Ronald A. Dye
University of Chicago

Proprietary and Nonproprietary Disclosures*

This paper examines the disclosure policies that arise from tension between the management objectives of protecting proprietary information and making value-increasing disclosures. The issues studied include the interaction between mandatory and voluntary disclosures, the dependence of disclosure policies on the costs of disclosure and the public's knowledge of firms' production technologies, and conditions under which firms have incentives not to disclose nonproprietary information.

I. Introduction

This paper presents a theory for the disclosure policies adopted by managers endowed with proprietary and nonproprietary information. Proprietary information is defined as information whose *disclosure* reduces the present value of cash flows of the firm endowed with the information. This theory explains selective disclosure of management's information and the effects of changes in financial reporting requirements on firms' voluntary disclosure policies. Most existing theories of disclosure[1] conclude that, when credible announcements of private information are feasible, full disclosure is optimal.[2] These theories posit that managers possess only one type of private information. I show that, when managers are endowed with both proprietary and

* Comments by Joel Demski, Sanford Grossman, Richard Leftwich, Michael Mussa, Katherine Schipper, Abbie Smith, workshop participants at the University of British Columbia, the University of California at Berkeley, the University of Chicago, Northwestern University, the University of Pittsburgh, Stanford University, and an anonymous referee are gratefully acknowledged. Research support was provided by the Institute of Professional Accounting at the University of Chicago.

1. See Grossman and Hart 1980; Grossman 1981; and Milgrom 1981. Verrecchia (1983) illustrates how selective disclosure can occur in a univariate setting.

2. An unimportant exception being the situation in which investors cannot take positions on markets prior to disclosure, in which case disclosure can have adverse risk-sharing effects.

(*Journal of Business*, 1986, vol. 59, no. 2, pt. 1)

0021-9398/86/5902-0007$01.50

nonproprietary information, nondisclosure or partial disclosure may be optimal even if credible announcements of all information can occur. Contrary to the usual conjecture that, as mandatory reporting requirements become more detailed, voluntary disclosures may decline,[3] I also establish that increasing mandatory reporting requirements increases the incentives for voluntary disclosure.

The results of this paper depend on the assumption that managers can make verifiable announcements regarding their private information. Since many studies assume that such announcements are not credible without being accompanied by costly signals,[4] this assumption deserves some comment. In practice, managers acquire private information about their firms by reading and drafting budgets, marketing and financial plans, internal accounting reports, etcetera. Managers could convert much of their private information into public information by releasing these reports. Even if the information sought by the market to evaluate a firm were only some summary statistic of its manager's private information, the believability of the manager's announcements of such a statistic would be enhanced by the release of supporting documents. This observation is the basis for both the assumption that management disclosures may be made credibly and the assumption that some information that might be disclosed is proprietary.

The intuition behind the complementarity of mandatory and voluntary disclosures is simple. Suppose a manager's private information is summarized by real numbers x and y and that there are initially no financial reporting requirements. If disclosures must be truthful, then the manager's disclosure options are (1) no disclosure, (2) disclosure of x, (3) disclosure of y, and (4) disclosure of x and y. Now suppose that the manager must disclose x to comply with financial reporting requirements. In this case fewer conditions have to be satisfied for the optimal disclosure to be full disclosure: with x disclosed mandatorily, full disclosure is optimal if option 4 is preferred to option 2, whereas with no mandatory disclosures, full disclosure is optimal only if option 4 is preferred to all options 1, 2, and 3.

Intuition for the nondisclosure or partial disclosure of information is also straightforward. Incomplete disclosure occurs because some items in the manager's array of private information are proprietary and

3. See Gonedes 1980; and Verrecchia 1982.

4. See Spence 1974. Though this paper is about disclosures and not signaling, it is worth noting that this observation alters the perspective on, but does not completely eliminate the desirability of, signaling activities when credible disclosures of information are possible. Signaling becomes a substitute for disclosure, undertaken to convey information about the firm's earnings' generating power without divulging the information itself. In a model of both disclosure and signaling, whether signaling about or the direct disclosure of information would occur depends on a comparison of the efficiency of the signal in communicating the information and the cost of the information's disclosure.

because disclosure of even nonproprietary information may partially reveal proprietary information if there are known statistical interdependencies between the two types of private information. In this situation, disclosure of good news may assuage investors' concerns regarding the firm's future earnings prospects while, at the same time, worsening these prospects by divulgence of proprietary data. A value-maximizing manager will be unlikely to make potentially damaging disclosures, even regarding nonproprietary data, unless the effect of the disclosure on the firm's value is dramatic.

The paper proceeds as follows. Section II introduces the model used to characterize a firm's value-maximizing disclosure policy. Value-maximizing policies are studied not only because of the traditional emphasis placed on value-maximization but also because, as argued in Section II, such policies have good incentive effects on managers. Section III contains a characterization of these value-maximizing policies and presents some comparative static results. In Section IV, it is shown that value-maximizing policies are sometimes identical to the disclosure policies investors would implement absent managerial incentive problems. Conclusions occupy Section V.

II. Value-maximizing Disclosure Policies

This section introduces the model. I begin by outlining the chronology of events, and then I discuss the nature of the information the manager possesses and the effect of its disclosure on investors' perceptions regarding the firm's cash flows. Disclosure policies are defined formally, and the relationship between market-clearing prices and the manager's disclosure policy is discussed. Finally, the concepts of credible and optimal disclosure policies are introduced and studied.

It is assumed to be common knowledge that a firm's manager receives new information, superior to that available to all investors, about his firm's expected cash flows on a particular date.[5] The manager's information is summarized by realizations x and y of random variables $\tilde{x}$ and $\tilde{y}$, with density $f(x, y)$ and joint sample space Ω. Trading in the firm's shares occurs both before the manager receives the information (on market 1) and after the manager has had an opportunity to make announcements regarding the information he has received (on market 2). (See fig. 1.)

5. The assumption that the manager's private knowledge concerns only the mean of the firm's cash flows is made for expositional convenience. Below, I indicate how investor's demands for the firm's shares change with changes in their perception of the firm's expected cash flows. If the manager had private information about other moments of the distribution of cash flows, then additional assumptions would have to be made regarding investors' asset demand functions. Also, an investigation of a firm's disclosure policy when investors are unsure of the time of arrival of the manager's information may be found in Dye (1985).

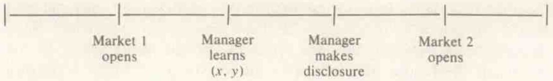


FIG. 1

The manager must make truthful disclosures: if (x, y) is the realization of $(\tilde{x}, \tilde{y})$, the manager can announce x, y, (x, y), or nothing, but he cannot make any disclosure inconsistent with (x, y).[6] If disclosures were costless, the disclosure that $(\tilde{x}, \tilde{y}) = (x, y)$ would cause investors and the manager to revise their expectations of the mean of the present value of the firm's cash flows to $E[\tilde{e}|x, y]$, where

$$\inf_{(x,y)\in\Omega} E[\tilde{e}|x, y] = 0$$

and $E[\tilde{e}|x, y]$ is increasing in x and y.[7] Disclosures are not costless, however. The random variable $\tilde{y}$ is assumed to be proprietary information: its disclosure reduces expected cash flows by \$$\bar{c}$. The variable x is assumed to be nonproprietary relative to $\{x, y\}$; that is, disclosure of x, given that y is disclosed, has no effect on the distribution of cash flows.[8] However, disclosure of x by itself may reveal something about y because of a statistical relationship between $\tilde{x}$ and $\tilde{y}$. For example, competitors of a firm may have no intrinsic interest in its fixed costs, x, but they may seek to infer its marginal costs, y, by exploiting the (typically inverse) relationship between fixed and marginal costs. In this example, the disclosure of fixed costs by itself may have cash flow effects, to the extent that such a disclosure improves the ability of its competitors to make inferences about the firm's marginal costs. Consequently, I assume disclosure of x alone can also reduce cash flows, but by an amount $\underline{c} \geq 0$, which is less than the loss $\bar{c}$ attributable to the disclosure of y. To summarize, given x and y, the manager's perception of the expected present value of the firm's cash flows conditional on

6. This assumption will be satisfied if announcements of $\tilde{x}$ and $\tilde{y}$ are audited or if substantial penalties can be imposed on the manager if he is detected lying, provided the probability of such detections is nonzero.

7. $E[\tilde{e}|x, y]$ is the expected present value of the firm's cash flows, conditional on x and y, *net* of the manager's compensation, assuming that no disclosures occur.

8. This requirement could be taken as the defining condition of nonproprietary information: a variable x is not proprietary relative to a set H of variables (which includes x) initially only privately known by the manager if, on the disclosure of all variables in H other than x, the subsequent release of x results in no further reduction in the present value of the firm's cash flows. This definition of nonproprietary information allows for the possibility that the disclosure of exclusively nonproprietary information affects the firm's cash flows, and it emphasizes the dependence of the classification of information (as nonproprietary) on the other information privately held by the firm's manager.

TABLE 1 **Expected Present Value of Cash Flows Net of Disclosure Costs**

No disclosure	$E[\tilde{e}\vert x, y]$
Disclosure of x alone	$E[\tilde{e}\vert x, y] - \underline{c}$
Disclosure of y alone	$E[\tilde{e}\vert x, y] - \bar{c}$
Disclosure of x and y	$E[\tilde{e}\vert x, y] - \bar{c}$

what he discloses is given in table 1.[9] Disclosure of x, but not y, is referred to as partial disclosure, whereas disclosure of y or x and y is referred to as full disclosure since it will be clear in what follows that y will never be disclosed without x also being disclosed.[10] In the text I confine attention to the relatively restrictive case in which the costs of disclosure are exogenous and independent of the realization of $(\tilde{x}, \tilde{y})$.[11] In notes 20 and 25 below, I indicate the qualifications necessary to generalize these results to the case of variable disclosure costs.

Now consider disclosure policies. A disclosure policy can be described in two equivalent ways: either as a function d indicating the disclosure corresponding to each realization of x and y or as three sets D^N, D^P, and D^F indicating where there is no disclosure, partial disclosure, or full disclosure, respectively. This can be described formally as follows.

Definition 1. A disclosure policy is a function $d: \Omega \rightarrow X \cup \Omega \cup \{\phi\}$ where, for every (x, y) in Ω, the disclosure $\bar{d}$ made in accordance with the policy $d(\cdot)$ belongs to the set $\{\phi, x, (x, y)\}$, with ϕ symbolizing the case of no disclosure. The sets of no, partial, and full disclosure are given by, respectively,

9. The costs associated with any disclosure are assumed to be common knowledge, though the expected cash flows corresponding to any incomplete disclosure are known only to the manager; to assume otherwise would be tantamount to assuming that the realizations of $(\tilde{x}, \tilde{y})$ are publicly known, which would be inconsistent with an incomplete disclosure.

10. If y, but not x, is disclosed and investors believe that managers adopt a value-maximizing disclosure policy, then investors will infer that x belongs to

$$D = \{x \vert E[\tilde{e}\vert y, \tilde{x} \in D] - \bar{c} \geqslant E[\tilde{e}\vert x, y] - \bar{c}\} = \{x \vert E[\tilde{e}\vert y, \tilde{x} \in D] \geqslant E[\tilde{e}\vert x, y]\}.$$

For D to be well defined, $D = \{\inf X\}$. This is the standard solution to the "lemons" problem: investors believe that the failure to disclose x must mean that x takes on the worst realization possible. Hence, unless $x = \inf X$, the manager will always disclose x if he elects to disclose y. If $x = \inf X$, the disclosure—or nondisclosure—of x is moot.

11. In a more general analysis, the cost of disclosing x might be affected not only by which x is disclosed but also by what disclosure policy the firm is believed to have adopted since the inferences about y a competitor can draw from observing x depend on the firm's disclosure policy. A consequence of taking disclosure costs to be exogenously given as assumed here is that such interactions between the firm's disclosure policy and its disclosure costs are ignored or, alternatively, that such interactions are unimportant, as might be reasonably assumed when the losses from disclosure arise from governmental responses to disclosures (e.g., loss of tariff protection on announcement of high earnings).

$$D^N = \{(x, y) \in \Omega | d(x, y) = \phi\},$$
$$D^P = \{(x, y) \in \Omega | d(x, y) = x\},$$
$$D^F = \{(x, y) \in \Omega | d(x, y) = (x, y)\}.$$[12]

The remainder of this section is concerned with the interaction between disclosure policies and stock prices. This interaction places natural restrictions on the disclosure policies subsequently studied.

