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Measuring Investment Performance in a Rational Expectations Equilibrium Model*

In this paper we study the problem of measuring investment performance when superior performance is identified with superior information. We do this in the context of a full equilibrium model of securities markets, in which agents have diverse and asymmetric private information and behave optimally. It is shown that the traditional risk-return measures are inappropriate in this context, and we develop alternative statistical procedures for making valid performance inferences.

I. Introduction

Much research has been concerned with evaluating the performance of managed portfolios (e.g., mutual funds) and examining whether portfolio managers display, in some sense, "superior performance." Superior performance would seemingly turn on the possession of superior information that is utilized in managing portfolios. In general, a fund's asset holdings (or portfolio weights) might not be observable to an outsider. Whether they are or not, however, they are likely to change over time. This happens especially when better performance is a result of superior information, since managers will change their portfolio composition on obtaining private information. Since such changes must be taken into account, the problem of measuring performance of managed portfolios is inherently different from that of evaluating the performance of individual assets, groups of assets, or, generally, portfolios with constant and known composition.

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In this paper, we identify superior performance with superior information and study the problem of detecting and measuring it within the context of a full equilibrium model of securities markets. Our analysis captures the ideas that diversity of beliefs and differences in information are important ingredients of speculative markets and that they are of crucial importance in the context of the performance measurement problem.

Most of the traditional work on the evaluation of investment performance has been concerned with "risk-return" measures, where the main idea is to use the mean returns on the asset or portfolio after some adjustment or "correction" for risk.[1] Risk is usually measured by the standard deviation of returns or the volatility (beta) of the portfolio. Although developed primarily to evaluate mutual funds, the issue of informational asymmetries, which is surely a major factor in the existence of these institutions, was not addressed formally in the early literature on risk-return measures. Indeed, they are the main tool for measuring the performance of individual securities.[2] The question arises, however, whether these measures are appropriate for managed portfolios when there are informational asymmetries both between the managers and the observer evaluating them and among market participants.

The basic problem that upsets the intuition behind the risk-return measures when informational asymmetries exist is that, as a result of the changes of composition in the manager's portfolio (made in response to the private information he or she obtains), the "risk" of the portfolio is not a well-defined concept and might be misleading if calculated literally. For each informative signal he receives, the manager forms his own posterior distribution of assets' returns, which is unknown to others and varies over time (depending on what the information happens to be). The "true" and relevant risk actually borne by him might now change over time even if other parameters are fixed or stationary. Since the private information itself is usually not available to an outsider, the observed returns come from a complicated distribution (from the observer's point of view) involving the joint prior distribution over the possible private signals and assets' returns. It is not clear, therefore, that there exists a single parameter, estimable from the data, that would serve as an appropriate measure of risk in a risk-return analysis.

Recent studies have begun to recognize the importance of explicitly modeling information asymmetries in the analysis of performance evaluation.[3] There has also been some research to examine the appro-

1. See, e.g., Treynor (1965), Sharpe (1966), Jensen (1968, 1969), Fama (1972), and, more recently, Lehmann and Modest (1984).

2. See, e.g., Brown and Warner (1980).

3. See, e.g., Merton (1981), Bhattacharya and Pfleiderer (1983), and Connor and Korajczyk (1984).

priateness of risk-return measures under information asymmetries.[4] In past analyses, however, the equilibrium model assumed that individuals possessed homogeneous information, but, somewhat paradoxically, it was possible for particular (possibly many) portfolio managers whose performance is being examined to have superior or different beliefs. None of these analyses was performed in an equilibrium model which involves heterogeneous beliefs and rational behavior throughout the market, as in the model we employ below.

A coherent theory of performance measurement requires, first, modeling the environment in which portfolio managers (and the rest of the market) operate and the rules by which they choose their actions. The model we use is one of a noisy rational expectations equilibrium. Traders in this model possess diverse pieces of private information, and they can vary both in their risk attitudes and in the precision of their private information. Private information is aggregated and partially revealed by equilibrium prices, and traders have rational expectations in the sense that they choose optimal portfolios given their private information and the public equilibrium prices. Since the distributions of private information signals are not the same across agents in our model, superior performance on the basis of better information is a natural concept.

After defining and characterizing superior performance, we examine whether the traditional methods are appropriate for measuring it in our context and determine that they are not. For example, the reward-to-variability ratio can be a decreasing function of the precision of the information possessed by the trader, so that the better his information is, the lower the reward-to-variability ratio. Similar results hold for the reward-to-volatility ratio and for measurements based on securities market line analysis. These results are similar to Dybvig and Ross (1984*a*). Using the framework of the model, we then formulate the performance problem as one of classical statistical inference about some unknown parameters in the distribution of observable data. This enables us to develop alternative statistical procedures.[5]

A recurrent theme in our analysis is the distinction between various conditional and unconditional distributions (e.g., of the observed re-

4. See Mayers and Rice (1979), Roll (1979), Dybvig and Ross (1984*a*, 1984*b*), and Bhattacharya and Pfleiderer (1984). It is worth stressing that Roll's (1978) famous criticism of the securities market line as a performance measurement tool is not based on information issues. It applies to portfolios with fixed and known composition and to individual assets. The model suggested by him to explain systematic deviations from the line is one where the index itself is mean variance inefficient.

5. As in all the past studies of performance measurement, we confine ourselves here to exogenous evaluation, where the test itself does not affect the observations. The situation is different if the manager is an agent in a principal-agent situation where one is concerned with rewards to performance, sharing rules, and incentives. Some of these issues are discussed in Bhattacharya and Pfleiderer (1984), Heckerman (1975), and Sharpe (1981).

turns on the manager's portfolio). Unconditional distributions and their moments will usually not be appropriate for performance measurement, while some conditional distributions will be useful and will give rise to the regression models we discuss. The key problem in performance evaluation is that the observer has a *different* information set than that of an informed manager. Statistical tests of performance based on strictly coarser information are problematic, mainly because some parameters are not well behaved under "iterated expectations." However, since the statistical inference is made ex post, the observer can partially control for the unknown private information by conditioning on the realized state of the world (e.g., functions of the realized returns on assets).[6]

The remainder of the paper is organized as follows. Section II describes the model. In Section III we characterize superior performance. Certain welfare comparisons are shown to be equivalent to familiar statistical comparisons. One interesting result is that a given signal distribution is preferred by every agent to another, based on an ex ante calculation (namely, before the private information and equilibrium prices are observed), if and only if it produces ex post (i.e., conditional) lower generalized variance of assets' payoffs. (The generalized variance of a random vector is defined by the determinant of its variance-covariance matrix.) In Section IV we examine the usefulness of traditional risk-return measures to measure performance in the context of the model, show that they are not useful, and explain the intuition behind these results.

Section V formulates the performance measurement problem as one of classical statistical inference and develops some statistical tools for making the inference. We show that risk tolerance and precisions of information can be separated (identified) as parameters in the distribution of the sets of observables we examine. Certain regression models arise naturally in the context of our model, and we discuss their application. Section VI provides concluding remarks.

II. The Model

The model we will use is a rational expectations equilibrium capital asset pricing model, which was developed in Admati (1983, 1985) and is basically a generalization of Hellwig (1980).[7] The central features of the model are that it has many risky assets and a large number of agents who possess diverse pieces of private information. The equilibrium

6. The role of conditional distributions in the general context of asymmetric information models was also suggested in Grossman (1978).

7. Hellwig (1980) analyzes the case of a single asset, and similar (single asset) models are discussed and applied in Diamond and Verrecchia (1981), Diamond (1983), Pfleiderer (1982), and Verrecchia (1982).

concept allows agents to use current prices, which reflect relevant information of other agents, as information about the future value of the assets. Since the equilibrium process is noisy, prices are not sufficient statistics (as they are in Grossman [1976]), so private information is useful in equilibrium. This creates differences in the beliefs among traders. The model also has a large (infinite) number of investors, so individual agents do not influence the equilibrium prices. A brief description of the model follows. For more details see Admati (1983, 1985).

There are two periods; trading takes place in the first period and consumption in the second. The investment opportunities include a riskless asset and n risky assets. The riskless asset, which is the numeraire, pays off R units, and risky asset i pays off $\tilde{\mathbf{F}}_i$ units of a single consumption good. Prior to trading, each agent j receives a private information signal $\tilde{\mathbf{Y}}_j$, which is informative about the random vector, $\tilde{\mathbf{F}}$. The signal and the unknown true payoff vector are related through

$$\tilde{\mathbf{Y}}_j = \tilde{\mathbf{F}} + \tilde{\boldsymbol{\epsilon}}_j, \tag{1}$$

where $\tilde{\boldsymbol{\epsilon}}_j$, the error vector affecting agent j's information, is independent of both $\tilde{\mathbf{F}}$ and $\tilde{\boldsymbol{\epsilon}}_k$ for $k \neq j$.

