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Taxes, Capital Structure, and Bondholder Clienteles*

Tax clienteles for corporate bonds are identified in a competitive financial equilibrium. Under an asymmetric corporate income tax with interest preceding principal, state-contingent bonds with higher coupon yields are issued by firms with higher pretax cash inflows per dollar of nondebt tax shield, purchased by investors in lower tax brackets, and implicitly priced at lower marginal tax rates. On these bonds all investors earn tax-induced surpluses. By contrast, all other bonds have implicit tax rates typically equal to the corporate rate and yield for many investors no tax-induced surpluses.

I. Introduction

Despite much recent progress, financial theory has yet to explain the connection, if any, between the U.S. tax code and many observed characteristics of corporate bonds.[1] Empirically, why is there no single tax rate equating yields on corporate bonds with yields on tax-exempt municipal bonds of apparently identical risks and maturities?[2] Specifically, why does the spread in yields between corporate and municipal bonds divided by the yield on corporates, a conventional measure of the marginal tax rate on corporates, decrease with increasing risk and maturity? More generally, why are different types of bonds evidently held by clienteles of investors in

* We gratefully acknowledge the helpful comments of George Constantinides, Kose John, Merton Miller, Jim Scott, Marti Subrahmanyam, and participants in seminars at Baruch, Chicago, Columbia, Houston, Michigan, NYU, Rutgers, USC, Washington, Wharton, the American Finance Association, the Western Finance Association, and the Johnson Symposium on Taxes and Finance at Wisconsin. George Constantinides was especially helpful.

1. The most notable contribution is Miller (1977). Other recent papers include Chen and Kim (1979); Kim, Lewellen, and McConnell (1979); DeAngelo and Masulis (1980*a*, 1980*b*); Taggart (1980); Kim (1982); Auerbach and King (1983); and Talmor, Haugen, and Bornea (1985). Black (1971) anticipates an equilibrium with clienteles.

2. Schaefer (1982*a*) identifies tax-induced clienteles of bondholders in the British gilt market. Van Horne (1982) reports that implicit tax rates vary over time as the supply of discount bonds changes. Additional evidence is cited in Trczinka (1982).

(*Journal of Business*, 1985, vol. 58, no. 2)

0021-9398/85/5802-0005$01.50

different tax brackets? Also, which bonds are held by various clienteles? Finally, which firms have competitive advantages in supplying securities attractive to specific clienteles?

In small part, these questions are answered in this paper. Here, in a single-period, state-preference model, firms fix their capital structures and investors pick their portfolios. Equilibrium exists in the competitive capital market when the supplies of securities by firms equal the demands by investors. The resulting tax equilibrium has the following properties. For each state, firms with higher pretax cash inflows per dollar of nondebt tax shield issue state-contingent bonds solely to investors in lower tax brackets. Implicitly priced at lower marginal tax rates, these bonds yield positive, tax-induced surpluses to all buyers. By contrast, all remaining bonds are implicitly priced at the highest marginal rate—typically, the corporate rate. On these bonds many investors earn no tax-induced surpluses.

This equilibrium has a simple, intuitive explanation. By assumption, interest expenses precede corporate taxes, while firms pay taxes only on corporate income exceeding interest expense plus nondebt tax shields. Given a higher pretax, state-contingent cash inflow per dollar of nondebt tax shield, a firm can then promise a higher state-contingent coupon and still deduct all interest expenses from taxable income. Consequently, such firms have a competitive advantage in issuing bonds with higher state-contingent coupons. Moreover, payments of interest precede repayments of principal; hence, bonds with sufficiently higher coupons also have higher coupons per dollar of bond price. These bonds attract investors in lower tax brackets who face binding personal constraints on short sales. As a result, such bonds are priced in equilibrium at lower implicit tax rates. Finally, because investors in lower tax brackets can switch costlessly to bonds with higher implicit tax rates and realize tax-induced surpluses, these investors must earn surpluses in equilibrium. By contrast, bondholders in the highest tax brackets, who buy bonds implicitly priced at the highest tax rates, cannot switch and therefore do not earn tax-induced surpluses.

This paper extends still further the growing literature on tax equilibria. In a major contribution, Miller (1977) shows that personal taxes on interest income offset corporate deductions of interest expenses to produce a tax equilibrium with no optimal capital structure for any single firm. For this result, Miller requires restrictions on tax arbitrage by investors, corporate income taxed at a single rate without limitations on deductible interest expenses, and bonds paying only interest and no principal. By contrast, DeAngelo and Masulis (1980*a*) construct a tax equilibrium with interior optimal capital structures. For this equilibrium, they must modify one of Miller's assumptions and limit the capacity of corporations to deduct interest expenses against income taxed at a single rate. In this paper, an additional assumption is altered;

now, interest is distinguished from principal. In particular, payments of interest are both deductible for firms and taxable to individuals, while repayments of principal are neither deductible nor taxable.[3]

All novel results in this paper follow from the above distinction between interest and principal. Because interest and principal are taxed differently, bonds with different ratios of interest to principal are no longer perfect substitutes, as in both Miller (1977) and DeAngelo and Masulis (1980*a*). Instead, with limitations on tax arbitrage by both firms and investors, familiar from DeAngelo and Masulis, some firms and some investors benefit by issuing or purchasing specific bonds. Consequently, clienteles of firms are categorized by their capacities to issue bonds with higher coupon yields and still deduct all interest expenses, while investors are categorized by their tax rates. As a result, many different bonds are priced in equilibrium at different implicit tax rates.

The model is specified in Section II. By assumption, the capital market is both perfectly competitive and complete, thereby separating any effects of taxes from considerations of risk. Firms issue state-contingent securities seeking to maximize their market values, while investors pick their portfolios to maximize expected utilities. Neither firms nor investors can engage in unlimited tax arbitrage. Specifically, firms cannot trade nondebt tax shields, whereas investors cannot borrow or sell short either stocks or bonds. Also, firms pay proportional taxes on corporate income net of interest expenses and nondebt tax shields, while investors pay proportional taxes at different rates on interest income. Finally, corporate payments have the following priorities, from first to last: interest expenses, income taxes, repayment of principal, and returns to stockholders. Throughout, repeated references to the Internal Revenue Code help to motivate the model's more critical assumptions.

Properties of the tax equilibrium are derived in Section III. As previously indicated, these properties include a ranking of implicit tax rates on corporate bonds and a separation of bondholders into tax-induced clienteles. Roughly, the derivation runs as follows. First, demands for bonds by investors are shown to satisfy a plunging condition. Constrained by limited personal wealth and precluded from short sales, each investor with a sufficiently low tax bracket buys bonds seeking to maximize his tax-induced surplus per dollar invested. Next, the optimal supplies of corporate coupons and, thereby, corporate bonds are computed. Given no trading in nondebt tax shields, each optimal supply curve is infinitely elastic to the coupon where the corporate deductibility of interest expenses is lost, and infinitely inelastic thereafter.

3. Interest distinguished from principal and the priority of corporate payments are also the primary focus of Talmor et al. (1985).

Finally, the demand, supply, and clearing conditions are combined to produce the properties of the tax equilibrium.

Empirical implications are discussed in Section IV. Most important, firms with higher pretax cash inflows per dollar of nondebt tax shield have greater tax-induced incentives to issue bonds with higher coupon yields. In turn, these bonds attract distinct clienteles of investors in lower tax brackets. As a result, such bonds are priced in equilibrium at lower implicit tax rates. Moreover, firms with more variable pretax cash inflows per dollar of nondebt tax shield have greater tax-induced incentives to issue more complex corporate bonds collectively resembling portfolios of state-contingent bonds. Surprisingly, these properties persist even when a firm cannot issue a portfolio of bonds which substitutes for its optimal state-contingent securities.

Section V contains the conclusion and discussion of possible extensions. All technical details appear in the Appendix.

