

Detecting Spot Price Forecasts in Futures Prices*

I. Introduction

Futures markets are often described as having two important social functions. First, they facilitate the transfer of commodity price risk, and, second, they provide forecasts of commodity prices. The evidence that futures markets transfer price risk is irrefutable. However, there is some debate about the markets' forecasting ability. In particular, forecasts based on the current spot price are often as good as those based on the futures price.

Some economists cite a failure to detect superior forecast power in futures prices as evidence of market inefficiency (see, e.g., Leuthold 1974; and Martin and Garcia 1981). There are at least two other explanations. First, there may be nothing for the futures market to forecast. If the current spot price equals the true expectation of the future spot price, the futures market cannot provide a better forecast. Second, a superior futures market forecast may be obscured by the unexpected component of the realized spot price. The true expectation of the future spot price is unobservable; one must approximate this expectation with the actual future spot price.

This paper identifies commodity characteristics that should be related to the accuracy of spot price forecasts in futures prices. Researchers frequently compare forecasts based on futures prices with forecasts based on current spot prices. Futures prices cannot provide reliably better forecasts unless the variance of the expected spot price changes is large relative to the variance of the actual spot price changes. This relative variance is related to a number of factors, including the importance of seasonals in production and the cost of storage. The evidence in a recent paper by Fama and French is consistent with these predictions.

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This approximation reduces the researcher's ability to detect forecast power.¹

In this paper I identify commodity characteristics that should be related to differences in forecast power across commodities. In Section II, I examine both the factors that generate predictable spot price changes and the factors that obscure the market's forecasts of these price changes. Section III uses these results to interpret the evidence on forecast power in Fama and French (1985). The last section contains a summary and some conclusions.

II. Factors Affecting Forecast Power

If the current spot price equals the expectation of the future spot price, the futures price cannot provide a better forecast of the future spot price. Equivalently, the futures market cannot predict changes in the spot price unless the spot price is expected to change.

Predictable spot price changes can be generated by both anticipated and unanticipated changes in supply and demand. The seasonal production of agricultural commodities is probably the most important source of anticipated changes in supply. One might expect futures prices to predict correctly a drop in the spot price of corn over a harvest and to predict an increase in the price between harvests.

Demand and supply shocks can also generate expected spot price changes. Suppose there is a sudden permanent increase in the demand for copper. This will cause an immediate increase both in the current spot price and in the expected future spot price. However, since the producers of copper will respond to these price shifts, the change in the expected price will be smaller than the change in the current price. In other words, after the unexpected increase in demand is (instantaneously) incorporated in the current price, the spot price is expected to fall back toward its original level. A positive demand shock leads to a negative expected change in the spot price.

A. Theoretical Model

To develop these concepts further, I use a two-period model of consumption, production, and storage.² This model is described by three equations. First, the total stock of the commodity available in the first period (time t) is divided between consumption and storage:

$$X(t) = D_t[S(t), u(t)] + I\{S(t), E_t[S(T)]\}. \quad (1)$$

1. Risk premiums can also reduce the accuracy of futures price forecasts. This topic is not addressed here. For evidence about risk premiums in futures prices, see Fama and French (1985).

2. This discussion draws on the work of Kaldor (1939), Blau (1944-45), Working (1948, 1949), Brennan (1958), and Telser (1958).

In this equation, $X(t)$ is the stock available at t , $D_t[S(t), u(t)]$ is the amount consumed at t when the spot price is $S(t)$, and $I\{S(t), E_t[S(T)]\}$ is the amount stored between t and T when the current spot price is $S(t)$ and the expected spot price is $E_t[S(T)]$. The random variable $u(t)$ shifts the demand function up and down. This demand shock is revealed before the consumption and inventory decisions are made in the first period.

The second equation in the model says that the quantity consumed in the second period, $D_T[S(T), u(T)]$, is equal to the amount produced between t and T , $Q\{E_t[S(T)], v(t), v(T)\}$, plus the amount stored from t to T :

$$D_T[S(T), u(T)] = Q\{E_t[S(T)], v(t), v(T)\} + I\{S(t), E_t[S(T)]\}. \quad (2)$$

Both the production function and the second-period demand function are affected by shocks that occur between t and T . Notice that no inventory is carried out of the second period.