The market 2 value of the firm after a disclosure is made is assumed to equal the expected present value of cash flows net of disclosure costs, conditional on what is disclosed and what disclosure policy investors believe the manager has implemented. All subsequent derivations depend only on the assumption that investors' demands for the firm's shares and, hence, the equilibrium price of the firm are strictly increasing functions of these expectations, and so this analysis does not hinge on investors' risk neutrality. The equilibrium price of the firm on market 1 is assumed to increase in the expected present value of cash flows net of costs associated with disclosures anticipated to be made before market 2 opens, or, equivalently, the market 1 price of the firm is assumed to increase in its expected market 2 price. No further specification of the linkage between market 1 and market 2 prices is required in this analysis.

Shareholders are assumed to care about disclosure policies only insofar as the price of the firm varies with changes in this policy.[13] Since different disclosure policies result in different trajectories for the price of the firm and therefore necessitate evaluating tradeoffs between current and future stock prices, shareholders may not hold unanimous views concerning the desirability of different disclosure policies. This is essentially the same problem as lack of shareholder unanimity concerning investment policies. To resolve this problem, it should be remembered that a firm's disclosure policy and its management's compensation are typically interdependent since management's willingness to implement a particular policy depends on its effect on their compensation. This interdependence arises in part because disclosures may influence the firm's price, which in turn may affect management's compensation. Since this paper is about disclosure policies and not management compensation per se, I shall make an assumption regarding

12. I distinguish notationally here between a disclosure *policy* and a particular disclosure by use of the symbols $d(\cdot)$ and $\underline{d}$; the difference between a disclosure policy and a particular disclosure made in accordance with that policy is the same as the difference between a function and the evaluation of the function at a particular point.

13. In a multifirm version of this model, neglect of the effect of this firm's disclosure policy on the prices of other firms would be justified if its disclosure policy had only second-order effects on the other firms' prices, as would presumably be the case if the disclosure losses arose from government sanctions specifically directed at the firm making the disclosures.

management compensation (which is consistent with some stylized facts [see Antle and Smith 1984]) and then investigate the disclosure policies managers are willing to implement, conditional on this assumption. Specifically, I assume that management's compensation is strictly increasing in the price of the firm at all times. Such a contract may give managers an incentive to disclose proprietary information. While there are other compensation schemes that eliminate this incentive (e.g., wage contracts with penalties for disclosure), it is implicitly assumed here that the agency costs of such contracts overwhelm the expected losses arising from proprietary disclosures.

The assumption that the manager's contract increases in the price of the firm imposes restrictions on what disclosure policies shareholders can get the manager to implement. Since investors cannot observe what disclosure policy the manager chooses—although they can see particular disclosures the manager makes—they cannot force the manager to adopt a disclosure policy that would result in the manager failing to make a value-maximizing (i.e., compensation-maximizing) disclosure on occasion. That is, if $P(\bar{d}|d)$ denotes the market 2 value of the firm when the manager makes disclosure $\bar{d}$ and investors believe that disclosure policy $d(\cdot)$ is employed, then investors should not expect the manager to implement any disclosure policy that is not credible in the sense of definition 2, which follows.

DEFINITION 2.[14] A disclosure policy $d(\cdot)$ is credible if, for every (x, y) in Ω,

$$P(d(x, y)|d) = \max \{P(\phi|d), P(x|d), P((x, y)|d)\}.$$

If a disclosure policy $d(\cdot)$ is not credible and investors believed the manager implemented $d(\cdot)$, then there exists some (x, y) and some truthful disclosure that result in a greater market 2 price of the firm than would obtain by disclosing $d(x, y)$, so investors' beliefs would not be fulfilled.

When credible disclosure policies exist, they may not be unique. The assumption that the manager's compensation increases at all times implies that if one credible disclosure policy $d^1(\cdot)$ resulted in uniformly higher prices for the firm than another credible policy $d^2(\cdot)$, then the manager would not implement $d^2(\cdot)$. Consequently, among the credible disclosure policies, we should expect managers to implement only "optimal" policies, as specified in definition 3.

DEFINITION 3. A credible disclosure policy d is optimal if no other credible policy d' exists satisfying

$$P(d'(x, y)|d') \geq P(d(x, y)|d)$$

14. Credible disclosure policies are sequentially rational for managers with payoff functions (defined in table 2 below) to implement (see Kreps and Wilson 1982).

for all (x, y) in Ω, with strict inequality occurring on a set of positive probability.

Since the market 1 price of the firm increases in its expected market 2 price, the failure to implement an optimal policy implies the existence of another policy that produces uniformly higher prices for the firm over time, which is inconsistent with the manager's incentives.

The discussion of credible and optimal disclosure policies will be complete on specifying the dependence of the market 2 price of the firm, $P(\bar{d}|d)$, on the disclosure $\bar{d}$ made and the disclosure policy d investors perceive the manager has adopted. Recall that the market 2 price of the firm equals the expected present value of the firm's cash flows net of disclosure costs and that the manager is confined to truthful disclosures. Consequently, if full disclosure occurs when $(\tilde{x}, \tilde{y}) = (x, y)$,

$$P((x, y)|d) = E[\tilde{e}|x, y] - \bar{c},$$

regardless of what disclosure policy d investors believe the manager has implemented. Also recall that the manager's compensation increases in the price of the firm at all times. Thus if the manager discloses only $\bar{d} = x$, investors must conclude that $\tilde{y}$ (which has not been disclosed) belongs to the set

$$D_x \equiv \{y| \begin{cases} \text{(i)} & P(x|d) \geq E[\tilde{e}|x, y] - \bar{c} \\ \text{(ii)} & P(x|d) \geq P(\phi|d)\}, \end{cases}$$

that is, the market 2 price of the firm must be highest by disclosing only x rather than (x, y) or nothing. For investors' perceptions to be correct, D_x must satisfy

$$P(x|d) = E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}; \qquad (1)$$

that is, the market 2 price of the firm, given the disclosure of x alone, must equal the expected value of cash flows net of disclosure costs conditional on the sole disclosure of x. In equation (1), $P(x|d)$ is defined in terms of itself; Appendix A provides sufficient conditions for $P(x|d)$ and D_x to be well defined.

Finally, consider the specification of the market 2 price of the firm, $P(\phi|d)$, when no disclosure occurs. If investors expected that no disclosure would occur with positive probability, then $P(\phi|d)$ equals $E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D^N]$, where D^N is the "no disclosure" set consistent with disclosure policy d:

$$D^N = \{(x, y)|P(\phi|d) \geq P((x, y)|d), P(\phi|d) \geq P(x|d)\}.$$

But if investors expect that the manager always engages in partial or full disclosure, then further specification of investors' "off-equilibrium" beliefs is required when the manager makes no disclosure. Since

$$\inf_{(x,y)\in\Omega} E[\tilde{e}|x, y] = 0,$$

TABLE 2 Market 2 Firm Value Conditional on Disclosures and Disclosure Policies

Full disclosure	$P((x, y)\|d) = E[\tilde{e}\|x, y] - \bar{c}$
Partial disclosure	$P(x\|d) = E[\tilde{e}\|x, \tilde{y} \in D_x] - \underline{c}$
No disclosure	$P(\phi\|d) = E[\tilde{e}\|(\tilde{x}, \tilde{y}) \in D^N]$ (or zero, if D^N is empty)

I have chosen to set $P(\phi|d)$ equal to zero in such cases, although this choice is not uniquely acceptable. The only effect of decreasing the value assigned to $P(\phi|d)$ is to expand the circumstances under which credible disclosure policies exist.[15]

The market 2 price of the firm conditional on what disclosure is made and what disclosure policy investors believe the manager has adopted is summarized in table 2.

III. Characterization of Credible Disclosure Policies

This section contains a characterization of credible and optimal disclosure policies and provides sufficient conditions for such policies to exist. This characterization of these policies is shown to have several implications. These include: (*a*) disclosure policies that require all nonproprietary information to be disclosed are nonoptimal; (*b*) the manager's incentives for complete disclosure increase with the disclosure of the manager's nonproprietary information through mandatory reports; and, more generally, (*c*) the manager's disclosure policy changes with the information investors have prior to disclosure.

Theorem 1 below identifies restrictions on the density $f(x, y)$ of $(\tilde{x}, \tilde{y})$ and on the conditional expected cash flows $E[\tilde{e}|x, y]$ that ensure that the graphs of credible and optimal disclosure polices take one of two simple forms: either there is no partial disclosure, in which case the no- and full-disclosure sets are separated by a downward-sloping boundary, or else partial disclosure does occasionally occur, in which case the sets of no, partial, and full disclosure are determined by two intersecting curves, one downward sloping (tracing out those points for which the value of the firm is the same with no disclosure and full disclosure) and one upward sloping (tracing out those points for which the value of the firm is the same with partial disclosure and full disclosure).

15. This assignment does not affect the characterization of credible disclosure policies when such policies exist; i.e., if d is a credible policy when $P(\phi|d)$ is assigned the off-equilibrium value K, then d is a credible policy when $P(\phi|d)$ is assigned the off-equilibrium value $K' < K$. It may also be useful to note that off-equilibrium beliefs do not require separate specification when $\tilde{x} = x$ is unexpectedly disclosed, i.e., when $x \notin d(\Omega)$: given the manager's incentives to make value-enhancing disclosures, the price of the firm in market 2 when only $\tilde{x}$ is disclosed must be as specified in (1), regardless of whether $x \in d(\Omega)$.

Before proceeding with the theorem (the proof of which may be found in App. A), I present some notational conventions, as well as some restrictions on the distributions of $\tilde{x}$ and $\tilde{y}$, that are presumed to hold throughout the paper.

The support X of the random variable $\tilde{x}$ with density $g(x)$ is assumed to take the form $X = (x_L, \bar{x})$, where $x_L < \bar{x}$.[16] The support of $\tilde{y}$, conditional on $\tilde{x} = x$, is the set $Y(x) \equiv \{y|f(y|x) > 0\}$, where $f(y|x)$ is the density of y given x. I assume $Y(x) = (y_L, \bar{y}(x)]$, where $y_L < \bar{y}(x)$ for all x.[17] If $\bar{y}(x)$ is infinite for one x, it is assumed infinite for all x in X. If $\bar{y}(x)$ is not infinite, $\bar{y}(x)$ is assumed to be continuously differentiable and increasing in x.[18] Subscripts denote partial derivatives with respect to the subscripted variable.

THEOREM 1.[19] Suppose

i) $E[\tilde{e}|x, y]$ is nonnegative, with zero as its greatest lower bound, bounded on Ω, differentiable, and strictly increasing in x and y,

ii) $E[\tilde{e}|x]$ is continuous and strictly increasing and its greatest lower bound is less than $\underline{c}$, and

iii) $E[\tilde{e}|x, y] - E[\tilde{e}|x, \tilde{y} \leq y]$ strictly increases in y and is continuously differentiable in (x, y).

Then

a) credible and optimal disclosure policies exist;

b) no disclosure policy that entails that something always be disclosed is credible;

c) if, in addition to the above hypotheses, $f_x(y|x)/f(y|x)$ is (weakly) increasing in y, and $(\partial/\partial x)\, E[\tilde{e}|x, y]$ is (weakly) declining in y, then $D_x = (y_L, \underline{y}(x)] \cap Y(x)$ where $\underline{y}(x)$ is continuously increasing in x and there exists a positive constant K^* such that every credible disclosure policy takes one of two forms (illustrated in figs. 2 and 3 below);

*c*i) if

$$K^* > E\left[\tilde{e}|\bar{x}, \tilde{y} \in (y_L, \underline{y}(\bar{x})]\right] - \underline{c},$$

16. The support of a random variable is the set of points at which its density is positive.

17. In the proof of theorem 1, I occasionally use the notation $(y_L, \underline{y}]$. When $\underline{y}$ is finite, this has its usual meaning, but when $\underline{y}$ is infinite, this notation is intended to mean $\{y|y > y_L\}$. Also, since either x_L or y_L is allowed to equal $-\infty$, expressions such as $E[\tilde{e}|x_L]$ should be interpreted as shorthand for

$$\lim_{x \to x_L} E[\tilde{e}|x].$$

18. The upper boundary of the support $Y(x)$ of $f(y|x)$ is assumed to be increasing and continuously differentiable to insure that the function $\underline{y}(x)$, which defines the sets D_x through $D_x = (y_L, \underline{y}(x)] \cap Y(x)$, is continuously increasing. This property of the sets D_x simplifies the characterization of the sets of no, partial, and full disclosure.

