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The Value of the Tax Subsidy on Risky Debt*

This paper examines the relationship between leverage and the value of the firm, when non-debt-related tax shields are available and the corporate tax is levied as an income tax. In a previous paper, De Angelo and Masulis argue that, in the presence of non-debt-related tax shields, the relationship between debt and firm value is concave, resulting in an interior optimal capital structure when there is a tax-induced differential in the cost of corporate debt. Their result derives from the fact that they assume the payment of debt principal and non-debt-related depreciation charges are deductible. However, the former deduction is consistent with a wealth tax, while the latter is consistent only with an income tax. We show that if the interest alone is deductible, the debt–firm value function is convex, resulting in corner solutions to the capital structure problem.

I. Introduction

The effect of corporate taxes on the market value of a levered firm continues to be a central issue in recent contributions in finance theory (e.g., Miller 1977; DeAngelo and Masulis 1980; Kim 1982; Modigliani 1982). In these and other studies (e.g., Krause and Litzenberger 1973; Scott 1976; Brennan and Schwartz 1978; Kim 1978), the relationship between market value and capital structure is established by formulating a tax subsidy function that specifies the partial effect of debt on the expected tax savings at the corporate level under the existing U.S. tax code. A working assumption in most of these studies is that both principal and interest are tax deductible. This assumption is made in the spirit of the original Modigliani and Miller (1963) formulation in which debt is taken to be perpetual and riskless. Indeed, it is well known that the tax shield provided by the deduction of principal in a single-period framework has the same value as the tax shield from interest deductions in the case of perpetual and riskless debt. Under this assumption, it is shown that in the absence of non-debt-

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related tax shields the tax subsidy associated with issuing any amount of debt is constant on a per unit basis. The introduction of firm-specific non-debt-related tax shields such as depreciation (DeAngelo and Masulis 1980) increases the number of states in which the tax shield of debt is redundant and thereby is said to generate a diminishing marginal tax subsidy of debt, which would result in an interior optimal capital structure on the basis of the tax code alone.

Consideration of alternative tax shields such as depreciation simultaneously with the deduction of principal (as in DeAngelo and Masulis [1980] and Kim [1982]) is internally inconsistent, however. This is readily observed by considering the difference between two alternative tax regimes: a wealth tax and an income tax. A wealth tax is imposed on shareholders' net worth which is the difference between the current value of the firm and the current value of its debt ("principal"). For a wealth tax, principal is deductible, but there is no depreciation deduction. Alternatively, an income tax is imposed on periodic income. For income tax, there is a depreciation deduction, but principal is not deductible. Thus, a deduction of both principal and depreciation is inconsistent with either tax system. To emphasize this point further, assume that an investment is totally financed by debt. Allowing for the deduction of both principal and depreciation amounts to a double deduction of the initial investment outlay.

Also, for risky debt, the equivalency between the deduction of interest in perpetuity and the deduction of principal in a single period breaks down because future cash flows are truncated—an event that is associated with bankruptcy. Therefore, the assessment of the tax subsidy of risky debt must be based on an analysis of corporate taxation in bankruptcy.

In view of these criticisms, it is safe to conclude that the only consistent way to analyze the tax subsidy of risky debt when non-debt-related deductions are present is by adhering to an income tax framework that allows for the deduction of interest but not principal and includes explicit references to the tax status of the firm in bankruptcy. As is shown in this paper, this modification of the tax environment has far-reaching implications regarding the nature of the tax subsidy function associated with debt. We find that in general the *marginal* value of the tax subsidy *increases* with the amount of debt employed. The intuitive explanation for this surprising result is that as the firm increases its financial leverage, debt becomes increasingly risky. Tax authorities determine the nondeductible portion of the promised payments ("principal") by the market value of the debt at issuance. As more debt is issued and the probability of partial payments increases, the magnitude of the promised payments (principal plus interest) must increase relative to the initial market value.[1] Consequently, the *ex-*

1. Baron (1975) recognized this property of risky debt when he examined the bounds

pected amount of interest deduction and the associated after-tax expected cash flows to owners of the firm *increase* with the proportion of debt in the capital structure. Of course, the probability of default on the promised payment also increases, and this second effect acts to reduce the value of the tax subsidy at the margin. As it turns out, however, the first effect dominates, and so the marginal tax subsidy function increases as additional debt is issued.