II. The Model

The model is specified in this section. Although greatly simplified, it captures some key characteristics of the capital market: firms facing asymmetric taxes on corporate income, bonds yielding both tax-exempt principal and taxable interest, and investors paying personal taxes at different rates. Given the emphasis here on taxes, repeated references to the Internal Revenue Code help to justify the model's novel approximations. Subsequently, this model is used to construct a tax equilibrium with state-contingent bonds.

By assumption, each corporation controls only its capital structure. Initially, each firm $j = 1, \ldots, J$ can issue stock and state-contingent bonds to finance a fixed investment outlay. After one period, investment by firm j produces the net cash inflows ξ_{jk} in states $k = 1, \ldots, K$. Finally, all firms are liquidated. Denote by c_{jk} the total coupon promised by firm j on the state-contingent bond (j, k) secured solely by the corporate assets generating cash inflows in state k. Because these nonrecourse bonds have no unsecured claim on other revenues of the firm, owners of bond (j, k) receive neither interest nor principal in any other state $k' \neq k$, $k' = 1, \ldots, K$.

This reliance on what may seem to be highly stylized, state-contingent bonds is motivated by the focus here on taxes.[4] With state-contingent securities, the effects of taxes on personal portfolios can be separated from considerations of risk. As a result, tax-induced clienteles of bondholders can be isolated and identified. Moreover, under the assumptions of the model, which include symmetric information

4. DeAngelo and Masulis (1980*b*) also study state-contingent bonds, but with both principal and interest deductible from corporate income and taxable as personal income.

and no transaction costs, corporations have both the capacity and incentive to promise different coupons c_{jk} in at least some different states. Debentures, by contrast, are portfolios of state-contingent bonds with identical promised payments in all states: $c_{jk} = c_j$, $k = 1, \ldots, K$. Consequently, the model cannot arbitrarily preclude state-contingent bonds by restricting firms to debentures.

Returns on corporate bonds reflect the priority of corporate payments. For reasons explained below, this priority is modeled as follows: (1) interest expenses, (2) corporate income taxes, (3) repayment of principal, (4) returns to stockholders. Assuming that each firm j has a fixed, nontransferable, nondebt tax shield δ_j, its taxable corporate income is $\max(0, \xi_{jk} - c_{jk} - \delta_j)$. On this taxable income, corporate taxes are assessed at the constant rate τ_c. In the revenue code, initial discounts (premia) on corporate bonds are amortized and included with coupons when computing taxable corporate and personal incomes, so that no generality is lost by assuming that each firm j issues each state-contingent bond (j, k) at its face value b_{jk}.[5] In this case, the after-tax corporate cash inflow pledged by firm j in state k to bond (j, k) is

$$x_{jk} = \min[\xi_{jk}, (1 - \tau_c)\xi_{jk} + \tau_c(c_{jk} + \delta_j)]. \tag{1}$$

Thus, if state k occurs, then firm j pays in sequence the taxable interest $\min(\xi_{jk}, c_{jk})$, its corporate taxes $\tau_c \max(0, \xi_{jk} - c_{jk} - \delta_j)$, the tax-exempt return of principal $\min(b_{jk}, x_{jk} - c_{jk})$, and finally the tax-exempt return on equity $\max(x_{jk} - b_{jk} - c_{jk}, 0)$.

This priority of corporate payments is designed to approximate as simply as possible actual patterns of payment. To see this, focus first on the presumed priority of interest over principal. Under the Internal Revenue Code, this priority depends on the circumstance. For solvent firms, "the general rule in the United States courts with regard to the application of a partial payment on debt is that it shall be first applied to the liquidation of the interest due before any part is applied to the principal."[6] Although for bankrupt firms principal is always paid first, most accrued interest on multiperiod bonds is paid before bankruptcy is declared.[7] Hence, without amortization of principal, any default on

5. Under sec. 171 of the Internal Revenue Code (I.R.C.) amortization of a discount (premium) on taxable bonds is treated like interest.

6. I.R.C. sec. 61 (1984 Federal Taxes [Prentice-Hall] para. 7157); 1984 Standard Federal Tax Reporter (Commerce Clearing House) 12,231 adds the interpretation: "In the absence of allocation by either party, the general rule is that payments are in reduction of interest on the theory that, in justice to the creditor, partial payments should be applied to that portion of the debt that does not bear interest, that is, the unpaid interest." See also n. 18, below.

7. 1984 Standard Federal Tax Reporter (CCH) para. 12,231 states: "In cases of insolvency and bankruptcy, . . . all claims of equal rank share on the basis of principal only, unless and until it becomes apparent that there will be more than enough assets to pay the principal of all assets in full."

accrued interest is usually small relative to a default on principal. Moreover, an amortizing bond is equivalent to a portfolio of bonds with different maturities and no amortization. Consequently, for corporate bonds effectively protected against payouts to stockholders, the above priority approximates actual patterns of payment.[8]

The presumed priority of corporate income taxes over repayments of principal has a similar interpretation. In practice, almost all corporations pay estimated taxes quarterly as they accrue. Hence, if bankruptcy occurs, firms typically have outstanding tax liabilities that are small relative to taxes previously paid. As a result, corporate income taxes effectively precede repayments of principal, even though under the Bankruptcy Code, sections 502 and 724, secured senior claims have priority over both tax liens and unsecured claims for income taxes.

Moreover, the model's fixed, nondebt tax shields have implications for the supply of corporate bonds similar to those of the Internal Revenue Code. Currently, the code does permit some transfers of nondebt tax shields. For example, under recently restricted "safe-harbor" leasing provisions, some investment tax credits and depreciation tax shields can be traded.[9] Also, subject to limitations, corporate losses can be carried backward and forward in time.[10] Nevertheless, with limited capacity to shelter corporate income under nondebt tax shields, the supply curves of corporate bonds have infinite elasticity within restricted ranges, similar to the results in Section III.[11] These supply conditions, when combined with the demands by investors, produce the major properties of the subsequent equilibrium.

Investors hold all securities issued by corporations. Denote by z^b_{ijk} and z^s_{ij} the fraction bought by investor i, $i = 1, \ldots, I$, of bond (j, k) and stock j, respectively. These securities have the respective market

8. Different authors have assumed different priorities of corporate payments in default. Under a wealth tax with principal treated like interest, both Kraus and Litzenberger (1973) and DeAngelo and Masulis (1980*a*, 1980*b*) assume that bondholders have priority over the IRS. Under an income tax with principal distinguished from interest, Kim (1978) assumes the opposite. In Scott (1976) secured bondholders precede the IRS, which precedes unsecured bondholders. Assuming that the IRS precedes all bondholders, three papers distinguish principal from interest. Baron (1975) assumes that interest precedes principal; Turnbull (1979) assumes the opposite; while Talmor et al. (1985) consider both cases.

9. Under the Tax Equity and Fiscal Responsibility Act of 1982, safe-harbor leasing benefits were repealed for all leases entered into after December 31, 1983. On prior leases the lessor may not reduce its income tax liability, including the corporate minimum tax, by more than 50% through safe-harbor benefits. Also, the Act restricts the amount of interest, ACRS (accelerated cost recovery system) deductions, net operating loss carrybacks, and investment tax credits that may be claimed on safe-harbor property.

10. Under I.R.C. sec. 172, net operating losses are first carried backward 3 years and then forward 15 years. See DeAngelo and Masulis (1980*a*) for further discussion. Cordes and Shefferin (1981) argue that carryover provisions increase for corporations the marginal tax benefit of debt to 36 cents from 32 cents without carryovers.