The per capita supply vector of the risky assets, which is denoted by $\tilde{\mathbf{Z}}$, is assumed to be a random vector, independent of both $\tilde{\mathbf{F}}$ and the $\tilde{\boldsymbol{\epsilon}}_j$'s. This randomness is the source of "noise" in the model, and it can be interpreted in a variety of ways. A possible interpretation is that it arises from liquidity motivated trades (e.g., for life-cycle reasons) which cannot be predicted by speculators in the model.[8] Equilibrium occurs when the markets clear while all agents create their portfolios optimally. Optimal behavior is defined according to the rational expectations hypothesis; each agent maximizes the expected utility of second-period wealth given private information and equilibrium prices.

To obtain a closed form equilibrium, it is assumed that agents have exponential utility functions over final consumption. The risk aversion coefficients a_j can be different across agents. Furthermore $(\tilde{\mathbf{F}}, \tilde{\mathbf{Z}}, \tilde{\boldsymbol{\epsilon}}_j)$ are jointly normal vectors with mean vectors $\overline{\mathbf{F}}$, $\overline{\mathbf{Z}}$, and $\mathbf{0}$ and variance-covariance matrices $\mathbf{V}$, $\mathbf{U}$, and $\mathbf{S}_j$, respectively. The matrices $\mathbf{V}$, $\mathbf{U}$, and $\mathbf{S}_j$ are assumed to be positive definite. We will refer to $\mathbf{S}_j^{-1}$ as the

8. The assumption of unknown total supply is standard in models of noisy rational expectations. See Grossman and Stiglitz (1980), Anderson and Sonnenschein (1982), and the references in n. 7 above. It seems reasonable that liquidity needs motivate some trade. Diamond and Verrecchia (1981) interpret the noise in supply as arising from uncertainty about the aggregate endowment. Alternatively, there may be other unknown parameters (for example, concerning the preferences of other agents) which prevent perfect revelation of information by prices.

"precision matrix of agent j."[9] Note that the $\mathbf{S}_j$'s can be different across agents and that no special assumptions are made concerning their structure. Assuming that there is a large number of agents, Admati (1983, 1985) solved for an equilibrium price vector that takes the form[10]

$$\tilde{\mathbf{P}} = \mathbf{A}_0 + \mathbf{A}_1\tilde{\mathbf{F}} - \mathbf{A}_2\tilde{\mathbf{Z}}, \tag{2}$$

where the vector $\mathbf{A}_0$ and the matrices $\mathbf{A}_1$, $\mathbf{A}_2$ depend on the aggregate parameters of the economy. In particular, they depend on $\bar{a}$, the average risk tolerance coefficient (which is the harmonic mean of the risk aversion coefficients) and on a weighted average of the precision matrices where the weights are the risk tolerance coefficients (i.e., where the coefficient of agent j is $1/a_j$). This weighted average is a measure of the precision of private information in the market, and it is denoted by $\mathbf{Q}$. (It turns out that in equilibrium $\mathbf{Q} = \mathbf{A}_2^{-1}\mathbf{A}_1$.)

More relevant for our purposes here, however, is the closed form solution for the beliefs and optimal demand functions of agents in the model. Consider an agent who receives a private signal $\tilde{\mathbf{Y}}$ with precision $\mathbf{S}^{-1}$ (we suppress the index j here). Since $(\tilde{\mathbf{F}}, \tilde{\mathbf{Y}}, \tilde{\mathbf{P}})$ are jointly normal, the conditional distribution of $\tilde{\mathbf{F}}$ given $\tilde{\mathbf{Y}}$ and $\tilde{\mathbf{P}}$ is normal. Let lowercase letters denote particular realizations of random vectors (for example, $\mathbf{y}$ is a realization of $\tilde{\mathbf{Y}}$). Then the expectation of $\tilde{\mathbf{F}}$ given $\mathbf{y}$ and $\mathbf{p}$ is a linear function of $\mathbf{y}$ and $\mathbf{p}$, and the conditional variance-covariance matrix is constant (independent of $\mathbf{y}$ and $\mathbf{p}$). This matrix takes a particularly simple form, namely,

$$\text{var}(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p}) = (\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}^{-1})^{-1}. \tag{3}$$

Individual demand functions can be shown to be

$$\mathbf{D}(\mathbf{y}, \mathbf{p}) = \frac{1}{a}[\text{var}(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p})]^{-1}[\mathbf{E}(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p}) - R\mathbf{p}] \tag{4}$$

and, by the properties of jointly normal variables, this is a linear function of $\mathbf{y}$ and $\mathbf{p}$. The explicit expression of this function is

$$\mathbf{D}(\mathbf{y}, \mathbf{p}) = \mathbf{G}_0 + \mathbf{G}_1\mathbf{y} - \mathbf{G}_2\mathbf{p}, \tag{5}$$

9. The model is more general, as it can allow some agents to receive information of lower dimension or to be completely uninformed. This will be reflected in blocks of zeros in the matrix $\mathbf{S}^{-1}$ corresponding to assets on which the agent is uninformed. In particular, an uninformed agent has $\mathbf{S}^{-1} = 0$.

10. The notion of equilibrium used here slightly modifies the usual definition to suit the framework of an infinite economy. This equilibrium can also be viewed as the limit of linear rational expectations equilibria of finite-agent economies as the number of agents grows. The particular solution used here is unique within the class of linear equilibrium price functions (with $\mathbf{A}_2$ nonsingular). These issues are discussed in detail in Admati (1985).

where

$$\mathbf{G}_0 = \frac{1}{a}\mathbf{G}$$

for

$$\mathbf{G} = \frac{1}{a}[\mathbf{I} - \mathbf{Q}\mathbf{U}^{-1}(\mathbf{I} + \bar{a}\mathbf{Q}\mathbf{U}^{-1})^{-1}]\,(\mathbf{V}^{-1}\overline{\mathbf{F}} + \mathbf{Q}\mathbf{U}^{-1}\overline{\mathbf{Z}}), \tag{6}$$

$$\mathbf{G}_1 = \frac{1}{a}\mathbf{S}^{-1}, \tag{7}$$

and

$$\mathbf{G}_2 = \frac{1}{a}(\mathbf{K} + \mathbf{R}\mathbf{S}^{-1})$$

for

$$\mathbf{K} = \mathbf{A}_2^{-1} - \mathbf{R}\bar{a}\mathbf{Q}. \tag{8}$$

This demand function has a very attractive form, since its coefficients are simple functions of the characteristics of the agent: both $\mathbf{G}$ and $\mathbf{K}$ depend only on economy-wide parameters, while $\mathbf{G}_1$, the coefficient of the private signal, is independent of the economy-wide parameters and depends only on the agent's risk aversion and the precision of his information. In the limiting case of an uninformed agent, for whom $\mathbf{S}^{-1} = \mathbf{0}$, we have $\mathbf{D}(\mathbf{p}) = (\mathbf{G} + \mathbf{K}\mathbf{p})/a$.

The above model provides us with a concrete framework for the analysis of performance evaluation. The beliefs held by agents in the model are diverse, since they depend on private signals, and, more important, asymmetric, since some agents might have more precise signals (in a sense to be defined). In addition, the fact that agents may vary in their risk attitudes enriches the setup, since both risk attitudes and the parameters of the private information affect the observables that are typically available for performance evaluation (e.g., the return on an agent's portfolio). Finally, we are able to focus on the asymmetric information aspect in a context where agents are rational and behave optimally; in particular, they use the public price vector as a source of information about what other agents know.

III. Evaluation of Private Information Signals

What constitutes "superior performance" in the model? Equivalently, what does it mean to be "better informed"? Here we are simply interested in comparing the variance-covariance matrices $\mathbf{S}_j$ (or the precision matrices $\mathbf{S}_j^{-1}$). Better information should lead to superior performance in some sense, and, since our eventual goal is to measure performance, it is important to examine the relationships between better information and the distributions of potentially observable variables.

Consider a specific agent in the model with a coefficient of risk aversion a and an initial wealth level W_0. Let $\tilde{W}(\mathbf{S})$ be the random terminal wealth (i.e., consumption) of the agent if his precision matrix is $\mathbf{S}^{-1}$. (The rate of return on his portfolio is $\tilde{W}(\mathbf{S})/W_0$, so our discussion applies to rates of return with the normalization $W_0 = 1$. Note that the demand for the risky assets is independent of W_0, since utility functions are exponential.) We will evaluate this agent's signal by comparing $\mathbf{S}$ to other positive definite matrices.

Suppose there is only one risky asset. Then it is intuitively clear that lower variance of the error term $\tilde{\boldsymbol{\epsilon}}$ in the private signal should correspond to better information. For the multi-asset case, one can apply the following analogous partial order for symmetric matrices: $\mathbf{A} \geqslant \mathbf{B}$ if $\mathbf{A} - \mathbf{B}$ is positive semidefinite. If $\mathbf{A}$ and $\mathbf{B}$ are variance-covariance matrices of the random vectors $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$, respectively, then $\mathbf{A} \geqslant \mathbf{B}$ is equivalent to the statement that for every vector $\mathbf{t}$ the random variable $\mathbf{t}\tilde{\mathbf{X}}$ has variance larger or equal to that of the random variable $\mathbf{t}\tilde{\mathbf{Y}}$. The next theorem explores this order between the variance-covariance matrices of the error terms in two signals. It will be shown that it coincides with some meaningful relations between parameters of the distribution of the conditional distribution of $\tilde{W}(\mathbf{S})$ given $\mathbf{f}$ and $\mathbf{p}$.

