2\sigma^2\right)}. \tag{20}$$

Expression (20) is the approximated analytical solution for hedging. It displays, unlike expression (12), the exact relationships between the parameters involved in the determination of the optimal hedging ratio. Substituting expression (20) into Eqs. (18) and (19) yields the analytical solutions for investment and debt levels.

4. The properties of the hedging strategy

This section demonstrates and discusses some properties of the optimal hedging strategy implied by the approximated analytical solution.

Proposition 3. *The optimal hedging ratio, h^*, is a decreasing function of the parameter α for any σ^2 lower than the critical value $\sigma^{*2}=(-(a-c)^2)/(3\alpha^2 ac)$.*

Proof. The sign of the hedging strategy as a function of α is given by the sign of the factor multiplying the ratio $(\alpha)/(V_0)$ of the RHS in Eq. (20). The expression $((a-c)^2+3a^2\alpha^2\sigma^2)$ on the numerator is always positive as it is a sum of squares. The expression $(a-c)$ on the denominator is always negative by the definitions of the parameters in Eqs. (13) and (14). The expression $(1+w-cV_0-(sc)/(a))$ on the numerator is positive for values of the parameters a, b, r and c consistent with the elasticity of the product to the investment calculated in I_0, $e_{\mathrm{I}}=(aI_0{}^2+sI_0)/((a)/(2)I_0{}^2+sI_0+k)$. This can be seen by taking the expectations at time 0 of the optimal investment level from Eq. (18): the expected level of investment is $I_0^{\mathrm{e}}=(b-(1+w)+cV_0)/(c-a)$; the expression $(1+w-cV_0-(sc)/(a))$ is hence positive if $I_0^{\mathrm{e}}(c-a)-s<-(sc)/(a)$, i.e. $I_0^{\mathrm{e}}<-(s)/(a)=I_0^{\mathrm{e}*}$. This upper bound condition to the expected investment is not binding for values of the parameters consistent with a positive elasticity of the product to the investment: substituting $I_0^{\mathrm{e}*}=-(s)/(a)$ into the expression for e_{I}, it turns out that $e_{\mathrm{I}}=0$. Hence, the ratio that multiplies the parameter α is negative whenever the expression $((a-c)^2+3ac\alpha^2\sigma^2)$ at the denominator is positive, i.e. whenever $\sigma^2<\sigma^{*2}=(-(a-c)^2)/(3\alpha^2 ac)$. □

This proposition confirms the general result that the higher the correlation between investment opportunities and the internal funds of the firm, the lower the amount of fluctuations hedged, but it also adds a limit: if the volatility of the internal fund fluctuations is too high, the hedging ratio is no longer a decreasing function of α. The maximum critical value of the variance, σ^{*2}, depends on the concavity of the profit function, expressed by the parameters a and c, and on the absolute value of the correlation parameter, α. A correlation parameter closer to 0 allows for a greater volatility of the internal funds to be hedged. In other words, the more idiosyncratic the investment risk of the firm is, the greater is the bearable volatility of the hedgeable shock, ε, for a linear hedging strategy to be feasible.

Another property of the hedging strategy concerns its dependence on the volatility of the variable to be hedged.

Proposition 4. *Provided that $\sigma^2<\sigma^{*2}$, the higher the variance of the variable to be hedged, σ^2, is, the higher is the sensitivity of the hedging ratio, h^*, to the parameter α. Therefore, the optimal hedging ratio is a decreasing function of the variance if $\alpha>0$ and an increasing function if $\alpha<0$.*

Proof. Everything else being constant, a higher variance, σ^2, increases the value of the numerator of expression (20), as $3a^2\alpha^2\sigma^2>0$, while decreasing the value of the denominator, as $3ac\alpha^2\sigma^2<0$. Hence, for any value of α, the higher the variance, σ^2, the farther the value of h^* from 1 (full hedging). Thus, h^* is more sensitive to the correlation parameter α. □

This proposition can be illustrated by comparing the dotted and the continuous curves in Fig. 1, the dotted line showing the hedging function corresponding to a greater level of the variance σ^2. This property of the optimal hedging strategy is not an intuitive result: when the relation between investment opportunity and internal funds is positive, one might expect more hedging, and not less hedging, as a higher internal fund riskiness implies a higher investment risk, thus a greater incentive to hedge to stabilise profits. The result of Proposition 4, however, is not surprising if one thinks that the increased risk of internal fund fluctuations, being related to the volatility of the return on investment, does not have a big effect on the risk of raising debt, as any wider extra investment needed will be financed by a proportionally wider extra amount of internal finance. The result confirms that the firm maximizing its expected profit from a concave profit function is not concerned with hedging against the market value fluctuations by themselves (transactional risk), but it is rather concerned with hedging against the risk of borrowing in connection with its investment opportunity.

The approximated analytical solution (Eq. (20)) for hedging specifies precisely how hedging is dependent on the firm's current cash flow. The following proposition applies.