19. If a hypothesis about a random variable is stated without qualifying its domain of application—e.g., $f_x(y|x)/f(y|x)$ is increasing—then the hypothesis is presumed to hold throughout the interior of the random variable's support.

then

$$D^N = \{(x, y) \in \Omega | K^* \geq E[\tilde{e}|x, y] - \bar{c}\},$$

D^P is empty, and D^F is the complement of D^N;
*c*ii) if

$$K^* \leq E\big[\tilde{e}|\bar{x}, \tilde{y} \in (y_L, \underline{y}(\bar{x})]\big] - \underline{c},$$

then there exists a unique x^* in X such that

$$D^N = \{(x, y) \in \Omega | \begin{cases} (a) & K^* \geq E[\tilde{e}|x, y] - \bar{c} \\ (b) & x^* \geq x\}, \end{cases}$$

$$D^P = \{(x, y) \in \Omega | (x, y) \notin D^N, y \leq \underline{y}(x)\},$$

and D^F is the complement of D^N and D^P; and

d) under the hypotheses of theorem 1*c*, among the credible policies, one that maximizes the exant probability of no disclosure is optimal.[20]

Before characterizing the disclosure policy further, I comment on the assumptions underlying theorem 1. Most of the density assumptions are standard. The one assumption that is not innocuous is condition iii, which assures that the sets D_x defined above are well defined. If $E[\tilde{e}|x, y]$ is of the form $h(x)y$ or $h(x) + y$ and $f(\tilde{y}|x)$ is exponential, uniform, or decreasing in $\tilde{y}$ with $f(y|x)$ bounded away from zero and infinity on its support, then this condition can be shown to be satisfied.[21] The assumption $E[\tilde{e}|x = x_L] < \underline{c}$ simply requires that, if x sufficiently close to x_L is observed, then the expected value of $\tilde{e}$ is less than the cost of disclosing x, so there would be no advantage to disclosing x by itself (though it may still be advantageous to disclose x in conjunction with y if y is sufficiently large). The assumption that $f_x(y|x)/f(y|x)$ is increasing in y is frequently interpreted to mean that x constitutes "good news" about y or that high realizations of x

20. In the general case in which $\underline{c}$ varies with x and $\bar{c}$ varies with y, theorem 1 becomes as follows.

THEOREM 1a. If, in addition to hypothesis i in theorem 1, $\underline{c}(x)$ and $\bar{c}(y)$ are continuously differentiable with $\bar{c}(y) > \underline{c}(x)$ for all (x, y) and $\underline{c}(x_L) > E[\tilde{e}|x_L]$, $E[\tilde{e}|x] - \underline{c}(x)$ increases continuously in x, $E[\tilde{e}|x, y] - E[\tilde{e}|x, \tilde{y} \leq y] - \bar{c}(y)$ strictly increases in y and is continuously differentiable, then the conclusions *a* and *b* of theorem 1 are valid. If, further, $E[\tilde{e}|x, \tilde{y} \leq y] - E[\tilde{e}|x, y] - \underline{c}(x)$ is differentiable and increasing in x, then the conclusions *c* and *d* of theorem 1 are valid. (If $\underline{c}(x)$ is constant, this last condition is implied by the hypotheses of theorem 1*c* in the text.)

The proof of this theorem follows the proof of theorem 1; details of the argument are available on request.

21. In cases in which $E[\tilde{e}|x, y] = h(x)y$ or $E[\tilde{e}|x, y] = h(x) + y$, where $h(x) \geq 0$, condition iii of theorem 1 is equivalent to $(\partial/\partial y)\ E[\tilde{y}|x, \tilde{y} \leq y] < 1$. The following proposition provides necessary and sufficient conditions for a nonnegative random variable $\tilde{y}$ with density $f(y)$ and distribution function $F(y)$ to satisfy this last condition. For all $y > 0$, $(d/dy)\ E[\tilde{y}|\tilde{y} \in [0, y]] < 1$ if and only if $\log \int_0^y F(y)dy$ is a concave function of y. This result follows by elementary calculus.

are indicative of high realizations of y (see Lehmann 1959; and Milgrom 1981); it is equivalent to the assumption that the family of densities $\{f(y|x)|x \in X\}$ possesses the monotone likelihood ratio property (see Milgrom 1981). The additional assumption in theorem 1c that $(\partial/\partial x)\ E[\tilde{e}|x, y]$ is weakly declining in y is satisfied whenever $E[\tilde{e}|x, y]$ is additively separable in x and y. This condition implies that, although increases in x constitute good news about cash flows, this good news aspect of high values of x is decreasing in y.[22]

The basic idea behind the proof of theorem 1 is "bootstrapping": first, attention is confined to disclosure policies for which something is always disclosed. The regions of partial and full disclosure are characterized for such disclosure policies through the sets D_x defined above. Second, disclosure policies are considered that provide the added option of disclosing nothing. For no disclosure to be desirable given realization (x, y) of $(\tilde{x}, \tilde{y})$, of course the price K of the firm with no disclosure must be higher than the price of the firm with partial disclosure $\{E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}\}$ or full disclosure $\{E[\tilde{e}|x, y] - \bar{c}\}$, that is, (x, y) must belong to the set

$$D^N(K) \equiv \{(x', y')|K \geq E[\tilde{e}|x', y'] - \bar{c},\ K \geq E[\tilde{e}|x', \tilde{y} \in D_{x'}] - \underline{c}\}.$$

However, investors who value the firm at its expected cash flows will pay only $E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D^N(K)] \equiv H(K)$ when the manager makes no disclosure and the manager's "no disclosure" set is believed to be $D^N(K)$. For an equilibrium to exist, $H(K)$ must equal K, in which case a credible disclosure policy can be constructed. The proofs of theorems 1a and 1b involve verifying the existence of a fixed point of $H(K)$. Such a fixed point clearly exists if $H(\cdot)$ is continuous, $H(0)$ is positive, and $H(K)$ is less than K for some K. Continuity of $H(\cdot)$ follows from the continuity of $E[\tilde{e}|x, y]$ and the existence of a density for the joint distribution of $(\tilde{x}, \tilde{y})$; $H(0) > 0$ follows from the assumption that

22. Some examples of random variables that satisfy the distributional assumptions of theorem 1 follow. Suppose $E[\tilde{e}|x, y] = x + y$. Let $f(y|x)$ be exponential with mean $x + 1$, i.e.,

$$f(y|x) = \frac{1}{(x + 1)} e^{-y/(x+1)},\ y > 0.$$

Condition iii of theorem 1 is equivalent here to the following condition:

$$\frac{d}{d\underline{y}} E\Big[\tilde{y}|x, \tilde{y} \in [0, \underline{y}]\Big] < 1$$

for all x, y. This condition is readily verified for exponential distributions. Note that $f_x(y|x)/f(y|x) = (y - 1)/(x + 1)$ is increasing in y. If $\tilde{x}$ has a (marginal) density that is twice continuously differentiable and its support is a finite interval in IR_+, then the hypotheses of theorem 1 hold for some $\underline{c}$, $\bar{c}$. As another example, let $E[\tilde{e}|x + y] = x + y$ and let $f(y|x)$ be distributed uniformly on $(0, x + 1) \equiv Y(x)$. Note that, for y in the interior of $Y(x)$, $f_x(y|x)$ is zero, so f_x/f is (trivially) increasing. Again, condition iii of theorem 1 is easily verified. Thus if the marginal distribution of x satisfies the conditions listed in the previous example, then the hypotheses of theorem 1 are satisfied for $\frac{1}{2} < \underline{c}$.

$\underline{c} > E[\tilde{e}|x_L]$; and $H(K) - K$ is negative for all large K because the unconditional mean of cash flows is finite. The characterizations of disclosure policies given in theorems 1c and 1d depend on having D_x be a subset of $D_{x'}$ for $x' > x$. The set D_x expands with x provided a sufficient number of monotonicity conditions—such as $f_x(y|x)/f(y|x)$, $E[\tilde{e}|x, y] - E[\tilde{e}|x, \tilde{y} \leq y]$, and $E[\tilde{e}|x, y]$ increasing in y; $E[\tilde{e}|x, y]$, $E[\tilde{e}|x]$ increasing in x; and $(\partial/\partial x)E[\tilde{e}|x, y]$ declining in y—hold.

This theorem has several implications, explained in points 1–7 below.

1. With positive probability, the manager's nonproprietary information is not disclosed under an optimal policy. This runs counter to the assertion that any nonproprietary information that investors know that a manager has must be disclosed. This assertion is true (with some additional qualifications [see Grossman 1981; Milgrom 1981; and Dye 1985]) when the manager is endowed only with nonproprietary information and he cannot commit to following a disclosure policy prior to receiving information since in that case the manager must distinguish the information he has from the worst information he possibly could have. But when the manager is endowed with both proprietary and nonproprietary information, the price of the firm may be highest on occasion with no release of information. The firm's value may not cascade downward with the manager's failure to disclose nonproprietary information since investors' demands for the firm's shares are based on their estimates of all the manager's information (and not simply the nonproprietary components of his information).

While the proof of theorem 1 relies on the assumption that the cost of disclosing nonproprietary information is positive (in order to satisfy the condition $E[\tilde{e}|\tilde{x} = x_L] < \underline{c}$), it is possible to construct examples for which the nondisclosure of nonproprietary information may occur with positive probability even if $\underline{c} = 0$; such an example is sketched in Appendix C. In that example, most of the density of $(\tilde{x}, \tilde{y})$—in what turns out to be the no-disclosure set D^N—is positive in the neighborhood of a curve described by the solution to $K_o + \bar{c} = E[\tilde{e}|x, y]$, for some constant K_o. The region $\{(x, y)|\, y$ belongs to $D_x\}$ lies substantially below this curve. Consequently, an announcement that $\tilde{x}$ equals x for $(\tilde{x}, \tilde{y}) = (x, y)$ is D^N produces a lower price for the firm than does the absence of an announcement—since the announcement $\tilde{x} = x$ reveals that the realization of $\tilde{y}$ is low, whereas no announcement is indicative of a high value for $\tilde{y}$. The fact that the manager is endowed with both proprietary and nonproprietary information may explain why nonproprietary information is not always disclosed, even if investors know the manager has the information and its disclosure is costless.

2. Though theorem 1 demonstrates that the manager's nonproprietary information is not disclosed with positive probability when he follows an optimal policy, a policy of absolutely no disclosure is typi-

cally not credible. A necessary and sufficient condition for such a policy to be credible is

$$1 = \mathrm{pr}(\{(x, y)|E[\tilde{e}] \geqslant \max\{E[\tilde{e}|x, y] - \bar{c}, E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}\}).$$

If $\bar{c}$ and $\underline{c}$ are sufficiently large, then this condition will be satisfied since $E[\tilde{e}|x, y]$ is bounded. When this condition holds, the optimal disclosure policy is a policy of no disclosure.

3. Under the hypotheses of theorem 1c, the sets corresponding to an equilibrium disclosure policy are depicted in figures 2 and 3. (The figures are drawn using the hypotheses of theorem 1c and the following additional assumptions:

$$x_L = y_L = 0,$$

$$\bar{x} < \infty, \bar{y}(x) \equiv \bar{y} < \infty,$$

$$E[\tilde{e}|x, \bar{y}] - E[\tilde{e}|x] > \bar{c} - \underline{c},$$

for all x.) The "northeastern" boundary of the no-disclosure set consists of those (x, y) satisfying $E[\tilde{e}|x, y] = K^* + \bar{c}$ for some positive constant K^*. It is downward sloping since $E[\tilde{e}|x, y]$ increases in x and y. For points below (above) this curve, no (full) disclosure is preferred

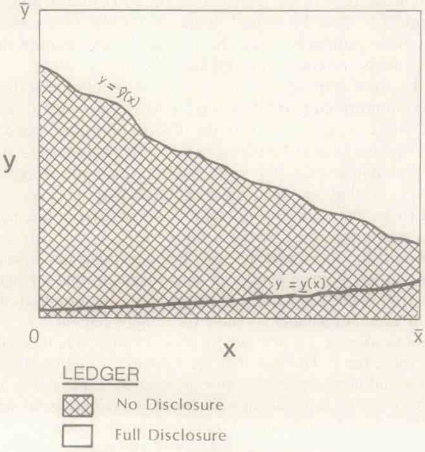


FIG. 2.—$\hat{y}(x)$ is defined by $K + \bar{c} \equiv E[\tilde{e}|x, \hat{y}(x)]$; $\underline{y}(x)$ is defined by $\bar{c} - \underline{c} \equiv E[\tilde{e}|x, \underline{y}(x)] - E[\tilde{e}|x, \tilde{y} \leqslant \underline{y}(x)]$.