THEOREM 1: For a fixed a and W_0, the following are equivalent:

(i) $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite (i.e., $\mathbf{S}_1 \geqslant \mathbf{S}_2$). (9)

(ii) For all $\mathbf{f}$ and $\mathbf{p}$, $E[\tilde{W}(\mathbf{S}_2)|\mathbf{f}, \mathbf{p}] \geqslant E[\tilde{W}(\mathbf{S}_1)|\mathbf{f}, \mathbf{p}]$. (10)

(iii) For all $\mathbf{f}$ and $\mathbf{p}$, $\mathrm{var}[\tilde{W}(\mathbf{S}_2)|\mathbf{f}, \mathbf{p}] \leqslant \mathrm{var}[\tilde{W}(\mathbf{S}_1)|\mathbf{f}, \mathbf{p}]$. (11)

(iv) For all $\mathbf{f}$ and $\mathbf{p}$, $-E\{\exp[-a\tilde{W}(\mathbf{S}_2)|\mathbf{f}, \mathbf{p}]\} \geqslant -E\{\exp[-a\tilde{W}(\mathbf{S}_1)|\mathbf{f}, \mathbf{p}]\}$. (12)

(v) For all $\mathbf{f}$ and $\mathbf{p}$, and for every monotone and concave utility function u,

$$E\{u[\tilde{W}(\mathbf{S}_2)|\mathbf{f}, \mathbf{p}]\} \geqslant E\{u[\tilde{W}(\mathbf{S}_1)|\mathbf{f}, \mathbf{p}]\}. \tag{13}$$

If $\mathbf{S}_1 - \mathbf{S}_2$ is positive definite, then for $\mathbf{f}$ and $\mathbf{p}$ such that $\mathbf{f} - R\mathbf{p} \neq \mathbf{0}$ the inequalities in (10)–(12) are strict.

PROOF: It is easy to see that $\tilde{W}(\mathbf{S}) = W_0R + \tilde{\mathbf{D}}(\mathbf{S})(\tilde{\mathbf{F}} - \mathbf{R}\tilde{\mathbf{P}})$, so from (5)–(8),

$$\begin{aligned} \tilde{W}(\mathbf{S}) = W_0R &+ \frac{1}{a}(\tilde{\mathbf{F}} - \mathbf{R}\tilde{\mathbf{P}})(\mathbf{G}_0 + \mathbf{K}\tilde{\mathbf{P}}) \\ &+ \frac{1}{a}(\tilde{\mathbf{F}} - R\tilde{\mathbf{P}})\mathbf{S}^{-1}(\tilde{\mathbf{Y}} - R\tilde{\mathbf{P}}). \end{aligned} \tag{14}$$

Hence, for every realization of $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{P}}$, the conditional distribution of $\tilde{W}(\mathbf{S})$ given $(\mathbf{f}, \mathbf{p})$ is normal. Using the fact that $\tilde{\boldsymbol{\epsilon}}$ is independent of $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{P}}$, we have

$$E[\tilde{W}(\mathbf{S})|\mathbf{f}, \mathbf{p}] = W_0R + \frac{1}{a}(\mathbf{f} - R\mathbf{p})(\mathbf{G}_0 + \mathbf{K}\mathbf{p}) + \frac{1}{a}(\mathbf{f} - \mathbf{R}\mathbf{p})\mathbf{S}^{-1}(\mathbf{f} - \mathbf{R}\mathbf{p}), \tag{15}$$

and

$$\mathrm{var}[\tilde{W}(\mathbf{S})|\mathbf{f}, \mathbf{p}] = \frac{1}{a^2}(\mathbf{f} - R\mathbf{p})\mathbf{S}^{-1}(\mathbf{f} - R\mathbf{p}). \tag{16}$$

From the formula for the moment generating function of normal variables

$$E\{\exp[-a\tilde{W}(\mathbf{S})|\mathbf{f}, \mathbf{p}]\} = -\exp[-aW_0R - (\mathbf{f} - R\mathbf{p})(\mathbf{G}_0 + \mathbf{K}\mathbf{p}) - \frac{1}{2}(\mathbf{f} - R\mathbf{p})\mathbf{S}^{-1}(\mathbf{f} - R\mathbf{p})]. \tag{17}$$

The equivalence of (9)–(12) follows from the above, since $\mathbf{G}_0$ and $\mathbf{K}$ do not depend on $\mathbf{S}$. Also, from the above calculations and the definition of positive definiteness it is obvious that if $\mathbf{f} - R\mathbf{p} \neq \mathbf{0}$ the inequalities in (10)–(12) are strict.

Now, (10) and (11) (and [12]) are clearly implied by (13). Together they imply (13) because of the normality of the conditional distribution of $\tilde{W}(\mathbf{S})$ given $(\mathbf{f}, \mathbf{p})$. All expected utility maximizers are concerned only with the mean and variance of its distribution, and those with monotone and concave function clearly prefer higher mean and lower variance. Q.E.D.

Before we discuss the implications of this result for performance measurement, it is interesting to consider whether a similar result holds if we condition on realizations of the private signal $\tilde{\mathbf{Y}}$ (and the price vector) instead of on the realization of $\tilde{\mathbf{F}}$ (and the prices), that is, evaluate the distribution of the signal from the point of view of the agent after it has already been observed. One may wonder whether a more precise signal in the sense of (9) implies that for every $\mathbf{y}$ and $\mathbf{p}$ the agent expects higher returns, lower variance of returns, and higher level of expected utility. It turns out that these questions cannot be answered independently of the particular realizations of $\mathbf{y}$ and $\mathbf{p}$. A more precise signal distribution indeed makes the agent receiving it better off, but this is so in an ex ante sense and not ex post given a fixed specific signal. To see this, imagine having just received a piece of news. If the news is bad (but nothing can be done to change it), then you would prefer that it actually came from an unreliable source. Good news, on the other hand, is best coming from a reliable source. The situation is more complicated here because the reliability of the source affects actions taken on the basis of the information: a rational agent scales the coefficients in his demand function according to the precision of the information, and this shift of the portfolio choice works to undermine imprecise signals, whatever they are. In other words, in acting on the signal the agent responds differently to it depending on its precision (and not only its given value). Still, the effect of the information itself (and its precision) on the posterior distribution of future wealth might make an agent worse off if a given signal value came from a more precise distribution. This is not difficult to verify formally.[11]

Turning to theorem 1 and its implications, note first that (12) and (13) are statements about the ex post utility derived from the investment strategy of agents in the model who receive signals with the given parameters. The calculation is performed from the point of view of someone who does not himself observe the signal. This could correspond, for example, to a principal-agent situation, where the agent is privately informed and manages the principal's funds according to the model. If the principal has any monotone concave utility function and calculates his utility ex post conditional on $\mathbf{f}$ and $\mathbf{p}$, then, from the equivalence of (9) and (13), he will (on average) always obtain a higher utility level when the manager has precision $\mathbf{S}_2$ than when he has $\mathbf{S}_1$ if $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite. Conversely, if any principal does better with the $\mathbf{S}_1$ agent than with the $\mathbf{S}_2$ agent for every $\mathbf{f}$ and $\mathbf{p}$, then $\mathbf{S}_1 - \mathbf{S}_2$ must be positive semidefinite. Moreover, by the law of iterated expectations, (13) implies that $\mathbf{S}_2$ is preferred to $\mathbf{S}_1$ on the basis of ex ante expected utility calculation. Hence:

COROLLARY 1: If $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite, then for any fixed a and W_0 and for every monotone and concave utility function u, $E\{u[\tilde{W}(\mathbf{S}_2)|\mathbf{p}]\} \geqslant E[\tilde{W}(\mathbf{S}_1)|\mathbf{p}]$ for every $\mathbf{p}$, and $E\{u[\tilde{W}(\mathbf{S}_2)]\} \geqslant E\{u[\tilde{W}(\mathbf{S}_1)]\}$.

(Note that the implication in the corollary goes in one direction only. We will characterize the relations between ex ante utility levels in theorem 2 below.)

Theorem 1 has some other implications that are important for performance measurement. The equivalence of (9) and (10)–(11) shows that parameters of the conditional distribution of returns given $\mathbf{f}$ and $\mathbf{p}$ can be useful in trying to assess how well informed an agent is. In particular, simple integration of (10) shows that in our model the unconditional

11. We have that $E[\tilde{W}(\mathbf{S})|\mathbf{y}, \mathbf{p}] = W_0R + [E(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p}) - R\mathbf{p}][\mathrm{var}(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p})]^{-1}[E(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p}) - R\mathbf{p}]/a$. The regression coefficients in $E(\tilde{\mathbf{F}}|\mathbf{y}, \mathbf{p})$ are complicated functions of $\mathbf{S}$, but since the demand function is a simple function of $\mathbf{S}$ it is more convenient to use the alternative formula $E[\tilde{W}(\mathbf{S})|\mathbf{y}, \mathbf{p}] = [\mathbf{G}_0 + \mathbf{Kp} + \mathbf{S}^{-1}(\mathbf{y} - R\mathbf{p})](\mathbf{V}^{-1} + \mathbf{S}^{-1} + \mathbf{QU}^{-1}\mathbf{Q})^{-1}[\mathbf{G}_0 + Kp + \mathbf{S}^{-1}(\mathbf{y} - R\mathbf{p})]$. The reader can verify that the above expression is not, uniformly over $\mathbf{y}$ and $\mathbf{p}$, larger for $\mathbf{S}_2$ than for $\mathbf{S}_1$ where (9) holds. For $n = 1$ this can be done by differentiation with respect to $\mathbf{S}$. Results for the variance and expected utility are almost identical.

expected return on the portfolio of a better-informed investor (in the sense of [9]) is higher.