The third equation is an equilibrium condition that ties equations (1) and (2) together. Agents store the commodity between t and T if the expected capital gain, $E_t[S(T)] - S(t)$, exceeds the sum of the interest cost, the marginal storage cost, and the cost of bearing commodity price risk. The interest cost, which reflects the opportunity cost of the capital invested in the commodity, is measured by the spot price at t times the interest rate between t and T , $S(t)r(t, T)$. The risk associated with storing the commodity between t and T arises because $S(T)$ is not known at t . This risk is identical to the risk associated with holding a long futures contract. The cost of bearing this risk equals the spot price at t times the commodity risk premium, $S(t)p(t, T)$.

The marginal storage cost equals the physical storage cost, including things like rental charges and insurance premiums, minus the marginal convenience yield. This convenience yield arises because inventory can have productive value. For example, a mill operator might store wheat because this allows him to smooth his production of flour. I assume that the physical storage cost is a constant fraction of the current spot price and that the marginal convenience yield is a function of the inventory level. At low inventory levels, the convenience yield is relatively high. As the inventory rises, the marginal convenience yield falls at a decreasing rate. Therefore, the marginal storage cost, $S(t)c(I)$, rises at a decreasing rate as inventory rises; $\partial c/\partial I > 0$ and $\partial^2 c/\partial I^2 < 0$.

In equilibrium, agents store the commodity until the expected capital gain equals the sum of the interest cost, the marginal storage cost, and the cost of bearing the commodity price risk:

$$E_t[S(T)] - S(t) = S(t)[r(t, T) + c(I) + p(t, T)]. \quad (3)$$

This inventory condition ties the expected spot price at T to the actual spot price at t :

$$\frac{E_t[S(T)]}{S(t)} = 1 + r(t, T) + c(I) + p(t, T). \quad (4)$$

B. Seasonal Factors

Equation (4) highlights the relation between seasonal variation in the price of agricultural commodities and variation in the marginal storage costs. Between harvests, when the inventory is relatively high, the marginal storage cost is also high, and the spot price is expected to increase. On the other hand, there is relatively little storage across the harvest. In this case, the marginal storage cost is low and the spot price is expected to fall.

Since the spot prices in different periods are linked together by storage, the magnitude of the expected seasonal variation is affected by the characteristics of the marginal storage cost function. If the marginal storage cost is relatively sensitive to changes in the inventory, the cyclical price pattern will be very pronounced. If the marginal storage cost is not very sensitive to the inventory level, the seasonal pattern will be less pronounced. Thus there should be large seasonal variation in the spot prices for agricultural commodities with relatively high marginal convenience yields at low inventory levels and low convenience yields at high inventory levels.

Naturally, the magnitude of the expected seasonal variation in the spot price is also affected by the importance of seasonality in the demand or production function. Since seasonal factors have little effect on the demand or supply of metals, one does not observe seasonal variation in metals prices. In contrast, one does expect to observe seasonality in the prices of agricultural commodities and animal products since they are subject to seasonal production. Moreover, commodities like corn, which only has one relatively short harvest each year, should exhibit more seasonality than commodities like wheat, which has two longer harvests each year.

C. Demand and Supply Shocks

Unanticipated changes in demand and supply can also lead to differences between the current and expected spot prices. Shocks affecting the current demand or supply will cause large expected price changes if their effect on the expected spot price is small relative to their effect on the current spot price. Equivalently, these shocks will cause large forecastable price changes if the elasticity of the expected spot price with respect to unexpected changes in the current spot price is close to zero. The magnitude of the forecastable price changes falls as this elasticity approaches one.

In the Appendix, I show that the spot price elasticity is equal to

$$\frac{dE_t[S(T)]}{dS(t)} \frac{S(t)}{E_t[S(T)]} = \frac{1}{1 - S(t) \frac{\partial c}{\partial I} \left\{ \frac{\partial D_T}{\partial S(T)} - \frac{\partial Q}{\partial E_t[S(T)]} \right\}} \quad (5)$$

Since the partial derivatives of the production and storage cost functions are positive, while the derivative of the demand function is negative, this elasticity is between zero and one.

