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Fully revealing equilibria with suboptimal investment

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Abstract

Myers and Majluf [Myers, S.C., Majluf, N.S., 1984. Corporate financing and investment decisions when firms have information that investors do not have. Journal of Financial Economics 13, 187–221.] showed that mispriced securities can lead managers with private information to invest inefficiently. It seems plausible that this problem would disappear in a fully revealing equilibrium, since information asymmetries are resolved and securities are priced correctly. In fact, Constantinides and Grundy [Constantinides, G.M., Grundy, B.D., 1989. Optimal investment with stock repurchase and financing as signals. Review of Financial Studies 2, 445–465.] claim that, in their model, any fully revealing equilibrium has efficient investment. This claim is incorrect, as infinitely many inefficient equilibria exist for the very example they work out. The inefficient outcomes survive the standard signaling-game equilibrium refinements. There are also examples that have fully revealing equilibria with inefficient investment but none with efficient investment. © 2000 Elsevier Science B.V. All rights reserved.

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Myers and Majluf (1984) recognized that private information in the hands of a firm's managers can create havoc with the firm's investment policy when capital must be raised externally. If managers act in the interest of current shareholders, perhaps because they themselves own shares, they may make socially inefficient investment decisions. Issuing new securities is unpalatable when the securities are

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undervalued in the market because the new investors get a bargain at the expense of existing shareholders. If the mispricing is severe enough, managers will pass up good investment projects rather than raise new capital. This is socially costly because wealth-increasing projects are foregone.

However, suppose that asymmetric information is the only friction in the capital markets and firms' equilibrium financing choices allow investors to *infer* the inside information. Securities are priced correctly in such an equilibrium, so it would seem that incentives to invest inefficiently would disappear. With frictionless capital markets, the only potential inefficiencies are from distorted investment policy, so signaling holds the promise of equilibria in which no value is dissipated, as in Ross (1977), Bhattacharya (1980), Heinkel (1982) and others. In fact, Constantinides and Grundy (1989) (CONSTANTINIDES AND GRUNDY, 1989) claim that any fully revealing equilibrium in their model is nondissipative — firms invest efficiently and the Myers and Majluf (1984) problem is avoided. Unfortunately, this claim is incorrect, and signaling is not enough to inspire confidence that investment policy will be efficient. For a large class of problems in the Constantinides and Grundy, 1989 model, including the example they analyze, fully revealing equilibria with efficient investment are always accompanied by infinitely many fully revealing equilibria with inefficient investment.

In the model, investors have conjectures about how privately informed managers will behave, with respect to both real investment and financing, and these conjectures determine the market prices of securities. The prices of securities in turn affect management choices about what securities to issue and how much to invest in production. In a fully revealing equilibrium with inefficient investment, management choices confirm investor conjectures, and all securities are priced correctly. Capital markets are as ‘‘efficient’’ as one could possibly hope, in the sense that security prices correctly reflect equilibrium investment policies. However, capital markets are quite inefficient in such an equilibrium in that the perverse conjectures of investors, as reflected in market prices of securities, induce managers to invest suboptimally.

Section 1 presents the model, and Section 2 analyzes the specific example examined by Constantinides and Grundy, 1989. Concluding comments are in Section 3.

1. The model

The model is taken from Constantinides and Grundy, 1989. We begin with an all-equity firm with one share outstanding. The firm's type is denoted $\theta \in \Theta$, and θ is the manager's private information. Since the firm's production function is parameterized by θ, only the manager knows the productivity of the firm's assets. The manager owns N shares, $0 < N < 1$, and outside shareholders hold the remaining $1 - N$ shares. All participants are risk neutral, and the riskless interest rate is zero. In the first of three periods, the manager announces the type of security the firm will issue to raise funds for investment, and announces the amount that the firm will invest in production. The characteristics of the security are summarized by a vector $\boldsymbol{s}$, e.g., face value for a straight bond issue, face value and conversion ratio for a convertible, etc., and the amount to be invested is denoted I. The manager's choices are limited to those $(\boldsymbol{s}, I)$ in some set A.

In the second period, the new security is sold to competitive bidders. If the amount raised exceeds I, the excess is used to repurchase shares from outside shareholders at the market price. If the excess cash is greater than the market value of all $1-N$ outside shares, the remainder is invested in a riskless security. If the amount raised is less than I, the cash is returned to the bidders, and no production occurs.

