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Dealing—Temporary Price Cuts—by Seller as a Buyer Discrimination Mechanism*

An explanation for temporary price cuts, usually called "deals," "sales," or "specials," is provided. We hypothesize that they are a mechanism that discriminates between more intense and less intense demanders. We further hypothesize a positive relationship between demand elasticity and holding costs. Buyers with more intense demand, with high holding costs, are those who will be buying the product at its regular price. The low-holding-cost buyers take advantage of deals by forward buying. The model is applicable to three relationships: (1) manufacturer-reseller, where the reseller may be a wholesaler or a retailer; (2) reseller-reseller, for example, wholesaler-retailer; and (3) retailer-consumer.

I. Introduction

This paper is concerned with temporary price cuts ("deals," "sales," and "specials") as a mechanism for the seller to price-discriminate between buyers with different intensities of demand. We analyze why a seller offers such price cuts, which buyers take advantage of them, and what the parameters are that determine the amount and frequency of the discount offered. In the remainder of this section, we briefly review the literature on price deals.[1] In section II, simple models of buyer and seller behavior are developed, and we show why and how a seller adopts a dealing strategy. Section III summarizes the key implications of the model and indicates how the price discrimination hypothesis could be tested in practice. Section IV suggests possible extensions of the model. Finally, Section V gives our conclusions.

In the marketing literature, the issue of who is "deal prone" has been extensively researched

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1. For a more detailed review of deals and similar price-cutting mechanisms, e.g., mail-in rebates, cents-off labels, coupons, etc., see Narsimhan (1982, chap. 2).

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(see, e.g., Webster 1965; Montgomery 1971; Blattberg et al. 1978). In the economics of information literature, price variations are sometimes explained as the equilibrium outcome of multiple buyers and sellers interacting in an environment of costly price information.[2] Other economic models abstract away from informational problems and assume a single firm. Specifically, Blattberg, Eppen, and Liebermann (1981; see also Eppen and Liebermann 1984) show that a retailer who faces some customers with higher inventory holding costs than his own costs and some consumers with lower inventory holding costs will offer periodic price deals as a mechanism for minimizing total inventory holding costs. Spatt (1981) shows that temporary price-cutting can be a means for a firm to expand its customer base by giving a lower price to new buyers of the product.[3]

The model that we develop in this paper is closer to Blattberg et al. (1981) than to Spatt (1981). We consider a single firm and buyers with either high or low inventory holding costs. We depart from their model by allowing price to affect demand of individual consumers, by allowing for different demand functions of heterogeneous consumers, and by not assuming that the seller is less efficient in inventorying the product than are the low-holding-cost buyers. Moreover, our rationale for dealing is price discrimination, not inventory cost transference of Blattberg et al.[4] This rationale, we argue, potentially applies to any seller-buyer relationship, for example, manufacturer-distributor, distributor-reseller, or retailer-consumer. It is limited, however, to situations where the seller has some monopoly power.

We make the following assumptions: (1) Buyers are heterogeneous in their demand parameters. (2) Customers differ in their ability to inventory a product. (3) Customers incur no transaction/shopping cost. (This assumption greatly simplifies the mathematics and is not essential to our argument.) (4) There is a positive correlation between consumption rates and holding cost (see below). (5) The seller is more efficient in stocking the product than buyers; specifically, holding cost for the seller is zero. (This assumption also both greatly simplifies the mathematics and is not essential to our argument.)

Assumption 4 is key to our analysis and results and requires some

2. The models that attempt to explain price dispersion as an equilibrium outcome when information on prices is costly make different assumptions concerning the search process, heterogeneity of search costs of consumers, and heterogeneity of technology across firms. See, e.g., Salop and Stiglitz (1977), Varian (1980), Burdett and Judd (1983), Carlson and McAfee (1983). See Rothschilld (1973) for an excellent discussion of the issues involved in this area.

3. Narasimhan (1984) generalizes this argument to include the discount offered as a decision variable.

4. Obviously, in practice, dealing may be profitable for other reasons as well. It may allow firms to discriminate between informed and uninformed buyers (see Varian 1980). Conventional wisdom in marketing suggests that retailers offer specials to build up store traffic, to generate new interest in an old product, to introduce a new product, etc.

discussion and justification. For some products there is a positive correlation between demand rates and holding costs. Consider, for example, two households, one with more members than the other. Since the larger household consumes more food and probably a greater variety of foods, it is likely to face a more severe refrigerator or pantry size constraint. Thus household size contributes to a positive correlation between rate of demand at a given price and inventory holding costs. A similar relationship might be due to geographical location. In high-cost and high-income areas, higher demand at a given price should be associated with higher inventory holding costs. This correlation could be particularly significant for a manufacturer who cannot discriminate openly between its suburban distributors (low-cost areas) and its urban distributors (high cost), but who could partially achieve such discrimination by offering temporary price cuts. Such price discrimination argument is the essence of our model. Our purpose is to show that when positive correlation between demand rates and customer inventory holding costs exists for a given product, sellers will find it profitable to deal as a means to price discriminate between buyers.