COROLLARY 2: If $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite, then for any a and W_0, $E[\tilde{W}(\mathbf{S}_2)|\,\mathbf{p}] \geq E[\tilde{W}(\mathbf{S}_1)|\mathbf{p}]$ for every $\mathbf{p}$, and $E[\tilde{W}(\mathbf{S}_2)] \geq E[\tilde{W}(\mathbf{S}_1)]$.

This result is consistent with what Verrecchia (1980) terms "the expected return hypothesis," namely, that increases in expected utility due to receipt of superior information will on average show as increases in expected returns.[12] Of course, there is no correction for risk here, and, moreover, the calculation assumes a fixed level of risk aversion, that is, the relationship does not apply to two agents with different risk attitudes. Since preferences are not generally observable, therefore, unconditional expected returns are usually not enough to indicate superior performance. This is somewhat mitigated in the next corollary, which concerns a "conditional" version of Sharpe's (1966) reward-to-variability ratio. Again, the proof follows immediately from inequalities (10) and (11).

COROLLARY 3: If $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite, then for all $\mathbf{f}$ and $\mathbf{p}$,

$$\frac{E[\tilde{W}(\mathbf{S}_2)|\mathbf{f},\,\mathbf{p}] - RW_0}{\sqrt{\mathrm{var}[\tilde{W}(\mathbf{S}_2)|\mathbf{f},\,\mathbf{p}]}} \geq \frac{E[\tilde{W}(\mathbf{S}_1)|\mathbf{f},\,\mathbf{p}] - RW_0}{\sqrt{\mathrm{var}[\tilde{W}(\mathbf{S}_1)|\mathbf{f},\,\mathbf{p}]}} \tag{18}$$

Inequality (18) compares the (conditional) reward-to-variability ratios for $\mathbf{S}_2$ and $\mathbf{S}_1$, defined as the expected excess rate of return on a portfolio per unit of standard deviation. The corollary verifies that better-informed agents in the sense of (9) will have better conditional ratios given every $\mathbf{f}$ and $\mathbf{p}$. Note that by (3) and (14), this measure is independent of the initial wealth and, more important, of the risk aversion coefficient, which makes it more useful for performance measurement. That is, (18) holds even if the manager whose precision is $\mathbf{S}_1^{-1}$ has one coefficient of risk aversion and the manager whose precision is $\mathbf{S}_2^{-1}$ has another. Unfortunately, the usual interpretation of the ratio treats the parameters unconditionally. Since the "law of iterated variance" (or standard deviation) is not as simple as that of expectations, the unconditional ratio is not well behaved, as we show in the next section.

Note finally that the statement $\mathbf{S}_1 \geq \mathbf{S}_2$, that is, (9), is independent of the parameters of the economy $\mathbf{V}$, $\mathbf{U}$, $\mathbf{Q}$, and $\bar{a}$, while the distribution of returns and agents' welfare generally depend on these parameters. Thus (9) has very strong implications for all economies of our kind. However, it is only a partial order, so it does not apply to any pair (or a finite set) of signals. The next theorem develops a complete order of the private information signals. This order relation, which is based on ex ante welfare comparisons and is obtained by integrating the (condi-

12. Verrecchia confirms the hypothesis for the exponential utility function in a model which is different in important ways from our model. In particular it assumes complete markets and does not involve information transmission by prices.

tional) expected utility over possible payoffs, employs another popular measure of variability for multivariate distributions known as the generalized variance of the distribution. In the following let $|\mathbf{A}|$ denote the determinant of a matrix $\mathbf{A}$.

THEOREM 2: For a fixed a and W_0 the following are equivalent:

$$\text{(i) } |\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_1^{-1}| \leqslant |\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_2^{-1}|. \tag{19}$$

$$\text{(ii) For all } \mathbf{p},\ -E\{\exp[-a\tilde{W}(\mathbf{S}_2)|\mathbf{p}]\} \geqslant -E\{\exp[-a\tilde{W}(\mathbf{S}_1)|\mathbf{p}]\}. \tag{20}$$

$$\text{(iii) } -E\{\exp[-a\tilde{W}(\mathbf{S}_2)]\} \geqslant -E\{\exp[-a\tilde{W}(\mathbf{S}_1)]\}. \tag{21}$$

PROOF: To calculate $E\{\exp[-a\tilde{W}(\mathbf{S})|\mathbf{p}]\}$ integrate (17) over $\tilde{\mathbf{F}}$ (given $\mathbf{p}$), or, equivalently, over $\tilde{\mathbf{x}}_\mathbf{p} = \tilde{\mathbf{F}} - R\mathbf{p}$ (given $\mathbf{p}$). Let $\bar{\mathbf{x}}_\mathbf{p} = E(\tilde{\mathbf{x}}_\mathbf{p}|\mathbf{p})$; $\mathbf{V}_\mathbf{p} = \text{var}(\tilde{\mathbf{x}}_\mathbf{p}|\mathbf{p})$. We have

$$E\{\exp[-a\tilde{W}(\mathbf{S})|\mathbf{p}]\} = \frac{\exp(-aW_0R)}{(2\pi)^{\frac{n}{2}}\sqrt{|\mathbf{V}_\mathbf{p}|}}\int \exp\Big[-\mathbf{x}_\mathbf{p}(\mathbf{G}_0 + \mathbf{K}\mathbf{p}) - \frac{1}{2}\mathbf{x}_\mathbf{p}\mathbf{S}^{-1}\mathbf{x}_\mathbf{p} - \frac{1}{2}(\mathbf{x}_\mathbf{p} - \bar{\mathbf{x}}_\mathbf{p})\mathbf{V}_\mathbf{p}^{-1}(\mathbf{x}_\mathbf{p} - \bar{\mathbf{x}}_\mathbf{p})\Big]d\mathbf{x}_\mathbf{p}, \tag{22}$$

but $\mathbf{G}_0 + \mathbf{K}\mathbf{p} = \mathbf{V}_\mathbf{p}^{-1}\bar{\mathbf{x}}_\mathbf{p}$ (this is the demand function of an uninformed agent), and therefore

$$E\{\exp[-a\tilde{W}(\mathbf{S})|\mathbf{p}]\} = \frac{\exp(aW_0R - \frac{1}{2}\bar{\mathbf{x}}_\mathbf{p}\mathbf{V}_\mathbf{p}^{-1}\bar{\mathbf{x}}_\mathbf{p})}{(2\pi)^{\frac{n}{2}}\sqrt{|\mathbf{V}_\mathbf{p}|}}\int \exp\Big(-\frac{1}{2}\mathbf{x}_\mathbf{p}(\mathbf{S}^{-1} + \mathbf{V}_\mathbf{p}^{-1})\mathbf{x}_\mathbf{p})\mathbf{x}_\mathbf{p}\Big)d\mathbf{x}_\mathbf{p} = \frac{\exp(aW_0R - \frac{1}{2}\bar{\mathbf{x}}_\mathbf{p}\mathbf{V}_\mathbf{p}^{-1}\bar{\mathbf{x}}_\mathbf{p})}{\sqrt{|\mathbf{V}_p(\mathbf{S}^{-1} + \mathbf{V}_\mathbf{p}^{-1})|}}. \tag{23}$$

Now, since $\mathbf{V}_p = (\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q})^{-1}$, it follows that (20 holds if and only if

$$|\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_1^{-1}| \leqslant |\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_2^{-1}|.$$

This proves the equivalence of (19) and (20). Since (19) is independent of $\mathbf{p}$, and (20) clearly implies (21), we get immediately that (21) is also equivalent to (19). Q.E.D.

Recall that from (3), $(\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}^{-1})^{-1}$ is the prediction variance-covariance matrix of $\tilde{\mathbf{F}}$ given $\mathbf{y}$ and $\mathbf{p}$. Thus every agent in the model prefers, both given any price vector $\mathbf{p}$ (but before observing the signal) and also a priori, before any information is observed, to receive a signal with precision $\mathbf{S}_2$ rather than $\mathbf{S}_1$ if and only if it will provide lower generalized prediction variance.[13] This order can be different for different economies (and it is also obtained for exponential utility functions only), but clearly if $\mathbf{S}_1 - \mathbf{S}_2$ is positive semidefinite (equivalently, for every $\mathbf{V}, \mathbf{U}, \mathbf{Q}$, $(\mathbf{V}^{-1} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_1^{-1}) - (\mathbf{V} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} + \mathbf{S}_2^{-1})$ is positive semidefinite), then (19)–(21) hold for every $\mathbf{V}, \mathbf{U}, \mathbf{Q}$, since (12) implies (20). Thus, this complete order agrees with (9) whenever (9) is applicable.