In the third and final period, the firm's projects pay off and the cash is distributed to securityholders. The firm's cash flow is x, with distribution function $F(x; I, \theta)$. In the examples presented here, the parameter θ orders F in the sense of first-order stochastic dominance, so one can take θ as a measure of firm quality. The payoff of the newly issued claim is $h(x; \boldsymbol{s})$, and limited liability requires

$$0 \le h(x; \boldsymbol{s}) \le x \tag{1}$$

for all x and $\boldsymbol{s}$. The residual, $x - h(x; \boldsymbol{s})$, goes to the remaining shareholders.

Since the manager knows θ, he or she knows that the intrinsic value of the terminal cash flow (assuming production occurs) is $V(I, \theta) = \int_X x\mathrm{d}F(x; I, \theta)$ and that the intrinsic value of the new security is $H(\boldsymbol{s}, I, \theta) = \int_X h(x; \boldsymbol{s})\mathrm{d}F(x; I, \theta)$. Investors do not know θ, but try to infer its value from the announced financing/investment plan $(\boldsymbol{s}, I)$. Let $G(\theta; \boldsymbol{s}, I)$ denote their posterior distribution, so the market value of the terminal cash flow is

$$\hat{V}(\boldsymbol{s}, I) = \int_\Theta V(I, \theta)\mathrm{d}G(\theta; \boldsymbol{s}, I) = \int_\Theta \int_X x\mathrm{d}F(x; I, \theta)\mathrm{d}G(\theta; \boldsymbol{s}, I),$$

and the market value of the existing shares is $\hat{V} - I$. The new security has market value

$$\begin{aligned}\hat{H}(\boldsymbol{s}, I) &= \int_\Theta H(\boldsymbol{s}, I, \theta)\mathrm{d}G(\theta; \boldsymbol{s}, I)\\ &= \int_\Theta \int_X h(x; \boldsymbol{s})\mathrm{d}F(x; I, \theta)\mathrm{d}G(\theta; \boldsymbol{s}, I),\end{aligned}$$

and the shares that remain after the repurchase have market value $\hat{V} - \hat{H}$.

When the manager announces a financing/investment plan $(\boldsymbol{s}, I)$, investors value the new security at $\hat{H}(\boldsymbol{s}, I)$. If $\hat{H} - I < 0$, the cash raised is insufficient to fund the investment plan, and all payoffs are zero. If $0 \le \hat{H} - I \le (1-N)(\hat{V} - I)$, the firm invests I in production and repurchases $(\hat{H} - I)/(\hat{V} - I)$ shares from outside shareholders. In this case, the portion of the residual cash flows belonging to the manager is

$$\frac{N}{1-\left[\hat{H}(\boldsymbol{s},I)-I\right]/\left[\hat{V}(\boldsymbol{s},I)-I\right]}=N\left(\frac{\hat{V}(\boldsymbol{s},I)-I}{\hat{V}(\boldsymbol{s},I)-\hat{H}(\boldsymbol{s},I)}\right).$$

Finally, if $\hat{H}-I$ is larger than $(1-N)(\hat{V}-I)$, the firm invests I in production, repurchases all $1-N$ outside shares, and invests the remaining cash in a riskless security.

The market's estimate of the value of the manager's stake is simply $N[\hat{V}(\boldsymbol{s}, I)-I]$ but since the manager knows the true cash flow distribution, he or she values the shares at

$$u(\boldsymbol{s},I;\theta)\equiv N\left(\frac{\hat{V}(\boldsymbol{s},I)-I}{\hat{V}(\boldsymbol{s},I)-\hat{H}(\boldsymbol{s},I)}\right)\left[V(I,\theta)-H(\boldsymbol{s},I,\theta)\right]. \qquad (2)$$

The expression $((\hat{V}-I)/(\hat{V}-\hat{H}))(V-H)$ in Eq. (2) is the intrinsic value per share. Because the manager owns N shares and knows θ, he or she maximizes intrinsic share value.[1] This intrinsic value depends on investor beliefs because investor beliefs determine the price of the new issue, which affects the number of outstanding shares and the terminal cash flow to the shareholders.[2]

The manager's strategy is a mapping ρ from Θ to the space of probability measures over A, and an equilibrium is a pair (ρ, G) that satisfies two conditions.

1. The manager's financing/investment choice is optimal: for each θ, $\rho(\theta)$ maximizes $E[u(\boldsymbol{s}, I; \theta)]$, taking $G(\cdot)$ as given.
2. Investor inferences $G(\theta; \boldsymbol{s}, I)$ satisfy Bayes' rule for all strategies $(\boldsymbol{s}, I)$ that the manager chooses.