II. The Model

Consider a monopolistic seller who has constant unit production costs, c. This seller faces two groups of buyers ($i = 1, 2$) whose rates of purchase, x_i, are the following functions of price P:

$$\begin{aligned} x_i &= \alpha_i' - \beta P = \beta(\alpha_i - P), && 0 < P < \alpha_i, \\ x_i &= 0 && P > \alpha_i, \end{aligned} \tag{1}$$

where $\alpha_1 > \alpha_2 > c$.

A fraction, θ, of the seller's potential customers are of type 1 (high-demand group) and $(1 - \theta)$ of type 2 (lower-demand group). If the seller seeks the constant price that maximizes his profits, he maximizes $\Pi_T = (P - c)\,[\theta x_1 + (1 - \theta)x_2]$ if $P < \alpha_2$ and maximizes $\Pi_1 = (P - c)\,\theta x_1$ if $\alpha_2 < P < \alpha_1$. The function Π_T is maximized at

$$P^* = \frac{1}{2}(c + \bar{\alpha}), \tag{2}$$

where $\bar{\alpha} = \theta\alpha_1 + (1 - \theta)\,\alpha_2$. The function Π_1 is maximized at

$$P_1^* = \frac{1}{2}(c + \alpha_1). \tag{3}$$

Equation (2) gives a local maximum of the seller's profit if $\frac{1}{2}(c + \bar{\alpha}) < \alpha_2$, or equivalently, $\theta < (\alpha_2 - c)/(\alpha_1 - \alpha_2)$. For this local maximum to be the global maximum, $\Pi_T(P^*)$ must be larger than $\Pi_1(P_1^*)$, which is true if $[(\bar{\alpha} - c)/2]^2 > \theta\,[(\alpha_1 - c)/2]^2$, or equivalently, $\theta < [(\alpha_2 - c)/(\alpha_1 - \alpha_2)]^2$. In sum, the seller's optimum constant price is P^* if $\theta <$

$[(\alpha_2 - c)/(\alpha_1 - \alpha_2)]^2$. Otherwise, the optimal constant price is P_1^*, which corresponds to ignoring the demand of group 2 because this group is too small and/or its demand is too weak.

Next, consider the strategy of periodically offering a temporary discount, d, below the regular price, P. Assume that this "deal" is offered in one out of every T periods. Typically, in the case of grocery items, the period is the week. Since most households shop at least once a week for groceries, the assumption of zero shopping cost is quite tenable: Shopping cost is a sunk cost that must be incurred each week.

The high-inventory-holding-cost consumers (group 1) take advantage of the deal only during the period when it is offered and do not stockpile goods bought on deal for consumption in later periods. Demand for members of this group during the deal period is $x_{1d} = \alpha_1 - P + d$ and during nondeal periods is $x_{1n} = \alpha_1 - P$, where P is the standard (nondeal) price and d is the discount offered on deal.[5] Low-inventory-cost consumers (group 2) buy enough on deal to satisfy consumption needs for S periods, $S \leqq T$. The rate of consumption during these S periods is determined by the average effective price of goods bought on deal which is equal to the deal price, $P - d$, plus the average inventory holding cost of a unit of goods bought on deal, $hS/2$. The inventory holding cost per unit per period of buyers of group 2 is denoted by h. The rate of consumption during these S periods for group 2 consumers is $x_{2S} = \alpha_2 - P + d - (hS/2)$.[6] After the inventory of goods bought on deal is exhausted, the rate of consumption for group 2 consumers is determined by the standard price and is given by $x_{2t} = \alpha_2 - P$.