11. Supply curves are infinitely elastic over all ranges, as in Miller (1977), only if the corporate tax is symmetric: losses must be credited at the same rate that income is taxed.

prices: b_{jk} and s_j. If an equilibrium is to exist in this model, unlimited tax arbitrage must be prevented, and thus some restrictions on personal borrowing, short sales, and noncorporate securities are required. Here, solely for notational simplicity, all personal borrowing, short sales, and noncorporate securities, including tax-exempt municipals, are precluded.[12] In this case, investor i allocates all his tradable wealth w_i among corporate securities, $z^b_{ijk}, z^s_{ij} \geq 0$, for $j = 1, \ldots, J$ and $k = 1, \ldots, K$, subject to the budget constraint:

$$w_i = \sum_j \left(\sum_k b_{jk} z^b_{ijk} + s_j z^s_{ij} \right). \tag{2}$$

Personal taxes are computed as follows. Ordinary income of investor i in state k is taxed at the constant marginal rate τ_{ik}, whereas all capital gains are untaxed.[13] Given a constant marginal tax rate $\tau_{ik} \geq 0$, no further generality is lost by ignoring all sources of ordinary income other than interest. Most important, all firms optimally pay no dividends. Moreover, with all bonds issued at face value and all interest paid first, bond (j, k) yields in state k the interest income $\min(c_{jk}, \xi_{jk})$. In this case, investor i realizes in state k the ordinary income:

$$y_{ik} = \sum_j \min(c_{jk}, \xi_{jk}) z^b_{ijk}. \tag{3}$$

Introduced for analytical convenience, this simple specification of taxes captures two key characteristics of tax equilibria: ordinary income taxed at higher rates than capital gains and different rates for different investors.

By assumption, the capital market is both perfectly competitive and complete. Accordingly, denote by p_k the initial market price of \$1.00 tax free in state k and zero in all other states. Given no personal borrowing, this state price is a unique constant, independent of the actions of any single firm or investor. Next, represent by $t_{jk} = T_{jk}(c_{1k}, \ldots, c_{Jk})$ the implicit tax rate of bond (j, k)—that is, the marginal tax rate of bondholders who implicitly price the bond's taxable interest. As indicated, this implicit tax rate can, but need not, depend on the k-contingent coupons c_{jk} of all firms $j = 1, \ldots, J$. Here, this holds because any changes in these coupons can, but need not, alter the clientele of investors in bond (j, k). However, in this complete, com-

12. With limited short sales and personal borrowing, bondholders still face a binding wealth constraint. This constraint is critical to Sec. III. For further discussion, see Auerbach and King (1983) and Schaefer (1982*a*). Also, with complete, state-contingent securities, municipal bonds substitute perfectly for tax-exempt stocks; hence, they add no new results in Sec. III.

13. Protopapadakis (1983) estimates that effective marginal tax rates on capital gains fluctuated between 3.4% and 6.6% during 1960–78. Also, heterogeneous, proportional personal taxes appear in Miller (1977) and DeAngelo and Masulis (1980*a*, 1980*b*).

petitive capital market, the implicit tax rate t_{jk} is determined independently of the coupons contingent on all other states, as well as the portfolios of individual investors.[14]

With this notation securities are priced as follows. Because all bonds are issued at face value and all interest is paid first, the market price of bond (j, k) is

$$b_{jk} = [\min(b_{jk} + c_{jk}, x_{jk}) - t_{jk}\min(c_{jk}, \xi_{jk})]\, p_k. \tag{4}$$

In (4) the bond's market price b_{jk} equals the current value of both principal and interest paid at maturity, $\min(b_{jk} + c_{jk}, x_{jk})$, minus all taxes on interest, $t_{jk}\min(c_{jk}, \xi_{jk})$, assessed at the implicit rate t_{jk}. Effectively, (4) is a definition of the implicit tax rate t_{jk}. That is, the state price p_k and bond price b_{jk} are determined in equilibrium by marginal stockholders in state k and investors in bond (j, k), respectively. In turn, the resulting state price p_k and bond price b_{jk} uniquely determine the implicit tax rate t_{jk} in (4). In addition, firm j has the market value

$$v_j = \sum_k [x_{jk} - t_{jk}\min(c_{jk}, \xi_{jk})]\, p_k. \tag{5}$$

Finally, the market value of the firm's stock equals the firm's market value minus the market values of all its state-contingent bonds.

III. Equilibrium

In this section, the financial equilibrium is determined from investors' demands for securities, firms' supplies of securities, and clearing conditions in the capital market. First, each investor's demand is derived from the solution to his portfolio problem. Next, each firm's supply is selected to maximize its initial market value, given the competing state-contingent coupons of all other firms. In equilibrium, the aggregate demand for each security then equals its aggregate supply. Here, the clearing conditions are

$$\sum_{i=1}^{I} z^b_{ijk} = 1 = \sum_{i=1}^{I} z^s_{ij}, \tag{6}$$

for all firms $j = 1, \ldots, J$ and states $k = 1, \ldots, K$.

Each investor solves a portfolio problem. To identify this problem, denote for investor i and state k the subjective probability π_{ik} and the corresponding state-contingent utility function U_{ik}. By assumption, all subjective probabilities π_{ik} are positive, whereas all state-contingent

14. In this complete capital market, the demands for bonds with coupons contingent on different states are independent, as shown in Sec. III. Thus, the implicit tax rate t_{jk} is independent of all state-contingent coupons $c_{j'k'}$ with $k' \neq k$, $j' = 1, \ldots, J$ and $k' = 1, \ldots, K$.

utility functions U_{ik} are increasing and strictly concave in the investor's terminal wealth. Using this notation, each investor i, $i = 1, \ldots, I$, solves the following problem: choose the fractional investments in state-contingent bonds $z^b_{ijk} \geq 0$ and stocks $z^s_{ij} \geq 0$, $j = 1, \ldots, J$ and $k = 1, \ldots, K$, to maximize

$$\sum_{k=1}^{K} \pi_{ik} U_{ik} \left\{ \sum_{j=1}^{J} [x_{jk} z^s_{ij} + \min(b_{jk} + c_{jk}, x_{jk})(z^b_{ijk} - z^s_{ij})] - \tau_{ik} y_{ik} \right\}, \quad (7)$$

subject to (2).[15] As indicated, each investor selects his portfolio so as to maximize the expected utility of his terminal wealth (7) subject to both his budget constraint (2) and constraints on short sales which preclude complete tax arbitrage.

The solution to this portfolio problem uniquely determines each investor's optimal demand for each security. As shown in Section I of the Appendix, the optimality conditions from (2) and (7) imply that each investor plunges in specific state-contingent bonds. In particular, investor i buys either no k-contingent bond or only k-contingent bonds issued by firms j_{ik}:

$$z^b_{ijk} = 0, j \neq j_{ik} \quad \in \arg\max[(t_{jk} - \tau_{ik}) \frac{c_{jk}}{b_{jk}} : j = 1, \ldots, J]. \quad (8)$$

If investor i realizes taxable income in state k, then he holds in (8) only bonds (j_{ik}, k) which maximize his tax-induced surplus $(t_{jk} - \tau_{ik})c_{jk}$ per dollar of bond price b_{jk}. Given his inability to sell short stocks yielding tax-exempt returns in state k, he thereby maximizes his after-tax returns from limited wealth invested in otherwise equivalent k-contingent bonds. Across states, investor i then allocates his wealth w_i in (2) to equalize his expected marginal utility per dollar invested.

By previous assumption, the market for tax-exempt stocks is complete. In this case, as shown in the Appendix, Section I, the optimality conditions from the previous portfolio problem categorize investors as follows. Consider an arbitrary state k; arbitrarily fix the coupons c_{jk}, $j = 1, \ldots, J$; and define the maximum k-contingent implicit tax rate $t_k \equiv \max(t_{jk}: j = 1, \ldots, J)$. If investor i has a marginal tax rate τ_{ik} in state k below the maximum rate t_k, $\tau_{ik} < t_k$, then he realizes returns in state k only from bonds with implicit tax rates t_{jk} above τ_{ik}. In other words, investor i holds no bonds (j, k) with $t_{jk} \leq \tau_{ik}$ and no stocks yielding returns in state k. Alternatively, if $\tau_{ik} = t_k$, then investor i may hold both bonds and stocks yielding returns in state k, but only bonds with the highest implicit tax rate t_k. Finally, if $\tau_{ik} > t_k$, then investor i holds no k-contingent bonds. Label these categories of investors k-

15. The constraints on short sales preclude the investor's maximization of present value. For example, a binding constraint on short sales of stock eliminates in the App., eq. (A5), proportionality between expected marginal utilities and state prices.

contingent bondholders, marginal investors, and stockholders. In general, a bondholder with respect to one state may be a marginal investor with respect to another and a stockholder for still a third.