In the spirit of the literature following Miller (1977), we analyze the implications of this result, assuming that investors are subject to personal taxes. To focus on the derivation of the corporate optimal supply of debt, we assume for simplicity that there is a uniform personal tax rate across all investors, which implies a horizontal demand curve. This assumption allows the derivation of an equilibrium relationship, which does not require imposing short sale restrictions to prevent tax arbitrage by investors.[2] Corporations are assumed to have access to non-debt-related tax shields. We demonstrate that the resultant equilibrium is characterized by *corner* solutions (either all equity or all debt) for the individual corporation, and a finite amount of debt for the corporate sector as a whole.

The paper is organized as follows. Section II discusses the existing U.S. tax code rulings regarding the deductibility of interest in bankruptcy and correspondingly presents the state-contingent after-tax cash flows to shareholders and bondholders. In Section III we derive the value of the levered firm and the associated marginal tax subsidy function. We then show the properties of bond market equilibrium under personal taxes and non-debt-related tax shields. Section IV concludes the paper.

II. Interest Deduction in Bankruptcy

Our analysis of the relationship between debt and corporate taxes is conducted within a single-period framework in which the firm issues debt at the beginning of the period to (partially) finance its investments. At the end of the period, the firm generates a single state-dependent cash flow, $\tilde{X}$. Define D as the market value of debt at issuance (principal), and R as the promised interest payment. Let I denote the amount of non-debt-related noncash tax deductions (e.g., depreciation charges associated with the original capital investment in the firm) and let τ denote the corporate tax rate. It is assumed that interest income from

on the value of the tax subsidy of debt under what we later refer to as the interest-first doctrine. Baron did not explore, however, the shape of the marginal function and does not allow for non-debt-related tax shields or for personal taxes.

2. For a thorough investigation of the demand for bonds in equilibrium under heterogeneous personal tax rates and for the characteristics of the resulting tax clienteles, see Park and Williams (1982).

bonds is fully taxable at the personal level at a uniform rate τ_p, but income from stocks is tax exempt.

The distribution of the realized cash flow, $\tilde{X}$, to bondholders and to stockholders depends on the relative magnitudes of D (a control variable) and I. Consider first the case $D > I$, as may be the situation when economic rents (or growth) exist. If we assume that debt is unsecured, the government's claim to taxes has priority over bondholders' claim for principal and interest. Bankruptcy occurs whenever the cash flows net of taxes are insufficient to meet the promised payments to bondholders ($R + D$). The condition for bankruptcy is given by $\tilde{X} - \tau(\tilde{X} - R - I) < R + D$. Under existing law, the corporation is subject to the corporate tax even when its cash flows are insufficient to produce returns to stockholders. Since under existing law the corporation is considered to be an individual entity, partial interest payments to bondholders continue to be a deductible expense when the amount of tax obligation is computed. Thus, in addition to I, the corporation may claim some part of the payment to bondholders as an interest deduction.

Note that bondholders receive only a fraction of the full promised payment in bankruptcy, so it is unclear how much is interest and how much is principal. As a general rule, partial payments made on interest-bearing notes are credited first to interest and then to principal.[3] Under this doctrine, the corporation retains the full interest shield so long as its payment exceeds the interest charge. However, in the case of bankruptcy where the payment is a lump-sum settlement of the entire debt, the doctrine that designates partial payments first to interest does not apply.[4] In this situation, the proceeds are deemed to apply first to principal, and any excess is then applied to interest. Thus the payment to the bondholders in bankruptcy is taken by the tax authorities to be $D + \alpha R$, where α is the fraction of the interest payment deemed to be received. The fraction α and the associated tax must be determined simultaneously.[5]

The interest-first and the principal-first doctrines have been respectively modeled in the literature by Baron (1975) and Turnbull (1979) for the case where there is no personal taxation. In the analysis that follows, we concentrate on the principal-first doctrine because it is applicable to the case of corporate bankruptcy. However, it can be shown that the central conclusion regarding the positive relationship between

