Suresh M. Sundaresan
Columbia University

Equilibrium Valuation of Natural Resources*

An equilibrium valuation model of natural resources is presented under two alternative market structures: pure monopoly and pure competition. The nature of the market structure is shown to be crucial in determining whether or not the rate of increase in spot price between successive discoveries is above or below the risk-free return. In the absence of discoveries, the optimal depletion policies and equilibrium spot prices are shown to be the same under both market structures. The equilibrium futures prices on the natural resource are also derived and discussed. The Hotelling valuation principle is shown to hold under uncertainty.

I. Introduction and Overview

The purpose of this paper is to provide a theoretical investigation of equilibrium spot and futures price behavior in a nonrenewable commodity market. The underlying commodity is nonrenewable in the sense that the reserve level of the commodity is expected to decline over time in the absence of successful discoveries of additional reserves. I wish to understand how the nonrenewable nature of the commodity influences the intertemporal extraction of the commodity, equilibrium spot price behavior, and the prices of derivative assets traded on the nonrenewable commodity. There is considerable disagreement among economists whether resources are exhaustible in the sense of resource constraints becoming binding on our society in the foreseeable future. Some economists have argued that natural resources are being depleted at

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too rapid a rate and, unless industrial production is curtailed, severe shortages will result in a century or so. Others have placed their faith in the market mechanism and have argued that as natural resources become scarce their prices will rise, triggering appropriate signals and incentives for the production of substitutes or superior technologies for recovering these resources from hitherto untapped sources. A lucid treatment of this dichotomy appears in Dasgupta and Heal (1979). In this paper both scenarios are accommodated: the one in which resources fluctuate randomly but decline over time with no additions through discoveries or production of substitutes, and the other in which reserves decline in the absence of discoveries but additions to reserves occur randomly via successful discoveries. In this context a successful discovery may be viewed as production of a perfect substitute. Alternative notions of scarcity of natural resources are not considered in this paper.[1]

The market structure of the resource is central to our analysis. Casual observations suggest that some natural resources markets are imperfectly competitive. The market power may take the form of cartels such as the Organization of Arab Petroleum-exporting Countries or national marketing boards. In such cases models of spot and futures markets based on competitive framework may be inappropriate. In this paper I examine two market structures: pure monopoly and pure competition. These are polar extremes. Models of imperfect competition other than monopoly present the usual problem of diverse choice of model specifications and equilibrium concepts and are not attempted in this paper. To the extent that in such models cooperative solutions come about, such outcomes may well be approximated by the pure monopoly case examined in this paper.

It will be useful to restate two basic results in the nonrenewable resources literature before we begin. They will serve as a useful frame of reference for the results of this paper. The first result is due to Hotelling (1931); the second to Weinstein and Zeckhauser (1975) and Stiglitz (1976). The Hotelling hypothesis states that under certainty, in the absence of extraction costs and competitive market conditions the price of a natural resource must rise at the market rate of interest to preclude arbitrage. The economic intuition behind this result is simple: if prices rise at a rate in excess of the risk-free rate, then one can immediately sell the riskless asset and invest the proceeds in the natural resource; otherwise, we can liquidate investments in the natural resource and buy the riskless asset. Thus even in a certain world,

1. Dasgupta and Heal (1979) provide an excellent discussion of "exhaustibility" of natural resources. Our model with discoveries would be classified as a renewable but exhaustible resource: the model with no discoveries would be classified as a nonrenewable exhaustible resource.

under the hypothesized conditions spot price must rise at the rate of interest.[2] To the extent that there are some natural resources markets that are imperfectly competitive, it is of interest to know whether a similar behavior is predicted by economic theory for the spot price behavior in such markets. This is the context of the principal implication of Weinstein and Zeckhauser (1975) and Stiglitz (1976): under certainty, with constant price elasticity of demand and zero cost of extraction, the equilibrium spot price trajectory of a natural resource is identical in both monopolistic and competitive regimes. The economic intuition behind this result is also straightforward: along the competitive path, prices rise at the rate of interest; and along the monopolistic path, marginal revenue rises at the rate of interest. However, with isoelastic demand functions and zero extraction costs, marginal revenue is proportional to price. Hence the price trajectories in both regimes are identical. A lucid exposition of this result appears in Stiglitz (1976).

