Gary D. Eppen
University of Chicago

Yehoshua Liebermann
Bar-Ilan University

Why Do Retailers Deal? An Inventory Explanation*

Price deals occur when a retailer offers an item for sale for a short period of time at prices below its going market rate. This paper demonstrates that, under certain conditions, price deals on nonperishable goods can benefit both the retailer and the consumer by transferring part of the inventory carrying cost from the former to the latter in return for an unusually low price. The analysis is based on the assumption that the consumer alters his purchasing, but not his consumption, behavior in the face of deals. The retailer faces an environment described by the assumptions of the classical EOQ model. The optimal behavior of both parties is derived as well as the implied characteristics of the system. Some empirical evidence that tends to support the theoretical argument is presented.

I. Introduction

Price deals, or price specials, are a well-established practice in retail sales promotion, and a topic of considerable interest to researchers (see Seligman and Love 1932; U.S. Federal Trade Commission 1932; Cassaday 1962; Hinkle 1965; Kuhn and Rohloff 1967; Tuch and Harvey 1972; Leed and German 1973; Kinberg and Rao 1975; Blattberg et al. 1978; Dobson, Tybout, and Sterhal 1978; Blattberg, Eppen, and Liebermann 1981). In these deals, items are offered for sale for a short period of time at prices below their going market rates. Supermarket chains, as well as independent retailers, heavily advertise their deals in local papers, by signs and displays at the store, and on television and radio. A natural question is, Why do many retailers prefer this strategy to other forms of price competition such as lower average prices? (This question is broached by Allvine, Teach, and Connelly [1976].) This paper demonstrates that, under certain conditions, price deals on nonperishable goods can benefit both retailer and consumer by transferring part of inventory holding cost from the former to the latter in return for an unusually low price. However, it does not deny that price deals may also reflect the influence of other considerations.

* We are indebted to Joe Bogus for comments on current practices. We, of course, are responsible for any errors.

(*Journal of Business*, 1984, vol. 57, no. 4)

0021-9398/84/5704-0005$01.50

Although the idea that inventory considerations play an important role in motivating deals has been mentioned in the literature (e.g., Frank and Massey 1967; Telser 1972; Kinberg, Rao, and Shakun 1974; Duffy 1977; Blattberg et al. 1978), we know of no other attempts to construct an explicit model portraying the interaction of retailer and consumers. The paper is organized into five sections. Section II presents the consumer model; Section III is devoted to the retailer model; and Section IV derives the implications of interacting the models of consumer and retailer. In the final section we present some empirical evidence that tends to support the theoretical argument. The paper concludes with a brief summary.

II. The Consumer

Each consumer consumes the product at a constant rate of r items per unit of time. Because there is a nonzero cost to holding inventory, under normal conditions a consumer chooses to hold none. That is, on each shopping trip he buys only the quantity required until his next trip. If, however, the retailer offers a deal, the consumer may be motivated to buy more units than he requires for current consumption and thus to hold inventory. Assume that a discount of D dollars per item is offered by the retailer and that the consumer's cost of carrying inventory is h times the area under the plot of the inventory on hand.

Then, the net cost to the consumer of buying q items on deal, $C(q)$, is given by

$$C(q) = hq^2/2r - Dq. \tag{1}$$

It follows that the optimal quantity that a consumer will buy on deal, say q^*, is $q^* = Dr/h$. This implies that, when faced with a single deal, a consumer will buy ahead for $t^* = D/h$ units of time.

III. The Retailer

A. The Retailer without Deals

Without deals, the demand faced by the retailer per unit of time is a constant, R. Assume that the retailer's situation is that described in the classical economic order quantity (EOQ) model, that is, price, P, is a known constant (except when deals are offered); the retailer must satisfy all demand; and delivery is instantaneous.