3. See Plunkett, 41 BTA 700, Dec, 11,045 (Acq.).

4. See Petit et al., 8 TC 228 Dec, 15,576 (Acq.); Warner Co., 11 TC 419, Dec, 16,603.

5. The tax is given by $\mathrm{TAX} = (\tilde{X} - I - \alpha R)\,\tau$ and the bondholders payment by $D + \alpha R = \tilde{X} - \mathrm{TAX}$. By substituting we find that $\alpha R = \tilde{X} - (D - I\tau)/(1 - \tau)$. Thus $\alpha = 0$ when $\tilde{X} = (D - I\tau)/(1 - \tau)$, and $\alpha = 1$ when $\tilde{X} = R + (D - I\tau)/(1 - \tau)$. In between these limits, the net cash flow accruing to bondholders is given by $\tilde{X} - (D - I\tau)/(1 - \tau) + D$.

the marginal tax subsidy and the amount of debt remains intact under the interest-first doctrine.[6]

Under the principal-first doctrine, the cash flows at the end of the period to equityholders, $\tilde{X}_E$, and to bondholders, $\tilde{X}_D$, for the case $D > I$, are given correspondingly by

$$\tilde{X}_E = \begin{cases} \tilde{X} - R - D - \tau(\tilde{X} - R - I) & ; \tilde{X} > d \\ 0 & ; \tilde{X} \leq d \end{cases} \tag{1}$$

and

$$\tilde{X}_D = \begin{cases} R + D - \tau_p R & ; d < \tilde{X} \\ D + (\tilde{X} - g)(1 - \tau_p) & ; g < \tilde{X} \leq d \\ \tilde{X} - \tau(\tilde{X} - I) & ; I < \tilde{X} \leq g \\ \tilde{X} & ; 0 < \tilde{X} \leq I, \end{cases} \tag{2}$$

where $d \equiv R + (D - \tau I)/(1 - \tau)$ and $g \equiv (D - \tau I)/(1 - \tau)$.

Now consider the case $D \leq I$. Bankruptcy occurs whenever $\tilde{X} \leq D + R$. In this case, no corporate taxes are paid as the amount of deduction at the corporate level, $I + R$, is more than sufficient to shelter $\tilde{X}$. The distribution of the cash flows when $D \leq I$ and for different realized values of $\tilde{X}$ is

$$\tilde{X}_E = \begin{cases} \tilde{X} - R - D - \tau(\tilde{X} - R - I) & ; R + I < \tilde{X} \\ \tilde{X} - R - d & ; R + D < \tilde{X} \leq R + I \\ 0 & ; \tilde{X} \leq R + D \end{cases} \tag{3}$$

$$\tilde{X}_D = \begin{cases} R + D - \tau_p R & ; R + D < \tilde{X} \\ \tilde{X} - \tau_p(\tilde{X} - D) & ; D < \tilde{X} \leq R + D \\ \tilde{X} & ; 0 < \tilde{X} \leq D. \end{cases} \tag{4}$$

III. Firm Valuation under Corporate and Personal Taxation

The valuation of the levered firm is based on the after-corporate-tax cash flows to stockholders and bondholders which appear in equations (1) through (4). It is assumed that capital markets are complete, so that all future cash flows can be expressed as an additive function of state-space prices. In most of what follows no specific assumptions will be made about the shape of the investors' utility function.

As described above, the tax treatment and hence the state-contingent cash flows to stockholders and bondholders depend on whether or not D exceeds I. Consequently, the value of the firm should be assessed separately for these two cases.

The total cash flow to the prior and residual claimants on the firm for

6. For more details, see n. 11.

the case $D > I$ is obtained by summing the cash flows accruing to stockholders and bondholders as presented in equations (1) and (2). So,

$$\tilde{CF} = \tilde{CF}_u + (\tau - \tau_p)\begin{Bmatrix} R \; ; d < \tilde{X} \\ \tilde{X} - g \; ; g < \tilde{X} \leq d \\ 0 \; ; \quad \tilde{x} \leq g \end{Bmatrix}, \tag{5}$$

where $\tilde{CF}_u$ is the cash flow to the shareholders of an identical non-levered firm and $\tilde{CF} \equiv \tilde{X}_E + \tilde{X}_D$. (Recall that $\tilde{X} = g$ is the point from which the firm begins to pay interest and $\tilde{X} = d$ is the point from which interest is paid in full.) Note from (5) that when $D > I$, taxation is symmetric: all the cash flows that are deductible for the corporation are simultaneously taxable for bondholders on personal account.