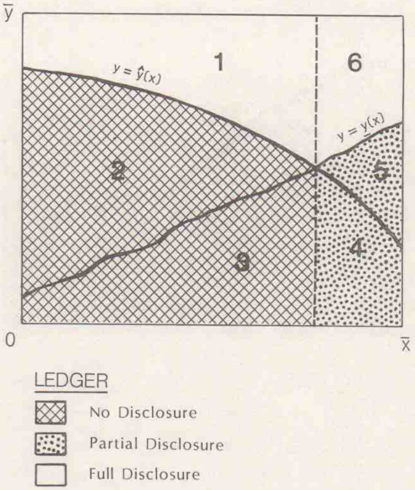


FIG. 3.—$g(x)$ is defined by $g(x) \equiv \max\{\underline{y}(x), \hat{y}(x)\}$

to full (no) disclosure. For (x, y) above (below) the function $y = \underline{y}(x)$, full (partial) disclosure is preferred to partial (full) disclosure. In figure 3, for points to the left (right) of the line $x = x^*$, no (partial) disclosure is preferred to partial (no) disclosure. Using these facts, it is straightforward to deduce the regions of no, partial, and full disclosure. Figure 2 (3) corresponds to case *c*i (*c*ii) of theorem 1*c*. As figure 2 illustrates, partial disclosure may be a zero-probability event. In figure 3, the region over which partial disclosure occurs consists of that portion of Ω below the line $y = \underline{y}(x)$, *not* in the no-disclosure set. In theorem 2*e* below, I indicate how changes in disclosure costs affect whether optimal policies entail a positive probability of partial disclosure.

4. Figures 2 and 3 illustrate that good news is more likely to be disclosed than bad news, that is, "large" (x, y) are more likely to be partially or fully disclosed than "small" (x, y). This result depends on the assumption that $f_x(y|x)/f(y|x)$ is increasing in y and illustrates the appropriateness of Milgrom's (1981) characterization of this condition (as x being good news about y). If this condition is not satisfied, there may exist (x, y) and (x', y') with $x' > x$ and $y' > y$, with partial

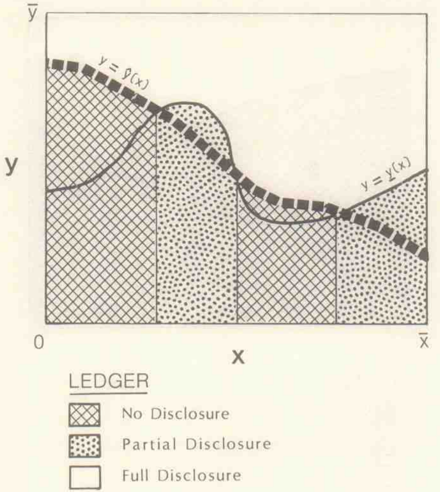


FIG. 4

disclosure occurring for (x, y) and no disclosure occurring for (x', y'). (See fig. 4.)[23] The assumption that the manager has an incentive to make value-increasing disclosures does not by itself assure that "high" (x, y) values are more likely to be disclosed than "low" (x, y) values.

5. Let N, P, and F represent, respectively, no, partial, and full disclosure, and read $N > P$ as "no disclosure produces a higher market 2 price for the firm than does partial disclosure"; the other relations are similar. Then for figure 3 we have $F > N > P$ in region 1, $N > F > P$ in region 2, $N > P > F$ in region 3, $P > N > F$ in region 4, $P > F > N$ in region 5, and $F > P > N$ in region 6. Notice that ranking any two of the three possible acts of disclosure implies nothing about the ranking of the third act since each conceivable ordering of preference among acts is represented as an event that occurs with nonzero probability. In particular note that there exist regions for which full disclosure is preferable to partial disclosure but for which full disclosure is not preferable to no disclosure. That is, the market 2 price of the firm with full disclosure $(E[\tilde{e}|x, y] - \bar{c})$ may exceed the price of the firm with partial

23. The details of an example in which $\underline{y}(x)$ is not monotonic and the optimal disclosure policy takes a form similar to that depicted in fig. 4 are available from the author.

disclosure ($E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}$), so that announcement of y, given that x has been disclosed, increases the price of the firm, in spite of the fact that the price of the firm if no disclosures had been made ($E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D^N]$) would be higher than its price with either partial or full disclosures. Thus even though investors may be better off in some cases with full disclosure than with partial disclosure, this does not imply that they are better off with full disclosure than with no disclosure whatsoever.

6. Though this paper contains no rationale for mandatory disclosures, the results of the paper can be used to illustrate some possible effects of exogenous changes in mandated disclosures on firms' voluntary disclosure policies. Explicitly, mandatory and voluntary disclosures are complements when mandatory disclosures consist of reports of a firm's nonproprietary information. Figure 3 illustrates the full disclosure set by means of the unshaded regions 1 and 6 when there are no mandatory disclosures. If $\tilde{x}$ must be disclosed through mandatory reports, full disclosure occurs whenever full disclosure yields a higher price for the firm than does partial disclosure, that is, for all y above $y = \underline{y}(x)$ (regions 1, 2, and 6). Mandatory disclosures of nonproprietary information may increase the circumstances under which full disclosure occurs. As was noted in Section I, this result follows from the observation that, as mandatory reporting requirements increase, the number of conditions that have to be satisfied for full disclosure to be the value-maximizing disclosure decline.[24]

7. The predictions of a voluntary disclosure theory are not independent of other reporting requirements. (This is also illustrated by point 6.) Consider the following hypothesis regarding the lower boundary $y = g(x)$ of the full-disclosure set (see fig. 3).

H_o^A. When full disclosure of (x, y) occurs, $y \geqslant g(x)$.

If there are no mandatory reporting requirements, the hypothesis is true. However, if all nonproprietary information must be disclosed in financial reports, then H_o^A is false since the full disclosure set then shifts (see point 6). Disclosure policies change as reporting requirements—and, more generally, as public information available about the firm (i.e., investors' "information sets")—change. Most hypotheses regarding disclosure policies must be conditioned on what investors know prior to the disclosure.

I conclude this section by considering the effects of exogenous changes in $\underline{c}$, $\bar{c}$ on the optimal disclosure policy. To present the results, the previous notation is parametrized by the costs of disclosure. For example, the region over which no disclosure occurs in an optimal

24. This is a partial equilibrium result. If the distributions of $\tilde{x}$ and $\tilde{y}$ shift when mandatory disclosures change, then the result may not hold. See Sec. V.

policy is now identified by the label $D^N(\underline{c}, \bar{c})$ when $(\underline{c}, \bar{c})$ denotes the disclosure costs. $D^P(\underline{c}, \bar{c})$ and $D^F(\underline{c}, \bar{c})$ are defined similarly.

The following theorem summarizes the comparative static results. Verbal interpretations of the results of this theorem succeed its proof.

THEOREM 2.[25] Under the hypotheses of theorem 1*c*,

a) if $(\underline{c}, \bar{c})$ is increased to $(\underline{c}', \bar{c}') \geq (\underline{c}, \bar{c})$, where $\bar{c} - \underline{c} \geq \bar{c}' - \underline{c}'$, then

$$D^N(\underline{c}, \bar{c}) \subset D^N(\underline{c}', \bar{c}');$$

b) if $(\underline{c}, \bar{c})$ is increased to $(\underline{c}', \bar{c}') = (\underline{c} + h, \bar{c} + h)$, then

$$D^P(\underline{c}', \bar{c}') = D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}') \subset D^P(\underline{c}, \bar{c}),$$

$$D^F(\underline{c}', \bar{c}') = D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}') \subset D^F(\underline{c}, \bar{c});$$

c) if $\bar{c}$ is increased to $\bar{c}' \geq \bar{c}$, with $\underline{c}$ held fixed, then

$$D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c}') \subset D^P(\underline{c}, \bar{c}');$$

d) if $\underline{c}$ is increased to $\underline{c}' \geq \underline{c}$, with $\bar{c}$ held fixed, then

$$D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}) \subset D^F(\underline{c}', \bar{c})$$

and

$$D^P(\underline{c}', \bar{c}) \subset D^P(\underline{c}, \bar{c}); \text{ and}$$

e) if $D^P(\underline{c}, \bar{c})$ is empty, $(\underline{c}', \bar{c}') \geq (\underline{c}, \bar{c})$, and $\bar{c} - \underline{c} \geq \bar{c}' - \underline{c}'$, then $D^P(\underline{c}', \bar{c}')$ is empty.

Theorem 2 contains the following implications.

a) As the cost of disclosing either x or y (or both) increases but the difference in the cost of disclosing x or y declines, the probability that no disclosure will occur increases.

b) If the cost of disclosing x and y increases by the same amount, the probability that partial disclosure occurs decreases, as does the probability that full disclosure occurs.

c) Conditional on the event that something is disclosed given costs $(\underline{c}, \bar{c}')$, the probability that partial disclosure occurs increases with the cost $\bar{c}'$ of full disclosure.

d) If the cost of disclosing x increases and the cost of disclosing y

25. In the general case in which $\underline{c}$ varies with x and $\bar{c}$ varies with y, theorem 2 becomes as follows.

THEOREM 2a. If all the hypotheses of theorem 1a are satisfied for the two pairs of cost functions $[\underline{c}(\cdot), \bar{c}(\cdot)]$ and $[\underline{c}'(\cdot), \bar{c}'(\cdot)]$, and if all other conditions listed in theorem 1a hold, then each of the conclusions of theorem 2 is valid provided the hypotheses of theorem 2 are required to hold uniformly in (x, y). For example, theorem 2a now reads: if $\underline{c}'(x) \geq \underline{c}'(x)$, $\bar{c}'(y) \geq \bar{c}(y)$, and $\bar{c}(y) - \underline{c}(x) \geq \bar{c}'(y) - \underline{c}'(x)$ hold for every (x, y), then $D^N[\underline{c}(\cdot), \bar{c}(\cdot)] \subset D^N[\underline{c}'(\cdot), \bar{c}'(\cdot)]$.

The proof of theorem 2a is similar to that of theorem 2; details are available on request.

is fixed, then the probability that partial disclosure will occur declines.

e) If there is no optimal disclosure policy where the probability of partial disclosure is positive given disclosure costs $(\underline{c}, \bar{c})$, then there is no optimal disclosure policy with partial disclosure given disclosure costs $(\underline{c}', \bar{c}') \geqslant (\underline{c}, \bar{c})$, where $\bar{c} - \underline{c} \geqslant \bar{c}' - \underline{c}'$; that is, if figure 2 depicts the optimal disclosure policy given $(\underline{c}, \bar{c})$, then the optimal disclosure policy given $(\underline{c}', \bar{c}')$ is depicted by a diagram similar to figure 2.

As the various conclusions of theorem 2 illustrate, how optimal disclosure policies change in response to changes in the costs of disclosure depends both on the absolute costs of disclosure and on the relative costs of the disclosure of proprietary and nonproprietary information.

IV. Alternative Motivation for Credible Disclosure Policies

The disclosure policies discussed in Sections II and III were motivated by concerns over management incentive problems: the disclosure policies that investors anticipated managers were willing to implement were restricted by the assumption that the only management compensation schemes that had acceptable incentive effects increased in the price of the firm. In this section, I consider briefly what disclosure policies investors would seek to implement when such incentive problems are absent, that is, when any disclosure policy can be implemented.

If shareholders derive utility from consumption only after the realization of $\tilde{e}$ becomes known, then the desirability of disclosures depends on the timing and completeness of markets that open before $\tilde{e}$ realizes its value. If no market opened prior to the disclosure of information (i.e., if market 1 did not exist), then costless disclosures are typically undesirable because of the adverse risk-sharing effects induced by such disclosures (see, e.g., Marshall 1984; and Wilson 1975). A fortiori, costly disclosures are undesirable in such situations. If market 1 were complete and disclosures were costless, then all shareholders would be indifferent among disclosure policies and market 2 would be inactive (Marshall 1974). For this case, the optimal disclosure policy in the presence of disclosure costs would be a policy of no disclosure. In the general case in which the market for the firm's shares prior to disclosure is incomplete and disclosures are costless, there exists a (pre–market 1) redistribution of endowments among shareholders so that no shareholder is made worse off by allowing complete disclosure, but, as the costs of partial or complete disclosure grow sufficiently large, this (weak) desirability of disclosures, even with endowment redistributions, disappears.

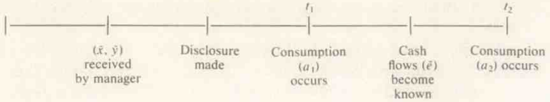


FIG. 5

If investors' consumption decisions depend on the disclosures made by the firm, then disclosures can be valuable even if no trading occurs in the firm's shares. As the following discussion indicates, disclosure policies chosen to account for the advantages deriving from the early resolution of uncertainty in such cases may appear to be quite similar to the value-maximizing disclosure policies presented in Section III.

Suppose that all individuals have the same preferences, initial wealths (W), and endowments in the shares of the firm (so no trading in the firm's stock occurs) and that consumption occurs as indicated in the time line in figure 5. For analytical convenience, further suppose that each shareholder's preferences for consumption are given by $U(a_1, a_2) = a_1 a_2$ and that cash flows are rescaled so that $\tilde{e}$ denotes the cash flows from the firm received by each shareholder. Since all individuals are identical, any one of them can be delegated the responsibility of selecting the firm's disclosure policy.