The intuition behind equation (5) is straightforward. Suppose the quantity demanded in period T is very sensitive to the price in period T (so that $\partial D_T / \partial S(T)$ is relatively large). Then a shock in the first period can have a large effect on both the inventory and the second period consumption without having a large effect on the expected spot price. The derivative of the production function plays a similar role. If the quantity produced is very responsive to changes in the expected price, a large change in the inventory can be offset without a large change in the expected price. Therefore, the magnitude of the expected price changes increases as the difference between these two partial derivatives, $\partial D_T / \partial S(T) - \partial Q / \partial E_t[S(T)]$, increases.

The partial derivative of the marginal storage cost also plays a sensible role in equation (5). Suppose the marginal storage cost is very sensitive to changes in the inventory level. Then $S(T)$ is insulated from shocks in period t ; large changes in the current spot price do not lead to large changes in either the inventory or the expected spot price. In other words, when $\partial c / \partial I$ is relatively large, the spot price elasticity is small.

Equation (5) implies that there should not be much forecast power in the futures price for precious metals. For example, for any reasonable inventory level, the marginal convenience yield from storing gold is essentially zero. Therefore, the marginal storage cost is also constant, and the spot price elasticity equals one. In other words, shocks to the current price are transmitted perfectly to the expected price; there is no variation in the expected price changes for the futures prices to forecast.

Equation (5) also makes predictions about the relation between inventory size and the magnitude of expected price changes. The sensitivity of the marginal storage cost to changes in the inventory decreases as the inventory increases ($\partial^2 c / \partial I^2 < 0$). Thus, when commodity stocks are low, changes in the inventory have a large effect on the marginal storage cost, and the spot price elasticity is close to zero. On the other hand, shocks in the first period have a more even effect on the current and expected spot prices when the inventory level is high and the marginal storage cost function is relatively constant. This implies that variation in the expected spot price changes for a particular

commodity should be inversely related to the inventory level. Moreover, to the extent that the inventory level is affected by the cost of storage, high-storage-cost commodities should exhibit more variation in their expected spot price changes than low-storage-cost commodities.

The model, as it is developed so far, ignores several potentially important factors. For example, the model assumes that demand shocks are completely transitory; a shock in the first period has no effect on the demand in the second period. Relaxing this assumption—so that a change in the first-period demand is positively related to a change in the second-period demand—increases the spot price elasticity. In effect, shocks in period t are transmitted through both the inventory and the demand function. Therefore, permanent demand shocks lead to smaller forecastable price changes.

The discussion also ignores production shocks in the first period. If these shocks change the amount of the commodity available at t , $X(t)$, they have the same effect as transitory demand shocks at t . The elasticity in equation (5) describes the response to these shocks. However, if the shocks affect the second-period production, $Q\{E_t[S(T)], v(t), v(T)\}$, transmission of the shock through the inventory is reversed. For example, suppose the shock at time t , $v(t)$, is positive. The immediate effect of this is to increase the expected output and to reduce the expected price. This leads to a reduction in both the inventory and the current price. In other words, the inventory transmits a change in $E_t[S(T)]$ back to $S(t)$.

In this case, the spot price elasticity is equal to

$$\frac{dE_t[S(T)]}{dS(t)} \frac{S(t)}{E_t[S(T)]} = 1 - \frac{S(t)}{E_t[S(T)]} S(t) \frac{\partial c}{\partial I} \frac{\partial D_t}{\partial S(t)}. \quad (6)$$

Since the slope of the demand function is negative, this elasticity is always greater than one; changes in $S(t)$ are associated with larger percentage changes in $E_t[S(T)]$. Equivalently, if a shock in the first period affects the second-period output, the expected spot price is affected more than the current spot price.

These production shocks will lead to large expected changes in the spot price if the elasticity in equation (6) is much greater than one. One factor affecting this elasticity is the slope of the first-period demand function. For example, suppose that the demand is perfectly inelastic so that $\partial D_t / \partial S(t)$ equals zero. Then output shocks have equal percentage effects on the current and the expected spot prices—and no effect on the inventory. In this case, there are no expected changes in the spot price to predict.