In a fully revealing equilibrium, there is at most one θ for which the manager chooses any particular $(\boldsymbol{s}, I)$. Therefore, the posterior distribution for any such $(\boldsymbol{s}, I)$ is just a mass point at the true θ, $\hat{V}(\boldsymbol{s}, I)$ is the true value $V(I, \theta)$, and $\hat{H}(\boldsymbol{s}, I)$ is the true value $H(\boldsymbol{s}, I, \theta)$. The equilibrium exhibits efficient investment if the chosen investment maximizes net present value, $V(I, \theta)-I$, thus circumventing

[1] The model does not address the issue of promoting efficient investment through a judicious choice of the manager's compensation scheme, as analyzed by Dybvig and Zender (1991), John and John (1993), and Persons (1994), for example.

[2] The formula in Eq. (2) applies in the case where the new issue raises enough cash to fund the investment but not so much that all the outside shares are repurchased (i.e., $0\le\hat{H}-I\le(1-N)(\hat{V}-I)$). For the case where all outside shares are repurchased, let $\delta=\hat{H}-I-(1-N)(\hat{V}-I)$ denote the amount invested in the riskless asset, so the firm's cash flow is $x+\delta$ rather than x. Then, the manager's expected payoff is

$$u(\boldsymbol{s},I;\theta)=\int_X(x+\delta-h(x+\delta;\boldsymbol{s}))\mathrm{d}F(x;I,\theta).$$

.

the Myers and Majluf (1984) problem. Proposition 1 of Constantinides and Grundy, 1989 claims that investment is efficient in any fully revealing equilibrium.

2. A counterexample with inefficient investment

This section examines the specific example worked out by Constantinides and Grundy, 1989 and shows that there are (infinitely) many *inefficient* fully revealing equilibria in addition to the efficient one they identify. The example has firm types located on the non-negative reals, $\Theta = [0, \infty)$, and has a Cobb–Douglas production function: $x = (\theta/\beta) I^{\beta}\epsilon$, where $0 < \beta < 1$ and ϵ is distributed uniformly on the interval (0, 2). The firm's expected cash flow is therefore $V(I, \theta) = E[x] = \theta I^{\beta}/\beta$, so the expected marginal product is $\theta I^{\beta-1}$. Setting the expected marginal product equal to 1 shows that the socially optimal investment choice is $I(\theta) \equiv \theta^{1/(1-\beta)}$. Financing choices are restricted to straight debt, so the manager chooses investment, $I \geq 0$, and the face value of newly issued debt, $K \geq 0$. Constantinides and Grundy, 1989 show that, assuming

$$N < \frac{1-\beta}{(1-\beta/2)^2}, \tag{3}$$

there is a fully revealing equilibrium in which firms invest the optimal amount.[3] Investor inferences in this equilibrium are that $\theta = I^{1-\beta}$ if $K = 2I/(2-\beta)$, and $\theta = 0$ otherwise. Given these investor beliefs, every manager with $\theta > 0$ will set $K = 2I/(2-\beta)$ — any other K yields a zero payoff because investors conclude that $\theta = 0$ — and choose I to maximize his or her payoff. The proof of Proposition 1 below shows that efficient investment results. That is, taking investor beliefs as given, managers maximize their own payoffs, and their privately optimal actions confirm investor beliefs about how they will behave. Managers could imitate higher types and issue overpriced bonds, but two forces prevent them from doing so. The first force is that value would be lost from the excessive investment, i.e., the firm would be taking some negative-NPV projects. The second force is that the firm would have to repurchase shares at the price investors *think* the shares are worth, namely, the value of the better firm. Since this is more than the actual value of the shares, wealth would be transferred to the selling shareholders from the shareholders who remain with the firm, including the manager.

For instance, when $\beta = 1/3$, the efficient investment level for firm θ is $I(\theta) = \theta^{3/2}$. In this equilibrium, investors expect firm θ to invest $I = \theta^{3/2}$ and issue debt with face value 20% higher than the amount invested, $K = 1.2I$.[4]

[3] Condition (3) ensures that firms do not repurchase all their outside shares.

[4] The market value of the debt is 8% higher than the amount invested; the extra 8% is spent repurchasing shares.

Managers, whose only goal is to maximize intrinsic value per share, choose to behave as investors expect. For instance, firm $\theta = 9$ chooses to invest $I = 27$ and issue debt with face value $K = 32.4$, and firm $\theta = 16$ invests $I = 64$ and issues debt with $K = 76.8$.