These two basic propositions serve as a reference point for this paper. I explicitly introduce uncertainty in the environment and study the optimal intertemporal extraction policy under both market structures. The optimal extraction policy is in turn used to determine the evolution of the reserves and the intertemporal equilibrium spot price behavior under both market structures. It is shown that in both cases the equilibrium spot price behavior is irrevocably linked to the market interest rate and the nature of the uncertain environment. Later the relationship between spot and futures prices is obtained using arbitrage arguments. The rest of this section addresses these issues in greater detail and provides a nontechnical overview about the nature of the analysis and results contained in this paper.

The paper is organized into five sections. First, I study a model of extraction and exploration of a nonrenewable resource. For illustrative purposes the framework is laid out in a monopolistic setting, although the procedure for a competitive setting is similar. The issue that is investigated is the simultaneous determination of optimal extraction and exploration policy. Two sources of uncertainty are modeled. The first source of uncertainty is not within the control of the producer. This is modeled through a smooth diffusion process. The other source of uncertainty is the actual amount of reserve that may become available in the future from exploratory activities. The probability of finding new reservoirs is assumed to be an increasing function of the effort spent in exploration. A success in exploration causes the existing reserves to jump by a possibly random amount. This source of uncer-

2. This result is not necessarily restricted to exhaustible resources. In fact, similar arbitrage relationships must hold for durables in competitive market settings. See Dasgupta and Heal (1979).

tainty is modeled through a Poisson process with a random jump.[3] The trade-offs faced by the producer in the choice of extraction policy and exploration policy are analyzed. In Section II, I analyze both monopolistic and competitive market structures under the assumption of value maximization, zero extraction costs, and isoelastic demand schedules. Explicit solutions to optimal extraction policy and equilibrium spot prices are obtained under both regimes in the presence of uncertain exogenous discoveries. It is shown that the extraction policy of the monopolist is conservative relative to that of the competitive producer. Along the optimal extraction path the evolutions of the reserve levels and the spot prices are derived under both regimes. It is shown that the rate of increase in the spot price is greater than the risk-free return in periods between discoveries in the competitive market regime. This confirms the results obtained by Dasgupta and Stiglitz (1981). The notion of competitiveness that leads to this result is also addressed. In the monopolistic market regime the rate of spot price increase in periods between discoveries is shown to be less than the risk-free rate of return.

In Section III the optimal extraction policy under uncertainty is investigated when there is no sudden increase in the supply due to discoveries. The central result is that when the demand schedules have a constant price elasticity, the optimal rate of extraction followed by a risk-averse monopolist and the rate of extraction followed by a number of price-taking risk-averse competitive producers are one and the same in the absence of extraction costs. This is a generalization of a result found by Weinstein and Zeckhauser (1975) and Stiglitz (1976). I also show that along the optimal extraction path both the reserve levels and spot prices have a lognormal distribution.

In Sections II and III, I derive the Hotelling valuation principle (see Miller and Upton 1983) under uncertainty. I show that, even in the presence of risk aversion, the utility value attached to the reserve is fully summarized by a function which depends only on the current spot price multiplied by the current reserves.

In Section IV, I discuss the equilibrium pricing of claims on nonrenewable resources for future delivery. The optimal extraction policy derived in Section III is used to determine the equilibrium value of

3. The choice of a mixed diffusion-jump process to model the reserve-level fluctuations deserve some motivation. In such a process random arrival of a Poisson event with a random amplitude can capture effectively the discovery pattern. Similar processes have been used by Pindyck (1980) and Arrow and Chang (1982) to study related problems. The mixed process can be thought of as capturing two sources of randomness: smooth variations over time due to uncontrollable variables—this being modeled by the Wiener process; and sudden changes in reserves due to discoveries—this captured by the Poisson events.

forward contracts and futures prices.[4] In conclusion I point out some empirical implications of the model and the possible directions for future research.