If there were no costs associated with disclosure, then each shareholder would prefer a policy of full disclosure since such a policy enhances each investor's consumption-investment decision. To provide a simple illustration of this claim, let $r = 1$ be the gross certain return on wealth W not invested in the firm at t_1 over the interval $[t_1, t_2]$, and suppose there are no limits on borrowing or lending. Then for any disclosure policy $d(\cdot, \cdot)$ we have the inequality

$$E\Big[\max_{a_1} E[a_1(W - a_1 + \tilde{e})|\tilde{x}, \tilde{y}]\Big] \geq E\Big[\max_{a_1} E[a_1(W - a_1 + \tilde{e})|d(\tilde{x}, \tilde{y})]\Big],$$

thus establishing that full disclosure is the expected utility-maximizing policy in the absence of disclosure costs. This simply illustrates the advantages associated with the early resolution of uncertainty (Kreps and Porteus 1979).

Now return to the case in which there are disclosure costs attending the disclosure of x or y. Shareholders now confront a trade-off: while there are advantages to having consumption-investment decisions depend on observations of $(\tilde{x}, \tilde{y})$, such observations reduce the cash flows the shareholder receives from the firm. If we let D^N, D^P, and D^F denote the regions of no, partial, and full disclosure, and if we let D_x be the set of y's for which shareholders choose to be informed of x, but not y, for (x, y) in D^P (note that D_x is not known at this point to be related to the set D_x defined in Sec. II), then the shareholder charged with choosing the firm's disclosure policy confronts the following problem. (The reason for defining D_x for all x in X will be apparent below.)

PROGRAM 1:

$$\max_{D^N, D^P, D^F, \{D_x | x \in X\}, a_1, a_1(\cdot), a_1(\cdot,\cdot)} E[a_1(W - a_1 + \tilde{e}) | (\tilde{x}, \tilde{y}) \in D^N] \mathrm{pr}[(\tilde{x}, \tilde{y}) \in D^N]$$
$$+ E\big[E[a_1(x)(W - a_1(x) + \tilde{e} - \underline{c}) | x, \tilde{y} \in D_x]\big] \mathrm{pr}[(\tilde{x}, \tilde{y}) \in D^P]$$
$$+ E\big[E[a_1(x, y)(W - a_1(x, y) + \tilde{e} - \bar{c}) | x, y]\big] \mathrm{pr}[(\tilde{x}, \tilde{y}) \in D^F].$$

The objective function of this program represents the shareholder's expected utility when disclosure costs (and cash flows) have been scaled on a per-shareholder basis. For $(x, y) \in D^N$, the shareholder incurs no disclosure costs, but his or her consumption decision is based only on the knowledge that (x, y) belongs to D^N. For $(x, y) \in D^P$, the shareholder learns $\tilde{x} = x$ and $\tilde{y} \in D_x$ and chooses his or her time t_1 consumption accordingly, but cash flows drop by $\underline{c}$ on x's disclosure. For $(x, y) \in D^F$, the shareholder's consumption at t_1, $a_1(x, y)$, is based on knowledge of the realizations of both $\tilde{x}$ and $\tilde{y}$ at the cost of having cash flows drop by $\bar{c}$.

A solution of this program is straightforward to characterize if we assume that $W - \bar{c} > 0$ and $\tilde{e} \geq 0$. If D^N, D^P, D^F, $\{D_x | x \in X\}$, a_1, $a_1(x)$, $a_1(x, y)$, is a solution, then

$$a_1 \in \arg_a \max E[a(W - a + \tilde{e}) | (\tilde{x}, \tilde{y}) \in D^N] \tag{i}$$

if D^N is a nonzero probability event,

$$a_1(x) \in \arg_a \max E[a(W - a + \tilde{e} - \underline{c}) | x, \tilde{y} \in D_x], \tag{ii}$$

and

$$a_1(x, y) \in \arg_a \max E[a(W - a + \tilde{e} - \bar{c}) | x, y], \tag{iii}$$

by the assumption that a_1, $a_1(\cdot)$, and $a_1(\cdot, \cdot)$ are part of the specification of a solution to program 1. Further, for almost all (x, y) in D^P, the following inequality must be satisfied:

$$\begin{aligned} &E\big[a_1(x)(W - a_1(x) + \tilde{e} - \underline{c}) | x, \tilde{y} \in D_x\big] \\ &\geq E\big[a_1(x, y)(W - a_1(x, y) + \tilde{e} - \bar{c}) | x, y\big]. \end{aligned} \tag{2}$$

If this inequality were strictly reversed on a subset $\hat{D}^P$ of D^P of positive probability, it is easy to show that the assumption that D^P is part of the specification of a utility-maximizing disclosure policy is contradicted.[26]

26. If inequality (2) is reversed on $\hat{D}^P$, then it is clear that the following disclosure policy provides strictly higher utility than does the optimum posited in the text: define $\bar{D}^F = D^F \cup \hat{D}^P$, $\bar{D}^P = D^P \setminus \hat{D}^P$, $\bar{D}^N = D^N$, and $\{\bar{D}_x = D_x \setminus \{y | \text{inequality (2) is reversed for some } (x, y)\} | (x, y') \in \bar{D}^P \text{ for some } y'\}$, leaving a_1, $a_1(\cdot)$, and $a_1(\cdot, \cdot)$ unchanged.

Solving for $a_1(x)$ and $a_1(x, y)$ in (ii) and (iii) above, we observe that inequality (2) is equivalent to

$$\{W + E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}\}^2 \geq \{W + E[\tilde{e}|x, y] - \bar{c}\}^2$$

or, since $\tilde{e}$, $W - \bar{c} \geq 0$,

$$E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \geq E[\tilde{e}|x, y] - \bar{c}, \tag{3}$$

for almost all $(x, y) \in D^P$ and $y \in D_x$. Similarly it can be shown that, if D^N has nonzero probability,

$$E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \geq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D^N], \tag{4}$$

for almost all $(x, y) \in D^P$ and $y \in D_x$. Furthermore, by including any (x, y) satisfying inequalities (3) and (4) (not already in D^P) in the partial disclosure set, it is clear that the objective function in program 1 does not decline, so the set of (x, y)'s satisfying (3) and (4) can be taken as defining an expected utility-maximizing partial disclosure set.[27] That is, a solution to program 1 exists for which partial disclosure is sought by the shareholder choosing the firm's disclosure policy given $(\tilde{x}, \tilde{y}) = (x, y)$ if and only if partial disclosure yields higher expected cash flows net of disclosure costs than either full disclosure or (if the no-disclosure set has nonzero probability) no disclosure.

By performing similar analyses on the no- and full-disclosure sets, I conclude that, if there is an expected utility-maximizing disclosure policy where the probability of no disclosure is positive, then there is an expected utility-maximizing disclosure policy determined by choosing that disclosure that maximizes the firm's expected cash flows net of disclosure costs among all conceivable disclosures for every realization of $(\tilde{x}, \tilde{y})$. That is, under these conditions, *there exists an expected utility-maximizing disclosure policy that is credible*. Further, it can be shown, under the hypotheses of theorem 1, that a solution to program 1 does exist where the probability of no disclosure is positive, and, in fact, under the hypotheses of theorem 1*c*, a solution to program 1 exists that is optimal in the sense of definition 2. In this event, theorem 1*c*i or theorem 1*c*ii characterizes a solution to program 1.

The reason that an expected utility-maximizing disclosure policy is "optimal" is that, conditional on the disclosures made, the expected utility of every shareholder is increasing in the expected cash flows of the firm net of disclosure costs, just as the price of the firm is increasing in expected cash flows in the model of Section II. An optimal disclosure policy is always expected utility maximizing whenever shareholders' expected utilities increase in the firm's expected net cash flows.

27. This is the reason for defining D_x for all x in X rather than defining D_x only for when there exists y with (x, y) in D^P: if (x, y) in Ω satisfies inequalities (3) and (4) for some x, then (x, y) should be included in D^P even if it was not included in the original definition of D^P.

This discussion provides justification for analyzing value-maximizing disclosure policies directly on the basis of shareholders' preferences and gives a rationale for studying credible disclosure policies without regard to the incentive effects of such policies.

V. Conclusions and Caveats

In the preceding sections, a simple model of disclosure was developed for the manager of a firm endowed with both proprietary and nonproprietary information. Disclosure policies were characterized, and conditions under which less than complete disclosure occurred were identified. The model analyzed has several testable implications, but, as was pointed out in Section III, tests of many of the results of this disclosure theory, as well as most others, will be difficult to implement. The problem is that most disclosure hypotheses are conditional on other information available to investors prior to disclosure.

How exogenous changes in mandatory reporting requirements affect firms' voluntary disclosure policies was also considered. That reporting requirements in practice do not change for exogenous reasons is important to remember in attempting to apply the paper's results to actual disclosure practices. For example, accounting standards might depend on what disclosures firms make voluntarily, and it is unclear a priori whether increases in voluntary disclosures induce more or less mandatory reporting requirements (a lack of voluntary disclosures could generate a demand for additional information through compulsory disclosures; the existence of many voluntary disclosures of a similar nature could generate a demand for codification). The complementary relation between mandatory reports and voluntary disclosures exhibited in Section III could be offset—or amplified—by such feedback effects.

Actual disclosures take place in an environment that is substantially more complex than the one studied here. Managers may be concerned with how disclosures about their firms' financial performance impinge on their reputations, influence their firms' negotiations with labor unions and other suppliers of inputs, affect their firms' relationships with governmental and other authoritative bodies (including accounting boards who have the jurisdiction to mandate additional disclosures), change the behavior of their competitors, etcetera. There is a common thread in these problems, however. There is tension between the managers' incentives to disclose information that reveals both their own and their firms' performance and the incentives to avoid the adverse reactions of parties external to the firm induced by disclosures. This paper has examined some of the elemental forces that contribute to this tension by examining the trade-offs involved in achieving value maximization and protecting proprietary information.

Appendix A
Proof of Theorem 1

Proof. The proof of theorem 1 proceeds in five steps: first, I show that, for any disclosure policy, the set of y's for which only x (but not (x, y)) is disclosed is well defined and independent of the specification of the no-disclosure set; second, I show that a credible disclosure policy, with a no-disclosure set having positive probability, exists; third, I show that no policy consisting of having something disclosed under all circumstances is credible; fourth, I show that disclosure policies must take one of the forms specified in conclusion *c;* finally, I prove theorem 1*d*.

First. Suppose that investors believe D^N is the set over which no disclosure occurs and that, if the manager discloses x, but not y, investors believe that $\tilde{y} \in D_x \subset Y(x)$. Given these beliefs, the actual subset $\hat{D}_x$ of $Y(x)$ for which x, but not y, is disclosed is given by

$$\hat{D}_x = \{y \in Y(x) | \begin{cases} (a) & E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \geqslant E[\tilde{e}|x, y] - \bar{c} \\ (b) & E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \geqslant E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D^N] \end{cases}\}$$

in the case in which D^N is a positive probability event; if D^N is a probability zero event, the right-hand side (RHS) of condition b in the definition of $\hat{D}_x$ is replaced by zero by appealing to the off-equilibrium specifications made in the text. This is the set of y's for which the price of the firm is higher by disclosing only x rather than disclosing x and y or nothing. Notice that condition b in either definition of $\hat{D}_x$ is superfluous since, although $\tilde{y}$ appears in b, the actual realization y does not. Thus whether or not D^N is a positive probability event,

$$\hat{D}_x = \{y \in Y(x) | E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \geqslant E[\tilde{e}|x, y] - \bar{c}\}. \tag{A1}$$

Lemma 1 below identifies sufficient conditions for $\hat{D}_x = D_x$.

Lemma 1. Under the hypotheses of theorem 1, for each x there exists a unique set D_x such that $\hat{D}_x = D_x$ in (A1). D_x is of the form

$$D_x = (y_L, \underline{y}(x)] \cap Y(x),$$

where $\underline{y}(x)$ is a continuous function. If $\underline{y}(x) < \bar{y}(x)$, then $\underline{y}(x)$ satisfies

$$\bar{c} - \underline{c} = E[\tilde{e}|x, \underline{y}(x)] - E[\tilde{e}|x, \tilde{y} \leqslant \underline{y}(x)]. \tag{A2}$$

Proof. Fix D_x.[28] Since $E[\tilde{e}|x, \tilde{y} \in D_x]$ is a constant and $E[\tilde{e}|x, y]$ increases in y, it follows that $\hat{D}_x$ defined through D_x by (A1) above is an interval, $\hat{D}_x = (y_L, \underline{y}]$.