However, what if investors have different expectations? For instance, suppose investors expect firms to invest only half the levels above. How would managers respond? The answer depends on what *financing* policy investors expect. If investors expect the same financing policy as above, managerial choices will confound their expectations. That is, this would not be an equilibrium. However, if investors expect investment of $(0.5)\theta^{3/2}$ *and* expect debt issues with face value $(0.6)4^{1/3}\theta^{3/2} \approx 1.905I$, managers choose precisely these policies. For instance, firm $\theta = 9$ invests only 13.5 and issues debt with $K \approx 25.72$ and firm $\theta = 16$ invests 32 and issues $K \approx 60.96$.

In addition to the efficient equilibrium analyzed by Constantinides and Grundy, 1989, there are fully revealing equilibria in which firms invest *inefficiently*, such as the one just described. In fact, Proposition 1 shows that there are infinitely many fully revealing equilibria with inefficient investment. The parameter ϕ, introduced in the proposition, captures the degree of underinvestment or overinvestment, with $\phi = 0.7$, for example, representing 30% underinvestment.

Proposition 1. *Consider the example of Constantinides and Grundy, 1989, and assume that Eq. (3) holds. There exist infinitely many fully revealing equilibria, parameterized by $\phi \in (\underline{\phi}, \overline{\phi}]$, with $0 \leq \underline{\phi} < 1 < \overline{\phi}$.*[5] *In equilibrium ϕ, firm θ invests $I = \phi\theta^{1/(1-\beta)}$ and issues debt with face value $K = 2\phi^{\beta-1}I/(2-\beta)$. Investor inferences are that $\theta = (I/\phi)^{1-\beta}$ if $K = 2\phi^{\beta-1}I/(2-\beta)$ and $\theta = 0$ otherwise.*

The equilibrium analyzed by Constantinides and Grundy, 1989 is the one with $\phi = 1$, so investment is socially optimal, but all the other equilibria have less desirable properties.

Corollary 1. *In each equilibrium $\phi \neq 1$ of Proposition 1, every firm except the worst ($\theta = 0$) invests a socially inefficient amount.*

To illustrate, return to the case $\beta = 1/3$, where optimal investment is $I(\theta) = \theta^{3/2}$, and suppose that management owns 10% of the shares. The solid lines in Figs. 1 and 2 illustrate the equilibrium $\phi = 1$, which exhibits socially optimal investment.

[5] The definitions of $\underline{\phi}$ and $\overline{\phi}$ are given in the Appendix. For β small enough, $\underline{\phi} = 0$; as β increases toward the point where Eq. (3) binds, $\underline{\phi}$ increases toward 1. For β small enough, $\overline{\phi}$ is close to 1; as β increases toward the point where Eq. (3) binds, $\overline{\phi}$ increases toward

$$\left[\left(1 + N + \sqrt{1-N}\right)/2\right]^{N(2-N-2\sqrt{1-N})^{-1}},$$

which takes a maximum value of e (for N close to 0).

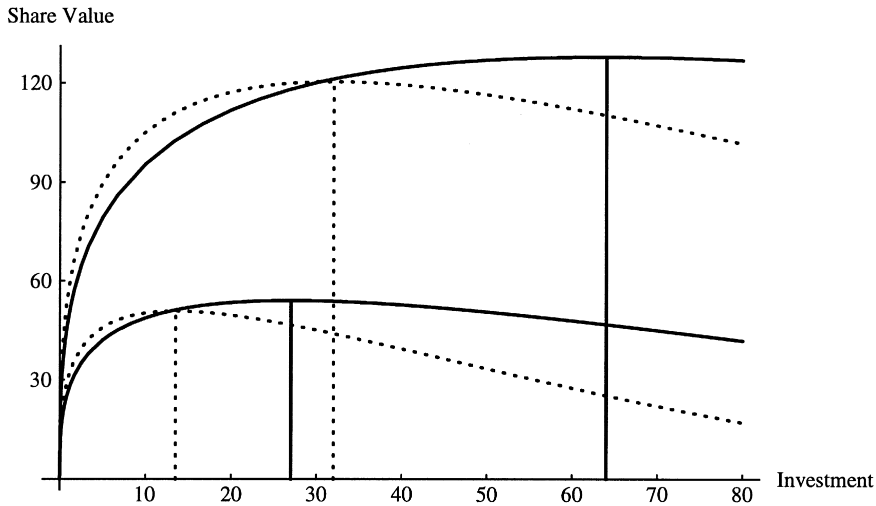

Fig. 1. The manager's choice of investment. The figure plots intrinsic share value as a function of investment I for two firm types, $\theta = 9$ and $\theta = 16$. The solid lines pertain to the case $\phi = 1$, where the manager chooses the socially optimal investment level. The dotted lines pertain to the case $\phi = 0.5$, where 50% underinvestment maximizes intrinsic share value.