II. A Model of Extraction and Exploration of a Natural Resource

I shall begin my analysis by constructing a model of an economy endowed with a resource whose level fluctuates in an uncertain manner through time. At time 0 the economy is endowed with $S(0)$ units of a nonrenewable resource, which is expected to decline in the absence of successful discoveries. The change in the resource level at time t is modeled through a mixed diffusion and jump process as described below:[5]

$$dS(t) = -R(S, t)dt + \sigma S dz(t) + g(S, t)dq(t). \tag{1}$$

In equation (1), $R(S, t)$ is the endogenous rate of extraction pursued by the producer(s). The stochastic process specified allows for both continuous (smooth) variations in the reserves and discontinuous random jumps in the reserves; in (1) σ is a scalar and $g(S, t)$ is a function of the reserves, $z(t)$ is a standard Wiener process, and $q(t)$ is a Poisson process assumed to be independent of $z(t)$. The following assumptions are made about the process $q(t)$: $\lambda(x)\Delta t + o(\Delta t)$ is the probability that $q(t)$ jumps once in the interval $(t, t + \Delta t)$, and the probability that $q(t)$ jumps more than once in the interval $(t, t + \Delta t)$ is $o(\Delta t)$. Thus the probability that $q(t)$ is constant in the interval $(t, t + \Delta t)$ is $1 - \lambda(x)\Delta t$. The variable x represents the exploration effort devoted by the producer(s). The magnitude of the jump $\tilde{\theta}$ is random with a density function $\phi(\theta)$.

It is possible to give an intuitive discussion of the stochastic process described above. The first source of uncertainty modeled by the

4. Since the important paper (Black 1976) in which Fischer Black clarified the structure of futures prices, forward prices, and commodity options, there have been a number of contributions in the area of futures research dealing with the equilibrium pricing of futures. Papers by Cox, Ingersoll, and Ross (1981), French (1981), and Richard and Sundaresan (1981) essentially address the issue of pricing futures contracts. Section IV develops a model for identifying the structure of futures prices when the underlying commodity is exhaustible, a feature not explicitly considered in the papers cited so far. The approach is consistent with the arguments of Black and Scholes (1973) as well.

5. This type of stochastic process has been used by Merton (1976) in an option pricing context. Deshmukh and Pliska (1980) have studied optimal consumption and exploration policies using a similar jump process. Their focus is on showing the existence of optimal policies and prices under mild assumptions. By assumption the interarrival time is exponentially distributed. If the interarrival time between two discoveries has an arbitrary distribution function, the state space will be further expanded. While this modification is likely to have strong empirical implications for the pattern of cash and future prices, analytically it becomes virtually intractable. I thank a referee who pointed out this to me.

Wiener term $z(t)$ is not within the control of the producer: this source of fluctuation may be best attributed to factors such as geological changes, errors in estimates, unanticipated loss of reserves, and the like. This specification implies that such fluctuations are greater, ceteris paribus, when reserves are higher. The other source of uncertainty in the economy refers to the actual amount of reserves that may become available in the future. This is modeled through a controlled Poisson process $q(t)$. The producer can invest resources in exploration x to increase the probability of success in locating additional reserves. The probability $\lambda(x)dt$ of finding new reserves is an increasing function of the discovery effort x. If the producer succeeds in locating a new reservoir, he suddenly increases the available reserves under his control. There may be technological limitations on the rate of exploration effort per unit of time. I assume that the physical limit on the rate of exploration is $M > 0$, so that $x \in [0, M]$. Furthermore, I assume that the probability of success in a unit interval of time $\lambda(x)$ depends only on the discovery effort. There is no provision in this model for the producer to learn about the available reserves using exploration activity.