IV. Traditional Performance Measures Revisited

In this section we examine whether traditional risk-return measures are appropriate for evaluating performance in the context of our model. If the measures are to be useful, they should at least detect that an informed agent has better information than an uninformed one. More than that, one would like them to be monotone in the precision of information, so that agents can be compared and ranked. (Ideally, a performance measure should be ex ante higher if and only if the agent has better information.) As we will see, neither properties will generally hold for risk-return measures. Our results here are similar to those in Dybvig and Ross (1984*a*).

The measures we consider involve moments of the distribution of the manager's returns and of the joint distribution of those returns and the returns on a benchmark portfolio. In applying them to our model, we take the point of view of an observer who is uninformed but observes prices. Thus, in calculating moments we employ the conditional distribution of the variables given prices alone. Similarly, the benchmark portfolio in our analysis will be mean variance efficient relative to an uninformed agent given (equilibrium) prices. This interpretation is natural in our context. First, since the uninformed agent forms his beliefs in response only to the prices he observes in the marketplace, the distribution he uses resembles best what might be thought of as the "market's perception." Second, this seems to represent, in our context, the actual application of the measures, in which estimates of unconditional moments are employed.[14]

It should be mentioned here that under heterogeneous beliefs the market portfolio is not generally viewed by agents as mean variance efficient (relative to the parameters implied by their various information sets), so it plays no important role in this model. However, be-

13. It is interesting that we obtained a complete order after only one integration, i.e., that (20) is independent of $\mathbf{p}$. The reason is that $E[W(\tilde{\mathbf{S}})|\,\mathbf{f}, \mathbf{p}]$ depends on $\mathbf{f}$ and $\mathbf{p}$ through $\mathbf{f} - \mathbf{R}\mathbf{p}$.

14. The distribution of variables unconditional on prices is not appropriate, since it views as random the current (equilibrium) prices, which are observable by agents when they trade (and therefore are not part of the uncertainty in the market).

cause every agent here is rational and because returns have joint normal distributions given any information agents may have, each agent can plot his own securities market line, based on the mean-variance efficiency of his portfolio relative to the distribution he uses. In particular, if betas are calculated against a portfolio on the efficient frontier of an uninformed agent, the securities market line that is obtained indeed reflects the equilibrium expected returns (given prices) of each asset and therefore of each constant (i.e., price measurable) combination of assets.

Sharpe (1966) proposed the following ratio as a measure of the performance of a portfolio:

$$\left(\frac{r}{v}\right) = \frac{E - R}{\sigma} \tag{24}$$

where E is the portfolio's expected rate of return, R is the riskless return, and σ is the standard deviation of the return on the portfolio. It is easy to see that this measure addresses the mean-variance efficiency of the portfolio at hand. With homogeneous beliefs about the parameters of returns distribution, if one portfolio displays a higher reward-to-variability ratio rather than another, then the first portfolio dominates the second in a mean-variance sense. This means that combinations of the first portfolio and the riskless asset achieve higher mean for the same variance than do combinations of the second asset and the riskless asset.[15]

Now, suppose an uninformed observer tries to evaluate the performance of a better-informed agent in our model. Will the manager appear to do well in a "mean-variance" sense? For simplicity assume that there is one risky asset. Let a_I be the risk aversion coefficient and W_0^I the initial wealth of the informed manager. As mentioned earlier, these parameters (especially a^I) might not be observable but, since both expected return and its standard deviation are inversely proportional to the product $a^I W_0^I$, the reward-to-variability ratio is independent of them. Denote $\tilde{x} = \tilde{F} - R\tilde{P}$; $\bar{x}_p = E(\tilde{x}|p)$; $\sigma^2 = \text{var}(\tilde{x}|p)$ and let $\tilde{r}^I$ be the return on the manager's portfolio, assuming he receives private information $\tilde{Y} = \tilde{F} + \tilde{\epsilon}$ with $\text{var}(\tilde{\epsilon}) = s^2$. Then

$$\tilde{r}^I = R + \frac{1}{a^I W_0^I}\left(\frac{\tilde{x}\bar{x}_p}{\sigma^2} + \frac{\tilde{x}(\tilde{x} + \tilde{\epsilon})}{s^2}\right). \tag{25}$$

Now

$$E(\tilde{r}^I|p) = R + \frac{1}{a^I W_0^I}\left(\frac{\bar{x}_p^2}{\sigma^2} + \frac{\bar{x}_p^2 + \sigma^2}{s^2}\right). \tag{26}$$

15. The measure indeed arises in the context of the likelihood ratio test of the mean-variance efficiency of a given portfolio (Ross 1980).

Calculating the variance of $\tilde{r}^I$ is more complicated:[16]

$$\operatorname{var}(\tilde{r}^I|p) = \left(\frac{1}{a^I W_0^I}\right)^2 \left\{\frac{\bar{x}_p^2}{\sigma^2} + \operatorname{var}\left[\frac{\tilde{x}(\tilde{x} + \tilde{\epsilon})}{s^2}\right] + \frac{2\bar{x}_p}{s^2\sigma^2}\operatorname{cov}[\tilde{x}, \tilde{x}(\tilde{x} + \tilde{\epsilon})]\right\}. \tag{27}$$

This simplifies to

$$\operatorname{var}(\tilde{r}^I|p) = \left(\frac{1}{a^I W_0^I}\right)^2 \left(\frac{\bar{x}_p^2}{\sigma^2} + \frac{2\sigma^4 + 4\sigma^2\bar{x}_p^2}{s^4} + \frac{\sigma^2 + 5\bar{x}_p^2}{s^2}\right), \tag{28}$$

so the value of Sharpe's measure is

$$\left(\frac{r^I}{v}|p\right) = \frac{\bar{x}_p^2 s^2 + (\bar{x}_p^2 + \sigma^2)\sigma^2}{\sigma\sqrt{\bar{x}_p^2 s^4 + \sigma^4(2\sigma^2 + 4\bar{x}_p^2) + s^2\sigma^2(\sigma^2 + 5\bar{x}_p^2)}}. \tag{29}$$

To see that the measure is not appropriate, consider the simple case $\bar{x}_p = \sigma^2 = 1$. The reward-to-variability ratio is actually monotone increasing in s^2, namely, the better-informed agents have a lower ratio. In particular, the uninformed agent has the best ratio relative to all informed agents.[17] This is not in any way a special case, as the reader can verify. There is simply no reason to expect the ratio to indicate superior information.

Intuitively, although better information implies higher expected returns (corollary 2), it also leads to a larger variance (unconditional and also given only prices). This is because the informed manager reacts to private information signals that are unavailable to the uninformed by making changes in the composition of his portfolio. Indeed, the changes are sharper the more accurate this information is. As a result, the observed returns appear to have relatively high variability when, in fact, the informed agent is choosing optimal (mean variance efficient) portfolios relative to the parameters implied by his information. The difficulty is that the uninformed observer does not know these parameters and cannot estimate them directly, since they depend on information he does not possess.

One way around the above problem is to control for the unknown information using more ex post observable variables. A possible candidate is the payoff $\tilde{F}$, which is realized in the second period. By corollary 3, this will make the ratio monotone decreasing in s^2 for every f and p. Note, however, that conditional moments cannot be estimated

16. The following fact (called Stein's lemma) is used in the calculation: If $(\tilde{X}, \tilde{Y})$ are jointly normal, then $\operatorname{cov}[\tilde{X}, f(\tilde{Y})] = Ef'(\tilde{Y})\operatorname{cov}(\tilde{X}, \tilde{Y})$, where $f'(\cdot)$ is the derivative of $f(\cdot)$ (assuming it exists).

17. This conclusion uses comparative statics results which are special to the case of one risky asset such as monotonicity of demand in the signal and in the price (see Admati 1983, 1985). The results are more ambiguous in the general multi-asset case, i.e., the ratio might be increasing in some precisions while decreasing in others.

by taking simple averages. The next section will explore more natural ways, which emerge from the overall joint distribution of the variables, to employ the ex post observables.

Our results concerning the reward-to-volatility ratio and securities market line analysis, which are based on the beta of a portfolio as a measure of risk, are similarly negative. The details of our analysis are provided in the Appendix.[18] In particular, we show that a better-informed agent may well plot further below the securities market line than a less well-informed agent, even if they have the same risk attitudes. Ranking or detecting superior information using this criterion is therefore hopeless in general. These results should not be surprising, given the above discussion, but it is worthwhile to explain them in the context of the recent controversy concerning the validity of securities market line analysis as a performance measure. The key observation is that, in general, informed agents in our model have timing ability with respect to the efficient portfolio used as a benchmark (i.e., that of an uninformed agent). Thus, justifications of securities market line analysis in the spirit of Mayers and Rice (1979) and Dybvig and Ross (1984*a*), which require that the agent has no information about the benchmark return, cannot be applied.