The spot price elasticity in (6) is also affected by the sensitivity of the marginal storage cost to changes in the inventory level ($\partial c / \partial I$). This derivative has the same effect here as it does in equation (5). When

small changes in the inventory are associated with large changes in the marginal storage cost, the current spot price is insulated from changes in the expected spot price. This means that, again, a relatively large value of $\partial c/\partial I$ leads to a large variation in the expected spot price changes.

Another important simplifying assumption of the model is that consumption, production, and storage stop in period T . Relaxing this assumption increases the spot price elasticity for demand shocks and for production shocks that affect the current output (eq. [5]). For example, if storage past the second period is included in the model, changes in the inventory between t and T will be spread between both consumption and this storage. Therefore, shocks in period t , which are transmitted through the inventory between t and T , will have a smaller effect on $E_t[S(T)]$. In other words, demand shocks and production shocks that affect the current output cause larger predictable changes in the spot price when multiple periods are considered in the model.

The spot price elasticity for production shocks that affect future output (eq. [6]) does not change when more than two periods are considered. Since storage, consumption, and production in other periods will react to production shocks, the effect of these shocks on $E_t[S(T)]$ is reduced. However, the inventory process transmitting this shock between t and T is unaffected, so the elasticity of $S(t)$ with respect to $E_t[S(T)]$ is also unchanged.

D. Detecting Price Forecasts

The discussion above has considered one factor that should be related to the forecast power of futures prices, namely, the magnitude of the variation in the expected spot price changes. The forecast power is also a function of the variation between the expected and the actual maturity spot prices. In fact, the potential forecast power in the futures prices is determined by the variance of the expected spot price changes relative to the variance of the actual spot price changes:

$$R(t, T) = \frac{\text{var}\{E_t[S(T)] - S(t)\}}{\text{var}[S(T) - S(t)]}. \quad (7)$$

If this ratio is close to zero, there is relatively little information available when futures traders make their forecast of the spot price change. The unexpected change in the spot price obscures whatever forecast is in the futures price. In contrast, when the relative variance of the expected spot price changes is large, the traders' potential forecast power is also large.

Within the framework of the two-period model, differences between the actual and the expected spot prices, $S(T) - E_t[S(T)]$, arise because of demand and production shocks in period T . As the variance of these time T shocks increases, holding everything else constant, the forecast

power of the futures prices will be reduced. For example, consider variation in the expected spot price changes that occurs because of seasonality in the production function. As the demand and production shocks become larger, both the seasonality in the actual spot prices and the predictions in the futures prices become less apparent.

When the expected spot price changes are generated by demand and production shocks, the ratio of the variance of the expected price changes to the variance of the actual price changes can be expressed as

$$R(t, T) = \frac{(1 - e)^2 \text{var}[w(t)]}{(1 - e)^2 \text{var}[w(t)] + \text{var}[w(T)]} \quad (8)$$

In this equation, $\text{var}[w(t)]$ is the variance of price shocks at time t , $\text{var}[w(T)]$ is the variance of price shocks at time T , and e is the spot price elasticity in either equation (5) or equation (6). (Equation [8] is derived in the App.) If the variance of shocks at t and T are equal, equation (8) simplifies to

$$R(t, T) = \frac{(1 - e)^2}{1 + (1 - e)^2}. \quad (9)$$

The relative variance in equation (9) falls as the price elasticity approaches one (from either direction). The intuition here is similar to the intuition in the previous section. If the price elasticity is close to one, price shocks are transmitted freely through the inventory; a 1% change in the current spot price generates a 1% change in the expected spot price. In this case, the unexpected change in the spot price dominates the expected change. The relative variance in equation (9) is close to zero, and there is little forecast power in the futures price. For example, since the marginal storage cost of gold is essentially constant for any reasonable inventory level, demand and supply shocks have nearly identical effects on the current and expected spot prices. Equivalently, the price elasticity is close to one, and the price changes are totally unpredictable.

On the other hand, if the price elasticity is close to zero, shocks that affect the current price have little effect on the expected price. In this case, the predictable price changes generated by demand and supply shocks represent a significant fraction of the total price changes, and the variance ratio is relatively large.