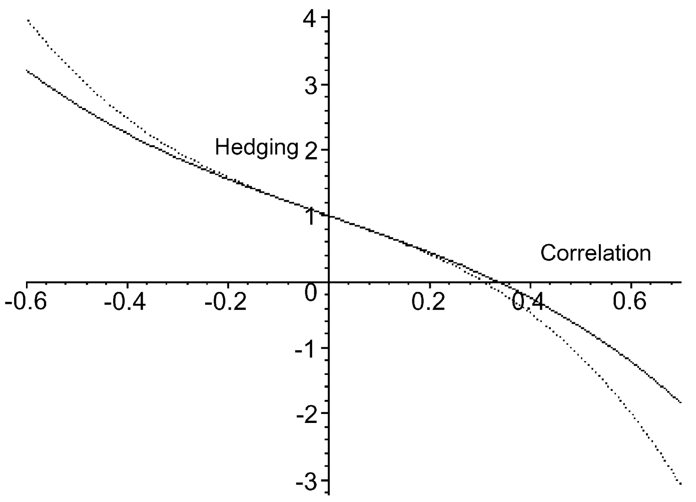

Fig. 1. Hedging strategy and correlation parameter. Optimal hedging strategy (h*) as function of the parameter capturing the correlation between shock to the internal funds and return on investment (α). The figure is derived from the approximated analytical solution of the model of hedging with bankruptcy costs, calibrated with plausible values of the parameters. Hedging equal to 0 indicates that the firm does not perform any hedging strategy; hedging equal to 1 indicates that the firm fully hedges against its internal fund fluctuations; hedging between 0 and 1 indicates that the firm adopts a strategy of partial reduction of internal fund risk exposure; negative hedging corresponds to a speculative strategy (increasing internal fund risk exposure); hedging greater than 1 corresponds to an over-hedging strategy (reversing the sign of internal fund fluctuations). The continuous and dotted lines indicate different sensitivity of the hedging strategy to the correlation parameter. The more sensitive curve (dotted line) corresponds to a higher volatility of the variable to be hedged and/or to a lower amount of internal funds available when the hedging decision is taken.

Proposition 5. *The higher the current cash flow of the firm, V_0, is, the lower is the sensitivity of the hedging ratio, h^*, to the parameter α. Therefore, the optimal hedging ratio is an increasing function of the current cash flow if $\alpha > 0$ and a decreasing function if $\alpha < 0$.*

Proof. From Eq. (20), the factors containing V_0 can be insulated: $((1 + w - cV_0 - (sc)/(a))/(V_0) = ((1 + w - (sc)/(a))/(V_0) - c$. This expression is positive (see Proof of Proposition 3) and is lower as V_0 is higher. Hence, the value of h^* is closer to 1 (full hedging) as V_0 is higher, for any value of α. Thus, h^* is less sensitive to the relation parameter α. □

Fig. 1 illustrates the hedging function corresponding to two different levels of internal funds available: the consequence of a higher current cash flow (continuous line) is a hedging ratio closer to the full hedging level for any value of the correlation parameter α. When the gap between investment and internal funds becomes lower, so does the marginal cost of debt. Hence, given the value of the correlation parameter, α, the incentive to substitute extra debt with extra internal funds is lower and the hedging strategy is closer to full hedging.

Finally, the approximated solution allows one to clarify what effect the hedging strategy may cause on investment and debt stabilisation.

Proposition 6. *If the marginal return on investment is more sensitive to the level of the investment than the marginal cost of debt to the level of the debt ($-a > c$), then the effect of the optimal hedging strategy is to stabilise the debt more than the investment.*

Proof. The impact of the hedging possibility on the investment and debt choice can be measured, respectively, by the difference between Eqs. (16) and (18), $I(\varepsilon) - I_h(\varepsilon) = hcV_0(\varepsilon - 1)$, and by the difference between Eqs. (17) and (19), $D(\varepsilon) - D_h(\varepsilon) = h\theta_\varepsilon aV_0(\varepsilon - 1)$. The latter impact is greater than the former if $|\theta_\varepsilon a| > |c|$. By construction, $c > 0$ and $a < 0$; hence, the previous condition becomes $|\theta_\varepsilon| > -(c)/(a)$. The latter inequality can be rewritten as $|\alpha(\varepsilon - 1)| > -(c)/(a) - 1$, the LHS being always greater or equal to 0, and the RHS being lower than 0 for $c < -a$. □

Proposition 6 states that under the sufficient condition that the second derivative of the investment function ($-a = f_{\mathrm{II}}$) is higher in absolute value than the second derivative of the debt function ($c = C_{\mathrm{DD}}$), the debt after hedging is stabilised more than the investment after hedging. In other words, provided that the contribution to the concavity of the profit function comes more from the concavity of the investment function than from the convexity of the cost function, hedging allows the firm to slightly sacrifice the probability of higher investment in order to drastically decrease the probability of a higher debt. Investment and debt decisions as function of the model's parameters are further analysed numerically in the next section.