Securities are supplied by firms. In the complete, competitive capital market, each firm supplies securities with sole objective of maximizing its initial market value (5), given the state-contingent coupons of all other firms. Inserting (1) into (5) then produces for each firm j, $j = 1, \ldots, J$, the following problem: Choose the coupons $c_{jk} \geq 0$, $k = 1, \ldots, K$, to maximize

$$\sum_{k} (1 - t_{jk})\xi_{jk}p_k + \sum_{c_{jk}<\xi_{jk}-\delta_j} [(t_{jk} - \tau_c)(\xi_{jk} - c_{jk}) + \tau_c\xi_{jk}]p_k + \sum_{\xi_{jk}-\delta_j \leq c_{jk} \leq \xi_{jk}} t_{jk}(\xi_{jk} - c_{jk})p_k, \tag{9}$$

with all k-contingent coupons $c_{j'k}$ of other firms $j' \neq j$ fixed. By controlling its coupons c_{jk}, $k = 1, \ldots, k$, firm j affects its market value v_j in (5) and thereby in (9). Simultaneously, these coupons determine the firm's supply of state-contingent bonds through the pricing equation (4) and, with corporate investment fixed, its supply of stock through its sources and uses of funds. Here, only technical convenience dictates that the firm controls its coupons. Equivalently, the firm could control each principal value b_{jk} and then allow the market to determine the corresponding coupon c_{jk}.

Each firm's optimal supply of coupons maximizes its market value (9). For firm j, the optimal supply of k-contingent coupons c^*_{jk} is computed in the Appendix, Section II, in three steps. First, the state-contingent bond price b_{jk} from (4) is inserted into the bondholder's maximand in (8). The resulting maximand, applicable to all bondholders, is then used, together with the clearing conditions (6), to identify properties of the implicit tax rate t_{jk} as a function of the corresponding state-contingent coupon c_{jk}. Finally, these properties are applied to the firm's maximand (9) to compute its optimal supply of state-contingent coupons. Surprisingly, this derivation does not depend, except through the implicit tax rate t_{jk}, on the supply of competing k-contingent coupons $c_{j'k}$ by distinct firms $j' \neq j$.

With state-contingent bonds, firms' optimal supplies of coupons are especially simple. For firm j, the optimal supply of k-contingent coupons c^*_{jk} is

$$\begin{aligned} c^*_{jk} &= 0 && t_{jk} > \tau_c \\ c^*_{jk} &\in [0, \xi_{jk} - \delta_j] && t_{jk} = \tau_c \\ c^*_{jk} &= \xi_{jk} - \delta_j && t_{jk} < \tau_c, \end{aligned} \tag{10}$$

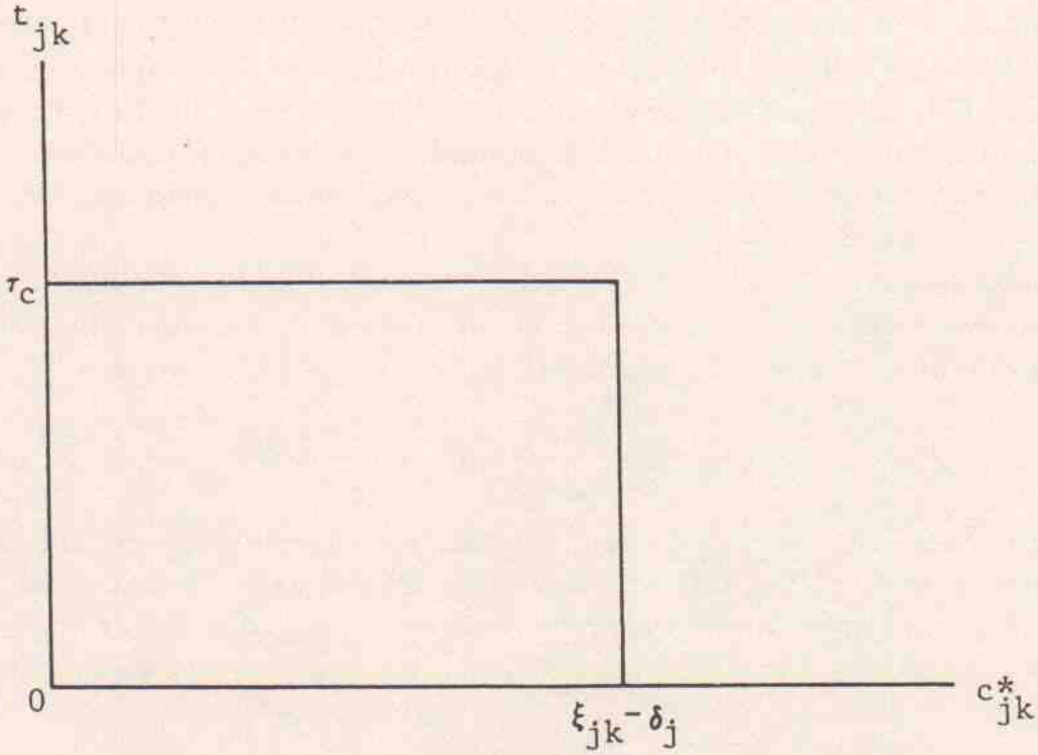

FIG. 1.—Supply of k-contingent coupons c^*_{jk} by firm j

independently of the k-contingent coupons $c_{j'k}$ from all remaining firms $j' \neq j$. This supply curve is drawn in figure 1. As indicated, the supply curve is horizontal at all coupons c_{jk} below $\xi_{jk} - \delta_j$, and vertical at $\xi_{jk} - \delta_j$. This is not surprising. For coupons c_{jk} below $\xi_{jk} - \delta_j$, taxable corporate income is positive in state k, so that the interest expenses c_{jk} are fully deductible. By contrast, for $c_{jk} \geq \xi_{jk} - \delta_j$, taxable corporate income is zero in state k, in which case no incremental interest expenses are deductible.

Of primary interest in this equilibrium is the relationship, if any, between firms' characteristics, implicit tax rates, and clienteles of investors. To identify this relationship, observe first from (10) that for each bond (j, k) with an implicit tax rate t_{jk} below the highest rate t_k, firm j optimally supplies the coupon $c^*_{jk} = \xi_{jk} - \delta_j$, independently of the k-contingent coupons $c_{j'k}$ from all remaining firms $j' \neq j$. For each such bond, the implicit tax rate in equilibrium is $t^*_{jk} = T_{jk}(c^*_{jk})$. Next, insert the optimal coupon c^*_{jk}, the resulting implicit tax rate t^*_{jk}, and the associated bond price b_{jk} from (4) into the investor's maximand in (8). Following the argument in Section II of the Appendix, this produces for investor i the implied maximand

$$\bar{U}^{ik}(c^*_{jk}, t^*_{jk}) = \frac{t^*_{jk} - \tau_{ik}}{\left[\frac{1}{(1 - \delta_j)/\xi_{jk}} - t^*_{jk}\right] p_k}, \tag{11}$$

with $\xi_{jk} > 0$. As previously indicated, each k-contingent bondholder i earns a tax-induced surplus: $\tau_{ik} < t^*_{jk}$. In this case, the implied maximand (11) increases in the tax rate t^*_{jk} and decreases in the ratio δ_j/ξ_{jk}. Consequently, all k-contingent bondholders prefer in (11) bonds (j, k) with higher implicit tax rates t^*_{jk} or lower nondebt tax shields per dollar of pretax cash inflow δ_j/ξ_{jk}.