I now turn to the demand side of the economy. Assume that the inverse demand curve can be summarized as $p = D[R(t)]$ where $R(t)$ is the rate of extraction at time t. The dependence of demand only on the rate of extraction is a nontrivial assumption. If the underlying resource is also durable, for instance, then the demand is likely to depend, among other things, on the inventories of available resource (from previous extractions) as well. Assume further that the resource, once extracted, must be consumed and cannot be stored. For a commodity such as gold the spot price will depend (in addition to the factors discussed so far) on the market anticipations of government auctions, as pointed out by Salant and Henderson (1978). My choice of the demand function focuses attention on the producer's ability to influence the spot prices directly by setting the rate of extraction at each instant. The demand function is assumed to obey $p'(R) < 0$ and $p''(R) > 0$. (Primes denote derivatives.)[6]

With respect to costs, two assumptions are made: the cost of extracting the resource is assumed to be zero, and the cost of exploration is $c(x)$. The cost function is assumed to satisfy $c'(x) > 0$ and $c''(x) > 0$. The role of zero extraction costs is discussed later in the paper.

The market structure is assumed to be monopolistic. In Section III, I consider both monopolistic and competitive market structures. The

6. I assume suitable regularity conditions to be satisfied by $p(\cdot)$. It should not be possible for the producer to cut the extraction rate to arbitrarily low levels and maximize profits. Usually the specification of a finite "choke price" $p(0) = \bar{p}$ is sufficient to preclude this.

objective of the monopolist is to maximize the expected discounted lifetime utility of profits.[7] Formally the monopolist's problem can be stated as

$$\max_{\{R,x\}} E\left\{\int_0^{\infty} e^{-\rho t} u[\pi(t)]dt\right\}, \tag{2}$$

where $u(\cdot)$ is an instantaneous concave utility function, ρ is a discount factor, and $E[\cdot]$ is an expectation operator. The rate of profit at t is denoted by

$$\pi(t) = pR - c(x). \tag{3}$$

At this stage it is useful to define the function

$$J(S) \equiv \max E_t\left\{\int_t^{\infty} e^{-\rho(v-t)} u[\pi(v)]dv\right\}. \tag{4}$$

The Bellman equation corresponding to the dynamic optimization problem facing the monopolist can now be written as[8]

$$\begin{aligned} 0 = \max_{\{R,x\epsilon(0,M)\}} \{&u(\pi) - \rho J - J_S R + \tfrac{1}{2} J_{SS}\sigma^2 S^2 + E_t\lambda(x) \\ &[J(S + g\tilde{\theta}) - J(S)]\}, \end{aligned} \tag{5}$$

where $E_t(\cdot)$ is an expectation operator with respect to $\phi(\theta)$ conditional on the state of the economy at t given by the reserve level $S(t)$ of the nonrenewable resource. The relevant first-order conditions of optimality are (for R and x, respectively):[9]

$$\begin{aligned} \psi_R &\equiv u'(\pi)[p + p'R] - J_S \leqq 0 \\ \psi_R R &= 0. \end{aligned} \tag{6a}$$

Let $\psi_x \equiv -u'(\pi)c'(x) + E_t\lambda'(x)[J(S + g\tilde{\theta}) - J(S)]$. Then

$$\text{if } \psi_x < 0 \text{ optimal } x = 0; \tag{7a}$$

$$\text{if } \psi_x > 0 \text{ optimal } x = M; \text{ and} \tag{7b}$$

$$\text{if } \psi_x = 0 \text{ optimal } x \in (0, M). \tag{7c}$$

7. The specification of the objective function deserves some explanation. Expected utility maximization is reasonably well accepted in the exhaustible resources literature. This objective function accommodates value maximization as a special case: linear utility. In this case the objective function becomes lifetime discounted expected profits maximization. We can select the discount rate to be the risk-free rate. Constantinides (1978) has shown that this is consistent with value maximization even when investors are risk averse so long as the drift term is suitably adjusted.