5. Investment and debt decisions

In this section, the approximated analytical solution for optimal debt, investment and risk management is validated by comparing it to numerically simulated nonclosed-form solutions. Firstly, a graphical comparison is made between hedging and nonhedging possibilities, showing the effects of hedging on the firm's financing and investing decisions. Secondly, some effects of changes in investment and cost function parameters on the optimal values are derived. Both exercises allow us to illustrate with examples the results developed in previous sections on optimal hedging, investment and debt.

The parameterisation of the model, which is used for both the approximated and the simulated solutions, is specified as follows: (i) the conditional distribution of the unhedgeable shock to the investment function, μ, is Uniform between 0 and 1; thus, the conditional probability of bankruptcy, given the realisation of the hedgeable shock to the internal funds, ε, is $g(\bar{\theta}|\varepsilon)=1/2(\bar{\theta}-\alpha(\varepsilon-1))$, and the cost function (Eq. (8)) becomes

$$C(D) = rD + bD\frac{1}{2}(\bar{\theta} - \alpha(\varepsilon - 1)); \tag{21}$$

(ii) the initial expected value of the physical product elasticity to the investment (β of Eq. (1)) is equal to 0.25, a value typically used in the Cobb–Douglas production function for the elasticity of the capital; (ii) the initial ratio between expected debt (D_0) and expected investment (I_0) is equal to 0.50 (i.e. the 50% of the new investment is financed by debt); (iii) the standard deviation of the shock to the internal funds is $\sigma=0.6$; (iv) the correlation parameter, α, is equal to 0.2 (i.e. positive correlation is assumed between investment opportunities and cash flow fluctuations); (iv) the initial interest rate on borrowing, r, is equal to 6%; and (v) the initial bankruptcy cost per unit of debt, b, is equal to 0.099, which corresponds to nearly 5% of the value of the investment when half of it is financed by debt, a value close enough to the estimated bankruptcy costs from empirical studies.[5] The scale parameter of the investment function, ω, is set according to all the other parameters.

The parameters of the approximated analytical solution (a, s, k, c, w, and z of Eqs. (13) and (14)) are obtained by carrying on the second-order Taylor expansion of the initial functions, Eqs. (1) and (21), around the expected equilibrium (I_0, V_0, D_0, $\varepsilon=1$). The numerical simulations of the nonclosed-form solution of the model are obtained by using a randomly generated sample of 1000 realisations of ε i.i.d. $N(1,\sigma^2)$ to mimic the expectations on the numerator and on the denominator of the nonclosed-form solution for hedging (Eq. (12)). The recursive computation of the optimal hedging ratio, h^*, is interrupted when the new value for h^* is within at least 10^{-5} of the previous one.

Figs. 2 and 3 represent, respectively, the investment and the debt choices as functions of the shock to the internal funds, ε. The functions are those taken from the approximated solution (Eqs. (16) and (18) for Fig. 2, and Eqs. (17) and (19) for Fig. 3). The dotted and the continuous lines illustrate the functions, respectively, before and after adopting the optimal hedging strategy h^*. Figs. 4 and 5 illustrate the same investment and debt choices of the firms derived from the numerical simulation (Eqs. (10), (3) and (12) for the hedging firm, Eqs. (10) and (2) for the nonhedging firm). The range of possible values of ε is wider

[5] Weiss (1990) estimates the bankruptcy costs as totalling 4% of the firm's debt plus the market value of equities. See also Branch (2001) for a survey.

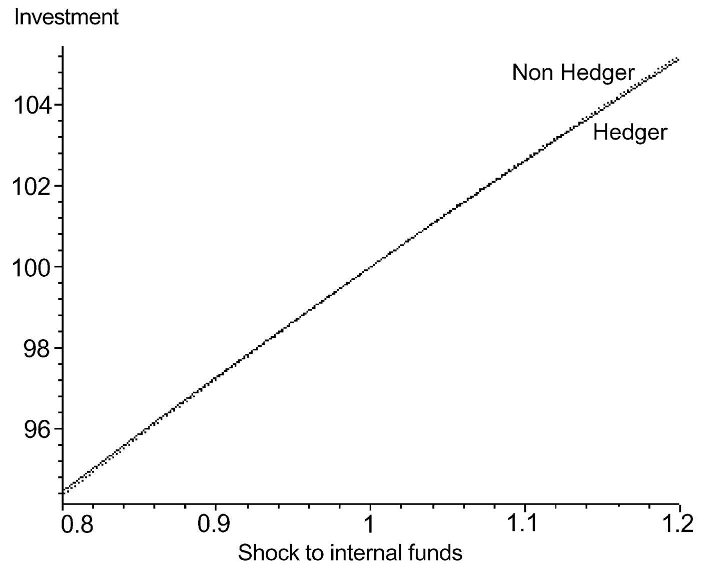

Fig. 2. Investment before and after hedging (approximated solution). Optimal investment decision as function of the shock to the internal funds (ε) around its expected value (1). The figure is derived from the approximated analytical solution of the model of hedging with bankruptcy costs, calibrated with plausible values of the parameters. The dotted and the continuous lines illustrate the optimal investment decision, respectively, before and after adopting the optimal hedging strategy, h^*.

in Figs. 4 and 5 than in Figs. 2 and 3 as the former figures include all the randomly generated values of ε, whereas the latter ones represent the investment and debt choices around the expected value of ε only. To compare the simulated solutions with the locally approximated ones, one should focus on the values of ε around the mean (i.e. around 1).