The ranking of implicit tax rates follows immediately from bondholders' preferences (11). Consider an arbitrary state k, and, solely for notational convenience, reorder all firms $j = 1, \ldots, J$ so that[16]

$$\frac{\delta_1}{\xi_{1k}} < \frac{\delta_2}{\xi_{2k}} < \ldots < \frac{\delta_J}{\xi_{Jk}}. \tag{12}$$

Also, recall that no k-contingent marginal investor buys any bond (j, k) with an implicit tax rate t^*_{jk} below maximum rate t^*_k. Because all k-contingent bondholders prefer in (11) bonds (j, k) with higher implicit tax rates t^*_{jk} or lower ratios δ_j/ξ_{jk}, the clearing condition (6) requires that

$$t^*_{1k} < t^*_{2k} < \ldots < t^*_{J_k k} < t^*_{J_k+1,k} = \ldots = t^*_{Jk} = t^*_k, \tag{13}$$

for some firm $J_k \in \{1, \ldots, J\}$. Here, bond (J_k, k) is the last k-contingent bond bought exclusively by k-contingent bondholders. By contrast, bond $(J_k + 1, k)$ may be bought by both k-contingent bondholders and marginal investors. Finally, any remaining bonds (j, k), issued by firms $j = J_k + 2, \ldots, J$, are bought exclusively by k-contingent marginal investors.

The ranking of implicit tax rates (13) has a simple explanation. For notational convenience, identify all bonds $(j, k), j = 1, \ldots, J_k$, as inframarginal bonds, and all remaining k-contingent bonds—those held mainly by k-contingent marginal investors—as marginal bonds. Were (13) not true for all inframarginal bonds, so that $t^*_{j-1,k} \geq t^*_{jk}$ for some firm $j \in \{1, \ldots, J_k\}$, then all k-contingent bondholders would prefer bond $(j - 1, k)$ over bond (j, k). Because no investor would buy bond (j, k), the clearing condition (6) could not hold. Hence, (13) must be true for all inframarginal bonds. By contrast, all remaining bonds are held partly by marginal investors. As previously verified, k-contingent marginal investors buy only bonds (j, k) with the highest implicit tax rate t^*_k. Consequently, (13) must hold in equilibrium.

The ranking of ratios (12) orders not only the implicit tax rates in (13), but also the marginal tax rates of all bondholders. In particular, whenever investor i, with the tax bracket τ_{ik}, weakly prefers inframar-

16. All bonds with identical ratios in (13) are easily shown to be perfect substitutes. Hence, removing from (13) all redundant bonds simplifies the subsequent exposition without sacrificing economic content. Of course, with redundant bonds, the number of k-contingent bonds may be less than J, thereby introducing into (13) a minor abuse of notation.

ginal bond (j, k) to bond $(j + 1, k)$, so that $\bar{U}^{ik}(c^*_{jk}, t^*_{jk}) \geq \bar{U}^{ik}(c^*_{j+1,k}, t^*_{j+1,k}) > 0$, then (11) has the property

$$\frac{\partial}{\partial \tau_{ik}} \left[\bar{U}^{ik}(c^*_{j+1,k}, t^*_{j+1,k}) - \bar{U}^{ik}(c^*_{jk}, t^*_{jk}) \right] > 0, \tag{14}$$

for all firms $j = 1, \ldots, J_k$. As indicated in (14), the tax-induced surplus (11) on bonds with higher implicit tax rates t^*_{jk} is relatively larger for investors in higher tax brackets. When combined with the clearing condition (6), this inequality guarantees that there exists some maximum marginal tax rate $\bar{t}_{jk}$ for all investors in each bond (j, k). In equilibrium, each investor i in inframarginal bond (j, k) then has a personal tax rate τ_{ik} satisfying

$$\bar{t}_{j-1,k} < \tau_{ik} \leq \bar{t}_{jk}, \tag{15}$$

for all firms $j = 1, \ldots, J_k$. If some k-contingent bondholders buy bond $(J_k + 1, k)$, then (15) also holds for that bond. In short, bondholders sort themselves into clienteles.

The maximum marginal tax rates of clienteles and the implicit tax rates of bonds are related. As previously verified, all bondholders earn tax-induced surpluses. Thus, all bonds bought exclusively by bondholders—that is, inframarginal bonds—have maximum tax rates below their implicit tax rates: $\bar{t}_{jk} < t^*_{jk}$ for all firms $j = 1, \ldots, J_k$ and all states $k = 1, \ldots, K$. In particular, the maximum and implicit tax rates on each inframarginal bond (j, k) have the relationship[17]

$$t^*_{jk} - \bar{t}_{jk} = (t^*_{j+1,k} - t^*_{jk}) \left[\frac{\dfrac{1}{1 - \delta_{j+1}/\xi_{j+1,k}} - t^*_{j+1,k}}{\dfrac{1}{1 - \delta_j/\xi_{jk}} - t^*_{jk}} - 1 \right]^{-1} > 0. \tag{16}$$

By contrast, all remaining bonds are held mainly by marginal investors. Because marginal investors earn no tax-induced surpluses, all marginal bonds have maximum tax rates equal to their implicit tax rates: $\bar{t}_{jk} = t^*_{jk} = t^*_k$ for all firms $j = J_k + 1, \ldots, J$ and states $k = 1, \ldots, K$. The above results are summarized in the following proposition.

Proposition: Arbitrarily fix the state k and adopt the notational convention (12). In a financial equilibrium satisfying the clearing conditions (6), the investors' problems (7), and the firms' problems (9), all implicit tax rates t^*_{jk}, $j = 1, \ldots, J$, are ordered by (13). Different

17. Bondholders with the $\bar{t}_{jk}$ are indifferent between bonds (j, k) and $(j + 1, k)$. Accordingly, evaluate (12) for each bond at this tax rate; equate the results; and rearrange terms. This yields (17).

inframarginal bonds attract distinct clienteles of bondholders, distinguished in (15) by their marginal tax rates. Also, all bondholders earn tax-induced surpluses, with the minimum surplus on each inframarginal bond determined by (16). Marginal investors, by contrast, buy only bonds implicitly priced at the highest tax rate t_k^* and earn no tax-induced surpluses.

Intuitively, these results can be understood as follows. Imagine that all bonds initially have identical implicit tax rates. Investors in lower tax brackets, constrained by their limited funds, obey (8) and search for the k-contingent bond with the largest tax surplus per dollar of bond price. Firms better suited to provide this surplus—that is, firms with lower ratios in (12)—successfully compete for these clienteles by maximizing their k-contingent coupon subject to the supply condition (10).[18] Because this forces firms to default subsequently on some k-contingent principal, k-contingent bondholders then receive the highest feasible fraction of taxable income in state k. This then maximizes the difference between the firm's k-contingent tax shield and its bondholders' k-contingent tax liability. In turn, investors compete for inframarginal bonds, bidding up prices and forcing down implicit tax rates. Eventually, investors in lower tax brackets bid k-contingent bonds away from investors in higher brackets. In the resulting equilibrium, all owners of inframarginal bonds must earn tax-induced surpluses, because bondholders can always switch to bonds with higher implicit tax rates. By contrast, owners of bonds with the highest implicit tax rates cannot switch, so that marginal investors earn in equilibrium no tax-induced surpluses.

The maximum implicit tax rates never exceed and often equal the corporate tax rate τ_c. To see this, examine figure 2. For each state k, the aggregate demand equals the sum of individual demands by k-contingent bondholders in firm J_{k+1} and k-contingent marginal investors in all firms $j = J_k + 1, \ldots, J$. As illustrated in figure 2, for k-contingent marginal bonds the equilibrium then has properties familiar from Miller (1977). In this case, the maximum implicit tax rate t_k^* equals the corporate tax rate τ_c; almost all firms have unused debt capacity for k-contingent marginal bonds; and no firm has an interior optimal capital structure with respect to k-contingent marginal bonds. If, instead, the aggregate demand and supply curves interest at $\Sigma_{j=J_k+1}^{J} (\xi_{jk} - \delta_j)$, then no firm has unused k-contingent debt capacity and the maximum implicit tax rate t_k^* satisfies $t_k^* \leq \tau_c$.

18. Consequently, to minimize their joint taxes, bondholders and firms agree that payments of interest precede repayments of principal. In this case, the priority of interest under I.R.C. sec. 61 is clear. 1984 Standard Federal Tax Reporter (CCH) para. 12,230 interprets the code as follows: "The matter of applying payments received on a loan toward reduction of the principal or toward interest generally concerns only the parties involved. . . . On the cash basis, however, the parties may agree how payments are to be applied."