Since

$$E[\tilde{e}|x, y] - E[\tilde{e}|x, \tilde{y} \leqslant y] \tag{A3}$$

is strictly, continuously increasing in y and equals

$$E[\tilde{e}|x, \bar{y}(x)] - E[\tilde{e}|x] \tag{A4}$$

28. If the manager discloses only x, then—since investors know that some realization of $\tilde{y}$ must have occurred—investors must form some conception of D_x, so D_x can be taken to be nonempty. In equilibrium, it may turn out that investors never expect only x to be disclosed, but the manager must be able to anticipate investors' reactions if x is the only disclosure. Thus D_x must be defined (and nonempty) for each x.

when $y = \bar{y}(x)$, it follows, when (A4) strictly exceeds $\bar{c} - \underline{c}$, that there exists a unique $\underline{y}$, denoted $y^*(x)$, solving

$$\bar{c} - \underline{c} = E[\tilde{e}|x, \underline{y}] - E[\tilde{e}|x, \tilde{y} \leq \underline{y}]$$

as (A3) equals zero when evaluated at $y = y_L$. In this case, $D_x = \hat{D}_x = (y_L, y^*(x)]$. If (A4) is less than or equal to $\bar{c} - \underline{c}$, then $D_x = \hat{D}_x$ if and only if $D_x = Y(x) = (y_L, \bar{y}(x)]$. In either case, $\hat{D}_x = D_x$ if and only if $D_x = (y_L, \underline{y}(x)] \cap Y(x)$, where

$$\underline{y}(x) = \begin{cases} y^*(x), x \text{ in } \mathbf{O} \equiv \{x \in (x_L, \bar{x})|E[\tilde{e}|x, \bar{y}(x)] - E[\tilde{e}|x] > \bar{c} - \underline{c}\} \\ \bar{y}(x), \text{ otherwise.} \end{cases}$$

The function $L(x, y)$, defined by $L(x, y) = E[\tilde{e}|x, y] - E[\tilde{e}|x, \tilde{y} \leq y]$ is by assumption continuously differentiable and strictly increasing, so the implicit function theorem can be applied, yielding $y^*(x)$ differentiable in x on the set $\mathbf{O}$. Using this fact, it is straightforward to show that $\underline{y}(x)$ is continuous,[29] completing the proof of the lemma.

It is important that the definition of D_x derived from lemma 1 does not depend on the set D^N on which investors believe the manager discloses nothing.

Second. I now show how to construct a credible disclosure policy for which nothing is disclosed on a set of positive probability. The problem is simply this: find a set $D \equiv D^N$ for which

$$D = \{(x, y) \in \Omega| \begin{cases} (a) & E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D] \geq E[\tilde{e}|x, y] - \bar{c} \\ (b) & E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D] \geq E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}\}. \end{cases}$$

The variable D is defined in terms of itself. To show that D is well defined, let

$$D(K) = \{(x, y) \in \Omega| \begin{cases} (a) & K \geq E[\tilde{e}|x, y] - \bar{c} \\ (b) & K \geq E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c}\} \end{cases} \tag{A5}$$

and

$$H(K) = E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K)]. \tag{A6}$$

From well-known results on the convergence of measures (see, e.g., Feller 1966, ch. 8), it follows that $H(K)$ is continuous in K. Obviously,

$$D(K') \supset D(K)$$

for $K' \geq K$ and

$$\lim_{K \to \infty} H(K) = E[\tilde{e}].$$

29. Obviously, the only points at which $\underline{y}(x)$ might not be continuous are on the boundary B of the set $\mathbf{O}$. Consider some x in $\bar{B}$. Clearly, $E[\tilde{e}|x, \bar{y}(x)] - E[\tilde{e}|x] = \bar{c} - \underline{c}$, so the definition of $y^*(x)$ can be extended to $\mathbf{O} \cup B$. (If x is in B, set $y^*(x) \equiv \bar{y}(x)$.) If $\{x_n\} \subset X$ is any sequence converging to x, $\{x_n\}$ can be decomposed into two (disjoint) subsequences $\{x_{n_1}\}$ and $\{x_{n_2}\}$ such that $\{x_{n_1}\} = \{x_n\} \cap \mathbf{O}$, $\{x_{n_2}\} = \{x_n\} \setminus \mathbf{O}$. Clearly,

$$\lim_{n' \to \infty} y^*(x_{n_1}) = y^*(x) = \bar{y}(x), \quad \lim_{n'' \to \infty} \bar{y}(x_{n_2}) = \bar{y}(x),$$

since y^* and $\bar{y}$ are continuous (on their respective domains), so $\lim \underline{y}(x_n) = \bar{y}(x) \equiv \underline{y}(x)$, so $\underline{y}$ is continuous on B and, therefore, on X.

Since $E[\tilde{e}]$ is finite, $H(K) - K$ is negative for K sufficiently large. I next claim that $H(0) > 0$. To prove this, it suffices to show that the (unconditional) probability of the event $D(0)$ is positive since $E[\tilde{e}|x, y]$ is positive for all x and y. To that end, notice that $D(0)$ contains the set

$$\{(x, y)|\bar{c} \geqslant E[\tilde{e}|x, y], \underline{c} \geqslant E[\tilde{e}|x]\} \tag{A7}$$

since $E[\tilde{e}|x, y]$ and $E[\tilde{e}|x, \tilde{y} \leqslant y]$ are increasing in y, with $E[\tilde{e}|x] = E[\tilde{e}|x, \tilde{y} \leqslant \bar{y}(x)]$. Since $\bar{c} > \underline{c} > E[\tilde{e}|x_L] > E[\tilde{e}|x_L, y_L]$ (see the hypotheses) and $E[\tilde{e}|x]$ and $E[\tilde{e}|x, y]$ are, respectively, continuous functions of x and x, y, it follows that there exist $\hat{x} > x_L$, $\hat{y} > y_L$, such that all (x, y) in $(x_L, \hat{x}) \times (y_L, \hat{y})$ belong to (A7). Since $f(x, y)$ is positive for (x, y) belonging to $(x_L, \bar{x}) \times (y_L, \bar{y}(x_L))$ (as $f(x, y) = f(y|x)g(x) > 0$ for $y \in Y(x) \supset Y(x_L)$ and $x \in (x_L, \bar{x})$), it follows that $D(0)$ is a positive probability event, so $H(0) > 0$. Thus $H(K) - K$ is a continuous function that is positive at zero and negative for large K. Therefore there exists some positive K, say K^*, satisfying

$$H(K^*) = K^*.$$

Since the set $D(K)$ expands in K, $\text{pr}[D(K^*)]$ is positive. The set $D(K^*) \equiv D^N$ constitutes an equilibrium set over which no disclosure occurs. Define D^P and D^F as follows. If $(x, y) \notin D^N$, and $y \in D_x$, then $(x, y) \in D^P$. If $(x, y) \notin D^N$ and $y \notin D_x$, then $(x, y) \in D^F$. This describes a credible disclosure policy d, with no disclosure occurring on a set of positive probability.

Third. Suppose investors believe the manager adopts a disclosure policy for which something is always disclosed. Then since

$$\inf_{(x,y) \in \Omega} E[\tilde{e}|x, y]$$

equals zero and $E[\tilde{e}|x_L] < \underline{c}$, it follows that there exists x, y with $E[\tilde{e}|x, y] - \bar{c} < 0$ and $E[\tilde{e}|x, \tilde{y} \leqslant y(x)] - \underline{c} \leqslant E[\tilde{e}|x] - \underline{c} < 0$, so given the assignment of off-equilibrium "beliefs," the price of the firm is highest by making no disclosure. Therefore the original disclosure policy is not credible.

Fourth. To show that $y(x)$ is increasing, note that equation (A2) above with the substitution of $y^*(x)$ for $\underline{y}$ is an identity in x for x in **O**. On differentiating it, we obtain

$$\begin{aligned}
&\frac{\partial}{\partial \underline{y}} \{E[\tilde{e}|x, \underline{y}] - E[\tilde{e}|x, \tilde{y} \leqslant \underline{y}]\}|_{\underline{y}=y^*(x)}\, y^{*\prime}(x) \\
&= E\left[E[\tilde{e}|x, \tilde{y}] \frac{f_x(\tilde{y}|x)}{f(\tilde{y}|x)} |x, \tilde{y} \leqslant y^*(x)\right] \\
&- E[\tilde{e}|x, \tilde{y} \leqslant y^*(x)]E\left[\frac{f_x(\tilde{y}|x)}{f(\tilde{y}|x)} |x, \tilde{y} \leqslant y^*(x)\right] \\
&+ E\left[\frac{\partial}{\partial x} E[\tilde{e}|x, \tilde{y}]|x, \tilde{y} \leqslant y^*(x)\right] - \frac{\partial}{\partial x} E[\tilde{e}|x, \underline{y}]|_{\underline{y}=y^*(x)},
\end{aligned}$$

$f_x(y|x)/f(y|x)$ is increasing in y. Also, $E[\tilde{e}|x, y]$ increases in y. Thus $E[\tilde{e}|x, \tilde{y}]$ and $f_x(\tilde{y}|x)/f(\tilde{y}|x)$ have nonnegative covariance. Therefore the first two RHS terms clearly sum to a nonnegative number. The sum of the last two terms is also nonnegative because of the assumption that $(\partial/\partial x)\, E[\tilde{e}|x, y]$ is *declining* in

y for each x. The coefficient of $y^{*\prime}(x)$ is positive by assumption. Therefore $y^*(x)$ increases in x.

The variable $\underline{y}(x)$ inherits monotonicity from $y^*(x)$ and $\bar{y}(x)$.

I now show that the characterization of the disclosure policy must be either of the form ci or $c$ii as described in the statement of theorem 1. First, recall that

$$K^* = E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K^*)],$$

where

$$D(K^*) = \{(x, y) \in \Omega| \begin{cases} (a) & K^* \geq E[\tilde{e}|x, y] - \bar{c} \\ (b) & K^* \geq E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)] - \underline{c}\}. \end{cases}$$

It can be shown[30] that $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)]$ increases in x. Therefore, either

i) $K^* \geq E[\tilde{e}|\bar{x}, \tilde{y} \leq \underline{y}(\bar{x})] - \underline{c}$, or
ii) there exists $x^* \in (x_L, \bar{x})$ such that

$$K^* = E[\tilde{e}|x^*, \tilde{y} \leq \underline{y}(x^*)] - \underline{c}$$

($x^* > x_L$ since $D(K^*)$ has positive probability of occurrence). In the first case,

$$D(K^*) = \{(x, y) \in \Omega | K^* \geq E[\tilde{e}|x, y] - \bar{c}\}.$$

Moreover, since $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)]$ increases in x, i implies that partial disclosure is never optimal, that is, that D^P is empty in case i. The set D^F is simply the complement of $D(K^*)$ in case i. In case ii,

$$D(K^*) = \{(x, y) \in \Omega| \begin{cases} (a) & K^* \geq E[\tilde{e}|x, y] - \bar{c} \\ (b) & x^* \geq x\}. \end{cases}$$

The variables D^P and D^F are defined in the obvious manner.

Fifth. If K and K', respectively, define disclosure policies (D^N, D^P, D^F) and (D'^N, D'^P, D'^F) through

$$H(K) = E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K)],$$

$$D(K) = D^N,$$

$$D^P = \{(x, y) \notin D^N | y \in D_x\},$$

and

$$D^F = \Omega \setminus (D^N \cup D^P)$$

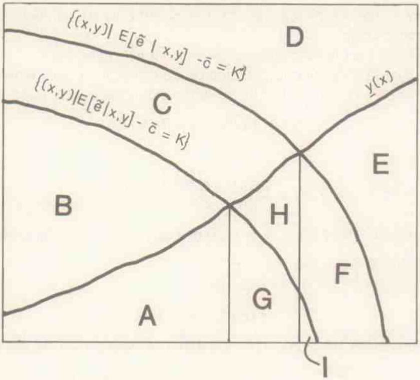
(and similarly for $K' > K$), and

$$P(d(x, y)) \equiv P(d(x, y)|d)$$

and

$$P'(d'(x, y)) \equiv P(d'(x, y)|d'),$$

30. If $x \in \mathbf{O}$, $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)] = E[\tilde{e}|x, \underline{y}(x)] - (\bar{c} - \underline{c})$, so $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)]$ inherits monotonicity from y^*. If x is the interior of the complement of $\mathbf{O}$, $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)] = E[\tilde{e}|x]$, which is increasing in x by assumption. If x is on the boundary of $\mathbf{O}$ and is the complement of $\mathbf{O}$, then $E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x)]$ is increasing from both the right and the left, so it is increasing throughout X.