Figs. 2–5 indicate that the effect of the risk management is to smooth the fluctuations of investment and debt. Given the profit function concavity, the negative events are weighted by the firm more than the positive ones. Under the parameterisation carried out in this section, the firm slightly sacrifices the probability of higher investment in order to drastically decrease the probability of a higher debt: the debt function after hedging is much flatter than the investment function after hedging, the latter's change being hardly perceptible. The illustrations confirm the result of Proposition 6: debt stabilisation is higher than investment stabilisation as in this setting the contribution to the concavity of the profit function comes more from the concavity of the investment than from the convexity of the cost functions ($-a>c$).

Tables 2 and 3 are obtained to work out how the optimal hedging, investment and debt decisions are sensitive to different values of the concavity parameters. Tables 2a and 3a report the results of the numerical simulation of the nonclosed-form version of the model. Tables 2b and 3b replicate the numerical experiment using the approximated analytical solution around the mean of ε. The tables illustrate the effects of changes in elasticity

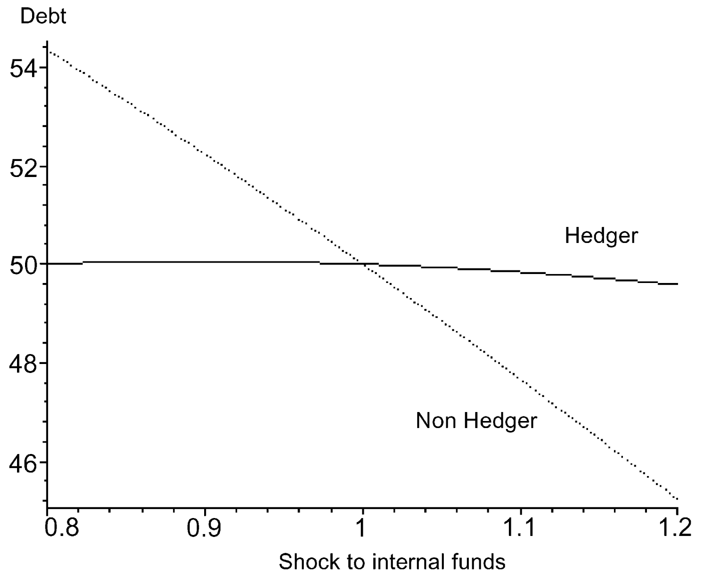

Fig. 3. Debt before and after hedging (approximated solution). Optimal debt decision as function of the shock to the internal funds (ε) around its expected value (1). The figure is derived from the approximated analytical solution of the model of hedging with bankruptcy costs, calibrated with plausible values of the parameters. The dotted and the continuous lines illustrate the optimal debt decision, respectively, before and after adopting the optimal hedging strategy, h*.

parameters (β for the investment function and r or b for the cost function, respectively) on the expected levels and the volatilities of investment and debt, as well as on the optimal hedging strategy.

Investment and debt volatilities are measured in the simulated solution by their standard deviations obtained from the randomly generated sample (σ_k with $k = I, D, I_h, D_h$), whereas in the approximated analytical solution, they are measured by an approximated angular coefficient, λ, computed as follows:

$$\lambda_k = \frac{k(\varepsilon_{\max}) - k(\varepsilon_{\min})}{\varepsilon_{\max} - \varepsilon_{\min}} = \frac{\Delta k}{\Delta \varepsilon},$$

with $k = I, D, I_h, D_h$, $\varepsilon_{\max} = 1.2$ and $\varepsilon_{\min} = 0.8$ (see extreme values of Figs. 2 and 3). Therefore, the volatilities of the analytical solutions are not numerically comparable to the standard deviations of the simulated solutions. However, they can be compared in qualitative terms: a rising or falling volatility is recognisable with both types of volatility measure.[6]

[6] Another difference between analytical approximation and numerical simulation is that, in the former, the equilibrium levels (I_k, D_k) of a hedging firm are, by construction, the same as the nonhedging firm, whereas, in the latter, they may differ. The reason is that, in the approximated analytical solution, the equilibrium is given as the starting point of the computation, whereas in the simulated nonclosed-form solution, the equilibrium is obtained as a solution, the starting point being the randomly generated sample of ε.