Equation (9) suggests that forecast power should be an increasing function of the forecast horizon. If there is a long period between t and T , shocks to the current price are dissipated before they can affect the expected price. Since the price elasticity is close to zero, equation (9) implies that the variance of the expected price changes should be relatively large. However, there is an offsetting effect. Increasing the time between t and T also increases the variance of the unexpected price

changes. Therefore, it is not clear how the forecast power should vary with the forecast horizon.

To summarize, seasonals in demand and supply may generate reliable forecast power in futures prices. Forecast power should be an increasing function of the importance of these seasonals and of the cost of storage. Demand and supply shocks obscure seasonal variation in the spot price. However, these shocks also generate predictable price changes. The relative magnitude of the expected spot price changes arising from demand and supply shocks is an increasing function of (a) the sensitivity of the marginal storage cost to changes in inventory; (b) the sensitivity of the quantity demanded to changes in price; (c) the sensitivity of the quantity produced to changes in the expected price; and (d) the cost of storage.

III. Empirical Evidence

In a recent paper, Fama and French (1985) study the futures price behavior for 21 commodities, including agricultural products, wood products, metals, and animal products, between 1965 and 1984. They provide both direct and indirect evidence about the forecast power in the futures prices for these commodities.

A. Predictions and Some Indirect Evidence

The analysis in Section II suggests that production seasonals may generate detectable forecast power in the futures prices for agricultural commodities and animal products. The magnitude of the seasonal forecast power should be related to the importance of seasonality in the production function. For example, seasonal price changes may be more important for corn than for wheat since corn has only one relatively short harvest each year while wheat has two longer harvests.

The magnitude of seasonal price variation should also be an increasing function of the marginal cost of storage. If the cost of storing a commodity is high, inventory will not be held between harvests unless the expected rate of growth in the spot price is also high.

Fama and French present indirect evidence that supports these predictions. They regress the relative basis, $[F(t, T) - S(t)]/S(t)$, against monthly seasonal dummies:³

$$\frac{F(t, T) - S(t)}{S(t)} = \sum_{m=1}^{12} \alpha_m d_m + e(t, T). \quad (10)$$

In this regression, $F(t, T)$ is the futures price at t for delivery of the commodity at T , and d_m is a dummy variable that equals one if the

3. Fama and French also include the nominal interest rate as an independent variable in their regression. Ignoring this variable does not affect the interpretation of the dummy variables.

futures contract matures in month m and zero otherwise.⁴ If futures traders are able to forecast seasonal variation in the price changes, this variation should be apparent in the basis. Therefore, the estimated dummy variable coefficients provide evidence about seasonal variation in the expected price changes.

The qualitative results from regression (10) are summarized in the first column of table 1. Fama and French find reliable evidence of seasonal variation in the basis for six of the 10 agricultural commodities they examine (coffee, corn, oats, orange juice, soybeans, and wheat) and for all five of the animal products (broilers, cattle, eggs, hogs, and pork bellies). As the model predicts, the evidence of seasonality is particularly strong for the animal products, where bulk and perishability imply high storage costs relative to value.

The model also predicts that storage costs are important in determining the magnitude of expected price changes generated by demand and supply shocks. At low inventory levels, prices in one period are insulated from shocks in another period, so there are large expected price changes. At higher inventory levels, the marginal storage cost is not very sensitive to changes in the inventory, and price shocks are transmitted freely from one period to another. Since storage costs that are high relative to value deter storage, forecast power should be an increasing function of these costs.

Again, indirect evidence supports this prediction. Fama and French report the standard deviations of the 2-month bases and spot price changes for the 21 commodities they consider. Their results are summarized in the second and third column of table 1. If futures prices contain rational forecasts of spot prices, the volatility of the basis should be directly related to the volatility of the spot price change. Therefore, the relative standard deviation of the basis provides an indirect measure of potential forecast power. If the volatility of the basis is small relative to the volatility of the spot price change, the futures price cannot provide a powerful forecast of the spot price. In contrast, if the relative standard deviation of the basis is large, one might expect to observe forecast power in futures prices.