Let $p(s)$ be the price of \$1.00 delivered if the state of nature is in the interval $[s, s + ds]$. From the linearity of the valuation operator it follows that the one-period interest rate on a risk-free asset, r^*, is defined by $\int_s p(s)ds = 1/(1 + r^*)$. As in Baron (1979), the measure $(1 + r^*)p(s)$ meets all the axiomatic requirements of a probability and thus can be interpreted as the "probability" associated with state s. (It will be equal to the true density for the case of risk neutrality.) This enables us to employ the following notation in our analysis:

$p(Z) =$ the price of \$1.00 delivered in all states for which $\tilde{X}(s) = Z$ $[p(z)dz] = p[s{:}X(s) = \tilde{X}(s) = Z]ds$;
$F(Z) =$ the cumulative "distribution" function at Z $[F(Z) = \int_{-}^{Z} p(X)dX]$;
$E_W^Z[g(\tilde{X})] =$ a partial expectation operator over the measure $p(\cdot)$ $\{E_W^Z[g(\tilde{X})] = \int^Z g(X)p(X)dX\}$, where the absence of superscript or subscript implies that the limit of integration is $\infty(-\infty)$;
$k =$ one-period discount factor, $k = 1/(1 + r^*)$;
$V_u =$ the value of the unlevered firm, $V_u = E(\tilde{CF}_u)$.

The assumption of a single personal tax rate, τ_p, guarantees that for each state s, $p'(s) = p(s)(1 - \tau_p)$, where $p'(s)$ is the state price of a taxable state-contingent security. Thus, the relative prices of debt and equity are established such that the after-personal-tax expected yields on debt and equity are equated.

The market value of debt, D, is determined by the state-contingent net cash flows to bondholders. For risk-free debt, $D = k([R(I - \tau_p) + D]$. For risky debt, its value in the region $D > I$ is given from (2) by

$$\begin{aligned} D = {} & [R(I - \tau_p) + D][k - F(d)] + E_0^d(\tilde{X}) \\ & - \tau E_I^g(\tilde{X} - I) - (g - D)[F(d) - F(g)] - \tau_p E_g^d(\tilde{X} - g). \end{aligned} \tag{6}$$

The first term on the right-hand side of (6) is the present value of the after-tax full promised debt payment. The second term in (6) is the value of the gross cash flow to bondholders in all states in which bankruptcy occurs ($\tilde{X} < d$). From this we subtract three terms. The first two terms represent the present value of corporate taxes paid by the bondholders in bankruptcy. The third expresses the personal taxes paid by bondholders in bankruptcy. Hence from equation (6), bond yields (prices) are grossed up (down) to reflect the personal tax rate, τ_p. Using integration by parts, expression (6) can be simplified to

$$D = \frac{(1 + r^*)}{r^*}\left[kR(1 - \tau_p) - \int_0^d F(X)dX + \tau \int_I^g F(X)dX + \tau_p \int_g^d F(X)dX\right]. \tag{6'}$$

The total value of the levered firm, V, is obtained by multiplying the state-contingent cash flows by the state prices and summing over all states. Hence from (5),

$$\begin{aligned} V &= V_u + (\tau - \tau_p)\{E_g^d(\tilde{X} - g) + R[k - F(d)]\} \\ &= V_u + (\tau - \tau_p)\left[kR - \int_g^d F(X)dX\right]. \end{aligned} \tag{7}$$

The valuation formula adds the market value of the tax shield to the value of the unlevered firm. The value of the tax shield is given by the product of (i) the differential tax rate between corporate and personal income and (ii) the sum of the present value of partial interest payments, $E_g^d(\tilde{X} - g)$, and the present value of full interest payments, $R[k - F(d)]$.