Let K = cost of placing an order and H = cost per item per unit time for the retailer to hold inventory and adopt the criterion of maximizing the average profit per unit of time over an infinite horizon. Note that, since P and R are constants, minimizing the average cost per unit of time maximizes the average profit per unit time. If k_N^* is the optimal time between orders and AC_N^* is the minimum average cost per unit of time when no deals are offered, the standard results (Eppen and Gould 1984) are that

$$k_N^* = \left(\frac{2K}{HR}\right)^{1/2} \tag{2}$$

$$AC_N^* = (2KHR)^{1/2}. \tag{3}$$

B. The Retailer with Deals

When a retailer offers a deal, some consumers will buy a quantity of the item at the reduced price but without changing their consumption pattern. This implies that the retailer will face a different temporal pattern of demand over (say) a year if he chooses to offer deals, with total annual demand remaining constant. Further, since holding inventory is costly to a retailer and delivery is instantaneous, a retailer will not place an order unless the inventory on hand is zero. Define the time between two consecutive moments at which the inventory on hand next hits zero as an inventory cycle. The following six assumptions are then made in regard to the retailer's dealing policy and the consumer's response.

1. The same policy is used in each inventory cycle.
2. A single deal is offered during each cycle.
3. A deal is instantaneous. The implication of this assumption is that all items sold on deal are removed from inventory instantaneously.
4. Consumers separate into two groups. The first group consists of consumers who have low inventory holding costs and respond to the deal by purchasing additional items and holding inventory. This group accounts for $\alpha(D)$ proportion of demand over any period of time. The second group consists of consumers who have high inventory holding costs and do not respond to the deal. They account for $1 - \alpha(D)$ proportion of demand over any period of time.
5. The function $\alpha(D)$ that determines the proportion of consumers that respond to a deal is assumed to be a monotone, nondecreasing function of D.
6. All consumers that respond to the deal have the same cost of carrying inventory, h.

We now derive the optimal dealing policy for the retailer on the (usual) assumption that he (the retailer) believes consumers will respond rationally. Some retailers may decide not to deal. For them, the optimal inventory policy is presented in Section III*A*. Those who decide to deal must determine: (1) the magnitude of the discount, D; (2) the quantity to order, Q, or, equivalently, the length of the inventory cycle, k; (3) the time during the inventory cycle at which the deal should be offered.

To determine the optimal values of these variables, we require the following two lemmas:

LEMMA 1: If the retailer decides to deal, then for any length of inventory cycle, k, it will be optimal to offer the deal at the beginning of each cycle.

PROOF: The rationale behind this proof is that it is optimal for the retailer to move inventory to the consumer as soon as possible. The proof is rather tedious in that a number of cases must be considered. Two main cases to consider are provided by the relationship between k, the length of the inventory cycle, and t^*, the optimal length of time for a consumer to hold inventory. Additional cases are provided by the location of the deal in the inventory cycle. In each case it must be shown that a deal at the beginning of the cycle is at least as good as other alternatives. This idea is so intuitively appealing that the proofs are omitted. Detailed proofs are available in (14).

LEMMA 2: If the retailer decides to deal, then for any k (cycle length) he will choose a value of D such that $t^* = k$.

PROOF: It clearly does not pay the retailer to choose $t^* > k$. If $t^* > k$, the consumers will buy all of their items on sale and a value of D that yields $t^* = k$ will accomplish this result. Any larger value of D is simply an unnecessary loss.

For $t^* \leq k$, the total cost, $TC(k, t^*)$, is given by the expression

$$TC(k, t^*) = K + D\alpha(D)t^*R + \frac{HR}{2}[k^2 - \alpha(D)t^{*2}]. \tag{4}$$

We know from Section II that $t^* = D/h$. Thus,

$$TC(k, t^*) = K + \frac{HR}{2}k^2 + R\alpha(ht^*)t^{*2}(h - H/2). \tag{5}$$

The minimization of $TC(k, t^*)$ depends crucially on the sign of $(h - H/2)$. If $(h > H/2)$, $TC(k, t^*)$ is minimized by setting $t^* = 0$. This implies that $D = 0$, that is, that the retailer does not offer a deal. If $h < H/2$, the last term in the expression for $TC(k, t^*)$ is negative and to minimize $TC(k, t^*)$ one makes the coefficient of this term as large as possible. Since $\alpha(ht^*)$ and t^{*2} are nondecreasing functions of t^*, we choose to set t^* equal to its maximum value, which is k. Larger values of t^* are ruled out by the first half of the proof. We now conclude that the optimal value of D is hk.