With these assumptions and specifications, it follows that average per period profits of the seller over the length of the deal cycle are given by

$$\begin{aligned}\Pi = \beta \Big\{ \theta \Big[\frac{T-1}{T} (\alpha_1 - P)(P - c) + \frac{1}{T} (\alpha_1 - P + d)(P - c - d) \Big] \\ + (1 - \theta) \Big[\Big(\frac{T - S}{T} \Big) (\alpha_2 - P)(P - c) \qquad (4) \\ + \frac{S}{T} (\alpha_2 - P + d - \frac{1}{2} hS)(P - c - d) \Big] \Big\}.\end{aligned}$$

But, if dealing is profitable for the seller, it must be that profits from group 2 during the first S periods of the cycle are higher than profits

5. We thus assume no dependence of current consumption on consumption in the previous period.

6. The average cost argument is not necessary since the formulation in continuous time leads to the same expression:

$$x_{2S} = \frac{1}{S} \int_0^S (\alpha - P + d - ht)\, dt = \alpha - P + d - hS/2.$$

from this group during the remaining $T - S$ periods; that is, $[\alpha_2 - P + d - (Sh/2)](P - c)$ must be larger than $(\alpha_2 - P)(P - c)$. Without loss of generality, therefore, S may be set equal to T, and the seller's average profit may be written as:

$$\Pi = \beta\left\{\theta\left[\frac{T-1}{T}(\alpha_1 - P)(P - c) + \frac{1}{T}(\alpha_1 - P + d)(P - c - d)\right]\right.$$
$$\left. + (1-\theta)(\alpha_2 - P + d - \frac{1}{2}hT)(P - c - d)\right\}. \tag{5}$$

The first-order conditions for the maximization of the seller's profits (assuming an interior solution with $d > 0$) are given by

$$\begin{aligned}\frac{\partial\Pi}{\partial P} &= \theta\left(-2P + \alpha_1 + c + \frac{2d}{T}\right) + (1-\theta) \\ &\quad \left(-2P + \alpha_2 + c + 2d - \frac{hT}{2}\right) = 0,\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial\Pi}{\partial d} &= -\frac{\theta}{T}\left(-2P + \alpha_1 + c + 2d\right) - (1-\theta) \\ &\quad \left(-2P + \alpha_2 + c + 2d - \frac{hT}{2}\right) = 0,\end{aligned} \tag{7}$$

$$\frac{\partial\Pi}{\partial T} = \frac{d\theta}{T^2}(-2P + c + \alpha_1 + d) - \frac{h(1-\theta)}{2}(P - c - d) = 0. \tag{8}$$

From (6) and (7) we obtain the result that the optimal standard price under the dealing strategy is the monopoly price when the seller serves only group 1; that is,

$$P = P_1^* = (\alpha_1 + c)/2. \tag{9}$$

This is sensible because under the dealing strategy only group 1 consumers ever pay the standard price, and the profit-maximizing price to charge this group is clearly the monopoly price P_1^*.

Replacing P by the expression for P_1^* allows one to simplify equation (7):

$$2fr = \alpha_1 - \alpha_2 - 2\left(d - \frac{hT}{4}\right) \tag{10}$$

where

$$f = \frac{\theta}{1-\theta}, \quad r = \frac{d}{T}.$$

The real price that group 2 consumers pay, $P - d + \frac{1}{2}hT$, must be less than P for these consumers to have the incentive to stockpile, implying that d must be larger than $\frac{1}{2}hT$ and a fortiori larger than $\frac{1}{4}hT$. Condition (10) thus imposes $\alpha_1 > \alpha_2$.

In addition, the contribution to seller's profit made by sales to group 2 must be positive. This contribution compensates for the opportunistic behavior of group 1 customers who, every T periods, take advantage of the deal. Referring to equation (5), the conditions are $P - c - d > 0$ and $\alpha_2 - P + d - ½hT > 0$. They imply $\alpha_2 > c$.

Lastly, we anticipate that the parameter h (inventory holding cost of group 2 buyers) should satisfy certain constraints. Intuitively, we expect that h must be sufficiently small for dealing to be profitable. Indeed, if h is too large, the discrimination mechanism via inventory holding costs will not be efficient in that stockpiling by group 2 will be very costly, and these buyers will want to pay less.

We already know that $d > ½ hT$, that is, $r = d/T > ½ h$. From condition (10) we see that $\alpha_1 - \alpha_2 > 2fr > 2f(½h)$. In other words, h must satisfy the condition

$$h < \frac{\alpha_1 - \alpha_2}{f} = \frac{1 - \theta}{\theta}(\alpha_1 - \alpha_2). \tag{11}$$

This condition is intuitive in that the upper bound on h is increasing with the size, $1 - \theta$, of the deal-prone segment and with the difference between high- and low-demand parameters.

In order to arrive at some further understanding of the nature of the solution to (7)–(8) and to uncover additional parameter constraints, we need to exploit the necessary conditions (7) and (8) more fully. Using the same notations as in equation (10), condition (8) can be rewritten as

$$\frac{2f}{h}r^2 = \frac{1}{2}(\alpha_1 - c) - d. \tag{12}$$

We have just seen that $r > ½h$. Equations (9) and (12) and condition $\alpha_2 - P + d - ½hT > 0$ with $d > 0$ thus imply

$$h < 2\frac{\alpha_2 - c}{f} = 2\frac{1 - \theta}{\theta}(\alpha_2 - c), \tag{13}$$

a condition of form similar to (11).