To explore the tax effect of corporate debt, consider the derivatives of V with respect to D. After algebraic manipulations, it is obtained from (6) and (7) that[7]

$$\begin{aligned} V_D = \frac{dV}{dD} &= \frac{\partial V}{\partial R}\frac{dR}{dD} + \frac{\partial V}{\partial D} \\ &= \frac{(\tau - \tau_p)}{(1 - \tau_p)}[kr^* + F(g)], \end{aligned} \tag{8}$$

which is positive for any $\tau > \tau_p$. Thus, it is established that the marginal tax subsidy is positive throughout the region $D > I$. This is true even

7. $\partial V/\partial R$ and $\partial V/\partial D$ are obtained directly by partially differentiating (7) with respect to the relevant variables. To evaluate dR/dD we totally differentiate (6) with respect to D. After rearranging,

$$dR/dD = \frac{kr^* + \frac{1}{(1 - \tau)}[(1 - \tau_p)F(d) - (\tau - \tau_p)F(g)]}{(1 - \tau_p)[1 - F(d)]} > 0.$$

Substituting the three derivatives and simplifying yields eq. (8). A similar procedure is employed in deriving eqq. (12a) and (12b) from (10) and (11).

though there are alternative tax shelters. Also note from (8) that the second derivative is

$$V_{DD} = \frac{d^2V}{dD^2} = \frac{(\tau - \tau_p)}{(1 - \tau)(1 - \tau_p)} p(g), \tag{9}$$

which is positive for any $\tau > \tau_p$.[8] This result may have important implications on the pre-Miller capital structure literature in terms of the required restrictions on debt-related-costs function (e.g., agency or bankruptcy costs) for generating interior solutions.

The result that the marginal tax subsidy of debt is rising, $V_{DD} > 0$ $(0 \leqslant \tau_p < \tau)$, might seem surprising at first glance. To see the rationale behind it, it should be noted that the sign of V_{DD} is determined essentially by the net of two counteracting effects. The first effect represents the added tax shield resulting from the increase in the promised interest rate as more debt is issued. This effect is positive for all levels of D and for any cash flow pattern. The second effect, which measures the decrease in the marginal tax advantage due to a higher probability of bankruptcy, is always negative. The result in (9), $V_{DD} \geqslant 0$, indicates that the increase in the probability of default and the associated loss of tax shelter generated by the marginal unit of debt is not sufficient to outweigh the added tax shelter from the increase in the deductible portion of the debt payment.

Consider now the case where non-debt-related tax shields exceed the level of debt $(D \leqslant I)$. From (4), the value of the debt is given implicitly by

$$\begin{aligned} D = {} & E_0^{R+D}(\tilde{X}) - \tau_p E_D^{R+D}(\tilde{X} - D) \\ & + [D + (1 - \tau_p)R][k - F(R + D)]. \end{aligned} \tag{10}$$

The value of the levered firm is equal in this case to

$$V = V_u + (\tau - \tau_p)kR - \tau \int_I^{I+R} F(X)dX + \tau_p \int_D^{R+D} F(X)dX. \tag{11}$$

By differentiating equations (10) and (11), the marginal tax subsidy function in the interval $D \leqslant I$ is

$$\begin{aligned} V_D = {} & \tau_p[F(R + D) - F(D)] + R_D\{\tau[k - F(R + I)] \\ & - \tau_p[k - F(R + D)]\} \end{aligned} \tag{12a}$$

8. Alternatively, DeAngelo and Masulis (1980) have calculated the marginal tax subsidy of debt by differentiating the value of the firm with respect to the total *promised* payment to debtholders ($D + R$ in our notation). Note, however, that the marginal tax subsidy *diminishes* when measured per unit of promised payment even when the tax effect of debt is *linear*, e.g., when $V = V_u + \tau D$. The proof is straightforward and follows from the fact that for risky debt, $d^2D/[d(D + R)^2] < 0$. Thus it appears that the DeAngelo and Masulis differentiation procedure generates their result of a negative second derivative even in the absence of non-debt-related tax shields.

where

$$R_D = \frac{kr^* + (1 - \tau)F(R + D) + \tau_p F(D)}{(1 - \tau_p)[k - F(R + D)]} > 0. \qquad (12b)$$

Since, in this region, interest expenses can be a redundant tax shelter for the corporation while fully taxed at the personal level, the sign of V_D in (12) could be either positive or negative. The existence of an interior maximum hinges, however, on the shape of V in this region; in particular, on the sign of V_{DD}.