Since $t^* = k$ whenever $h < H/2$, equation (5) can be written as

$$TC(k) = K + \frac{HR}{2}k^2 + R\alpha(hk)k^2\left(h - \frac{H}{2}\right), \tag{6}$$

which makes it clear that the retailer's problem has been reduced to selecting the single parameter k, that is, the cycle length.

The retailer's objective is to minimize the average cost per unit of time. Thus he wishes to select k to minimize

$$AC(k) = \frac{K}{k} + \frac{HR}{2}k - \alpha(hk)R\left(\frac{H}{2} - h\right)k. \tag{7}$$

To analyze this problem, first assume $\alpha(hk) = 0$, that is, no one buys the item at the sale price. The retailer's problem then reduces to the standard EOQ problem, that is, minimizing the sum of his ordering and inventory holding costs. In this case the optimal decision is k_N^* and the minimum average cost is AC_N^*. As the other extreme, assume $\alpha(hk) = 1$, that is, everyone buys the item on deal. In this case the retailer holds no inventory. His problem is to minimize the sum of the ordering cost plus the loss due to the deal. His cost per unit time is

$$AC(k) = \frac{K}{k} + \frac{RkD}{k}, \tag{8}$$

but, since $D = hk$, (8) becomes

$$AC(k) = \frac{K}{k} + (hR)k \tag{9}$$

and the optimal cycle length is $k_T^* = \sqrt{K/hR}$. Note that setting $\alpha(hk) = 1$ in (7) yields the same version of (9) as the one just derived by economic arguments.

Recall that $\alpha(D)$ is a general nondecreasing function of D (or k). We now approximate $\alpha(D)$ with $A(D)$, a function in which α remains constant over intervals. An example is provided in figure 1.

This approximation has several advantages: It yields easy analytic results. It is easily adapted to any $\alpha(D)$ function. The approximation can be made arbitrarily close simply by increasing the number of intervals. To illustrate assume that $A(D)$ has three segments, as shown in figure 1. The retailer's problem then can be stated as follows:

$$\min \begin{cases} \min\left[\frac{K}{k} + \frac{HR}{2}k - \alpha_1 R\left(\frac{H}{2} - h\right)k\right] \\ 0 < k \leq \frac{a_1}{h} \\ \min\left[\frac{K}{k} + \frac{HR}{2}k - \alpha_2 R\left(\frac{H}{2} - h\right)k\right] \\ \frac{a_1}{h} < k \leq \frac{a_2}{h} \\ \min\left[\frac{K}{k} + \frac{HR}{2}k - \alpha_3 R\left(\frac{H}{2} - h\right)k\right] \\ \frac{a_2}{h} < k. \end{cases}$$

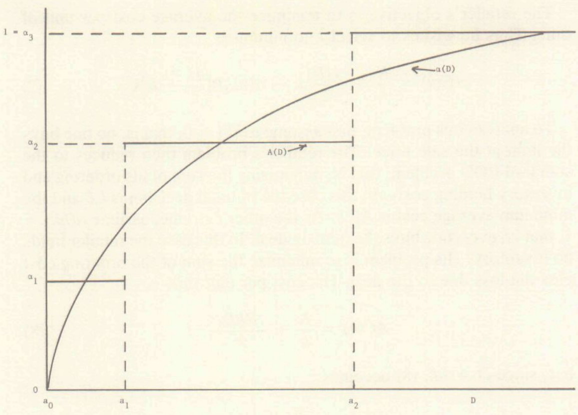

FIG. 1

This problem is closely analogous to the problem of all unit quantity discounts in the standard EOQ model. For those unfamiliar with inventory theory, we note that this problem consists of finding the minimum of each of three subproblems and then picking the smallest of these three. Fortunately, each subproblem is easy to solve. Indeed, each of the subproblems is simply one of finding the minimum of a convex function on a compact set. The first subproblem is illustrated in figure 2.