8. See Merton (1971) for a detailed derivation of the Bellman equation.

9. The controls must satisfy the restrictions that $R \geqq 0$ and $M \geqq x \geqq 0$.

In this section the existence and uniqueness of $J(\cdot)$ will be assumed. In later sections the existence of $J(\cdot)$ will be shown for specific assumptions on $u(\cdot)$.

Characterization of Optimal Extraction and Exploration Policies

If an interior solution were to exist for the extraction policy, then

$$u'(\pi)[p + p'R] = J_S. \tag{8}$$

Condition (8) simply says that the optimal rate of extraction will be such that the marginal-utility-weighted marginal revenue is equal to the shadow price of the scarce resource. From (8) I deduce that if $u(\cdot)$ is linear, the monopolist picks his extraction policy by setting the marginal revenue proportional to the shadow price. Under the same conditions a competitive firm will select the rate of extraction by setting the price equal to the shadow price. Equation (8) indicates that the optimal rate of extraction depends on (i) the risk aversion of the monopolist, (ii) price elasticity of demand for different levels of extraction, (iii) cost of discovery efforts, and (iv) the reserve levels. If $u(\cdot)$ is linear, then the discovery expenditures do not directly affect the optimal rate of extraction. For an interior solution of x it must be true that

$$u'(\pi)c'(x^*) = \lambda'(x^*)E_t[J(S + g\tilde{\theta}) - J(S)], \tag{9}$$

where x^* is the optimal policy. Equation (9) is intuitively reasonable. The discovery effort chosen optimally requires the marginal-utility-weighted marginal discovery expenditure to be set exactly equal to the expected increase in the lifetime utility due to a successful discovery weighted by the marginal increase in the probability of success due to one unit of extra discovery effort.

If $u'(\pi)c'(x) > \lambda'(x)E_t[J(S + g\tilde{\theta}) - J(S)]$, then there is no incentive for any discovery expenditure and the value of $x^* = 0$. On the other hand, if $u'(\pi)c'(x) < \lambda'(x)E_t[J(S + g\tilde{\theta}) - J(S)]$, then there is incentive for additional exploration. Indeed, given the technological constraints, the optimal discovery effort will be $x^* = M$. For the optimal discovery effort to be in the interval $(0, M)$, equation (7c) must hold. These are fairly intuitive conclusions. The problem facing the competitive market structure can also be posed in a similar manner. I do not present the details here. Rather, I proceed to explore some special cases under both monopolistic and competitive market structures.

In what follows in this section I assume that producers maximize the discounted value of expected profits. The discount rate is assumed to be the risk-free rate. This, however, is not as restrictive an assumption as it may seem to be at first glance. This objective function is equivalent to value maximization if producers are risk neutral. Indeed, one need not make this restrictive interpretation. As Constantinides (1978) has pointed out, risky cash flows can be evaluated by discounting the expected cash flow at a risk-free rate, once the drift term of the stochastic process is adjusted suitably. Thus the rest of the analysis is consistent with value maximization under risky situations. In order to sharpen my intuition, I assume:

A. Discoveries are purely exogenous and random. Furthermore, I assume that $\sigma^2 \equiv 0$, $g(S, t) \equiv S$, and θ is nonrandom.

I shall now state and prove a result characterizing the optimal extraction policy and equilibrium spot prices in a monopolistic regime where the monopolist maximizes the value.