(i) $D^N = A \cup B$

(ii) $D^F \cap D'^N = C$

(iii) $D^F \cap D'^F = D$

(iv) $D^P \cap D'^N = G \cup H$

(v) $D^P \cap D'^P = E \cup F \cup I$

FIG. A1

respectively, define the market 2 equilibrium prices of the firm given realization (x, y) of $(\tilde{x}, \tilde{y})$ under disclosure policies d and d', then,

i) $(x, y) \in D^N \subset D'^N$ implies

$$P\big(d(x, y)\big) = E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K)] \leqslant E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K')] = P'\big(d'(x, y)\big),$$

the inequality following from the fact that the only (x, y) in $D(K')$ not in $D(K)$ are "larger" than the elements in $D(K)$;[31]

31. Specifically, with

$$D(K; l) = D(K) \cap \{(x, y)|y = y_L + l(x - x_L)\}$$

for $l > 0$, it is easy to show that $(x', y') \in D(K'; l) \setminus D(K; l)$ implies $(x', y') > (x, y)$ for all $(x, y) \in D(K; l)$. This implies

$$E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K'; l)] \geqslant E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K; l)]$$

for all $l > 0$, which in turn implies

$$E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K')] \geqslant E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K)].$$

ii) $(x, y) \in D^F \cap D'^N$ implies

$$P(d(x, y)) = E[\tilde{e}|x, y] - \bar{c} \leq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D'^N] = P'(d'(x, y));$$

iii) $(x, y) \in D^F \cap D'^F$ implies

$$P(d(x, y)) = E[\tilde{e}|x, y] - \bar{c} = P'(d'(x, y));$$

iv) $(x, y) \in D^P \cap D'^N$ implies

$$P(d(x, y)) = E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} \leq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D'^N] = P'(d'(x, y));$$

and

v) $(x, y) \in D^P \cap D'^P$ implies

$$P(d(x, y)) = E[\tilde{e}|x, \tilde{y} \in D_x] - \underline{c} = P'(d'(x, y)).$$

From the fourth step, these constitute all conceivable cases (see fig. A1).

Thus D^N, D^P, D^F, is not optimal. As $K^* \equiv \sup\{K|H(K) = K\}$ (which is finite as $H(K) \leq E[\tilde{e}]$) belongs to $\{K|H(K) = K\}$ (by the continuity of $H(K) - K$), it follows that, with $D^{N^*} \equiv D(K^*)$ (and D^{P^*} and D^{F^*} defined in the obvious ways), D^{N^*}, D^{P^*}, and D^{F^*} describe an optimal policy.

Appendix B
Proof of Theorem 2

Proof.

a) By theorem 1*c*, for each pair $(\underline{c}, \bar{c})$ the set $D^N(\underline{c}, \bar{c})$ over which no disclosure occurs is defined by means of a constant $K = K(\underline{c}, \bar{c})$, where

$$K(\underline{c}, \bar{c}) = E\big[\tilde{e}|(\tilde{x}, \tilde{y}) \in D\big(K(\underline{c}, \bar{c}), \underline{c}, \bar{c}\big)\big],$$

$$D^N(\underline{c}, \bar{c}) \equiv D\big(K(\underline{c}, \bar{c}), \underline{c}, \bar{c}\big),$$

$$D(K, \underline{c}, \bar{c}) = \{(x, y) \in \Omega | \begin{cases} (a) & K + \bar{c} \geq E[\tilde{e}|x, y] \\ (b) & K + \underline{c} \geq E[\tilde{e}|x, \tilde{y} \in D_x(\underline{c}, \bar{c})]\end{cases}\},$$

$$D_x(\underline{c}, \bar{c}) = \{(x, y) \in \Omega | E[\tilde{e}|x, \tilde{y} \in D_x(\underline{c}, \bar{c})] - \underline{c} \geq E[\tilde{e}|x, y] - \bar{c}\}.$$

It is obvious that $D_x(\underline{c}, \bar{c})$ depends on $(\underline{c}, \bar{c})$ only through the difference $\bar{c} - \underline{c}$. In the next lemma, I show that the set of y's over which partial disclosure is superior to full disclosure expands with $\bar{c} - \underline{c}$ and that $E[\tilde{e}|x, \tilde{y} \in D_x(\underline{c}, \bar{c})]$ is monotonic in $\bar{c} - \underline{c}$.

An argument related to this one is spelled out in more detail in the proof of theorem 2a in App. B. If, say, $y_L = -\infty$, then this argument can be duplicated to show that

$$E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K'; \hat{y})] \geq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K; \hat{y})] \quad \text{(i)}$$

where

$$D(K; \hat{y}) \equiv \bigcup_{l \geq 0} D(K; l, \hat{y}) \equiv \bigcup_{l \geq 0} \{(x, y)|y = \hat{y} + l(x - x_L)\} \cap D(K).$$

Taking the limit of both sides of (i) as $\hat{y}$ converges to $-\infty$ proves the result, in case $y_L = -\infty$. The case in which $x_L = -\infty$ (or both x_L and y_L equal $-\infty$) is similar.

LEMMA 2. Under the hypotheses of theorem 1c, if $\bar{c} - \underline{c} > \bar{c}' - \underline{c}'$, then

$$D_x(\underline{c}, \bar{c}) \supset D_x(\underline{c}', \bar{c}')$$

and

$$E[\tilde{e}|x, \tilde{y} \in D_x(\underline{c}, \bar{c})] \geq E[\tilde{e}|x, \tilde{y} \in D_x(\underline{c}', \bar{c}')].$$

Proof. As in the proof of lemma 1,

$$D_x(\underline{c}, \bar{c}) = (y_L, \underline{y}(x; d)],$$

where $d = \bar{c} - \underline{c}$ and

$$\underline{y}(x; d) = \begin{cases} y^*(x; d), x \text{ in } \mathbf{O}(d) \equiv \{x \in (x_L, \bar{x}) | E[\tilde{e}|x, \bar{y}(x)] - E[\tilde{e}|x] > d\} \\ \bar{y}(x), \text{ otherwise,} \end{cases}$$

where $y^*(x; d)$ solves (for x in $\mathbf{O}(d)$)

$$d = E[\tilde{e}|x, y^*(x; d)] - E[\tilde{e}|x, \tilde{y} \leq y^*(x; d)]. \tag{B1}$$

It is easy to show that $y^*(x; d)$ is increasing in d for fixed x, by differentiating (B1). Since $\bar{y}(x)$ is independent of d, it is straightforward to show that $\underline{y}(x; d)$ is (weakly) increasing in d for each x. As

$$D_x(\underline{c}, \bar{c}) = (y_L, \underline{y}(x; \bar{c} - \underline{c})],$$

this shows that

$$D_x(\underline{c}, \bar{c}) \supset D_x(\underline{c}', \bar{c}')$$

for $\bar{c} - \underline{c} \geq \bar{c}' - \underline{c}'$. To complete this lemma's proof, notice that[32]

$$\frac{\partial}{\partial d} E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x; d)] = \underline{y}_d(x; d) \frac{\partial}{\partial \underline{y}} E[\tilde{e}|x, \tilde{y} \leq \underline{y}]|_{\underline{y}=\underline{y}(x;\, d)} \geq 0.$$

I now return to the proof of theorem 2a. For any fixed K, for $(\underline{c}', \bar{c}') \geq (\underline{c}, \bar{c})$ and $\bar{c}' - \underline{c}' \leq \bar{c} - \underline{c}$,

$$D(K, \underline{c}', \bar{c}') \supset D(K, \underline{c}, \bar{c}). \tag{B2}$$

To see this, take $(x, y) \in D(K, \underline{c}, \bar{c})$. Since

$$K + \bar{c}' \geq K + \bar{c} \geq E[\tilde{e}|x, y]$$

and

$$K + \underline{c}' \geq K + \underline{c} \geq E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x; \bar{c} - \underline{c})] \geq E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x; \bar{c}' - \underline{c}')]$$

(the last inequality follows from lemma 2 and the assumption that $\bar{c}' - \underline{c}' \leq \bar{c} - \underline{c}$), we conclude that

$$(x, y) \in D(K, \underline{c}', \bar{c}'),$$

thus, verifying (B2). I now claim that

$$E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}', \bar{c}')] \geq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}, \bar{c})]. \tag{B3}$$

32. These derivatives should be interpreted as one-sided derivatives, which exist because these functions are monotonic with respect to the variables being differentiated.

For a proof when x_L and y_L are finite, define

$$D(K, \underline{c}, \bar{c}, l) = D(K, \underline{c}, \bar{c}) \cap \{(x, y)|y = y_L + l(x - x_L)\}$$

for each $l > 0$. Then

$$(x', y') \in D(K, \underline{c}', \bar{c}', l) \setminus D(K, \underline{c}, \bar{c}, l)$$

implies

$$(x', y') \geq (x, y) \tag{B4}$$

for all

$$(x, y) \in D(K, \underline{c}, \bar{c}, l).$$

To see this, suppose there exists (x, y) in $D(K, \underline{c}, \bar{c}, l)$ with either $x > x'$ or $y > y'$. Then since $y = y_L + l(x - x_L)$ and $y' = y_L + l(x' - x_L)$, it follows that both $x > x'$ *and* $y > y'$. Since $E[\tilde{e}|x, y]$ increases in both x and y, we conclude that

$$K + \bar{c} \geq E[\tilde{e}|x, y] \geq E[\tilde{e}|x', y'].\tag{B5}$$

Also,

$$K + \underline{c} \geq E[\tilde{e}|x, \tilde{y} \leq \underline{y}(x; \bar{c} - \underline{c})] \geq E[\tilde{e}|x', \tilde{y} \leq \underline{y}(x'; \bar{c} - \underline{c})] \tag{B6}$$

since $E[\tilde{e}|x, \tilde{y} \leq y(x; \bar{c} - \underline{c})]$ is increasing in x. These inequalities (B5) and (B6) imply that $(x', \bar{y}')$ belongs to $D(K, \underline{c}, \bar{c}, l)$, contrary to assumption, so (B4) holds. Therefore,

$$E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}', \bar{c}', l)] \geq E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}, \bar{c}, l)],$$

so

$$\begin{aligned} &E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}', \bar{c}')] \\ &= \int_0^\infty E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}', \bar{c}', l)]\mathrm{pr}[D(K, \underline{c}', \bar{c}', l)]dl \\ &\geq \int_0^\infty E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}', \bar{c}', l)]\mathrm{pr}[D(K, \underline{c}, \bar{c}, l)]dl \\ &\geq \int_0^\infty E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}, \bar{c}, l)]\mathrm{pr}[D(K, \underline{c}, \bar{c}, l)]dl \\ &= E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}, \bar{c})]. \end{aligned}$$

If either of x_L or y_L are infinite, the preceding argument can be easily extended, by following the approach in note 30 above.

To complete the proof of theorem 2*a*, recall from theorem 1 that

$$H(K, \underline{c}, \bar{c}) \equiv E[\tilde{e}|(\tilde{x}, \tilde{y}) \in D(K, \underline{c}, \bar{c})]$$

is continuous in K with $H(0, \underline{c}, \bar{c}) > 0$ and that $H(K, \underline{c}, \bar{c})$ is bounded above by its horizontal asymptote $E[\tilde{e}]$. Further, under the hypotheses of theorem 1*c*, $H(K, \underline{c}, \bar{c})$ is increasing in K. A representative graph of $H(\cdot)$ along with the 45-degree line is drawn in figure B1. Figure B1 illustrates the possibility of several intersections of H with the 45-degree line. By (B3), $H(K, \underline{c}', \bar{c}') \geq H(K, \underline{c}, \bar{c})$

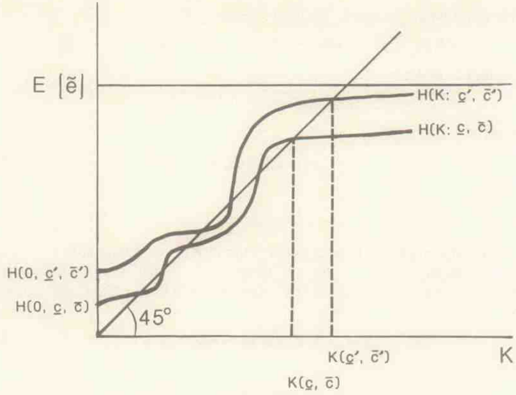


FIG. B1

for any K, so if $K(\underline{c}, \bar{c})$ is *any* intersection of $H(K, \underline{c}, \bar{c})$ with the 45-degree line (including, possibly, that K defining the optimal disclosure policy, given $(\underline{c}, \bar{c})$), there exists $K(\underline{c}', \bar{c}') \geq K(\underline{c}, \bar{c})$ with $H(K(\underline{c}', \bar{c}'), \underline{c}', \bar{c}') = K(\underline{c}', \bar{c}')$. Credible disclosure policies corresponding to these intersections of $\bar{H}$ with the line $K = K$ exist, with no disclosure sets defined by

$$D^N(\underline{c}, \bar{c}) = D\big(K(\underline{c}, \bar{c}), \underline{c}, \bar{c}\big)$$

and

$$D^N(\underline{c}', \bar{c}') = D\big(K(\underline{c}', \bar{c}'), \underline{c}', \bar{c}'\big).$$