The global minimum of the function

$$\frac{K}{k} + \frac{HR}{2}k - \alpha_1 R\left(\frac{H}{2} - h\right)k$$

occurs at the intersection of the K/k curve and the straight line; that is, a cycle length of k_1 would yield the minimum average cost if any value of k could be selected. However, in the first subproblem we are restricted to choosing a $0 < k \leq a_1/h$. Since $k_1 > a_1/h$ and the cost function is concave, the optimal value of k for this problem is $k = a_l/h$.

Generalizing the notation and analysis from the example in figure 2 provides a means of analyzing the general problem. In particular, let

1. $C_i(k) = \frac{K}{k} + \left[\frac{HR}{2} - \frac{\alpha_i R}{2}\left(H - 2h\right)\right]k;$

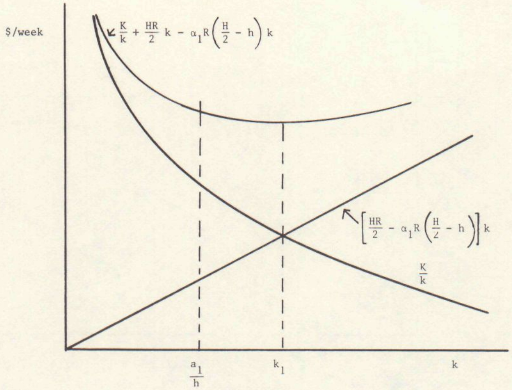

FIG. 2

that is, $C_i(k)$ is the cost if an α_i proportion bought the item on sale and a cycle length of k is selected.

2. Let k_i minimize $C_i(k)$;
simple calculus shows that

$$k_i = \left(\frac{2K}{HR - \alpha_i R(H - 2h)}\right)^{1/2}.$$

3. Let k_i^* minimize $C_i(k)$ over the range $a_i - 1/h < k \leq a_i/h$. By considering the shape of $C_i(k)$ in figure 2 we see that (*a*) If $a_i/h \leq k_i$ then $k_i^* = a_i/h$. (*b*) If $a_{i-1}/h < k_i \leq a_i/h$ then $k_i^* = k_i$. (*c*) If $k_i \leq a_{i-1}/h$ then $k_i^* = a_{i-1}/h$.

Thus, finding the optimal value of k for each subproblem is trivial.

One way to find the optimal cycle length when deals are used, say k^*, is to select the k_i^* which yields the minimum average cost per unit of time. In other words, k^* is the value of k_i^* which yields $\min_i C_i(k_i^*)$.

However, it may not be necessary to make all of these comparisons. Consider, for example, the situation illustrated in figure 3. Since α_i is a nondecreasing function of i, we know that $C_i(k_i) > C_{i+1}(k_{i+1})$. Also, $C_i(k_i^*) \geq C_i(k_i)$. Thus, if I is the largest i for which $k_i = k_i^*$, then $C_i(k_i^*) > C_I(k_I^*)$ for all $i < I$.