Expressing both d and T as functions of r (using eqq. [10] and [11]) and writing that the ratio d/T is r (by definition) leads to the following equation for r:

$$\frac{-2f}{h}r^3 + \frac{3f}{2}r^2 + \frac{1}{2}(\alpha_2 - c)r - \frac{h}{8}(\alpha_1 - c) = 0. \tag{14a}$$

There is no closed-form expression for the appropriate root of this cubic equation. However, it is possible to discuss the existence of the solution that is economically meaningful. The classical approach to the analysis of third-degree equations $ax^3 + bx^2 + cx + d = 0$ is to consider the change of variables $X = x + b/3a$ so as to rewrite the equation as $X^3 + pX + q = 0$. In our case, we obtain

$$R^3 - \frac{h}{4}\left(\frac{3h}{4} + \frac{\alpha_2 - c}{f}\right)R + \frac{h^2}{16}\left(\frac{\alpha_1 - \alpha_2}{f} - \frac{h}{2}\right) = 0 \quad (14b)$$

where $R = r - h/4$.

Because $p = -h/4\,(3h/4 + (\alpha_2 - c)/f)$ is negative, equation (14a–14b) has at least one real root. Moreover, as the left-hand side of (14a) is negative for $r = 0$ and positive for r approaching $-\infty$, this real root is negative. For the equation to have an economically meaningful root (i.e., a positive root), it must satisfy the well-known condition $27q^2 + 4p^3 < 0$, in which case it will in fact have two real positive roots. The issue then will be, Which of these two roots is the solution that has economic meaning? The condition $27q^2 + 4p^3 < 0$ is

$$-\frac{27}{16}\cdot\frac{\alpha_1 - c}{f}h^2 + \frac{9}{4}\frac{1}{f^2}\left[3\left(\frac{\alpha_1 - \alpha_2}{2}\right)^2 - (\alpha_2 - c)^2\right] \\ h - \left(\frac{\alpha_2 - c}{f}\right)^3 < 0. \quad (15)$$

Condition (15) is easier to understand if one uses the well-known trigonometric reparameterization of (14b), $R^3 + pR + q = 0, p < 0$; we rewrite its roots as $\rho \cos \theta$ where

$$\rho = \sqrt{-\frac{4}{3}p} = \sqrt{h\left(\frac{h}{4} + \frac{\alpha_2 - c}{3f}\right)}$$

and the θ's are the three roots of

$$\cos 3\theta = -\frac{4q}{\rho^3} = \frac{\sqrt{h}\left(\frac{\alpha_1 - \alpha_2}{f} - \frac{h}{2}\right)}{4\left(\frac{h}{4} + \frac{\alpha_2 - c}{3f}\right)^{1/2}}. \quad (16)$$

Condition (15) is nothing but $\cos^2 3\theta < 1$. However, in our particular case, we already know that the numerator of the right-hand side in (16) is positive (see condition [11]). As a result, the existence of positive roots simply requires $\cos 3\theta < 1$. This condition is

$$\sqrt{h}\left[\left(\frac{\alpha_1 - \alpha_2}{f}\right) - \frac{h}{2}\right] < 4\left(\frac{\alpha_2 - c}{3f} + \frac{h}{4}\right)^{1/2} \quad (17a)$$

or

$$2\left(\frac{\alpha_1 - \alpha_2}{fh}\right) - 1 < \left[4\left(\frac{\alpha_2 - c}{3fh}\right) + 1\right]^{1/2}. \quad (17b)$$

In general, this condition does not provide a simple restriction on h. However, in the special case $\alpha_1 - \alpha_2 \gg \alpha_2 - c$, the term $(\alpha_1 - \alpha_2)/f -$

$h/2$ in (17a) is large compared to $[(\alpha_2 - c)/3f + h/4]^{1/2}$. This is because $h < 2(\alpha_2 - c)/f$ is small compared to $(\alpha_1 - \alpha_2)/f$. As a result, an approximate condition derived from (17a) is

$$\sqrt{h}\left(\frac{\alpha_1 - \alpha_2}{f}\right) < 4\left(\frac{\alpha_2 - c}{3f}\right)^{3/2},$$

that is,

$$h < \frac{16}{27} \cdot \frac{\alpha_2 - c}{f} \cdot \left(\frac{\alpha_2 - c}{\alpha_1 - \alpha_2}\right). \tag{18}$$

Under the condition that $(\alpha_1 - \alpha_2)/(\alpha_2 - c)$ is large, condition (18) is more binding than (13) and a fortiori (11). It is worth noting that the quantities $(\alpha_1 - \alpha_2)/(\alpha_2 - c)$ and θ (i.e., f) were found earlier to determine whether optimal constant pricing should ignore the deal-prone segment or not. That the same quantities provide constraints on h is not surprising.