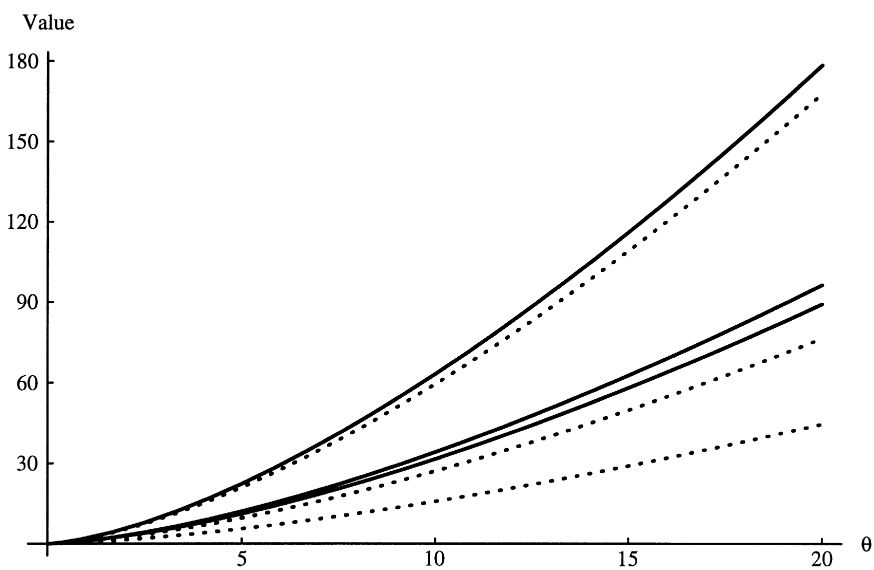

Fig. 2. Equilibrium share value, financing, and investment. The solid lines illustrate the efficient equilibrium presented by Constantinides and Grundy, 1989; the dotted lines illustrate an inefficient equilibrium constructed by setting $\phi = 0.5$. The highest of the three curves is the equilibrium share value, the middle curve is the value of the debt issue, and the lowest curve is the amount invested.

Fig. 1 shows the choice problem for the two firm types mentioned earlier, $\theta = 9$ and $\theta = 16$, by graphing intrinsic value per share as a function of investment, I. Because the manager owns 0.1 shares and knows θ, he or she maximizes intrinsic share value. The maximizing investments for the two firms are $I = 27$ and $I = 64$, respectively — the socially optimal amounts. The solid lines in Fig. 2 summarize this equilibrium for all firm types θ between 0 and 20. The highest curve is the equilibrium share value, $V - I$; the middle curve is the amount raised by the debt issue, H; and the lowest curve is the amount invested, I. The wedge between the two lower curves, $H - I$, is the amount spent repurchasing shares.

The dotted lines in Figs. 1 and 2 illustrate the equilibrium $\phi = 0.5$, in which each firm invests only half the efficient amount. Fig. 1 plots intrinsic share value for the two cases $\theta = 9$ and $\theta = 16$, and the maximizing investments are $I = 13.5$ and $I = 32$, respectively. Given investor beliefs in this equilibrium, these choices maximize intrinsic *share* value even though doubling the amount invested would maximize the intrinsic value of the *firm*, $V - I$. The dotted lines in Fig. 2 show the equilibrium share value, amount of financing raised, and amount invested for the $\phi = 0.5$ equilibrium. The dramatically larger wedge between the two lower curves means there is a huge increase in repurchases in this inefficient equilibrium. Why are more shares repurchased in this case? Since equilibrium investment is inefficiently low, managers could increase the true value of the firm $(V - I)$ by investing more in production and masquerading as a higher type. Larger repurchases are therefore necessary to prevent such mimicry: if a firm masquerades as a better type, it has to repurchase a lot of shares at the value of that better type. For similar reasons, the equilibria with overinvestment $(\phi > 1)$ exhibit less repurchasing than the efficient equilibrium.