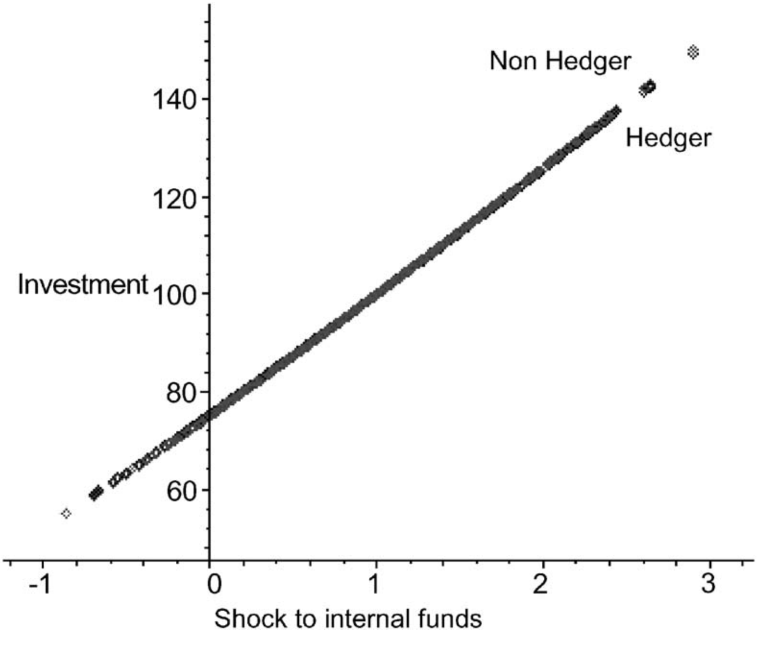

Fig. 4. Investment before and after hedging (numerical simulation). Optimal investment decision as function of a randomly generated shock to the internal funds (ε), normally distributed with mean equal to 1. The figure is obtained from numerical simulations of the nonclosed-form solution of the model of hedging with bankruptcy costs, calibrated with plausible values of the parameters. The bright and dark lines illustrate the optimal investment decision, respectively, before and after adopting the optimal hedging strategy, h^*.

The effects of an exogenous rise in the investment elasticity, e_I (from 0.25 to 0.35), on a hedging firm are the following (see Table 2): (i) increasing expected levels of investment and debt; (ii) increasing leverage ratio; (ii) increasing variability of the investment decision; (iii) decreasing value of the hedging ratio, h^*; (iv) increasing variability of the debt decision.

A rise in the investment function elasticity leads to a greater expected return on investment, therefore to an increase in the investment level. As the expected cash flow is not changed, the level of the debt rises up to the level where the marginal return on investment equals the marginal cost of debt. Given the value of the correlation parameter, α, the hedging strategy becomes less tight (h^* closer to 0), in order to let higher cash flow fluctuations supply more internal finance to higher investment fluctuations.

In the third example of Table 2, the investment function elasticity is so high ($e_I = 0.35$) that the hedging strategy changes sign (from positive to negative) and becomes speculative. Higher investment productivity can motivate the firm to raise dramatically the optimal investment and, consequently, the level of debt by the same amount, the expected internal cash flow being unchanged. Without the speculative strategy, the debt becomes an

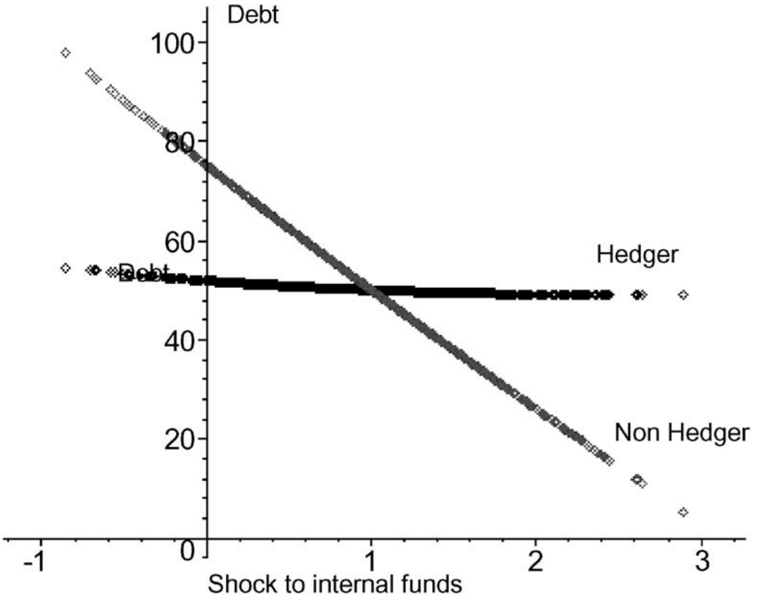

Fig. 5. Debt before and after hedging (numerical simulation). Optimal debt decision as function of a randomly generated shock to the internal funds (ε), normally distributed with mean equal to 1. The figure is obtained from numerical simulations of the nonclosed-form solution of the model of hedging with bankruptcy costs, calibrated with plausible values of the parameters. The bright and dark lines illustrate the optimal debt decision, respectively, before and after adopting the optimal hedging strategy, h^*.

increasing function of the shock ε ($\lambda_D > 0$). A speculative hedging strategy allows the firm to amplify the internal fund fluctuations in order to substitute virtually costless cash flow to costly debt and reverse the slope of the debt function ($\lambda_{Dh} < 0$). Hence, speculation occurs as the firm prefers to increase the cash flow volatility in order to reduce the risk of borrowing.