The results in table 1 indicate that potential forecast power does tend to increase with storage costs. While the standard deviations of the spot price changes are uniformly large, the standard deviations of the bases differ systematically across commodities. The lowest relative basis volatility is observed for the precious metals, where storage costs are trivial relative to value. The highest basis volatility is observed for some of the wood and animal products (lumber, broilers, eggs, and hogs), where bulk and perishability make storage expensive or impos-

4. In their tests, Fama and French use the maturing futures price to measure the spot price, $S(t) = F(t, t)$.

TABLE 1 Direct and Indirect Evidence of Forecast Power in the Basis

	Reliable Seasonals in the Basis	Standard Deviations		Forecast Power
		Basis $F(t, t + 2) - S(t)$	Change $S(t + 2) - S(t)$	
Animal products:				
Broilers	Yes	.058	.111	Strong
Cattle	Yes	.033	.110	Weak
Eggs	Yes	.132	.163	Strong
Hogs	Yes	.071	.129	Strong
Pork bellies	Yes	.019	.169	Good
Agricultural products:				
Cocoa	No	.040	.146	Weak
Coffee	Yes	.052	.150	Weak
Corn	Yes	.028	.099	Weak
Cotton	No	.024	.090	Good
Oats	Yes	.044	.117	Good
Orange juice	Yes	.049	.131	Good
Soybeans	Yes	.028	.122	Good
Soy meal	No	.041	.134	Good
Soy oil	No	.047	.135	Weak
Wheat	Yes	.033	.145	Weak
Wood products:				
Lumber	No	.069	.115	Weak
Plywood	No	.032	.098	Good
Metals:				
Copper	No	.025	.121	Weak
Gold	No	.006	.131	Weak
Platinum	No	.022	.162	Weak
Silver	No	.005	.185	Weak

NOTE.—This table is based on results in tables 3, 4, and 5 of Fama and French (1985). The standard deviations for the 3-month basis and spot price change are reported for platinum. Seasonals in the basis are detected by regressing the basis against monthly dummy variables:

$$\frac{F(t, T) - S(t)}{S(t)} = \sum_{m=1}^{12} \alpha_m d_m + e(t, T).$$

Forecast power is detected by regressing the change in the spot price against the basis:

$$S(T) - S(t) = a + b[F(t, T) - S(t)] + z(t, T).$$

sible. As a general characterization, the relative standard deviation of the basis increases with the cost of storage.

B. Regression Tests

Fama and French look for direct evidence of forecast power in futures prices by regressing the realized change in the spot price, $S(T) - S(t)$, against the basis, $F(t, T) - S(t)$:

$$S(T) - S(t) = a + b[F(t, T) - S(t)] + z(t, T). \quad (11)$$

Evidence that the slope coefficient b is positive implies that the basis observed at t contains information about the change in the spot price

from t to T . Equivalently, the futures price has power to forecast the future spot price.

The qualitative results from these regressions are summarized in the fourth column of table 1. This direct evidence provides more support for the model's predictions about the relation between storage costs and forecast power. As Fama and French (1985, p. 23) explain:

The regressions for eight commodities indicate that the basis $F(t, T) - S(t)$ has statistically reliable information about the future change in the spot price $S(T) - S(t)$ for most maturities ($T - t$). Four of these commodities are animal products (broilers, eggs, hogs and porkbellies) where perishability and bulk imply high storage costs relative to value. Bulk and/or perishability also characterize the remaining four commodities (cotton, oats, soybeans and soy-meal) where futures prices show consistent forecast power. Forecast power is not found in futures prices for the precious metals (gold, silver, and platinum) where storage costs are low relative to value and basis variances are low relative to variances of spot price changes.

There is less evidence that demand and supply seasonals translate into forecast power. The monthly dummy variable regressions (10) produce statistically reliable evidence of seasonals in the basis for 11 of the 21 commodities. The price change regressions (11) identify forecast power in the basis for only seven of these commodities. It appears that the unexpected component of the spot price changes obscures the expected seasonal component of the price changes in regression (11).

IV. Summary

The quality of spot price forecasts in futures prices has been a contentious issue for many years. Some researchers have concluded that forecasts based on the current price are as good as those based on the futures price. Futures prices cannot provide forecasts that are reliably better than the current spot price unless the variance of the expected spot price changes is a large fraction of the variance of the actual spot price changes. If the variation in the expected price changes is small relative to the variation in the actual price changes, forecasts that reflect all the available information may be hidden by the unexpected price changes.