Another way to explain the failure of the securities market line criterion, which is useful for our purposes, is to note that the conditional expectation (i.e., the theoretical regression) of the agent's return given those on the benchmark portfolio becomes a *nonlinear* function of the benchmark return. This is in part because the unconditional returns are not normally distributed if the agent has private information. Since securities market line analysis is based on the residuals in a linear regression of the manager's returns on those of the benchmark, it becomes inappropriate. Also note that, as with the standard deviation, the calculated beta of the portfolio, the ratio of covariance with the benchmark return and the variance of the benchmark return (both calculated given prices only), is not appropriate as a measure of risk since the true beta, which depends on the private information, is changing over time. Indeed, it should be viewed by an uninformed agent as a random variable, and part of the problem is that it is correlated with the benchmark returns. What is done in the traditional method is to estimate this regression coefficient as if it were constant.[19]

18. Although our model is different, especially because of its rational expectations aspect, the result here is similar in spirit and in some of the specifics to Dybvig and Ross (1984*a*). This is because we apply the measures using the conditional distribution given prices. Once prices are fixed, the situation is practically identical to the one they consider, at least as far as the observer and the manager are concerned.

19. Models of switching regressions coefficients, as in Kon and Jen (1979) or Kon (1983), test for changes in betas of managed portfolios and attribute such changes to timing ability. This procedure still ignores the dependence of the true betas and the benchmark returns. See Admati and Ross (1984) for further analysis of the interpretation and measurement of timing and selectivity.

V. The Statistics of Performance Evaluations

The problem of performance measurement is that of inference concerning what private information a specific agent (or agents) might have. In this section we provide a classical statistical inference formulation for the problem, and we illustrate how it can be applied. In doing so, we gain some useful insights.

Suppose the model of Section II is repeated over time. The model is determined by the exogenous vectors $\tilde{\mathbf{F}}$, $\tilde{\mathbf{Z}}$, and $\tilde{\boldsymbol{\epsilon}}_j$, $j = 1,2, \ldots$, which are assumed to form a stationary time series.[20] The performance problem can be then characterized by a set of sample observations, from which an observer wishes to infer the unknown precision matrices of specific agents. This might involve estimating a precision matrix, testing hypotheses in which it is compared to other known or unknown matrices (e.g., to a specific precision matrix or to that of another agent) or tested for some properties, such as its rank.

In general, and given our discussion in the above sections, there are many statistical inference problems that one might be interested in analyzing, and we will not try to treat them exhaustively. Rather, this section will suggest a general framework, as well as a variety of specific tools that may be employed in making the desired inference. To do this we will mostly (but not exclusively) be concerned with the basic inference problem of estimating the matrix **S**. Many interesting inference problems are likely to involve estimation of some function of **S** (or perhaps of several such matrices). Since **S** is a very large matrix in general, in many of the applications one can expect various restrictions on its elements. This should be straightforward to incorporate into our analysis.[21]

We consider three classes of inference problems, corresponding to three possible sets of available observations. The first case is when the time-series realizations of the private information signals are actually observable. Then we consider the case in which the signals are not observable, but the actual portfolio choices of the manager are available over time. Finally, we address the inference problem that arises when only observations of the realized returns on the manager's portfolio are available. In each case, in addition to the observation about the manager, we will take as observable the corresponding sequence of equilibrium prices (which are in the public information set) and the ex post realizations of the payoffs. Sample realizations will mostly be denoted by lowercase letters throughout this section.

20. A straightforward interpretation of $\tilde{\mathbf{F}}_t$ is as the sum of dividends and capital gains from period t to $t + 1$. The stationarity assumption we make is quite strong and needs further theoretical justification, which is beyond the scope of this paper. Indeed, this is so for most of the stationarity assumptions that are commonly made in empirical studies.

21. One problem that might require a somewhat different approach is the inference concerning the generalized prediction variance (see theorem 2). Anderson (1958, pp. 166–73) includes some procedures that are useful in this context.

Suppose, first, that the manager (truthfully) discloses the realizations of his private information signals, so that the sample consists of T i.i.d. observations $(\mathbf{y}_t, \mathbf{f}_t, \mathbf{p}_t)$, $t = 1,2, \ldots T$. Since the error terms themselves are in fact observable, one simply has a sample of i.i.d. multivariate normal variables with zero mean and unknown variance-covariance matrix. Procedures for estimating and testing hypothesis in this model are well known (see, e.g., Anderson 1958).

Note that if private information is directly observable ex post, then there is no need to make assumptions on utility function, optimization, the use of price information, and the market equilibrium. (This was referred to as the "nonparametric" case in Henriksson and Merton [1981].) Unfortunately, in practice an outsider might not have access (even ex post) to the private information of managers. In the rest of this section we assume that only some function of $\tilde{\boldsymbol{\epsilon}}$ is observable. In such cases it is necessary to be more specific about how private information is actually used to create the observables.

Suppose next that the manager's demand vector, $\mathbf{d}_t$, is observable over time, but not the private signal realizations. (Since prices are observable, this is equivalent to observing the portfolio weights and the initial wealth levels.) We know that

$$\tilde{\mathbf{D}}_t = \frac{1}{a}[\mathbf{G} + \mathbf{K}\tilde{\mathbf{P}}_t + \mathbf{S}^{-1}(\tilde{\mathbf{Y}}_t - R\tilde{\mathbf{P}}_t)], \tag{30}$$

where $\mathbf{G}$ and $\mathbf{K}$ are independent of $\mathbf{S}$. Thus, the sample $(\mathbf{d}_t, \mathbf{p}_t, \mathbf{f}_t)$, $t = 1, 2, \ldots, T$, comes from a multivariate normal distribution (which is the same over time). The matrix $\mathbf{S}$ affects this distribution through the mean vector of $\tilde{\mathbf{D}}_t$, its variance-covariance matrix and its covariance (matrices) with $\tilde{\mathbf{F}}_t$ and $\tilde{\mathbf{P}}_t$.[22]

Unlike the case where the private information signals were observed, it is not obvious a priori that the matrix $\mathbf{S}$ is identifiable.[23] From (30) it may seem that $\mathbf{S}$ cannot be separated from the coefficient of risk aversion a, so if a is unknown, classical estimation of $\mathbf{S}$ is impossible.[24] Fortunately, a and $\mathbf{S}$ are identifiable, as the following argument shows.

Consider the conditional distribution of $\tilde{\mathbf{D}}_t$ given $\mathbf{f}_t$ and $\mathbf{p}_t$. From joint normality, it is easy to see that this distribution is normal with parameters

$$\mathbf{E}(\tilde{\mathbf{D}}_t|\mathbf{f}_t, \mathbf{p}_t) = \frac{1}{a}[\mathbf{G} + \mathbf{K}\mathbf{p}_t + \mathbf{S}^{-1}(\mathbf{f}_t - R\mathbf{p}_t)] \tag{31}$$

22. If $\bar{\mathbf{Z}} = \mathbf{0}$ then the (unconditional) mean of $\tilde{\mathbf{D}}_t$ is actually independent of $\mathbf{S}$. It is easy to see that the variance-covariance matrix of $(\tilde{\mathbf{D}}_t, \tilde{\mathbf{P}}_t, \tilde{\mathbf{F}}_t)$ always depends on $\mathbf{S}$.

23. A parameter is identifiable if there is one-to-one correspondence between its values and the probability distribution that they induce.

24. Note that for some inference problems separating $\mathbf{S}$ and a might not be necessary (e.g., comparing managers who are assumed to have the same attitudes toward risk or whose coefficients of risk aversion are a multiple of each other).

and

$$\text{var}(\tilde{\mathbf{D}}_t|\mathbf{f}_t, \mathbf{p}_t) = \frac{1}{a^2}\mathbf{S}^{-1}. \tag{32}$$

Now, since $\tilde{\mathbf{P}}$ is not perfectly correlated with $\tilde{\mathbf{F}}$, $(a\mathbf{S})^{-1}$ is identifiable from the regression equation (31). Also, from (32), $(a^2\mathbf{S})^{-1}$ is identifiable. These two facts together imply that both a and $\mathbf{S}$ are indeed identifiable.

The estimation of (a and) $\mathbf{S}$, while conceptually straightforward, involves simultaneous estimation of the mean and variance-covariance matrix. Since $\mathbf{S}$ is a parameter affecting both the mean and variance-covariance matrix of the (conditional) distribution, it creates a constraint relating them to each other. This is not standard,[25] but it seems to capture an important property of the distribution at hand; the asset positions more correlated with future payoffs are also the more variable ones, since responses to more precise signals are sharper.