From the definition of $D(K, \underline{c}, \bar{c})$, it is clear that

$$D(K', \underline{c}, \bar{c}) \supset D(K, \underline{c}, \bar{c})$$

for $K' \geq K$. Therefore, applying (B2),

$$\begin{aligned} D^N(\underline{c}', \bar{c}') &= D\big(K(\underline{c}', \bar{c}'), \underline{c}', \bar{c}'\big) \supset D\big(K(\underline{c}, \bar{c}), \underline{c}', \bar{c}'\big) \supset D\big(K(\underline{c}, \bar{c}), \underline{c}, \bar{c}\big) \\ &= D^N(\underline{c}, \bar{c}), \end{aligned}$$

which completes the proof of theorem 2*a*.

b) From theorem 2*a*,

$$D^P(\underline{c}, \bar{c}) = D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c}) \supset D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}')$$
$$D^F(\underline{c}, \bar{c}) = D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c}) \supset D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}').$$

Take

$$(x, y) \in D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}').$$

Clearly, $y \in D_x(\underline{c}, \bar{c}) = D_x(\underline{c}', \bar{c}')$ by lemma 2. As $(x, y) \notin D^N(\underline{c}', \bar{c}')$, conclude $(x, y) \in D^P(\underline{c}', \bar{c}')$. Therefore,

$$D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}') \subset D^P(\underline{c}', \bar{c}').$$

To prove the reverse inclusion, take

$$(x, y) \in D^P(\underline{c}', \bar{c}') = D^P(\underline{c}', \bar{c}') \setminus D^N(\underline{c}', \bar{c}').$$

Thus, by theorem 2*a*, $(x, y) \notin D^N(\underline{c}, \bar{c})$. By definition of $D^P(\underline{c}', \bar{c}')$, $y \in D_x(\underline{c}', \bar{c}') = D_x(\underline{c}, \bar{c})$, so $(x, y) \in D^P(\underline{c}, \bar{c})$. Therefore,

$$D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}') = D^P(\underline{c}', \bar{c}').$$

The second claim in (2*b*) follows from simple set-theoretic manipulations:

$$D^F(\underline{c}', \bar{c}') = [D^P(\underline{c}', \bar{c}') \cup D^N(\underline{c}', \bar{c}')]^o. \tag{B7}$$

(Here, o is used to denote set complementation to avoid confusion between c's and the customary complementation symbol.) From the previous part of 2*b* we know

$$[D^P(\underline{c}', \bar{c}')]^o = [D^P(\underline{c}, \bar{c})]^o \cup D^N(\underline{c}', \bar{c}').$$

Substitute this into (B7) to obtain:

$$\begin{aligned} D^F(\underline{c}', \bar{c}') &= [D^P(\underline{c}, \bar{c})]^o \cap [D^N(\underline{c}', \bar{c}')]^o \\ &= D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}'). \end{aligned}$$

This completes the proof of 2*b*.

c) Take

$$(x, y) \in D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c}').$$

Then

$$y \in D_x(\underline{c}, \bar{c}) \subset D_x(\underline{c}, \bar{c}').$$

As $(x, y) \notin D^N(\underline{c}, \bar{c}')$, it follows that $(x, y) \in D^P(\underline{c}, \bar{c}')$, so

$$D^P(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c}') \subset D^P(\underline{c}, \bar{c}').$$

d) Take

$$(x, y) \in D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}).$$

Then

$$(x, y) \in D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}, \bar{c})$$

from theorem 2*a*. By definition of $D^F(\underline{c}, \bar{c})$, this implies $y \notin D_x(\underline{c}, \bar{c})$. By lemma 2, $y \notin D_x(\underline{c}', \bar{c})$, so $y \in D^F(\underline{c}', \bar{c})$, so

$$D^F(\underline{c}, \bar{c}) \setminus D^N(\underline{c}', \bar{c}) \subset D^F(\underline{c}', \bar{c}).$$

Finally, suppose $(x, y) \in D^P(\underline{c}', \bar{c})$. If

$$(x, y) \in D^N(\underline{c}, \bar{c}) \subset D^N(\underline{c}', \bar{c}),$$

a contradiction to $(x, y) \in D^P(\underline{c}', \bar{c})$ is obtained. Therefore, $(x, y) \in D^P(\underline{c}, \bar{c})$, so apply lemma 2 to conclude

$$D^P(\underline{c}', \bar{c}) \subset D^P(\underline{c}, \bar{c}).$$

e) From theorem 1, we know that an optimal disclosure policy, with positive probability, has no disclosure occur. Therefore we can confine the analysis to credible policies with a positive probability of no disclosure. Let $K(\underline{c}, \bar{c})$ be the K consistent with an optimal disclosure policy, given $(\underline{c}, \bar{c})$, and let $K(\underline{c}', \bar{c}')$ be consistent with an optimal disclosure policy, given $(\underline{c}', \bar{c}')$. By (B3), $H(K, \underline{c}', \bar{c}') \geq H(K, \underline{c}, \bar{c})$ for all K, so it follows that there exists $K \geq K(\underline{c}, \bar{c})$ with $H(K, \underline{c}', \bar{c}') = K$. By the proof of theorem 2*a*, it follows that $K(\underline{c}', \bar{c}') \geq K$ and therefore $K(\underline{c}', \bar{c}') \geq K(\underline{c}, \bar{c})$. By hypothesis,

$$\begin{aligned} K(\underline{c}, \bar{c}) + \underline{c} &\geq E[\tilde{e}|\bar{x}, \tilde{y} \leq \underline{y}(\bar{x}; \bar{c} - \underline{c})] \\ &\geq E[\tilde{e}|\bar{x}, \tilde{y} \leq \underline{y}(\bar{x}; \bar{c}' - \underline{c}')] \end{aligned}$$

(the first inequality from theorem 1*c*, the second from lemma 2), so

$$K(\underline{c}', \bar{c}') + \underline{c}' \geq E[\tilde{e}|\bar{x}, \tilde{y} \leq \underline{y}(\bar{x}; \bar{c}' - \underline{c}')].$$

Therefore, the optimal no disclosure set $D^N(\underline{c}', \bar{c}')$ is described by theorem 1*ci*, which completes the proof.

Appendix C

This appendix provides an outline for the construction of an example for which the optimal disclosure policy entails that nothing be disclosed with positive probability, even though $\underline{c} = 0$. Fix a positive constant $K_o > \bar{c}$, and let $\bar{y} > K_o + \bar{c}$ and $\bar{x} > K_o - \bar{c}$. Define the distribution of $\tilde{y}$ given x as follows. For $x < K_o - \bar{c}$,

i) if $y \neq K_o + \bar{c} - x$ and $y \in [0, \bar{y}]$, $\tilde{y}$ given x has a density $f^\epsilon(y|x) = \epsilon > 0$; and

ii) if $y = K_o + \bar{c} - x$, $\tilde{y}$ has a point mass $p^\epsilon(x) = 1 - \epsilon\bar{y}$.

For $K_o - \bar{c} \leq x < \bar{x}$, $f^\epsilon(y|x)$ is uniform on $[0, \bar{y}]$. Let the marginal distribution of $\tilde{x}$ be uniform on $[0, \bar{x}]$. Finally, let $E[\tilde{e}|x, y] = x + y$. For $x < K_o - \bar{c}$ and $\underline{y} < K_o + \bar{c} - x$, $E^\epsilon[\tilde{y}|x, \tilde{y} \leq \underline{y}\ ] = \underline{y}/2$ since the conditional distribution of $\tilde{y}$ given x is uniform in this region. Therefore, $\underline{y} = 2\bar{c}$, solves, for each $x < K_o - \bar{c}$,

$$\underline{y} - E^\epsilon[\tilde{y}|x, \tilde{y} \leq \underline{y}\ ] = \bar{c}. \tag{C1}$$

For $x \geq K_o - \bar{c}$, the conditional distribution of $\tilde{y}$ given x is uniform throughout $[0, \bar{y}]$, so $\underline{y} = 2\bar{c}$ solves (C1) for all $x \in [0, \bar{x}]$.

Recall that

$$\begin{aligned} D(K) &= \{(x, y)|K + \bar{c} \geq x + y,\ K \geq x + E^\epsilon[\tilde{y}|x, \tilde{y} \leq \underline{y}\ (x)]\} \\ &= \{(x, y)|K + \bar{c} \geq x + y,\ K - \bar{c} \geq x\} \end{aligned}$$

The graph of $D(K)$ for $K = K_o$ is illustrated in figure C1 below. $D(K_o)$ is the shaded region *including* its boundaries.

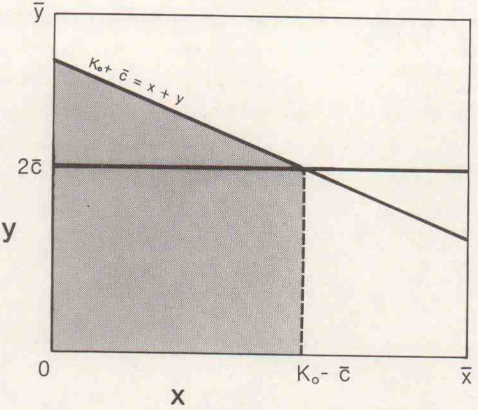


FIG. C1

The next thing to observe is that $E[\tilde{x} + \tilde{y}|(\tilde{x}, \tilde{y}) \in D(K_o)]$ exceeds K_o for sufficiently small ϵ:

$$E^{\epsilon}[\tilde{x} + \tilde{y}|(\tilde{x}, \tilde{y}) \in D(K_o)] > \frac{\int_o^{K_o - \bar{c}} [(K_o + \bar{c} - x)(1 - \epsilon\bar{y}) + x]dx}{\int_o^{K_o - \bar{c}} \{1 - \epsilon\bar{y} + \epsilon(K_o + \bar{c} - x)\}dx}.$$

The denominator of this expression equals $\text{pr}^{\epsilon}[D(K_o)]$ times $\bar{x}$. The numerator neglects those positive $\tilde{y} \neq K_o + \bar{c} - x$. As ϵ converges to zero, this expression converges to $K_o + \bar{c}$, which exceeds K_o.

It is clear (as in the proof of theorem 1) that, if K is sufficiently large, $E[\tilde{x} + \tilde{y}|(\tilde{x}, \tilde{y}) \in D(K)] - K$ becomes negative. So *if* $E[\tilde{x} + \tilde{y}|(\tilde{x}, \tilde{y}) \in D(K)]$ were continuous, an equilibrium no-disclosure set would exist where the probability of no disclosure is positive and defined by

$$K = H(K) = E[\tilde{x} + \tilde{y}|(\tilde{x}, \tilde{y}) \in D(K)]$$

for some $K > K_o$. Of course, $H(K)$ is not continuous in K as the example is presented. However, it is easy to modify this example in a way that preserves the continuity of $H(K)$ as well as the inequality $H(K_o) > K_o$ by making the distribution of $\tilde{y}$ conditional on x admit a density representation throughout its support that places a very high (conditional) density on the realizations of $\tilde{y}$ near the line $y = K_o + \bar{c} - x$. This completes the outline of the example.

References

Antle, R., and Smith, A. 1984. An empirical investigation into relative performance evaluation of corporate executives. Working paper. Chicago: University of Chicago.

Dye, R. 1985. Disclosure of nonproprietary information. *Journal of Accounting Research* 23 (Spring): 123–45.

Feller, W. 1966. *An Introduction to Probability Theory and Its Applications*. Vol. 2. New York: Wiley.

Gonedes, N. 1980. Public disclosure rules, private information production, decisions and capital market equilibrium. *Journal of Accounting Research* 18 (Autumn): 441–75.

Grossman, S. J. 1981. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics* 24 (December): 461–84.

Grossman, S., and Hart, O. 1980. Disclosure laws and takeover bids. *Journal of Finance* 35 (May): 323–34.

Kreps, D., and Porteus, E. 1979. Dynamic choice theory and dynamic programming. *Econometrica* 47 (January): 91–100.

Kreps, D., and Wilson, R. 1982. Sequential equilibria. *Econometrica* 50 (July): 863–94.

Lehmann, E. 1959. *Testing Statistical Hypotheses*. New York: Wiley.

Marshall, J. 1974. Private incentives and public information. *American Economic Review* 64 (June): 373–90.

Milgrom, P. 1981. Good news and bad news: Representation theorems and applications. *Bell Journal of Economics* 12 (Autumn): 380–91.

Spence, M. 1974. *Market Signalling: Informational Transfer in Hiring and Related Screening Processes*. Cambridge, Mass.: Harvard University Press.

Verrecchia, R. 1982. The use of mathematical models in financial accounting. *Journal of Accounting Research* 20, suppl.:1–42.

Verrecchia, R. 1983. Discretionary disclosures. *Journal of Accounting and Economics* 5 (December): 179–94.

Wilson, R. 1975. Informational economies of scale. *Bell Journal of Economics* 6 (Spring): 184–95.