Predictable spot price changes are generated by both anticipated and unanticipated changes in demand and supply. For example, production seasonals lead to anticipated changes in both the supply and the spot price of agricultural commodities. Unanticipated changes in demand and supply generate predictable spot price changes if they are not spread evenly across all time periods. For example, a sudden temporary shift in the demand for eggs might produce a large increase in the current spot price without having much effect on the expected spot

price. This positive demand shock leads to a negative expected change in the spot price.

Naturally, the magnitude of seasonal variation in the spot price increases with the importance of seasonality in the demand or production function. The seasonal variation is also affected by the sensitivity of the marginal storage cost function to changes in the inventory. If the convenience yield is high at low inventory levels and the physical storage cost is high at high inventory levels, the seasonal price pattern will be pronounced.

The sensitivity of the marginal storage cost function to changes in the inventory also affects the size of expected price changes generated by demand and supply shocks. If the slope of the marginal storage cost function is relatively large, the spot price in one period is insulated from shocks in another period. In this case demand and supply shocks generate large expected price changes. If changes in the inventory have little effect on the marginal storage cost, shocks are transmitted freely from one period to another, and there are no large expected price changes for futures prices to predict.

The sensitivity of the marginal storage cost function decreases as the inventory increases. At low inventory levels, a small change in the inventory produces a relatively large change in the storage cost, so demand and supply shocks generate large expected price changes. At higher inventory levels, the marginal storage cost function is relatively constant, so shocks generate smaller expected price changes. To the extent that the inventory level is affected by the cost of storage, the magnitude of the predictable changes in the spot price should be an increasing function of the cost of storage.

The evidence in Fama and French (1985) is consistent with these predictions. They test whether information in the basis, $F(t, T) - S(t)$, can be used to forecast changes in the spot price, $S(T) - S(t)$. Fama and French find no evidence of forecast power in the basis for metals. This is consistent with the predictions of the model since metals prices are not affected by demand or supply seasonals or by storage costs that are large relative to value. In contrast, they find reliable evidence that the bases for many animal products and agricultural products do contain power to forecast spot price changes. Again, this is consistent with the predictions of the model since these commodities are affected by production seasonals and by relatively large storage costs.

Appendix

I. Derivation of Equations (5) and (6)

Define $w(T)$ as the unexpected change in the spot price:

$$w(T) = S(T) - E_t[S(T)]. \quad (A1)$$

This forecast error is a function of the second-period demand and supply shocks, $u(T)$ and $v(T)$. These shocks are the only information revealed after the expectation is formed at t . Equation (2) can be rewritten as

$$D_T\{E_t[S(T)] + w(T), u(T)\} = Q\{E_t[S(T)], v(t), v(T)\} + H\{S(t), E_t[S(T)]\}. \quad (A2)$$

The total derivative of this equation is

$$\begin{aligned} & \frac{\partial D_T}{\partial S(T)} \{dE_t[S(T)] + \frac{\partial w(T)}{\partial u(T)} du(T) + \frac{\partial w(T)}{\partial v(T)} dv(T)\} + \frac{\partial D_T}{\partial u(T)} du(T) \\ &= \frac{\partial Q}{\partial E_t[S(T)]} dE_t[S(T)] + \frac{\partial Q}{\partial v(T)} dv(T) + \frac{\partial I}{\partial S(t)} dS(t) + \frac{\partial I}{\partial E_t[S(T)]} dE_t[S(T)]. \end{aligned} \quad (A3)$$

Since the production and inventory levels are not affected by $u(T)$, its net effect on the quantity demanded must also be zero:

$$\frac{\partial D_T}{\partial S(T)} \frac{\partial S(T)}{\partial u(T)} + \frac{\partial D_T}{\partial u(T)} = 0. \quad (A4)$$

In the same way, $v(T)$ must have identical effects on the quantity produced and the quantity demanded:

$$\begin{aligned} \frac{\partial Q}{\partial v(T)} &= \frac{\partial D_T}{\partial S(T)} \frac{\partial w(T)}{\partial v(T)} \\ &= \frac{\partial D_T}{\partial S(T)} \frac{\partial S(T)}{\partial v(T)}. \end{aligned} \quad (A5)$$