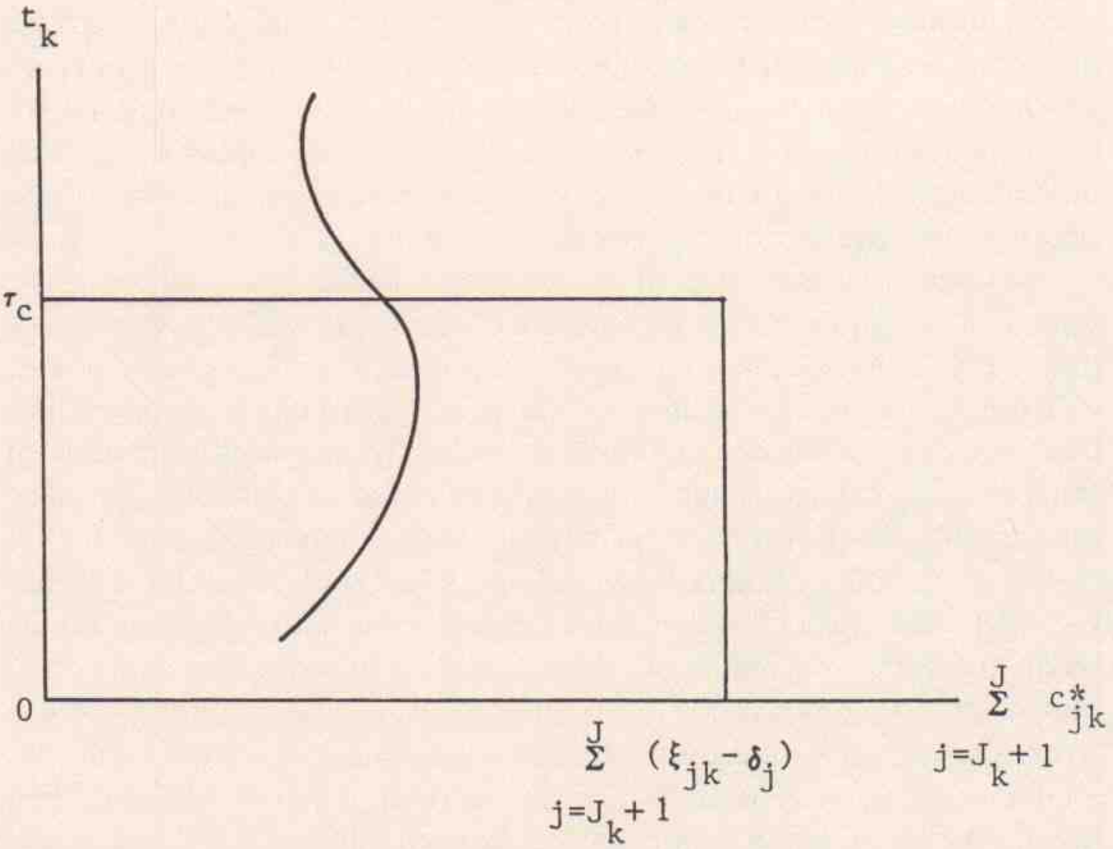

Fig. 2.—Aggregate demand and supply of k-contingent coupons with the maximum k-contingent implicit tax rate t_k.

In equilibrium, different firms can have very different capital structures with respect to different state-contingent securities. For either different firms and a specific state or a specific firm and different states, some bonds may be inframarginal, while others are marginal. For each inframarginal bond (j, k), firm j has a unique, optimal, k-contingent capital structure. In this case, the optimal coupon $c^*_{jk} = \xi_{jk} - \delta_j$ exhausts the firm's k-contingent debt capacity, eliminates its k-contingent tax liability, and thereby minimizes the taxes paid in state k by both the firm and its bondholders.[19] By contrast, for each marginal bond (j, k) with an implicit tax rate t^*_k equal to the corporate rate τ_c, the firm's k-contingent capital structure is largely irrelevant. Instead, any coupon $c^*_{jk} \in [0, \xi_{jk} - \delta_j]$ is optimal for this bond.

IV. Empirical Implications

Though derived for the technically convenient case of state-contingent debt, this equilibrium has empirical implications for more realistic corporate bonds. Specifically, it has implications for the demands, sup-

19. In this case, principal repayed equals δ_j. Thus, the tax here on interest only differs from the tax on both principal and interest in Miller (1977) and DeAngelo and Masulis (1980*a*, 1980*b*).

plies, and implicit tax rates of more complicated, composite securities in a complete market for taxable debt. Surprisingly, these properties persist even when a firm cannot issue a portfolio of bonds that substitutes for its optimal state-contingent securities. By contrast, with an incomplete market for taxable debt, the empirical implications of the previous tax equilibrium are much less precise.

Consider first the case of a complete market for taxable debt. Specifically, suppose that investors can replicate, without short positions, any taxable, state-contingent coupon c_{jk} from a portfolio of taxable corporate bonds. In this case, the previous argument again implies that investors in lower tax brackets optimally purchase portfolios of bonds paying relatively more interest per dollar of principal. Because interest effectively precedes principal, these bonds again have higher coupons per dollar of market price. In practice, such bonds have higher risk and lower ratings, other things equal. Alternatively, these bonds could accelerate payments of interest relative to repayments of principal—such as, until recently, original issue discount bonds.[20] Moreover, these bonds might include more valuable call options and less valuable conversion options. In equilibrium, such bonds are then bought by clienteles in lower tax brackets and priced at lower implicit tax rates.

Similarly, firms have tax-induced incentives to issue portfolios of bonds which substitute for their optimal state-contingent debt. Contingent on a state with a sufficiently higher pretax cash inflow per dollar of nondebt tax shield, a firm can promise a higher coupon per dollar of market price, reduce its corporate taxes, attract bondholders in lower tax brackets, and thereby reduce the total taxes paid in that state by both the firm and its bondholders. In practice, such a capital structure might be achieved by a mix of corporate bonds. Here, candidates include senior and junior debentures issued by the parent and its subsidiaries, and secured, possibly nonrecourse bonds, such as mortgages and equipment trust certificates. Moreover, with more variable pretax cash inflows per dollar of nondebt tax shield, firms have greater tax-induced incentives to issue securities collectively resembling state-contingent bonds. In practice, such firms may specialize in relatively risky technologies intensive in inputs other than capital.

These points are best illustrated by a simple example. Assume only two states, $k = 1, 2$, and order the states so that $\xi_{j1} < \xi_{j2}$ for an arbitrary firm j. If firm j optimally issues only marginal bonds contingent on state 2, then it offers any coupon c satisfying $0 \leq c_{j2} \leq \xi_{j2} - \delta_j$. In this case, the firm can attain its optimal capital structure by issuing a

20. Other features of original issue discount bonds are discussed in Pyle (1982) and Silver (1982). Silver's empirical evidence suggests that, for his sample period, original issue discount bonds had lower pretax yields to maturity than corresponding conventional bonds.

debenture with the coupon $c_j = c^*_{j1}$. However, if firm j optimally issues an inframarginal bond contingent on state 2, then it optimally offers the coupon $c^*_{j2} = \xi_{j2} - \delta_j$. Given $c^*_{j1} \leq \xi_{j1} - \delta_j < \xi_{j2} - \delta_j$ and $\delta_j > 0$, this requires a more complicated capital structure. Here, one optimal capital structure includes a debenture with the coupon $c_j = c^*_{j1}$, and a nonrecourse mortgage with the coupon $\xi_{j2} - \delta_j - c^*_{j1}$, secured solely by the assets generating returns in state 2. Alternatively, this second security could be issued by a subsidiary generating returns only in state 2.