Persons (1997) shows that the inefficient equilibria do not depend on ''unreasonable'' beliefs, in that the equilibria survive the standard equilibrium refinements.[6] He also shows that if one assumes that the manager chooses ex post how much to invest in production, rather than *committing* to a production level, the only equilibrium from Proposition 1 is one with underinvestment.[7]

As a last comment on this example, consider the most extreme overinvestment equilibrium, $\phi = \overline{\phi} = 1.08^{3/2} \approx 1.12$. In this equilibrium, the firm issues debt with market value $\hat{H} = 1.08(\overline{\phi})^{-2/3} I = I$, exactly the amount invested in production, so no firm repurchases any shares. This contradicts Proposition 2 of Constantinides and Grundy, 1989, which claims that in any fully revealing equilibrium, all firms

[6] Specifically, the equilibria satisfy the intuitive criterion and equilibrium domination (Cho and Kreps, 1987), forward induction (Cho, 1987), and divinity and universal divinity (Banks and Sobel, 1987). They are also perfect in the sense of Grossman and Perry (1986).

[7] It is the one with investment $[1 - \beta^2/(2-\beta)^2]^{1/(1-\beta)}\theta^{1/(1-\beta)}$ rather than the efficient level, $\theta^{1/(1-\beta)}$. When $\beta = 1/3$, for example, this equilibrium has about 6% underinvestment.

with positive stock prices repurchase some shares. In equilibrium $\overline{\phi}$, all firms except the worst one have positive stock prices ($\hat{V} - \mathrm{I} = 1.92\sqrt{1.08\theta^3} > 0$), but they repurchase no shares.[8]

Corollary 2. *In equilibrium $\overline{\phi}$ of Proposition 1, all firms except the worst ($\theta = 0$) have positive stock prices, but none of these firms repurchases shares.*

3. Concluding remarks

This paper shows that the problem of inefficient investment identified by Myers and Majluf (1984) can persist even in fully revealing equilibria, where information asymmetries are resolved and securities are priced correctly. This contradicts the claim of Constantinides and Grundy, 1989 that fully revealing equilibria always exhibit socially optimal investment. This is shown specifically for the example worked out by Constantinides and Grundy, 1989, which has infinitely many inefficient equilibria in addition to the efficient one they identify. The inefficient equilibria survive standard equilibrium refinements.

The problem of inefficient investment even when firm types are revealed is broader than this specific example, which assumes debt financing and a Cobb–Douglas production function. Persons (1997) shows the same result for a class of problems with more general production functions, more general sets of firm types, and unrestricted financing choices. For this class of problems, whenever there is an efficient fully revealing equilibrium, there is also a continuum of inefficient ones.

This provides a role for private financing. Suppose that, at some cost, management can convey its private information to a wealthy investor. If we are ‘‘stuck’’ in an inefficient equilibrium, firms are worth less than their potential value. If the communication cost is small enough, it will pay for the best firms, where the most value would be lost, to incur this cost and utilize private financing. Other firms would rely on public financing rather than incur the cost of private financing. In such an equilibrium, announcement returns would be consistent with the empirical

[8] Their claim assumed that $h(x; \boldsymbol{s})$ is non-decreasing in x, a condition satisfied by the debt contracts of this example. It is also true that some examples have fully revealing equilibria with *efficient* investment and no repurchases. Suppose in the example just examined that the set of types is $\Theta = \{0,0.1,1\}$ rather than $\Theta = [0,\infty)$. Then there is an equilibrium with debt financing in which each firm invests efficiently (investment is $\theta^{3/2}$:0, 0.0316 or 1) and issues just enough debt to finance the investment (face value is $(6-\sqrt{24})\theta^{3/2}$: 0, 0.0348 or 1.1010). No shares are repurchased, but the middle type does not imitate the good type because this would yield a zero payoff — all the cash generated would go to the debtholders.

evidence that private issues of debt and equity elicit more positive price reactions than public issues.[9]

Finally, the difficulty is not exclusively a problem of multiple equilibria. There are examples that have fully revealing equilibria with inefficient investment but have *none* with efficient investment.[10] In such cases, signaling actually *requires* inefficient investment, just the opposite of what was claimed by Constantinides and Grundy, 1989.

It is left for future research to determine whether restricting the problem along certain dimensions — the production technology, investment and financing choices, the set of firm types, or investor beliefs — will ensure efficient investment in all fully revealing equilibria, thus circumventing the Myers and Majluf (1984) problem. If such restrictions exist, characterizing them will advance our understanding of investment and financing behavior when management has superior information.