The exogenous rise in the cost function parameters (r rising up to 0.20 and b rising up to 0.8) generates the following effects (see Table 3): (i) decreasing expected levels of investment and debt; (ii) decreasing leverage ratio; (iii) decreasing variability of the investment decision; (iv) decreasing variability of after-hedging debt;[7] and (v) increasing hedging ratio, h^*.

Rising cost of debt leads to lower profits per unit of debt, therefore it induces the firm to reduce the scale of production, i.e. both investment and debt levels. As the expected cash flow is not changed, a greater proportion of internal cash flow is expected to finance the lower investment, thus the leverage ratio falls. As less external finance is needed for a lower investment, the hedging strategy becomes slightly tighter: lower

[7] Debt before hedging slightly increases. This is presumably the result of investment variability falling too slowly compared to the falling expected level of debt.

Table 2
Investment elasticity effects

(a) Simulated nonclosed-form solution				(b) Approximated analytical solution			
e_I	0.25	0.30	0.35	e_I	0.25	0.30	0.35
b	0.099	0.267	0.267	b	0.099	0.099	0.099
r	0.06	0.06	0.06	r	0.06	0.06	0.06
EI_h	99.47	137.48	199.81	EI	100	138.15	200.63
ED_h	50.23	88.60	151.55	ED	50	88.15	150.63
ED_h/EI_h	0.50	0.64	0.76	ED/EI	0.50	0.64	0.75
$h*$	0.469	0.215	− 0.226	$h*$	0.444	0.177	− 0.287
σ_{Ih}	15.20	22.47	35.12	λ_{Ih}	26.73	39.57	61.90
σ_{Dh}	0.88	1.37	2.26	λ_{Dh}	− 1.06	− 1.57	− 2.43
EI	99.48	137.49	199.80				
ED	50.91	88.91	151.23				
ED/EI	0.51	0.65	0.76				
σ_I	15.36	22.56	35.01	λ_I	27.20	39.78	61.51
σ_D	14.84	6.67	4.93	λ_D	− 22.80	− 10.22	11.51
Recursions	3	3	3				

Numerical simulation (a) and analytical approximation (b) of the solution for optimal hedging, investment and debt when the elasticity of the output to the investment rises. (a and b) report numerical results of the investment elasticity, e_I, changing from 0.25 to 0.35. The cost function parameters, b and r, are constant. $h*$ is the computed optimal hedging ratio, EI the expected investment and ED the expected debt. In (a), the expected investment and debt after hedging, which are different from those before hedging, are given, respectively, by EI_h and ED_h. (a and b) report the expected leverage ratios expressing the expected quote of investment which is financed by debt. The (unreported) standard deviation of the shock to the internal funds is equal to 0.6. The investment volatility before and after hedging are given by the standard deviations of the simulated values in (a) (σ_I and σ_{Ih}, respectively), and by approximated angular coefficients of the investment curves in (b) (λ_I and λ_{Ih}, respectively). The debt volatility before and after hedging are computed using the same methodology of the investment volatility and given by σ_D and σ_{Dh} in (a), and by λ_D and λ_{Dh} in (b). The last row in (a) reports the number of recursions before convergence.

fluctuations of the cash flow are thus sufficient to hedge against the risk of raising extra debt.

By reading Tables 2 and 3 through columns, it is possible to observe the pure effect of hedging already illustrated in Figs. 2–5 and generalised to different values of the profit function's concavity: a positive hedging ratio (h*>0) always reduces both investment and debt volatilities. However, it reduces the latter much more in proportion. In the special case of speculative behaviour ($e_I = 0.35$), the volatility of the internal cash flow increases after hedging ($h* < 0$) as well as the investment volatility.

Comparing Tables 2 and 3, it can be observed that the impact of the investment function elasticity on the firm's optimal decisions is much higher than the impact of the cost function elasticity. This result might illustrate the behaviour of a company facing both higher costs of borrowing and high profitability of its investment (for example, a high-tech company): the profitability of the investment is so high that the incentive to raise the levels of the investment and the debt prevails over the incentive to lower them and the hedging strategy may become speculative with the sole purpose of stabilising the debt around its expected level.