Unlike for the case where $D > I$, we are unable to prove analytically that the sign of V_{DD} in the region $D \leqslant I$ is always positive. However, our analysis indicates that in general $V_{DD} \geqslant 0$ and so any local extremum, when it exists, is necessarily a maximum. This is based on the following results:

1. We have conducted an extensive numerical simulation for the case of risk neutrality, where $\tilde{X}$ is normally distributed and where the underlying parameters I, r^*, $\sigma(\tilde{X})$, τ_p are drawn from a wide range of plausible values, given $E(\tilde{X})$ and τ. Let $E(\tilde{X}) = 100$, $\tau = .46$, and allow the other parameters to vary along the following ranges: $I = (0, 150)$, $r^* = (0.5, .20)$, $\sigma(\tilde{X}) = (10, 80)$, and $\tau_p = (0, \tau)$. The results of the numerical simulations are graphed in figure 1 for a representative firm with a given level of I under differing values of τ_p.[9] In all cases the valuation functions (7) and (11) are convex, with local maxima of the combined valuation function $V(D)$, at its extreme corners. Debt capacities (the amount of debt which equals the bound on the value of the firm) for the three representative cases depicted in figure 1 are indicated at the intersection of the curves with the 45-degree line by the three values for $\bar{D}$.[10] To illustrate the nature of the function, consider the case where

9. A detailed report of the numerical simulations in the paper is available from the authors on request.

10. From eq. (12) it is obtained that the initial slope of $V(D)$ (i.e., at the point $D = 0$) is

$$V_D(D = 0) = \frac{r^*}{(1 - \tau_p)}\{\tau[k - F(I)] - k\tau_p\} \gtreqless 0. \qquad (13)$$

The sign of $V_D(D = 0)$ depends on I in a simple manner. For given τ amd τ_p ($\tau_p < \tau$), companies with a low I portray $V_D(D = 0) > 0$; while for a large I, $V_D(D = 0) < 0$. From eqq. (6), (7), (10), and (11) it can also be verified that $V(D)$ is continuous throughout. However, eqq. (8) and (12) indicate that for any given value of τ_p there is a kink in the function $V(D)$ at the point where $D = I$. To see the kink in $V(D)$, note from eq. (12) that in the neighborhood of $D = I^-$,

$$V_D(D = I^-) = \frac{1}{(1 - \tau_p)}[(\tau - \tau_p)kr^* + \tau(1 - \tau_p)F(R + I) - (1 - \tau)\tau_p F(I)], \qquad (14)$$

while from (8),

$$V_D(D = I^+) = \frac{1}{(1 - \tau_p)}[(\tau - \tau_p)kr^* + (\tau - \tau_p)F(I)]. \qquad (15)$$

By subtracting one expression from the other, it is straightforward to show that $V_D(D = I^-) > V_D(D = I^+)$.

$\tau_p = \tau$. When $D \leq I$, there are states in which the interest deduction is redundant at the corporate level while treated as taxable income for the bondholder. These extra payments of taxes reduce the value of the levered firm so that $V < V_u$. However, when $D > I$, the interest payment is always nonredundant as a tax deduction to the firm. Since in this case the firm and the investors are taxed at the same rate, then $V = V_u$.

2. For the applicable tax rates ($\tau_p \leq \tau < .5$) the sign of V_D in the neighborhood of $D = \bar{I}$ is unambiguously positive (see eq. [ii] in n. 10). Thus if there is only one local extremum point it must be a minimum, so that the value of the firm is maximized at the corner. The remoteness of the possibility of an interior optimum is clearly seen by considering the requirements on the value function which must have at least two extremum points, and possibly three extremums—that is, when V_D ($D = 0$) < 0—to generate a local maximum. Also, even if such a maximum exists it must correspond to a value which exceeds the values at the corners to justify interior global optimum.

3. The condition for a local extremum requires particular values for τ_p. Setting $V_D = 0$ in (12a) yields that for a local extremum to hold it must be that

$$\tau_p \geqslant \tau \left[\frac{k - F(R + I)}{k - F(R + D)} \right] \equiv \tau^*. \tag{16}$$

That is, for values of $\tau_p \leq \tau^*$ there is no local extremum at all, so that by previous results $V(D)$ is maximized at the corner.

To conclude, in view of the analytical results for the case $D > I$, the simulation results for $D \leq I$, and the restrictions on the value function for having interior global maximum, we argue that this possibility is quite remote.