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Appendix A. Proof

Proof of Proposition 1. The Constantinides and Grundy, 1989 example has firm types $\Theta = [0, \infty)$, cash flows uniformly distributed on $(0, 2\theta I^{\beta}/\beta)$, and financing choices restricted to straight debt (face value denoted K). Fix some $\phi > 0$ and suppose that investors infer the firm type to be $\theta = (I/\phi)^{1-\beta}$ if $K = 2\phi^{\beta-1}I/(2-\beta)$, and $\theta = 0$ otherwise. The first part of the proof assumes that manager choices are consistent with these beliefs — firm type θ invests $\phi\theta^{1/(1-\beta)}$ and issues debt with face value $2\theta^{\beta-1}I/(2-\beta)$, and shows that there is an interval of ϕ values for which the debt proceeds are large enough to fund the investment and small enough that some outside shares remain outstanding. For all such ϕ, then,

[9] For example, see Eckbo (1986), Mikkelson and Partch (1986) and Asquith and Mullins (1986) for public issues and James (1987), Fields and Mais (1991) and Wruck (1989) for private issues.

[10] Persons (1997) provides an example like this.

the posited investment/financing strategies are feasible. The last part of the proof shows that these policies are in fact optimal for the manager.
The intrinsic value of the firm is

$$V(I,\theta) = E\left[\theta(1/\beta)I^{\beta}\epsilon\right] = \theta I^{\beta}/\beta \tag{4}$$

and the intrinsic value of the debt issue, assuming $K \leq 2\theta I^{\beta}/\beta$, is

$$\begin{aligned} H(K,I,\theta) &= \frac{1}{2\theta I^{\beta}/\beta}\left\{\int_0^K x\,\mathrm{d}x + \int_K^{2\theta I^{\beta}/\beta} K\,\mathrm{d}x\right\} \\ &= K - \frac{\beta K^2}{4\theta I^{\beta}} = K - \frac{K^2}{4V(I,\theta)}. \end{aligned} \tag{5}$$

[If $K > 2\theta I^{\beta}/\beta$, then $H(K, I, \theta) = V(I, \theta)$.] Given the inference rule of investors, the market value of any firm that chooses $K \neq 2\phi^{\beta-1}I/(2-\beta)$ is zero (as is the value of any debt the firm might issue). For firms that issue debt with face value $2\phi^{\beta-1}I/(2-\beta)$, investors infer that $\theta = (I/\phi)^{1-\beta}$; substitution into Eqs. (4) and (5) yields market value

$$\hat{V}(K,I) = \frac{I}{\beta\phi^{1-\beta}} \tag{6}$$

for the firm and

$$\hat{H}(K,I) = \frac{I}{\phi^{1-\beta}}\left[1 + \frac{\beta(1-\beta)}{(2-\beta)^2}\right] \tag{7}$$

for the debt issue.

The first observation, from Eq. (7), is that the debt issue is sufficient to finance the planned investment as long as

$$\phi \leq \overline{\phi} \equiv \left[1 + \frac{\beta(1-\beta)}{(2-\beta)^2}\right]^{\frac{1}{1-\beta}}.$$

As claimed in the proposition, $\overline{\phi}$ is strictly larger than 1. For the case $\phi = \overline{\phi}$, $\hat{H}$ is exactly equal to I, so no firms repurchase shares.
The next step is to show that for all ϕ greater than some $\underline{\phi} < 1$, the financing proceeds are small enough that some outside shares remain after the repurchase. If $\phi = \overline{\phi}$, no shares are repurchased, so we may assume $0 < \phi < \overline{\phi}$. Now define

$$\underline{\phi} = \begin{cases} 0 & \text{if } N \leq \left(\dfrac{1-\beta}{1-\beta/2}\right)^2 \\ \left\{\dfrac{1}{\beta}\left[1 - \dfrac{1}{N}\left(\dfrac{1-\beta}{1-\beta/2}\right)^2\right]\right\}^{\frac{1}{1-\beta}} & \text{otherwise.} \end{cases} \tag{8}$$

(Assumption (3) ensures that $\underline{\phi} < 1$.) The number of shares repurchased, say Q, is identical for each firm type:

$$Q = \frac{\hat{H} - I}{\hat{V} - I} = \frac{\dfrac{I}{\phi^{1-\beta}}\left[1 + \dfrac{\beta(1-\beta)}{(2-\beta)^2}\right] - I}{\dfrac{I}{\beta\phi^{1-\beta}} - I} = \frac{\left[1 + \dfrac{\beta(1-\beta)}{(2-\beta)^2}\right] - \phi^{1-\beta}}{\beta^{-1} - \phi^{1-\beta}}. \tag{9}$$