Conditions (15) or (17) are automatically satisfied, however, if the discriminant of the quadratic left-hand side of (15) is negative. This condition is

$$\left[\frac{3}{4}\left(\frac{\alpha_1 - \alpha_2}{\alpha_2 - c}\right)^2 - 1\right]^2 < \frac{4}{3}\,\frac{\alpha_1 - c}{\alpha_2 - c}.$$

This condition is obviously not satisfied when $(\alpha_1 - \alpha_2)/(\alpha_2 - c)$ is large. In this situation, that is, $(\alpha_1 - \alpha_2)/(\alpha_2 - c)$ is large, where for a finite θ the optimum constant price is to ignore the deal-prone segment, it is indeed intuitive that stringent conditions on h should be satisfied for dealing to be profitable. This is what we found with condition (18).

Going back to equations (14) and (16), if (15) is true, equations (14) and (16) have two positive roots. The question then is, Which root of equation (14a) or (14b) or (16) is the appropriate solution? In the general case, the answer to this question is difficult. In the special case where h is small and satisfies $h \ll 2(\alpha_2 - c)/f$, $h \ll (\alpha_1 - \alpha_2)/f$, and $h \ll (27/16)\,[(\alpha_2 - c)/f]\,[(\alpha_2 - c)/(\alpha_1 - \alpha_2)]$, equation (16) becomes $\cos 3\theta \sim 0$ whose three roots are $\theta_1 \sim \Pi/6$, $\theta_2 \sim \Pi/2$, and $\theta_3 \sim 5\Pi/6$, so that $\cos\theta_1 \sim \sqrt{3/2}$, $\cos\theta_2 \sim 0$, and $\cos\theta_3 \sim -\sqrt{3/2}$. Consequently, the approximate three roots of (14b) are $R_1 \sim \frac{1}{2}\sqrt{[(\alpha_2 - c)]/f\,h}$, $R_3 \sim -\sqrt{(\alpha_2 - c)/f\,h}$, and R_2 is of order higher than $h^{1/2}$. The roots of (14a) are thus $r_1 \sim \frac{1}{2}\sqrt{(\alpha_2 - c)h/f}$, $r_3 \sim -\frac{1}{2}\sqrt{(\alpha_2 - c)h/f}$, and $r_2 \sim \frac{1}{4}(\alpha_1 - c)/(\alpha_2 - c)h$, since $r_1 r_2 r_3 = -[h^2(\alpha_1 - c)]/16f$. The choice of $r = d/T$ is between the two positive roots r_1 and r_2. If one chooses r_2, equation (12) implies $d \sim \frac{1}{2}(\alpha_1 - c)$ and thus $T = d/r = [2(\alpha_2 - c)]/h$. It is easy to see that equation (10) is verified. However, the conditions $P - c - d > 0$ and $\alpha_2 - P + d - \frac{1}{2}hT > 0$ are not satisfied. In fact, $P - c - d = 0$ and

$\alpha_2 - P + d - \frac{1}{2} Th = 0$. The acceptable root is thus the larger one, r_1. It implies[7]

$$d^* \sim \frac{1}{2}(\alpha_1 - \alpha_2) \tag{19}$$

and

$$T^* \sim (\alpha_1 - \alpha_2)\sqrt{\frac{\theta}{(1-\theta)\,h\,(\alpha_2 - c)}}. \tag{20}$$

Equations (9) and (19) show that the real price paid by group 2 is $P - c - d \sim \frac{1}{2}(\alpha_2 - c)$. Indeed, as $h \to 0$, discrimination becomes perfect and real price charged to group 2 is the monopoly price for group 2. Since $hT \to 0$ as $h \to 0$, the rate of demand of group 2 is $\alpha_2 - P + d \sim \frac{1}{2}(\alpha_2 - c)$. Profits generated by group 2 are monopoly profits $(1-\theta)[(\alpha_2 - c)/2]^2$. Profits generated by group 1, when $h \to 0$, are $\theta\,[(\alpha_1 - c)/2]^2$. In other words, as $h \to 0$, the profits converge as expected toward profits under perfect discrimination:

$$\Pi_{\max} = \theta\left(\frac{\alpha_1 - c}{2}\right)^2 + (1-\theta)\left(\frac{\alpha_2 - c}{2}\right)^2. \tag{21}$$