Given this conclusion, then, for any given level of τ_p the firm in figure 1 adopts a corner solution to its capital structure problem. However, for some critical τ_p between zero and τ the solution changes from all debt to all equity. The level of τ_p at which the switch is made differs across firms depending on the level of I. If τ_p is greater than τ, debt is a losing proposition for all firms. For $\tau_p = \tau$, firms for which $I = 0$ are indifferent to issuing debt vis-à-vis equity. As τ_p is reduced below τ, an increasing number of firms will switch from all-equity to all-debt financing. Aggregating horizontally across firms, we obtain a downward sloping aggregate supply curve, which intersects the horizontal demand curve that lies at the level $r^*/(1 - \tau_p)$ reflecting the personal tax rate τ_p. In equilibrium, individual firms do not face the capital structure decision with indifference, but rather adopt debt or equity corner solutions depending on the firm-specific values for I. Although we are conducting our analysis in a very similar framework, our result differs fundamentally from that obtained by DeAngelo and Masulis

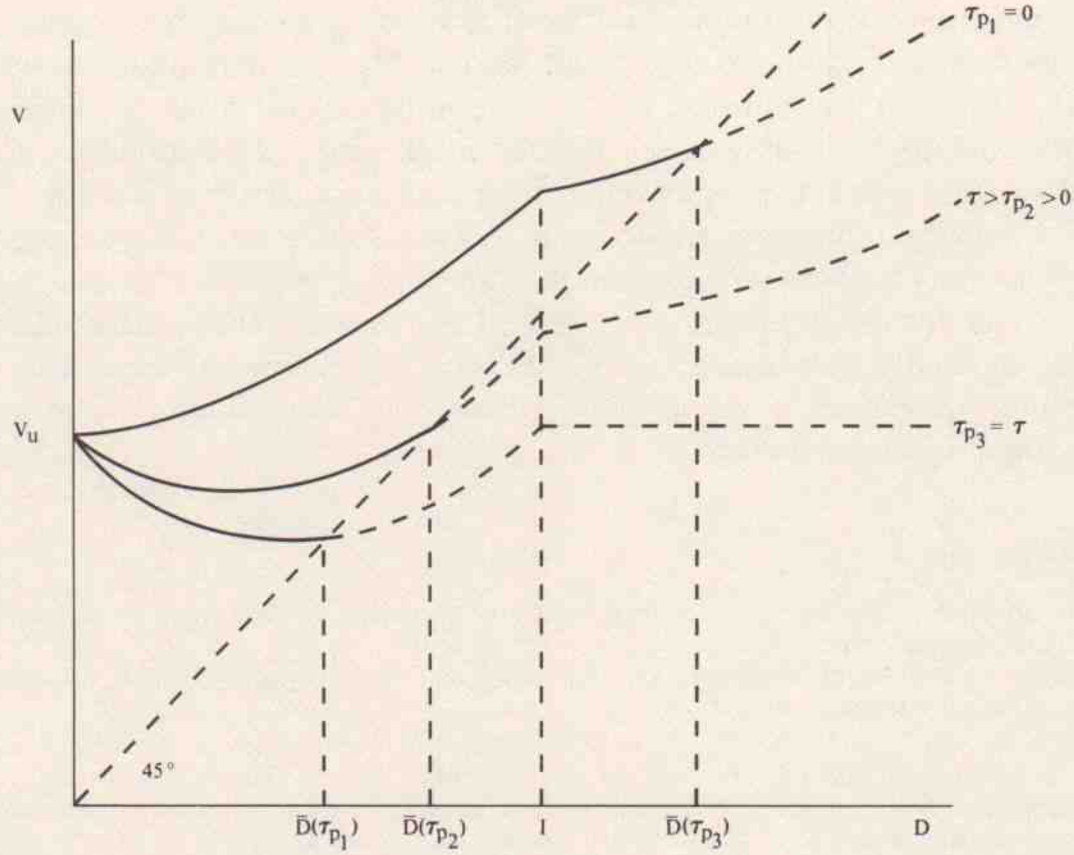

FIG. 1.—The value of the levered firm under $1 > 0$ and tax-induced yield differentials.