Even if firms cannot issue portfolios of bonds substituting for state-contingent claims, similar results hold under plausible conditions. For example, suppose that the revenue code prohibits corporate tax deductions for interest expenses c_{jk} exceeding some level γ_{jk}. As indicated, these constants $\gamma_{jk} \geq 0$ can, but need not, differ across firms j and states k. Assuming that corporations can collectively issue sufficient bonds to complete the market for taxable debt, bondholders again plunge as in (8). That is, investors in lower tax brackets buy, consistent with (8), portfolios of bonds with higher coupons per dollar of bond price. In turn, from the argument producing (10), each firm j optimally supplies a portfolio of bonds with the k-contingent coupons:

$$\begin{aligned} c^*_{jk} &= 0 & t_{jk} &> \tau_c, \\ c^*_{jk} &\in [0, \min(\gamma_{jk}, \xi_{jk} - \delta_j)] & t_{jk} &= \tau_c, \\ c^*_{jk} &= \min(\gamma_{jk}, \xi_{jk} - \delta_{jk}) & t_{jk} &< \tau_c, \end{aligned} \tag{17}$$

independent of the k-contingent coupons $c_{j'k}$ from all remaining firms $j' \neq j$.

Given these demands and supplies, the financial equilibrium retains most of its previous properties. Again, portfolios of bonds with coupons matching those of k-contingent, inframarginal bonds are bought in equilibrium by clienteles in lower tax brackets and thereby priced at lower implicit tax rates. Such bonds are again issued by firms able in (17) to promise higher state-contingent coupons c_{jk} and still deduct all interest expenses from corporate taxes. However, the identities of state-contingent, inframarginal bonds, and thereby the securities in bondholders' portfolios, may differ. In particular, if the maximum deductible coupon γ_{jk} is sufficiently small, then the state-contingent bond (j, k) must be a marginal bond. In this case, bondholders buy no portfolio that replicates in state k the returns on bond (j, k).

Several, realistic modifications of the model's assumptions have little impact on its major empirical implications. For example, adding personal borrowing constrained by collateral alters neither clienteles of bondholders nor implicit tax rates in equilibrium. Instead, only stockholders—that is, investors in higher tax brackets than corporations—borrow to exploit the tax deductibility of interest expenses. Similarly, with complications like the carryover of net operating losses, a corporation's tax rate decreases as its interest expenses increase. Although this generates a downward-sloping supply curve of coupons, much as in DeAngelo and Masulis (1980*a*), and thereby complicates clienteles of firms issuing bonds in equilibrium, it again leaves unchanged the ordering by tax rates of both bonds and bondholders.

Without a complete market for taxable debt, incentives to diversify portfolios and minimize taxes interact to confound both clienteles of bondholders and implicit tax rates on bonds. That is, with adverse selection, moral hazard, and transactions costs, corporations may collectively issue insufficient securities to complete the market for taxable debt. As a result, investors in lower tax brackets, who have on average smaller wealth and thereby greater risk aversion, may buy relatively more, less risky bonds. With interest effectively preceding principal, less risky bonds typically have lower coupons per dollar of market price. Thus, risk aversion confounds the previous negative relationship between implicit tax rates and coupons per dollar of bond price.

V. Conclusion

Prior to Miller (1977), the theory of corporate capital structure focused primarily on corporate taxes and the resulting supply of corporate securities. In his important paper, Miller introduced proportional personal taxes, different for different investors, and produced a tax equilibrium with no optimal capital structure for any single firm. By adding asymmetric corporate taxes, DeAngelo and Masulis (1980*a*) identified an equilibrium with an implicit tax rate on corporate bonds below the corporate rate and interior optimal capital structures for all firms. Here, with asymmetric corporate taxes and payments of interest distinguished from repayments of principal, many bonds are no longer perfect substitutes. Instead, different bonds are categorized in equilibrium by their clienteles of investors, implicit tax rates, and issuing firms.

The tax equilibrium of this paper has the following properties. State-contingent bonds with higher coupons per dollar of market price are issued by firms with higher state-contingent cash inflows per dollar of nondebt tax shield. These bonds are bought by investors in lower marginal tax brackets and thereby implicitly priced at lower marginal tax rates. On these bonds all investors earn tax-induced surpluses. By contrast, all remaining state-contingent bonds attract investors with the highest tax brackets among bondholders, have implicit tax rates commonly equal to the corporate rate, and yield for many investors no tax-induced surpluses.

Because the model is so simple, many extensions are possible. One major extension would include multiperiod bonds. What are some likely properties of a tax equilibrium with such bonds? Perhaps bonds with longer maturities are purchased by investors in lower tax brackets and priced at lower implicit tax rates.[21] To see this, suppose that a firm issues stock and either a multiperiod bond or a sequence of single-period bonds with one objective—maximizing its current market value. In either case, stockholders optimally default whenever a promised payment on the bond exceeds their firm's current cash inflow plus their stock's market value immediately after the payment. Assuming that the multiperiod bond has a coupon below the coupon plus principal on the current single-period bond, stockholders may optimally default later on the longer-term bond, and never sooner. Because interest effectively precedes principal, the longer-term bond then yields more interest relative to principal. If bondholders' returns are not altered too much by tax-induced options to realize capital gains or losses, then longer-term bonds have higher coupons per dollar of initial price and thus lower implicit tax rates.[22]

Appendix

I. Demand for Securities

Derivation of the Plunging Condition (8)

With $\delta_j \geq 0$, the presumed priority of corporate payments produces the effective coupon:

$$\min(c_{jk}, \xi_{jk}) = \min(c_{jk}, x_{jk}). \tag{A1}$$

Hence, set $c_{jk} \leq x_{jk}$ and rewrite (4) as follows:

$$\min(b_{jk} + c_{jk}, x_{jk}) = \frac{b_{jk}}{p_k} + t_{jk}c_{jk}. \tag{A2}$$

Next, differentiate (7) with respect to z^b_{ijk}; apply the associated Kuhn-Tucker condition; insert (A2); and rearrange terms. This yields at the unique solution to problem (7) the necessary condition:

$$(t_{jk} - \tau_{ik})\frac{c_{jk}}{b_{jk}} = \frac{\kappa_i + \lambda^b_{ijk}}{\pi_{ik}U'_{ik}} - \frac{1}{p_k}, \tag{A3}$$

$$\kappa_i > 0,\ \lambda^b_{ijk} \leq 0 \leq z^b_{ijk},\ \lambda^b_{ijk}z^b_{ijk} = 0, \tag{A4}$$

with the constants κ_i and λ^b_{ijk}. Here, the multiplier κ_i is associated with the budget constraint (2), while the multiplier λ^b_{ijk} is associated with the short-sale constraint on bond (j, k). For notational convenience, the latter constraint has been rewritten equivalently as $z^b_{ijk}b_{jk} \geq 0$. On the right side of (A3) only λ^b_{ijk} depends on j; hence, this multiplier satisfies $\lambda^b_{ijk} < 0$ for all $j \in \{1, \ldots, J\}$ not maximizing the left side of (A3). For all such j, (A4) requires that $z^b_{ijk} = 0$, yielding (8).

21. This relationship persists empirically. See, e.g., the citations in Trczinka (1982).
22. As shown in Constantinides and Ingersoll (1984), tax-induced options to realize gains or losses alter effective yields on bonds.