To find the optimal cycle length, k^*, the problem reduces to finding $\min_{I \leq i} C_i(k_i^*)$. It is thus clear that if $A(D)$ is used to approximate $\alpha(D)$, a simple algorithm exists for selecting the optimal cycle length k^*. Once

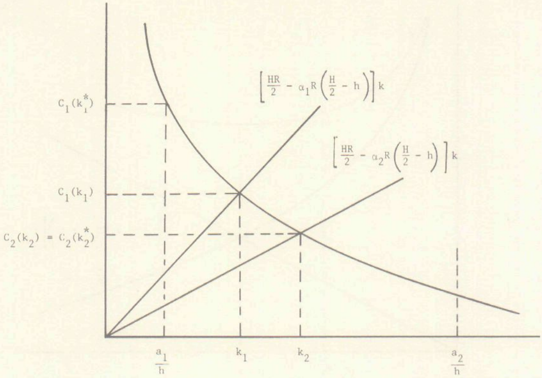

FIG. 3

k^* is available, the relationships $Q^* = R/k^*$ and $D^* = k^*h$ yield the optimal order quantity and the optimal discount. But, in addition, the model implies a number of interesting qualitative results, which are presented in the following section.

IV. Dealing Strategies

The decision to deal or refrain depends crucially on the relationship between the retailer's holding cost, H, and the deal-prone consumer's holding cost, h. Unless H is much larger than h (two times as large in the model), it does not pay for the retailer to offer a deal. Thus, deals on items such as paper goods, which cost little for the consumer to store and have a high opportunity cost to the retailer in terms of alternative use of shelf space, are attractive items to deal.

Recall that $AC_N^* =$ optimal average cost with no deal; $k_N^* =$ optimal cycle length with no deal. Now let $AC_D^* =$ optimal average cost with a deal; $k_D^* =$ optimal cycle length with a deal.

The analysis that produces the optimal value of the cycle length shows that when a retailer decides to deal:

1. His average cost will decrease, that is, $AC_N^* > AC_D^*$. This is not exactly a surprise, since otherwise he would not deal.
2. The inventory cycle will increase, that is, $k_D^* > k_N^*$. In other words, the inventory cycle of the no-deal model is a lower bound on the optimal inventory cycle.

3. The order quantity will increase. This follows directly from point 2 since $q^* = k^*R$.

These results are intuitively appealing. The effect of the sale is to reduce the amount of inventory that is held by the retailer every time an order is placed. Thus, when the retailer deals, a larger quantity must be ordered in order to obtain the appropriate balance between his ordering cost and the holding costs.

The magnitudes of the effects described above depend on the values of the various parameters h, H, K, and R. It is clear that the more rapidly $A(D)$ increases from zero to one the more important is the role of deals. In other words, deals are important if a high percentage of consumers will buy on deal for a small discount. In terms of the model, if $A(D)$ assumes a value of one for a cycle length less than k_N^*, the retailer is able to obtain the largest possible benefit from dealing.

The rate of demand R plays an interesting role in determining the optimal dealing policy. In particular, note that

1. As R increases AC_N^* increases and thus the opportunity to reduce inventory costs becomes more interesting.
2. As R increases, the quantity $\alpha R[(H/2) - h]k$ increases. Since this is the amount saved by dealing for a cycle of k weeks, it is clear that dealing becomes more important as R increases.
3. The expression for k_i can be written as

$$k_i = \left\{\frac{2K}{R[(1 - \alpha_i)\,H + 2\,\alpha_i h]}\right\}.$$

The denominator is obviously positive and increasing in R.

It follows that k_D^* decreases (or remains the same) as R increases. Finally, since $D^* = hk^*$ we see that as R increases, D^* will either decrease or remain the same. The bottom line is that more popular storable items offer greater opportunity for savings through dealing. They should be dealt more frequently with smaller discounts.

To conclude this section, we note that quantity discounts seem to dominate price deals as a mechanism for transferring inventory carrying costs from the retailer to the consumer. In particular, only the deal-prone consumers would have the benefit of the reduced price. The $1 - \alpha(D)$ proportion of nondealers would continue to buy at the regular price. The retailer thus reduces his loss due to the discount price and retains the transfer of inventory carrying costs. In view of this fact, it is natural to ask, If the transfer of inventory costs is important, why don't retailers use quantity discounts?