(1980) because under their tax system (deductibility of both principal and interest) the tax benefit of debt financing declines at the margin. This results in an interior solution to the capital structure decision. Under our treatment of the deductibility of interest the marginal tax benefits increase at the margin so that value is maximized at either no debt or all debt financing, as portrayed by figure 1.[11]

IV. Concluding Remarks

This paper presents an integrated analysis of the tax subsidy function of debt financing in the context of an income tax system, based on the existing U.S. tax code. It is emphasized that only an income tax (unlike a wealth tax) is consistent with the notion of risky debt and allows for noncash deductions such as depreciation. Our analysis considers the

11. The analysis thus far has been based on the principal-first criterion. In order to examine the robustness of the results with respect to the choice of tax doctrine under bankruptcy, we have replicated the analysis while assuming the interest-first doctrine. The results are that whenever $D \leq I$, there is no difference between the two tax doctrines. As for the case $D > I$, the shape of V_D is generally indeterminant. However, by solving both numerically for a normal distribution and analytically for a uniform distribution, it can be shown that the marginal tax subsidy is piecewise convex also under the interest-first doctrine.

implications of tax-induced differential returns that emerge as investors face personal taxes on interest income. We find that in the context of a (partial) equilibrium, tax considerations cause firms to finance investments by issuing either all debt or all equity. An optimal capital structure is obtained, however, for the corporate sector as a whole.[12]

Contrary to previous studies, our analysis shows that tax arguments alone are insufficient to explain interior capital structures even when tax-induced differential returns, as well as non-debt-related tax shields, are explicitly considered. Another type of imperfection (e.g., bankruptcy or agency costs) is required in order to produce an interior optimal capital structure.

References

Baron, D. P. 1975. Firm valuation, corporate taxes, and default risks. *Journal of Finance* 30 (December): 1251–64.

Baron, D. P. 1979. Investment policy, optimality, and the mean-variance model. *Journal of Finance* 34 (March): 207–32.

Brennan, M. J., and Schwartz, E. S. 1978. Corporate income taxes, valuation and the problem of optimal capital structure. *Journal of Business* 51 (January): 103–14.

Cooper, I., and Franks, J. R. 1983. The interaction of financing and investment decisions when the firm has unused tax credits. *Journal of Finance* 38 (May): 571–83.

Cordes, J. J., and Sheffrin, S. M. 1981. Taxation and the sectoral allocation of capital in the U.S. *National Tax Journal* 34 (December): 419–32.

Cordes, J. J., and Sheffrin, S. M. 1983. Estimating the tax advantage of corporate debt. *Journal of Finance* 38 (March): 95–105.

DeAngelo, H., and Masulis, R. W. 1980. Optimal capital structure under corporate and personal taxation. *Journal of Financial Economics* 7 (March): 3–29.

Kim, E. H. 1978. A mean-variance theory of optimal capital structure and corporate debt capacity. *Journal of Finance* 33 (March): 45–63.

Kim, E. H. 1982. Miller's equilibrium, shareholder leverage clienteles, and optimal capital structure. *Journal of Finance* 37 (May): 301–19.

Krause, A., and Litzenberger, R. H. 1973. A state-preference model of optimal financial leverage. *Journal of Finance* 28 (September): 911–22.

Miller, M. H. 1977. Debt and taxes. *Journal of Finance* 32 (May): 261–75.

Modigliani, F. F. 1982. Debt, dividend policy, taxes, inflation and market valuation. *Journal of Finance* 37 (May): 255–73.

Modigliani, F. F., and Miller, M. H. 1963. Corporate income taxes and the cost of capital: A correction. *American Economic Review* 53 (June): 433–43.

Park, S. Y., and Williams, J. 1982. Taxes, capital structure, and bondholders clienteles. Unpublished manuscript. (November, rev. February 1984).

Scott, J. H. 1976. A theory of optimal capital structure. *Bell Journal of Economics* 7 (Spring): 33–54.

Turnbull, S. M. 1979. Debt capacity. *Journal of Finance* 34 (September): 931–40.

Warren, A. C., and Auerbach, A. J. 1982. Transferability of the tax incentives and the fiction of safe-harbor leasing. *Harvard Law Review* 95 (June): 1752–86.

12. Our analysis, as well as that of DeAngelo and Masulis, explicitly assumes the existence of imperfections on the mechanism for transfer of redundant tax shelters of all types from one firm to another. Cooper and Franks (1983) and Warren and Auerbach (1982) provide detailed analyses of factors that inhibit the ability of firms to market redundant deductions. For supporting empirical evidence, see Cordes and Sheffrin (1981, 1983).