Q is decreasing in ϕ (to zero at $\phi = \overline{\phi}$), which means that for every $\phi > 0$,

$$Q < \frac{\left[1 + \dfrac{\beta(1-\beta)}{(2-\beta)^2}\right]}{\beta^{-1}} = \beta\left[1 + \frac{\beta(1-\beta)}{(2-\beta)^2}\right].$$

This justifies setting $\underline{\phi} = 0$ in the first part of Eq. (8) because $N \le ((1-\beta)/(1-\beta/2))^2$ implies

$$1 - N \ge 1 - \left(\frac{1-\beta}{1-\beta/2}\right)^2 = \beta\left[1 + \frac{\beta(1-\beta)}{(2-\beta)^2}\right] > Q.$$

When N is larger, the second part of Eq. (8) applies. Substituting into Eq. (9) and then simplifying shows that $\phi > \underline{\phi}$ implies

$$Q < \frac{\left[1 + \dfrac{\beta(1-\beta)}{(2-\beta)^2}\right] - \dfrac{1}{\beta}\left[1 - \dfrac{1}{N}\left(\dfrac{1-\beta}{1-\beta/2}\right)^2\right]}{\dfrac{1}{\beta} - \dfrac{1}{\beta}\left[1 - \dfrac{1}{N}\left(\dfrac{1-\beta}{1-\beta/2}\right)^2\right]} = 1 - N,$$

as required. Thus, for all $\phi \in (\underline{\phi}, \overline{\phi}]$, the proposed investment and financing policies are feasible and the formula in Eq. (2) gives the manager's payoff (since all excess cash is used to repurchase shares).

The last step is to verify that it is in the manager's interest to choose $I = \phi\theta^{1/(1-\beta)}$, as investors expect. It is certainly in the manager's interest to set $K = 2\phi^{\beta-1}I/(2-\beta)$ since any other choice yields a payoff of zero. Observe also that the manager will never choose $K > 2\theta I^{\beta}/\beta$, since this is the maximum possible cash flow. Such a high level of debt would make the equity worthless — $V - H$ would be zero, even though $\hat{V} - \hat{H}$ might be positive. This means that

$$\frac{2\phi^{\beta-1}I}{(2-\beta)} \le \frac{2\theta I^{\beta}}{\beta}, \quad \text{or} \quad I \le \phi\left[\frac{(2-\beta)\theta}{\beta}\right]^{\frac{1}{1-\beta}}. \tag{10}$$

Since the manager's payoff is zero, if I is set to zero or to the upper bound in Eq. (10), but is positive in between, there must be an interior choice of I that is optimal. The optimal investment is determined by substituting $K = 2\phi^{\beta-1}I/(2-\beta)$ into Eq. (5), then substituting Eqs. (4)–(7) into Eq. (2). The substitution yields

$$u(K,I;\theta) = N\frac{\dfrac{I}{\beta\phi^{1-\beta}} - I}{\dfrac{I}{\beta\phi^{1-\beta}} - \dfrac{I}{\phi^{1-\beta}}\left[1 + \dfrac{\beta(1-\beta)}{(2-\beta)^2}\right]}$$

$$\times\left\{\frac{\theta I^{\beta}}{\beta} - \frac{2I}{(2-\beta)\phi^{1-\beta}} + \frac{\beta}{4\theta I^{\beta}}\frac{4I^2}{(2-\beta)^2\phi^{(1-\beta^2)}}\right\}$$

$$= C_0\left[\beta^2 I^{2-\beta} - 2\beta(2-\beta)\theta\phi^{1-\beta}I + (2-\beta)^2\theta^2\phi^{2(1-\beta)}I^{\beta}\right],$$

where C_0 is a positive constant. Differentiating with respect to I yields

$$C_1\left[\beta I^{1-\beta} - 2\theta\phi^{1-\beta} + (2-\beta)\theta^2\phi^{2(1-\beta)}I^{-(1-\beta)}\right], \tag{11}$$

which has roots

$$I = \phi\left[\frac{(2-\beta)\theta}{\beta}\right]^{\frac{1}{1-\beta}} \quad \text{and} \quad I = \phi\theta^{\frac{1}{1-\beta}}.$$

(C_1 is again a positive constant.) The first root is the upper boundary given in Eq. (10), which yields the manager zero — it is a global minimum. The second root is the maximizer, so $I = \phi\theta^{1/(1-\beta)}$ is the manager's optimal choice. Because management choices confirm investor beliefs, the proposition is proved.
Corollary 1 follows because efficient investment requires $I = \theta^{1/(1-\beta)}$. Corollary 2 follows from the observation above that $\hat{H} \equiv I$ in the equilibrium $\overline{\phi}$.

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