Therefore, equation (A3) simplifies to

$$\left\{ \frac{\partial D_T}{\partial S(T)} - \frac{\partial Q}{\partial E_t[S(T)]} \right\} dE_t[S(T)] = \frac{\partial I}{\partial S(t)} dS(t) + \frac{\partial I}{\partial E_t[S(T)]} dE_t[S(T)]. \quad (A6)$$

Holding the risk premium fixed, the total derivative of equation (3) is

$$dE_t[S(T)] - \frac{E_t[S(T)]}{S(t)} dS(t) = S(t) \frac{\partial c}{\partial I} \left\{ \frac{\partial I}{\partial S(t)} dS(t) + \frac{\partial I}{\partial E_t[S(T)]} dE_t[S(T)] \right\}. \quad (A7)$$

Combining equations (A6) and (A7) yields

$$dE_t[S(T)] - \frac{E_t[S(T)]}{S(t)} dS(t) = S(t) \frac{\partial c}{\partial I} \left\{ \frac{\partial D_T}{\partial S(T)} - \frac{\partial Q}{\partial E_t[S(T)]} \right\} dE_t[S(T)]. \quad (A8)$$

This simplifies to equation (5) in the text.

$$\frac{dE_t[S(T)]}{dS(t)} \frac{S(t)}{E_t[S(T)]} = \frac{1}{1 - S(t) \frac{\partial c}{\partial I} \left\{ \frac{\partial D_T}{\partial S(T)} - \frac{\partial Q}{\partial E_t[S(T)]} \right\}}. \quad (5)$$

Equation (6) describes the effect of shocks at time t on production and demand at time T . These shocks do not affect the stock available at time t , $X(t)$, or the time t demand shock, $u(t)$, so the total derivative of equation (1) is

$$0 = \frac{\partial D_t}{\partial S(t)} dS(t) + \frac{\partial I}{\partial S(t)} dS(t) + \frac{\partial I}{\partial E_t[S(T)]} dE_t[S(T)]. \quad (A9)$$

Combining equations (A7) and (A9) yields

$$dE_t[S(T)] - \frac{E_t[S(T)]}{S(t)} dS(t) = -S(t) \frac{\partial c}{\partial I} \frac{\partial D_t}{\partial S(t)} dS(t). \quad (\text{A10})$$

This simplifies to equation (6):

$$\frac{dE_t[S(T)]}{dS(t)} \frac{S(t)}{E_t[S(T)]} = 1 - \frac{S(t)}{E_t[S(T)]} S(t) \frac{\partial c}{\partial I} \frac{\partial D_t}{\partial S(t)}. \quad (6)$$

II. Derivation of Equations (8) and (9)

Suppose the current and expected spot prices are equal before the demand and supply shocks are observed at t :

$$E_{t-}[S(T)] = E_{t-}[S(t)]. \quad (\text{A11})$$

Define the price shock at t as the unexpected change in the spot price:

$$w(t) = S(t) - E_{t-}[S(t)]. \quad (\text{A12})$$

Then the expected change in the spot price from t to T is approximately

$$E_t[S(T)] - S(t) = (e - 1)w(t). \quad (\text{A13})$$

In this expression, e is the elasticity of the expected spot price with respect to the current spot price in either equation (5) or equation (6).

The realized change in the spot price equals the expected change plus the price shock at time T :

$$S(T) - S(t) = E_t[S(T)] - S(t) + w(T). \quad (\text{A14})$$

Using equations (A13) and (A14), the ratio of the variance of the expected spot price change relative to the realized spot price change is

$$R(t, T) = \frac{\text{var}\{E_t[S(T)] - S(t)\}}{\text{var}\{S(T) - S(t)\}} = \frac{(1 - e)^2 \text{var}[w(t)]}{(1 - e)^2 \text{var}[w(t)] + \text{var}[w(T)]}. \quad (8)$$

If the variances of the price shocks are equal, this simplifies to

$$R(t, T) = \frac{(1 - e)^2}{(1 - e)^2 + 1}. \quad (9)$$

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