Categories of Investors

By assumption, the market for tax-exempt stocks is complete, even with the short-sale constraints $z^s_{ij} \geq 0, j = 1, \ldots, J$. In this case, the notation is greatly simplified, without further loss in generality, by orthogonalizing the returns on stocks as follows: $x_{jk} = 0$ for all $j \notin \{J'_k, \ldots, J'_{k+1} - 1\}$ with $J'_1 = 1$ and $J'_{k+1} = J + 1$. Consequently, all firms effectively issue state-contingent stocks; that is, stocks $j \in \{J'_k, \ldots, J'_{k+1} - 1\}$ yield returns only in state k. With this simplification, (4), (5), and (A1) imply that

$$s_j = \max(x_{jk} - b_{jk} - c_{jk}, 0)p_k, \tag{A5}$$

for all $j \in \{J'_k, \ldots, J'_{k+1} - 1\}$. Differentiate (7) with respect to z^s_{ij}; apply the associated Kuhn-Tucker condition; and insert (A5). After simplification, this yields at the unique solution to problem (7) the necessary condition:

$$\pi_{ik}U'_{ik} = (\kappa_i + \lambda^s_{ij})p_k \tag{A6}$$

$$\lambda^s_{ij} \leq 0 \leq z^s_{ij},\ \lambda^s_{ij}z^s_{ij} = 0, \tag{A7}$$

with the additional multiplier λ^s_{ij} associated with the short-sale constraint on stock j. Again, for notational convenience, this constraint has been rewritten equivalently as $z^s_{ij}s_j \geq 0$. Finally, insert (A6) into (A3) to give

$$(t_{jk} - \tau_{ik})\frac{c_{jk}}{b_{jk}} = \frac{\lambda^b_{ijk} - \lambda^s_{ij}}{\pi_{ik}U'_{ik}}. \tag{A8}$$

As defined in the text, k-contingent bondholders buy no bonds (j, k) with $t_{jk} \leq \tau_{ik}$ and no stocks $j \in \{J'_k, \ldots, J'_{k+1} - 1\}$. To see this, observe from (8) that if $\tau_{ik} < t_k$, then $z^b_{ijk} > 0$ only if $\tau_{ik} < t_{jk}$. For such bonds (j, k) with $z^b_{ijk} > 0$, (A4) and (A8) imply that $\lambda^s_{ij} < \lambda^b_{ijk} \leq 0$, so that $z^s_{ij} = 0$ from (A7).

Also, as defined in the text, k-contingent marginal investors buy no bonds (j, k) with $t_{jk} < t_k$. To verify this, again note from (8) that if $\tau_{ik} = t_k$, then $z^b_{ijk} > 0$ only if $t_{jk} = t_k$.

Finally, as defined in the text, k-contingent stockholders buy no bonds (j, k), $j = 1, \ldots, J$. In this last case with $\tau_{ik} > t_k$, (A7) and (A8) require that $\lambda^b_{ijk} < \lambda^s_{ij} \leq 0$, so that $z^b_{ijk} = 0$ from (A4).

II. Supply of Coupons

Implied Maximand of Bondholders

Using (A1), rewrite (4) as follows:

$$b_{jk} = \begin{cases} (1 - t_{jk})x_{jk}p_k & x_{jk} < c_{jk} \\ (x_{jk} - t_{jk}c_{jk})p_k & c_{jk} \leq x_{jk} < b_{jk} + c_{jk} \\ \dfrac{1 - t_{jk}}{1 - p_k}c_{jk}p_k & b_{jk} + c_{jk} \leq x_{jk}. \end{cases} \tag{A9}$$

From (1) and (A9), the inequality $b_{jk} + c_{jk} \leq x_{jk}$ holds if and only if $c_{jk} \leq c^0_{jk}$, where c^0_{jk} satisfies

$$c^0_{jk} \equiv x_{jk} - b_{jk} = (1 - \tau_c)\,\xi_{jk} + \tau_c(c^0_{jk} + \delta_j) - \frac{1 - t_{jk}}{1 - p_k}c^0_{jk}p_k. \tag{A10}$$

Inserting (9a) into the bondholder's maximand in (8) yields the implied maximand

$$\bar{U}^{ik}(c_{jk}, t_{jk}) \equiv \begin{cases} \dfrac{t_{jk} - \tau_{ik}}{p_k} \dfrac{1 - p_k}{1 - t_{jk}} & 0 \le c_{jk} \le c_{jk}^0 \\ \dfrac{t_{jk} - \tau_{ik}}{p_k} \left[\dfrac{(1 - \tau_c)\xi_{jk} + \tau_c \delta_j}{c_{jk}} + \tau_c - t_{jk} \right]^{-1} & c_{jk}^0 \le c_{jk} \le \xi_{jk} - \delta_j \\ \dfrac{t_{jk} - \tau_{ik}}{p_k} \left[\dfrac{\xi_{jk}}{c_{jk}} - t_{jk} \right]^{-1} & \xi_{jk} - \delta_j \le c_{jk} \le \xi_{jk}, \end{cases} \tag{A11}$$

with

$$c_{jk}^0 = \frac{(1 - \tau_c)\xi_{jk} + \tau_c \delta_j}{1 - \tau_c + (\tau_c - t_{jk})p_k}(1 - p_k)$$

from (A10). The critical coupon c_{jk}^0 has the following interpretation. When $0 \le c_{jk} \le c_{jk}^0$, both principal and interest are fully paid in state k; whereas, when $c_{jk} \ge c_{jk}^0$ principal is not completely paid.

Properties of the Implicit Tax Rates t_{jk}

Because each bondholder i has the implied maximand (A11), properties of the implicit tax rates t_{jk} follow from properties of (A11) and the clearing conditions (6). Accordingly, examine (A11). Also, arbitrarily fix the k-contingent coupons $c_{j'k}$ of all remaining firms $j' = 1, \ldots, J$ with $j' \neq j$. On the initial interval $0 \le c_{jk} \le c_{jk}^0$, (A11) is independent of the coupon c_{jk} and increasing in the implicit tax rate t_{jk}. Thus, the clearing condition (6) requires that $t_{jk} = t_k$ for any bond (j, k) with $0 \le c_{jk} \le c_{jk}^0$. Were this condition violated for some coupon c_{jk} below c_{jk}^0, then no investor would buy bond (j, k). By contrast, on the second interval $c_{jk}^0 \le c_{jk} \le \xi_{jk} - \delta_j$, the maximand (A11) is increasing in both c_{jk} and t_{jk}. In this second case, the clearing condition (6) requires that the implicit tax rate t_{jk} decreases in the coupon c_{jk}. Again, were this condition violated, then no investor would buy bond (j, k). Finally, for $\xi_{jk} - \delta_j \le c_{jk} \le \xi_{jk}$, the maximand (A11) decreases in c_{jk} for fixed $t_{jk}c_{jk}$ and increases in $t_{jk}c_{jk}$ for fixed c_{jk}, since $\tau_{ik} < t_{jk}$ from Section I of this Appendix. Consequently, on this final interval the clearing condition (6) requires that $t_{jk}c_{jk}$ increases in the coupon c_{jk}, once again for the same reason.

Derivation of the Supply Curve (10)

Denote by A_{jk} all terms in (9) independent of c_{jk}. Using this notation, rewrite (9) as

$$\begin{aligned} &A_{jk} + (\tau_c - t_{jk})c_{jk}p_k && 0 \le c_{jk} \le \xi_{jk} - \delta_j, \\ &A_{jk} - t_{jk}c_{jk}p_k && \xi_{jk} - \delta_j \le c_{jk} \le \xi_{jk}. \end{aligned} \tag{A12}$$

For the final interval $\xi_{jk} - \delta_j \le c_{jk} \le \xi_{jk}$, the above results guarantee that $t_{jk}c_{jk}$ increases in c_{jk}, so that (A12) decreases in c_{jk}. This insures that $c_{jk}^* \le \xi_{jk} - \delta_j$. Next, consider the initial interval $0 \le c_{jk} \le c_{jk}^0$, on which $t_{jk} = t_k$ from the above results. Here, (A12) is increasing in c_{jk} if $t_k < \tau_c$, constant if $t_k = \tau_c$, and decreasing if $t_k > \tau_c$. By contrast, t_{jk} is decreasing in c_{jk} on the intermediate interval $c_{jk}^0 \leq c_{jk} \leq \xi_{jk} - \delta_j$ from the previous results. Collectively, these properties guarantee for all $0 \leq c_{jk} \leq \xi_{jk} - \delta_j$ that $t_{jk} \leq t_k \leq \tau_c$; in which case, (A12) increases in c_{jk} on the intermediate interval $c_{jk}^0 \leq c_{jk} \leq \xi_{jk} - \delta_j$. These properties of the maximand (A12) are independent of the remaining k-contingent coupons $c_{j'k}$, $j' = 1, \ldots, J$ with $j' \neq j$. This produces the supply curve (10).

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