Given the joint normality, it is natural to use (31) and (32) as a multivariate linear regression model.[26] Let us write

$$\mathbf{d}_t = \mathbf{H}_0 + \mathbf{H}_1\mathbf{f}_t + \mathbf{H}_2\mathbf{p}_t + \mathbf{u}_t. \tag{33}$$

Then the error terms of this regression model are $(a\mathbf{S})^{-1}\tilde{\boldsymbol{\epsilon}}_t$, which are i.i.d. and independent of the regressors $(\mathbf{f}_t, \mathbf{p}_t)$. Therefore regression theory implies that the OLS estimators $\hat{\mathbf{H}}_i$ of $\mathbf{H}_i$, $i = 0, 1, 2$, are consistent, that is,

$$P\text{lim}_{T\to\infty}\hat{\mathbf{H}}_0 = \frac{1}{a}\mathbf{G}, \tag{34}$$

$$P\text{lim}_{T\to\infty}\hat{\mathbf{H}}_1 = \frac{1}{a}\mathbf{S}^{-1}, \tag{35}$$

and

$$P\text{lim}_{T\to\infty}\hat{\mathbf{H}}_2 = \frac{1}{a}(\mathbf{K} - R\mathbf{S}^{-1}). \tag{36}$$

Also, the usual estimator of the errors' variance-covariance matrix, $\hat{\boldsymbol{\Sigma}}$, is a consistent estimator of $a^{-2}\mathbf{S}^{-1}$. As the sample size grows to infinity, $\hat{\boldsymbol{\Sigma}}$ becomes a scalar multiple of $\hat{\mathbf{H}}_1$, where this scalar is the risk-aversion coefficient of the manager. However, when a, $\mathbf{G}$, and $\mathbf{K}$ (in addition to the elements of $\mathbf{S}$) are unknown, optimal estimation in the regression model (33) must be done under the constraint that $\hat{\mathbf{H}}_1 = a\hat{\boldsymbol{\Sigma}}$,

25. A similar situation arises in statistical inference concerning the mean-variance efficiency of a given portfolio (see Ross 1980; Kandel 1984).

26. An equivalent model is obtained by using $\mathbf{p}_t$ and $\mathbf{x}_t = \mathbf{f}_t - R\mathbf{p}_t$ as regressors. $\mathbf{S}$ is then a parameter only through the coefficient of $\mathbf{x}_t$. This model is more convenient if $\mathbf{K}$ is known.

and one cannot generally rely on the OLS procedures for efficient estimation.[27]

In summary, the above specification suggests that in this case one should use the regression of the manager's observed actions (namely, asset holdings) on the public information available when these actions were taken (in the model these are current prices) and on the ex post observations of the variables that the manager is trying to predict (namely, the future payoffs). The error terms in the private information signal of the manager around the true values serve as the error terms in the regression.

The above specification may seem somewhat backward; intuitively, it may seem natural to regress the ex post observation of the "state of the world" on the manager's actions and the public information to see whether the manager's actions explain (ex post) what eventually happened. Indeed, since $(\tilde{\mathbf{D}}_t, \tilde{\mathbf{F}}_t, \tilde{\mathbf{P}}_t)$ are jointly normal, the theoretical regressions of any subset of them on the rest is linear, so other specifications are valid as well. However, the regression suggested above (that of $\mathbf{d}_t$ on $\mathbf{f}_t$ and $\mathbf{p}_t$) is the most natural in our model: First, the coefficients (and residual variance-covariance matrix) of this specification are the simplest functions of $\mathbf{S}$. Second, some other specifications are prone to problems such as multicollinearity. Consider, for example, regressing $\mathbf{f}_t$ on $\mathbf{d}_t$ and $\mathbf{p}_t$, as may seem intuitive. If the manager is poorly or partly informed (i.e., receives information that is very imprecise or of rank smaller than n), $\tilde{\mathbf{D}}_t$ becomes a linear function of $\tilde{\mathbf{P}}_t$, so the regressors will tend to be multicollinear. This regression is, therefore, not recommended. The intuition is simple: The actions of a manager who does not have superior information are highly correlated with public information. Under the null hypothesis of no private information, therefore, the manager's actions and the public information are problematic as joint regressions.

Suppose, finally, that neither the signal observations nor the choices of the manager are available for performance measurement. In addition to the sample $(\mathbf{p}_t, \mathbf{f}_t)$ then, only the time series of the manager's initial and final wealth levels, W_{0t} and W_{1t}, $t = 1,2, \ldots T$, are observable. (This corresponds to observing the returns on the manager's portfolio, given by W_{1t}/W_{0t}.) Since $W_{1t} = W_{0t} + \mathbf{d}_t(\mathbf{f}_t - R\mathbf{p}_t)$, we have that the $\mathbf{d}_t(\mathbf{f}_t - R\mathbf{p}_t)$ are observable. Denote $\tilde{\mathbf{X}}_t = \tilde{\mathbf{F}}_t - R\tilde{\mathbf{P}}_t$. It will be convenient to use the sample observations $(\mathbf{x}_t, \mathbf{p}_t, \mathbf{d}_t\mathbf{x}_t)$ in our discussion.

As indicated in Section IV, the returns on the portfolio of an in-

27. A constrained maximum likelihood estimation could be employed. (See Frankel [1982] for a similar econometric model, in the context of international finance.) Similarly, one can perform likelihood ratio tests concerning various hypotheses. The statistics for the (Neyman-Pearson) test of whether $\mathbf{S}$ is equal to one prespecified matrix or another, for example, is some linear combination of the sample mean and the sample variance-covariance matrix. This is not surprising, since the hypotheses concern simultaneously the mean and the variance-covariance matrix of the distribution at hand.

formed manager might pose difficulties as elements in a statistical analysis of performance. For example, they are not normally distributed—$\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t$ are quadratic functions of (dependent) normal variables, which results in a complicated distribution (see Johnson and Kotz 1972). However, the conditional distribution of $\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t$ given $\mathbf{x}_t$ and $\mathbf{p}_t$ is normal, since, given $\mathbf{x}_t$, it is a fixed linear combination of normal variables. (This is not true, as we saw in Sec. IV, of the conditional distribution of $\tilde{\mathbf{X}}_t\tilde{\mathbf{D}}_t$ given $\mathbf{p}_t$, which is the ex ante distribution of the manager's payoffs given the information of an uninformed agent. The ex post observation of $\tilde{\mathbf{F}}_t$ is therefore crucial here.) This suggests the following approach. Suppose the manager receives private information $\tilde{\mathbf{Y}}_t = \tilde{\mathbf{F}}_t + \tilde{\boldsymbol{\epsilon}}_t$. Then

$$\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t = \frac{1}{a}\,[(\mathbf{G} + \mathbf{K}\tilde{\mathbf{P}}_t)\tilde{\mathbf{X}}_t + (\tilde{\mathbf{X}}_t + \hat{\boldsymbol{\epsilon}}_t)\mathbf{S}^{-1}\tilde{\mathbf{X}}_t]. \tag{37}$$

Now, since $E(\tilde{\mathbf{X}}_t\mathbf{S}^{-1}\tilde{\boldsymbol{\epsilon}}|\mathbf{x}_t, \mathbf{p}_t) = 0$, we get the following regression model:

$$E(\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t|\mathbf{x}_t, \mathbf{p}_t) = \frac{1}{a}\,(\mathbf{G}\mathbf{x}_t + \mathbf{x}_t\mathbf{K}\mathbf{p}_t + \mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t), \tag{38}$$

where the residual variance is

$$\operatorname{var}\left(\frac{1}{a}\,\tilde{\boldsymbol{\epsilon}}\mathbf{S}^{-1}\tilde{\mathbf{X}}_t|\mathbf{p}_t, \mathbf{x}_t\right) = \frac{1}{a^2}\mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t. \tag{39}$$

This is still a linear multiple regression model where the regressors are $(\mathbf{x}_t)_i$, $(\mathbf{x}_t)_i(\mathbf{p}_t)_j$ and $(\mathbf{x}_t)_i(\mathbf{x}_t)_j$ for all pairs of indices i and j (i.e., the elements of $\mathbf{x}_t$ and all their cross-products with themselves and with the coordinates of $\mathbf{p}_t$). The elements of $(a\mathbf{S})^{-1}$ can then be estimated as the relevant coefficients in this regression. It is easy to verify, again, that $\mathbf{S}$ is an identifiable parameter of this model, since the risk tolerance coefficient affects the variance nonlinearly.

Unfortunately, unlike regressions of jointly normal variables on each other, where the residual variances are independent of the regressors, the conditional variance of $\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t$ given $\mathbf{x}_t$ and $\mathbf{p}_t$, given in (39), is a quadratic function of the specific $\mathbf{x}_t$. Thus, although the errors in (38) are (conditionally) independent of each other and of the regressors, and are normally distributed with mean zero, they are heteroscedastic. Ordinary least squares estimation will still provide unbiased and consistent estimators, but it is not efficient.[28]

28. Note that by using a simple transformation the above model becomes what is known as a nonlinear model with additive errors: Let $\mathbf{U}_t^* = (\sqrt{\mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t/a})\tilde{\boldsymbol{\epsilon}}_t$. Then, conditional on x_t, $\mathbf{U}_t^*$ is normally distributed with mean zero and variance one. Thus, the conditional distribution of $\mathbf{U}_t^*$ given $\mathbf{x}_t$ is independent of $\mathbf{x}_t$. Multiplying both sides of (38) by $\sqrt{\mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t/a}$ and rearranging yields $^{-}{}_a^1(\mathbf{x}_t\mathbf{d}_t) - \mathbf{G}_0 - \mathbf{x}_t\mathbf{K}\mathbf{p}_t - \mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t/\sqrt{\mathbf{x}_t\mathbf{S}^{-1}\mathbf{x}_t} = \mathbf{U}_t^*$. While the errors in the right-hand side are i.i.d., the left-hand side is a nonlinear function of $\mathbf{S}$ and the other unknown parameters. Maximum likelihood estimation can be applied either directly to (37) or to the nonlinear equation above.