When h is not zero, smaller profits than $\Pi_{\max}$ are achieved. Appendix A shows that profits achieved are

$$\begin{aligned}\Pi(h) = {} & \theta\left(\frac{\alpha_1 - c}{2}\right)^2 + (1-\theta)\left(\frac{\alpha_2 - c}{2}\right)^2 \\ & - \left(\frac{\alpha_1 - \alpha_2}{2}\right)\sqrt{h\theta(1-\theta)(\alpha_2 - c)},\end{aligned} \tag{22}$$

where only the lower-order term approximation (term in $\sqrt{h}$) is considered in addition to the constant term $\Pi_{\max}$ (that is independent of h). Finally, Appendix B shows that the second-order sufficient conditions for a local maximum of Π at P_1^*, d*, and T^* given by equations (9), (19), and (20) are all satisfied under the assumption that h is small.

In sum, we have established that price discrimination between two groups of buyers by the seller is possible and profitable if the high demanders also have high inventory holding costs. The low demanders must have an inventory holding cost sufficiently small that their forward buying on "deal" more than compensates for the opportunistic behavior of the high demanders who, when the deal is offered, do not stockpile but purchase for current consumption at the reduced price.

7. Equation (9) gives the optimal regular price, P_1^*, in the general case. Equations (19) and (20) are only approximations when h is small: $d/[\frac{1}{2}(\alpha_1 - \alpha_2)] \to 1$ and $T/[\alpha_1 - \alpha_2)/\sqrt{\theta/[(1-\theta)\,h\,(\alpha_2 - c)]}] \to 1$ as $h \to 0$.

III. Model Implications and How to Test the Price Discrimination Hypothesis

We now state observations that are consistent with our model, discuss the model's implications and indicate how the price discrimination hypothesis could be tested.

First, given a heterogeneous population, we should observe some consumers to stockpile when a deal is offered, and others less so—or even not at all. This is the same conclusion as Blattberg et al. However, as stated earlier, our justification of dealing by the seller is different from theirs: price discrimination versus inventory cost transference.

In order to test the inventory cost transference hypothesis, one would want to identify a correlation between amount of dealing activity and seller inventory costs: everything else constant, higher inventory cost sellers would deal more. Also Blattberg et al. would predict that bulkier items should be dealt more often and with a larger discount. By contrast, our model justifies dealing not as a cost savings measure but as a demand stimulation measure. A test of our model is thus the observation of increased average demand (and increased revenue) under dealing over the constant price strategy. Because our model postulates two groups of buyers only, optimal behavior by the seller implies no sales trough after the deal period. Adding a third group with $h' > h$ would create a trough after the deal period.

Ehrenberg's wide experience with purchase data is consistent with our theory of deals. He wrote (1980, p. 17.2): "Sales levels often respond strongly to temporary price cuts . . . . However, sales usually decrease again to more or less the pre-promotion level once the promotion is over." Elsewhere (Ehrenberg 1972, p. 128), he also espouses a view consistent with the notion of population heterogeneity that is at the core of our model: "the analysis showed that the extra sales due to a consumer promotion arose mainly from attracting extra buyers."

Ehrenberg's observation of how a promotion works and our model are consistent with the notion that consumers who buy on a deal are different from the regular price buyers; a consumer who normally buys on deal is less likely to buy at the regular price.

A major difference between our model and the inventory transference model is that only products for which the postulated correlation between demand and holding costs exists should be the object of dealing. Casual evidence suggests that some items are heavily dealt while others are almost never dealt.

Finally, the assumption of heterogeneity of the population indicates that products that cater to specific, narrowly defined segments are less likely to be dealt, for example, imported foods and drinks. This assumption also suggests that retailers with a heterogeneous clientele are more likely to deal. Large grocery stores will deal more than convenience stores and exclusive stores.

All these implications can be tested using panel data or scanner data from stores. Further, as we extend the model along the lines suggested in the next section, it is possible to uncover a richer set of implications.

IV. Model Limitations and Possible Extensions

The idealized situation of two customer groups with different demand functions and inventory holding cost, that is, h finite versus $h \to +\infty$, should be extended to the case where the demand parameters and the inventory holding cost as well as the shopping costs parameters (the latter are presently assumed to be zero) are distributed over the population of consumers.

Another important issue is that of uncertainty. Specifically, would it be more profitable for the retailer to randomize the timing of the deals? The issue here is whether the retailer can minimize the amount bought on deal by customers who would buy at the regular price and thus increase his profits. Our model assumes that demand in one period is independent of demand in previous periods. In this case, the predictability of deals is not an issue. However, in the likely case of demand interdependencies over time, predictability of deals might lead consumers who would buy at the regular price to time their consumption with the deal pattern—if the latter is predictable. Offering deals at random might minimize this opportunistic behavior.