Table 3
Cost of debt effects

(a) Simulated non closed-form solution

e_I	0.25	0.25	0.25	0.25	0.25	0.25
b	0.099	0.099	0.099	0.099	0.4	0.8
r	0.06	0.1	0.15	0.20	0.06	0.06
EI_h	99.47	94.80	89.48	84.66	98.43	97.45
ED_h	50.23	45.52	40.18	35.30	49.18	48.19
EI_h/ED_h	0.50	0.48	0.45	0.42	0.50	0.49
$h*$	0.469	0.494	0.522	0.548	0.474	0.479
σ_{Ih}	15.20	14.51	13.74	13.03	14.62	14.09
σ_{Dh}	0.88	0.81	0.74	0.67	1.27	1.64
EI	99.48	94.81	89.49	84.67	98.44	97.46
ED	50.91	46.24	40.92	36.10	49.87	48.89
EI/ED	0.51	0.49	0.46	0.43	0.51	0.50
σ_I	15.36	14.69	13.92	13.23	14.87	14.42
σ_D	14.84	15.51	16.28	16.98	15.33	15.78
Recursions	3	3	3	2	5	7

(b) Approximated analytical solution

e_I	0.25	0.25	0.25	0.25	0.25	0.25
b	0.099	0.099	0.099	0.099	0.15	0.20
r	0.06	0.1	0.15	0.20	0.06	0.06
EI	100	95.30	89.95	85.10	98.99	98.04
ED	50	45.30	39.95	35.10	48.99	48.04
EI/ED	0.5	0.47	0.44	0.41	0.49	0.49
$h*$	0.444	0.470	0.510	0.527	0.450	0.455
λ_{Ih}	26.73	25.47	24.04	22.75	26.47	26.23
λ_{Dh}	– 1.06	– 1.01	– 0.96	– 0.91	– 1.03	– 1.00
λ_I	27.20	25.95	24.54	23.25	27.17	27.14
λ_D	– 22.80	– 24.05	– 25.46	– 26.75	– 22.83	– 22.85

Numerical simulation (a) and analytical approximation (b) of the solution for optimal hedging, investment and debt when the expected cost of debt rises. (a and b) report numerical results of the effect of either a rising interest rate (r going from 6% to 20%), the bankruptcy cost being constant, or a rising bankruptcy cost (b going from the 10% to the 80% of the debt level), the interest rate being constant. The investment elasticity, e_I, is constant. $h*$ is the computed optimal hedging ratio, EI the expected investment and ED the expected debt. In (a), the expected investment and debt after hedging, which are different from those before hedging, are given, respectively, by EI_h and ED_h. (a and b) report the expected leverage ratios expressing the quote of expected investment which is financed by expected debt. The (unreported) standard deviation of the shock to the internal funds is equal to 0.6. The investment volatility before and after hedging are given by the standard deviations of the simulated values in (a) (σ_I and σ_{Ih}, respectively), and by approximated angular coefficients of the investment curves in (b) (λ_I and λ_{Ih}, respectively). The debt volatility before and after hedging is computed using the same methodology of the investment volatility and given by σ_D and σ_{Dh} in (a), and by λ_D and λ_{Dh} in (b). The last row in (a) reports the number of recursions before convergence.

Overall, the numerically simulated solution is a successful validation of the approximated analytical solution. Firstly, Figs. 2 and 3 from the analytical approximation replicate the investment and debt functions of Figs. 4 and 5 around the expected equilibrium ($\varepsilon = 1$) well. Secondly, Tables 2b and 3b replicate quite accurately the numerical results reported in Tables 2a and 3a, as the hedging ratios and the expected values of both investment and debt are sensibly close to those computed by the numerical simulations and also the alternative volatility indicators are consistent with each other. The approximated hedging ratios tend to slightly undervalue the simulated ones, the error slightly rising when the investment elasticity rises, presumably because of the greater levels and volatilities involved in the computation. Even though volatilities are not numerically comparable, it can be observed that the effects of hedging (columns) and the effects of changing parameters (lines) in the simulated solutions are the same as in the approximated ones.[8] The properties and the implications of the approximated analytical solution, therefore, can be used with reasonable confidence for values of the variables around their expected levels.

6. Conclusion

The aim of this work was to derive the effects of hedging availability on the investment and financing decisions of a firm facing costly bankruptcy. This paper has built on a framework by Froot et al. (1993), in which the investment opportunities are partially correlated to the fluctuations of the internal finance sources, the external finance is increasingly costly, and corporate hedging is a tool to coordinate investment and financing decisions.

The convexity of the external finance cost function, rather than assumed as in Froot et al. (1993), has been derived as a result of expected undiversifiable bankruptcy costs. It has been demonstrated that the firm has a convex cost structure in the presence of bankruptcy costs when a higher leverage implies that (i) bankruptcy is more likely and (ii) the marginal increment in the probability of bankruptcy is nonnegative. Under these sufficient conditions, the firm is motivated to hedge against shortfalls in cash flow availability and unexpected rises in funding costs, thereby reducing default probability and mitigating the incentive to underinvest.