It is interesting to note that under some conditions the above specification is similar to the well-known Treynor and Mazuy (1966) quadratic regression. Suppose (without loss of generality) that the first asset corresponds to the market portfolio and that we are interested only in timing ability, namely, whether the manager has information that enables better predictions of the market return. Then the matrix $\mathbf{S}^{-1}$ is assumed to have only one nonzero element, corresponding to the first asset, and the relevant regression coefficient in our specification is that of the squared excess payoff (i.e., $(\mathbf{x}_t)_1^2$). (The controversial procedure suggested by Treynor and Mazuy [1966] and critically examined by Jensen [1972] has recently been studied by Bhattacharya and Pfleiderer [1983] in the context of a model with private information where equilibrium is determined according to the CAPM.) The regression equation (38) now becomes

$$\mathbf{E}(\tilde{\mathbf{D}}_t\tilde{\mathbf{X}}_t|\mathbf{x}_t, \mathbf{p}_t) = \frac{1}{a}[\mathbf{G}\mathbf{x}_t + \mathbf{x}_t\mathbf{K}\mathbf{p}_t + (\mathbf{x}_t)_1^2/s_1], \tag{40}$$

where s_1 is the variance of the error term in the signal concerning the first asset return, that is, the market timing information. Hypotheses concerning the particular assets on which a manager claims to have information, as well as about the covariance of the errors in his predictions (e.g., that they are independent or are related in a specific way) can similarly be incorporated.[29]

Our analysis in this section indicates that certain regression models arise naturally in the analysis of performance evaluation under information asymmetries and rational expectations. This is encouraging, since regression models are convenient for applications and are robust with respect to the exact parametric specification. However, in most cases these models will have a special structure, that should be exploited to improve straightforward OLS regressions. In particular, if one uses observed returns only, they might involve heteroscedastic errors or other nonlinearities. If the actual compositions of the managed portfolio over time are observable, then the inference problem is simpler, but it still involves a relation between the mean and the variance of the observed asset demands.

VI. Concluding Remarks

Virtually all work on performance evaluation has been conducted in the context of equilibrium models (mainly the CAPM) that assume

29. In all the above analysis we have actually suppressed a constraint that might create difficulties in using least squares estimators to estimate $\mathbf{S}^{-1}$, namely, that it must be a positive semidefinite matrix. Without other restrictions, this may introduce nonlinear constraints on some regression coefficients. Note that if estimation is done through the variance-covariance matrix of the error terms (in the case of observed holdings), then the usual variance estimators will indeed be positive semidefinite.

homogeneous beliefs. Such analyses always involve the implicit assumption that fund managers, who might not share these homogeneous beliefs, do not affect equilibrium. This is somewhat disturbing, since there is no reason to believe either that the bulk of the market has the same beliefs or that the combined effect of managed portfolios on equilibrium is zero. In contrast, the model we have employed includes many agents with heterogeneous and asymmetric information. No special provisions are therefore required for an agent's information (and, therefore, their beliefs) to be different from those held by others. Since information asymmetries exist throughout the market, the performance of every agent or group of agents can be evaluated. In addition, while agents have diverse and asymmetric private information, they are perfectly rational and use optimally all the information available to them, including prices.

Using the model we have been able, in a conceptually straightforward manner, to define and characterize superior performance and identify it with superior information. We then examined the robustness of existing concepts and measures with respect to our specification. We found that the traditional risk-return measures are not generally useful for the performance problem considered here. Since the basic intuition behind these measures fails under information asymmetries, the negative results we obtained seem general and robust and are not an artifact of the model we employed. As in Dybvig and Ross (1984*a*), the key to it is simply that an informed agent reacts to information that is private and that is generally correlated with ex post observables as well as with benchmarks created on the basis of coarser information. As a result, the returns on the managed portfolio may well seem mean variance inefficient or plot below the securities market line of an outside observer.

We have also used the model to develop a theoretical framework and some specific tools for making valid performance evaluations. Our analysis of alternative measures provides useful insights. In particular, it suggests that if the actions (portfolio choices) of the manager are observable ex post, then those actions should be regressed on the public information available when these choices were made and on the ex post observable returns. This linear regression was shown in our model to have a particular special structure; namely, there is a relationship between some regression coefficients and the variance-covariance matrix of the residuals. If only returns are available then inference is complicated by nonlinear terms. This can partially be overcome by using quadratic terms in a linear regression, in the spirit of Treynor and Mazuy's quadratic model, but some statistical difficulties such as heteroscedasticity still remain.

A final comment on the application of our results. The model we have employed above is a 2-period model, where trade and consumption occur only once. While we believe that many elements of the analysis of Section V are characteristic of the performance evaluation problems in general, our analysis is still preliminary. To develop a complete statistical model that is consistent with observation of a time series, a multiperiod model is required, from which the properties of such time series can be derived. This is a difficult task, particularly in the context of rational expectations equilibrium models. Alternatively, as is done in the empirical work based on the 2-period CAPM, one can postulate that some variables of the model follow a stationary time series (e.g., rates of return) and proceed. Since such an assertion is not based on an explicit intertemporal model, it merely represents a statistical hypothesis concerning the time series at hand that is tested jointly with any other relevant hypotheses. In any case, care should be taken in interpreting the results of any such application.

Appendix

In this Appendix we provide the details of applying the reward-to-volatility ratio and securities market line analysis to the model. Using the general notation of Section IV, let $\tilde{r}^u$ be the return on the benchmark, with the normalization that $a^u = W_0^u = 1$. Then we have

$$\tilde{r}^u = R + \frac{\tilde{x}\bar{x}_p}{\sigma^2}, \tag{A1}$$

$$E(\tilde{r}^u|p) = R + \frac{\bar{x}_p^2}{\sigma^2}, \tag{A2}$$

$$\operatorname{var}(\tilde{r}^u|p) = \frac{\bar{x}_p^2}{\sigma^2}, \tag{A3}$$

and

$$\operatorname{cov}(\tilde{r}^I, \tilde{r}^u|p) = \frac{1}{a^I W_0^I}\left[\frac{\bar{x}_p^2}{\sigma^2} + \frac{\bar{x}_p^2(s^2 + 2\sigma^2)}{\sigma^2 s^2}\right] + \frac{2\bar{x}_p^2}{s^2}. \tag{A4}$$

Therefore,

$$\beta_p = \frac{\operatorname{cov}(\tilde{r}^I, \tilde{r}^u|p)}{\operatorname{var}(r^u|p)} = \frac{s^2 + 2\sigma^2}{a^I W_0^I s^2}. \tag{A5}$$

Let us now examine first the reward-to-volatility ratio. For every price p, it is given by

$$\left(\frac{r}{\beta}|p\right) = \frac{E(\tilde{r}^I|p) - R}{\beta p} = \frac{\bar{x}_p^2 s^2 + (\bar{x}_p^2 + \sigma^2)\sigma^2}{\sigma^2(s^2 + 2\sigma^2)}. \tag{A6}$$

It can be easily verified that

$$\frac{\partial}{\partial s^2}\left(\frac{r}{\beta}|p\right) = \frac{\bar{x}_p^2 - \sigma^2}{(s^2 + 2\sigma^2)^2}, \tag{A7}$$

which is ambiguous in sign. Since $\bar{x}_p$ is decreasing in p, we know that for prices in some interval $(p, \bar{p})$ the reward-to-volatility ratio is decreasing in s^2, while for prices outside it is increasing. This means that sometimes better information implies a higher reward-to-volatility ratio and sometimes a lower ratio.

Now consider applying securities market line analysis. Given p, the securities market line plotted by the uninformed will predict that the expected return on a portfolio whose return is given by (39) is

$$E_{SML}(\tilde{r}^I|p) = R + \beta_p[E(\tilde{r}^u|p) - R] = R + \left(\frac{s^2 + 2\sigma^2}{a^I W_0^I s^2}\right)\frac{\bar{x}_p^2}{\sigma^2}. \tag{A8}$$

Thus the differential return is calculated as

$$E(r^I|p) - E_{SML}(r^I|p) = \frac{\sigma^2 - \bar{x}_p^2}{a^I W_0^I s^2}. \tag{A9}$$

This is, again, ambiguous in sign. In fact, it has the same sign as the derivative of Treynor's ratio, namely, there is an interval $(p, \bar{p})$ such that the differential return is negative for every price in the interval and positive outside. Also note that if the differential return is negative for some price then it gets smaller if s^2 is smaller; that is, better-informed agents plot even further below the securities market line.

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