V. Conclusion

A simple framework has been developed that formalizes some key characteristics of buyer response to deals offered by a seller. The model was simplified in order to ensure tractability but proved to be rich enough to lead to interesting implications. These include: (1) some buyers stockpile, others do not; (2) there is a pattern of periodic peaks over the normal level of sales; (3) deal-prone buyers buy more on deals than they do at the regular price; (4) some products are more likely than others to be dealt; and (5) some selling establishments are more likely than others to offer deals. This already rich set of empirically testable implications will be significantly strengthened when extensions (customer shopping costs, uncertainty, etc.) are incorporated.

Appendix A

Profits Achieved under Dealing When h Is Small: Approximation Limited to the Term in h of Lowest Order

When h is small, $r = d/T$ is approximately as $\frac{1}{2}\sqrt{(\alpha_2 - c)/f}$. The term of lowest order in h is $\sqrt{h}$. In order to approximate d, the profit function, it is necessary to know the order of the term following this first term (of order ½). To determine the order of the next term, let us write $r = A\sqrt{h} + B\sqrt{h} \cdot \epsilon$, where $A = ½ \sqrt{(\alpha_2 - c)/f}$ and $\epsilon \to 0$ as $h \to 0$. The unknown coefficient B, independent of h, must satisfy the cubic equation $r^3 - (3h/4)r^2 - h/4f(\alpha_2 - c)r + h^2/16f(\alpha_1 - c) = 0$. In other words,

$$(A^3h^{3/2} + 3A^2Bh^{3/2}\epsilon + 3AB^2h^{3/2}\epsilon^2 + B^3h^{3/2}\epsilon^3) - \frac{3}{4}(A^2h^2 + 2ABh^2\epsilon + B^2h^2\epsilon^2)$$
$$- \frac{\alpha_2 - c}{4f}(Ah^{3/2} + Bh^{3/2}\epsilon) + \frac{\alpha_2 - c}{16f}h^2 = 0.$$

This equation is satisfied if

$$A^3h^{3/2} - \frac{\alpha_2 - c}{4f}Ah^{3/2} = 0,$$

which gives the expression above for A, and if

$$3A^2Bh^{3/2}\epsilon - \frac{3}{4}A^2h^2 - \frac{\alpha_2 - c}{4f}Bh^{3/2}\epsilon + \frac{\alpha_1 - c}{16f}h^2 = 0.$$

This last equation implies that ϵ is $\sqrt{h}$. This is because the terms $h^{3/2}\epsilon$ and h^2 must cancel one another out. In other words, r can be written as $A\sqrt{h} + e$ where e is of order 1 (e = constant $\cdot\, h$). Equation (12) in the main text then implies

$$d = \frac{1}{2}(\alpha_1 - \alpha_2) + e', \tag{A1}$$

where e' is of order ½ (e' = constant $\cdot \sqrt{h}$).

Finally,

$$T = \frac{d}{r} = \frac{½(\alpha_1 - \alpha_2) + e'}{A\sqrt{h} + e} = (\alpha_1 - \alpha_2)\sqrt{\frac{\theta}{(1 - \theta)(\alpha_2 - c)h}} + C, \tag{A2}$$

where C is a constant independent of h. The profit function of the seller is (see eq. [5] in main text):

$$\Pi = \beta\{\theta[(\alpha_1 - P)(P - c) + \frac{d}{T}(2P - c - \alpha_1 - d)]$$
$$+ (1 - \theta)[(\alpha_2 - P + d)(P - c - d) - \frac{1}{2}hT(P - c - d)]\},$$

with $P = ½(\alpha_1 + c)$ and d and T given by (A1) and (A2).

$$\Pi = \beta\left\{\theta\left[\left(\frac{\alpha_1 - c}{2}\right)^2 - \frac{d^2}{T}\right] + (1 - \theta)\left[\left(\frac{\alpha_2 - c}{2} + e'\right)\right.\right.$$
$$\left.\left.\left(\frac{\alpha_2 - c}{2} - e'\right) - \frac{1}{2}hT\left(\frac{\alpha_2 - c}{2} - e'\right)\right]\right\}$$
$$= \beta\left\{\theta\left[\left(\frac{\alpha_1 - c}{2}\right)^2 - \frac{d}{T}d\right] + (1 - \theta)\left[\left(\frac{\alpha_2 - c}{2}\right)^2 - e'^2\right.\right.$$
$$\left.\left.- \frac{1}{2}hT\left(\frac{\alpha_2 - c}{2} - e'\right)\right]\right\}.$$