The answer lies in the importance of a single unit price in chain operations. With thousands of items, a large proportion of part-time employees, and a cash business, it is essential to have a simple system of control. A firm unit price system forms the basis for such systems.

To illustrate this point, consider two policy changes that most retail chains have adopted:

1. Simple discount schemes where items are priced (for example) two for 79¢ but a customer could buy a single item unit for 40¢ have been discontinued.
2. The employee benefit of buying products at cost plus 10% (or some such figure) has been largely discontinued. The difficulty of control and the opportunity for employee fraud make it unattractive.

In summary, the systems costs of quantity discounts outweigh the direct advantages of inventory cost transfer.

V. Empirical Findings

It is useful and frequently revealing to subject theoretical economic analysis (like the one in this paper) to empirical testing. A great deal of casual empiricism supports the general assumptions behind the model in this paper and, indeed, provided the motivation for its construction. In particular: January white sales are an institution in Chicago. It is a common practice for households to anticipate their bedding and towel needs for a year and to purchase these items during the sales. It is also obvious that income plays an important role in the "buy and store" behavior of consumers. Affluent families are more apt to buy and store items than low-income families who cannot afford to tie up much of their assets in stocks of goods. This behavior holds over a wide spectrum of goods from easily stored items like paper products and liquor to perishable goods, like meat and frozen foods that require special care. Although it is easy to find such examples, they are simply casual observations.

More rigorous tests of the model require data on detailed consumer behavior. What follows is an analysis of data provided by the *Chicago Tribune* consumer panel. Although these data are not ideal, they are usable for the purpose at hand, which is to show that consumers respond to deals by stockpiling but subsequently abstain from purchasing the deal item in postdeal periods. These interrelated predictions are based on the claim that "deal offers" on storable items will affect consumers' purchasing patterns rather than their consumption patterns.

Both predictions will be tested for a single product category, facial tissue. The data set includes purchasing records made by consumers over a 3½-year period (1/1958–6/1961). Each purchasing record contains information about the number of units purchased, the price paid, the brand name, the package size, and the store where the purchase was made. Consumers were also requested to note a deal purchase by a special code. From observing the data it is clear, however, that very frequently the deal code was not recorded.

The fact that the deal code is not useful makes it necessary to use a different method for identifying deal transactions. For this purpose the following procedure is employed. For each size/brand/store combination the regular (nondeal) price for given periods of time was determined by noting the most frequently reported price. Deal transactions were then defined as transactions on which the differential between the regular price and the price reported paid is positive and exceeds a predetermined limit. (This process accounts for small variations in the regular price.) Since store codes in the data refer to aggregated groups of stores, only deal transactions offered by chain stores are regarded as such. The reason for this modification is the centralized pricing and dealing policies used by chains but not by other aggregated stores (e.g., independent food stores).

To test the first prediction (consumers' stockpiling), the mean number of units bought per deal and per nondeal transaction is measured and the difference is calculated. Results for a sample of two nationally distributed brands of the 400-tissue size category are reported in table 1. Because of interdependent observations (one consumer makes more than one purchase), the standard statistical test for difference between means is not employed. However, standard errors are reported (in parentheses).

The second prediction is that the interval between purchases should increase after an item is purchased on deal. This prediction was also tested with data on the 400-tissue size of facial tissue. The results are reported in table 2. Because of the dependent observation problem (one consumer has at least four intervals), no formal test is run for the

TABLE 1 **Consumer Responses to Deals on Facial Tissue**

Mean No. of Units Bought per Deal Transaction	No. of Deal Transactions	Mean No. of Units Bought per Nondeal Transaction	No. of Nondeal Transactions	Differential between Mean No. of Units Bought per Deal and Nondeal Transaction
1.99 (.03)	7.33	1.24 (.01)	2,510	+.75

NOTE.—For the 400-tissue size category of the brands of Kleenex and Scotties as purchased at the chains of A & P, National, Jewel, Kroger, and Walgreen's.