The general nonclosed-form solution for hedging, derived by Froot et al. (1993), does not appear to be fully informative on precisely how the model's parameters and variables are connected to the hedging strategy of the firm. In order to take more information out of the general solution, an approximated analytical solution has been derived (Section 3). The analytical solution contributes to the analysis on how the sensitivity of the optimal hedging strategy to the correlation parameter depends on the riskiness of the variable to be hedged,

[8] The approximated solution actually adds information about the direction of the relation between shock to the internal funds, ε, and either investment or debt decisions. It can be noticed that, when the optimal hedging strategy becomes negative, i.e. speculative ($e_I = 0.35$), the hedging strategy reverses the slope of the debt function from positive to negative.

as well as on the level of the internal funds available when the hedging decision is taken. From the analytical solution, hedging is confirmed to be a decreasing function of the correlation parameter, as in Froot et al. (1993); however, it appears to be not feasible when internal fund fluctuations are highly risky. Moreover, if the internal fund fluctuations are positively correlated to the investment return, the optimal hedging strategy appears to be a decreasing function of the internal fund volatility and an increasing function of the internal funds available at the time when the hedging decision is taken. Finally, the hedging strategy results in debt stabilisation rather than investment stabilisation when the concavity of the profit function originates more from the investment function concavity than from the cost function convexity.

The approximated analytical solution has been validated by comparing it to numerical simulations of the nonclosed-form optimal hedging strategy. Two exercises have been carried out by using both approximated and simulated solutions. In the first exercise, the implications of the hedging strategy on both investment and debt decisions have been derived and illustrated graphically. It appears that, for reasonable values of the parameters, the hedging decision stabilises the debt decisions much more than the investment ones. In other words, the firm hedges against the internal fund fluctuations in order to reduce the debt fluctuations. In the second exercise, the effects of changing elasticities have been obtained, for hedging and nonhedging firms, by changing the values of either production or cost function parameters. The effect of a rise in the investment profitability on levels and volatilities appears to be much stronger than the opposite effect of a rise in the cost of debt. This result may illustrate how a high-tech firm, which is also high risk, can be motivated to raise debt despite its higher cost. It may also show that, when the investment is highly profitable, the firm can even adopt a speculative strategy, which amplifies its internal fund fluctuations, in order to reduce the probability of raising extra debt.

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Appendix A. Nonclosed-form simulation procedure

Nonclosed-form solutions for the hedging firm are simulated as follows:

(i) a random sample of 1000 realisations of ε_i i.i.d. $N(1,\sigma^2)$ is generated, with $i=1\ldots1000$;
(ii) a starting value is assigned to h_j, taken arbitrarily from the approximated analytical solution, say, $h_j=h_0$;

(iii) given h_j, for each realisation ε_i, a realisation of the investment, I_i, is derived by solving numerically the following implicit function:

$$I_i = V_0[h_j + (1-h_j)\varepsilon_i] + \frac{I}{2(1-\beta)r}(2r-b) + \frac{I}{2(1-\beta)r}\left[(2r-b)^2 - \frac{8r\omega}{b}I^{-\beta}\left((\alpha(\varepsilon-1)+1)\omega I^{-\beta} - (1+r) - \frac{b}{r}\frac{(1-\beta)}{\omega I^{1-\beta}}I - \frac{b}{2}\alpha(\varepsilon-1)\right)\right]^{1/2}$$

which is simply derived by substituting the first-order condition (Eq. (10)) into the hedger's budget constraint (Eq. (3));

(iv) given h_j, for each realisation ε_i, a numerical value for D_i is also found by substituting the computed I_i into the first-order condition (Eq. (10));

(v) given h_j, for each realisation ε_i, both numerator, $n_i = (-f_{\mathrm{I}}C_{\mathrm{DD}})/(\theta_\varepsilon f_{\mathrm{II}} - C_{\mathrm{DD}})$, and denominator, $d_i = (-\theta_\varepsilon f_{\mathrm{II}}C_{\mathrm{DD}})/((\theta_\varepsilon f_{\mathrm{II}} - C_{\mathrm{DD}}))$, of the hedging function, Eq. (12), are computed by substituting the numerical values of ε_i, I_i, D_i and h_j into θ_ε, f_{I}, f_{II}, C_{DD} (see expressions (1), (8) and (4));

(vi) given 1000 realisations of both n_i and d_i, their mean values are calculated: n_j and d_j;

(vii)Substituting n_j and d_j into the formula for hedging, a new value for h is computed:

$$h_{j+1} = 1 + \frac{\alpha}{V_0}\frac{n_j}{d_j};$$

(vii) if $h_{j+1} - h_j \geq 10^{-5}$, the computation restarts from (iii) by using h_{j+1}; if otherwise, the calculation ends.

For the nonhedging solution, the same generated sample sub (i) is used to compute numerical values for I_i and D_i by combining the f.o.c. (Eq. (10)) and the nonhedger budget constraint (Eq. (2)).

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