The term $d/T \cdot d$ is small and of order ½. The term $-\frac{1}{2} hT[(\alpha_2 - c)/2]$ is of the same order. The term e'^2 is of higher order, namely, one and thus can be neglected. The same is true of the term ½ hTe'. In sum, the approximation of the profit function limited to the term of lowest order in h ($\sqrt{h}$, that is) is

$$\Pi = \beta\left[\theta\left(\frac{\alpha_1 - c}{2}\right)^2 + (1-\theta)\left(\frac{\alpha_2 - c}{2}\right)^2 - \frac{1}{2}(\alpha_1 - \alpha_2)\sqrt{\theta(1-\theta)(\alpha_1 - c)h}\right].$$

Appendix B

Sufficient Second-Order Conditions for a Local Maximum, When *h* Is Small

The sufficient conditions for a local maximum at the point (P^*, d^*, T^*) are $\partial^2\Pi/\partial P^2$, $\partial^2\Pi/\partial d^2$ and $\partial^2\Pi/\partial T^2 < 0$ at this point, while the two-dimensional determinants,

$$\begin{vmatrix} \partial^2\Pi/\partial P^2 & \partial^2\Pi/\partial P\partial d \\ \partial^2\Pi/\partial P\partial d & \partial^2\Pi/\partial d^2 \end{vmatrix}$$

and the other two involving P and T and d and T, respectively, are positive at the same point. Finally, the three-dimensional determinant

$$\begin{vmatrix} \partial^2\Pi/\partial P^2 & \partial^2\Pi/\partial P\partial d & \partial^2\Pi/\partial P\partial T \\ \partial^2\Pi/\partial P\partial d & \partial^2\Pi/\partial d^2 & \partial^2\Pi/\partial d\partial T \\ \partial^2\Pi/\partial PdT & \partial^2\Pi/\partial d\partial T & \partial^2\Pi/\partial T^2 \end{vmatrix}$$

should be negative at this point.

In the case where h is small and terms of order 1 or higher are neglected, these conditions are respectively at the local point (P^*, d^*, T^*):[8]

$$\frac{\partial^2\Pi}{\partial P^2} = -2 < 0, \quad \frac{\partial^2\Pi}{\partial d^2} = \frac{-2\theta}{T} - 2(1-\theta) \sim -2(1-\theta) < 0, \quad \frac{\partial^2\Pi}{\partial T^2} = \frac{-2d^2\theta}{T^3} < 0.$$

The two-dimensional determinants are

$$\begin{vmatrix} \partial^2\Pi/\partial P^2 & \partial^2\Pi/\partial P\partial d \\ \partial^2\Pi/\partial P\partial d & \partial^2\Pi/\partial d^2 \end{vmatrix} = (-2)\left[\frac{-2\theta}{T} - 2(1-\theta)\right] - \left[\frac{2\theta}{T} + 2(1-\theta)\right]^2 \sim 4\,\theta(1-\theta) > 0$$

$$\begin{vmatrix} \partial^2\Pi/\partial P^2 & \partial^2\Pi/\partial P\partial T \\ \partial^2\Pi/\partial P\partial T & \partial^2\Pi/\partial T^2 \end{vmatrix} = \frac{4d^2\theta}{T^3} - \left[\frac{-2\theta d}{T^2} - \frac{h(1-\theta)}{2}\right]^2 \sim \frac{4d^2\theta}{T^3} > 0$$

$$\begin{vmatrix} \partial^2\Pi/\partial d^2 & \partial^2\Pi/\partial d\partial T \\ \partial^2\Pi/\partial d\partial T & \partial^2\Pi/\partial T^2 \end{vmatrix} \sim \frac{4\theta\,(1-\theta)^2}{T^3} > 0.$$

8. The symbol ~ represents mathematical equivalence; e.g., $g(h) \sim \text{constant} \cdot h$ means $g(h)/h \to \text{constant}$ as $h \to 0$.

Finally, the three-dimensional determinant is

$$\begin{vmatrix} -2 & \dfrac{2\theta}{T} + 2(1-\theta) & -\dfrac{2\theta d}{T^2} - (1-\theta)\dfrac{h}{2} \\ \dfrac{2\theta}{T} + 2(1-\theta) & -\dfrac{2\theta}{T} \quad 2(1-\theta) & \dfrac{2\theta d}{T^2} + (1-\theta)\dfrac{h}{2} \\ \dfrac{-2\theta d}{T^2} - (1-\theta)\dfrac{h}{2} & \dfrac{2\theta d}{T^2} + (1-\theta)\dfrac{h}{2} & \dfrac{-2d^2\theta}{T^3} \end{vmatrix} = -4(1-\theta)\,\theta\,\frac{2d^2\theta}{T^3} < 0.$$

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