TABLE 2 **The Time between Purchases in Days**

Mean Time for Intervals Following a Deal Purchase	No. of Deal Intervals	Mean Time for Intervals Following a Nondeal Purchase (Days)	No. of Nondeal Transactions	Difference between Means
58.66 (1.77)	1,629	51.70 (1.22)	45.02	6.96

difference between the means. However, standard errors are reported (in parentheses).

The partial empirical findings seem to support the consumer part of the model. As a response to deals on facial tissue consumers increased the average number of units bought per transaction by 60.5%. Correspondingly, they abstained from purchasing in postdeal periods which is reflected in an increase of 13.4% in their average interpurchase time interval following nondeal purchases. Recalling that these results reflect the behavior of the entire panel population makes them more supportive to the model. It seems reasonable to believe that corresponding numbers for low-holding-cost consumers would show still stronger stockpiling and postdeal effects.

References

Allvine, Fred C.; Teach, Richard D.; and Connelly, John, Jr. 1976. The demise of promotional games. *Journal of Advertising Research* 16 (October): 79–84.

Blattberg, Robert; Buesing, Thomas; Peacock, Peter; and Sen, Subrata. 1978. Identifying the deal prone segment. *Journal of Marketing Research* 15 (August 1978): 369–77.

Blattberg, Robert C.; Eppen, Gary D.; and Liebermann, Yehoshua. 1981. A theoretical and empirical evaluation of price-deals for consumer non-durables. *Journal of Marketing* 45 (Winter): 116–29.

Cassady, Ralph, Jr. 1962. *Competition and Price Making in Food Retailing*. New York: Ronald.

Dobson, Joe A.; Tybout, Alice M.; and Sterhal, Brian. 1978. Impact of deal and deal retraction on brand switching. *Journal of Marketing Research* 15 (February 1978): 72–81.

Duffy, Brian. 1977. Getting the most from your trade deals. *Product Marketing* 6 (January): 24–29.

Eppen, Gary D., and Gould, F. J. 1984. *Introductory Management Science*. Englewood Cliffs, N.J.: Prentice-Hall.

Frank, Ronald E., and Massy, William F. 1967. Effects of short-term promotional strategy in selected market segments. In *Promotional Decisions Using Mathematical Models*, ed. Patrick J. Robinson. Boston: Allyn & Bacon.

Hinkle, Charles R. 1965. The strategy of price deals. *Harvard Business Review* 43 (July–August): 75–85.

Kinberg, Yoram, and Rao, Ambar G. 1975. Stochastic models of price promotions. *Management Science* 21 (April): 897–907.

Kinberg, Yoram; Rao, Ambar G.; and Shakun, Melvin F. 1974. A mathematical model for price promotions. *Management Science* 20 (February): 948–59.

Kuehn, Alfred A., and Rohloff, Albert C. 1967. Consumer response to promotions. In *Promotional Decisions Using Mathematical Models*, ed. Patrick J. Robinson. Boston: Allyn & Bacon.

Leed, Theodore W., and German, Gene A. 1973. *Food Merchandising: Principles and Practices*. New York: Chain Store Age.

Lieberman, Yehoshua. 1978. A theory of price deals in supermarkets. Ph.D. dissertation, University of Chicago.

Seligman, E. R. A., and Love, R. A. 1932. *Price Cutting and Price Maintenance*. New York: Harper & Bros.

Telser, Lester G. 1972. *Competition, Collusion, and Game Theory*. Chicago: Aldine.

Tuck, R. T. J., and Harvey, W. G. B. 1972. Do promotions undermine the brand? *ADMAP* (January 1972), pp. 30–33.

U.S. Federal Trade Commission. 1932. *Chain Stores: Chain-Store Leaders and Loss Leaders*. Washington D.C.: Government Printing Office.