LEMMA 1: Under the assumptions stated in A and an isoelastic demand schedule $P = R^b$ where $b \in (-1, 0)$, the optimal extraction policy of the monopolist is

$$R(S) = \left\{ \frac{\mathrm{r} - \lambda[(1 + \theta)^{1+b} - 1]}{-b} \right\} S \equiv \Delta_m S,$$

and the equilibrium spot price is given by

$$P(S) = \left[\frac{r - \lambda\{(1 + \theta)^{1+b} - 1\}}{-b} \right]^b S^b \equiv \Delta_m^b S^b,$$

where r denotes the risk-free rate. The proof of this lemma is in Appendix A. Q.E.D.

The monopolist's extraction policy function is increasing in the risk-free rate. If θ is positive, then the extraction policy function is a decreasing function in the mean arrival rate of discoveries. This result is somewhat counterintuitive. As more discoveries are made, the stock under the control of the monopolist increases. Instead of releasing the stock into the spot market and driving the spot price down, the monopolist restricts his extraction rate and sells it at a higher price than competitive considerations would dictate. Furthermore, the monopolist gets all the additional stocks resulting from discoveries. This amount is increasing and directly proportional to the reserve level S so long as $\theta > 0$. Hence the monopolist has the incentive to cut his extraction rate so as to maximize the benefits from discoveries. Later, in a competitive setting, I will show that this incentive disappears.[10] Substituting the optimal extraction policy into the reserve level dynamics, yields

$$ds(t) = -\Delta_m S dt + \begin{cases} \xrightarrow{\lambda dt} S\theta \\ \xrightarrow[1 - \lambda dt]{} 0. \end{cases}$$

In periods between discoveries the reserve levels decline at a rate Δ_m. This rate is less than the certainty rate $(-r/b)$. "Arrival" of additional

10. I thank Douglas Breeden for suggesting this interpretation. Additional restrictions are needed to assure that $\Delta_m > 0$.

reserves causes the reserves to jump back up to a higher level. A sample path of the reserve level shown in figure 1 illustrates the spot price behavior predicted on my theory in a monopolistic regime. The reserves deplete from their initial level of $S(0)$ at a rate Δ_m until T_1, when additions via discovery occur, causing the reserves to rise. This pattern is repeated at random time intervals. Consequently the equilibrium spot price evolves as shown in figure 2. In periods between discoveries the spot price rises at a rate less than the risk-free rate of return.

I now turn to a competitive market structure. The competitive problem can also be solved using stochastic optimal control method. But in order to highlight the economic intuition, I follow a different approach. The next lemma and theorem characterize the problem under a competitive market structure.

LEMMA 2: Under the assumption stated in A, the optimal extraction policy $R(S, t)$ satisfies the relationship given by

$$R(S, t) = \frac{rP(S) + \lambda\{P(S) - P[S(1 + \theta)]\}}{-P_S}. \tag{10}$$

In (10) r is the equilibrium risk-free rate of return. The proof of this lemma follows from the requirement that at equilibrium the expected return from holding a unit of the resource must be equal to the risk-free rate of return. By generalized Itô's lemma,

$$E\left(\frac{dP}{P}\right) = \left(-\frac{PSR}{P} + \lambda\left\{\frac{P[S(1 + \theta)]}{P} - 1\right\}\right)dt \overset{e}{=} rdt, \tag{11}$$

where $\overset{e}{=}$ denotes the equilibrium requirement. Rearranging (11) yields (10). Q.E.D.

Lemma 2 suggests a natural way to solve for the optimal aggregate extraction policy and equilibrium spot price. Note that, once the demand schedule faced by the producers is posited, the problem is fully specified. Let the price elasticity of demand be a constant given by $|\eta(R)| = 1/|b|$ and let $|\eta(R)| \in (1, \infty)$. This implies a spot price specification $P(R) = R^b$ where $-1 < b < 0$. With these assumptions it is possible to solve endogenously for the extraction policy and the equilibrium spot price.

THEOREM 1: When the demand schedule facing the competitive producers is isoelastic, the optimal aggregate extraction policy is

$$R(S, t) = \left\{\frac{-r}{b} + \frac{\lambda}{-b}[1 - (1 + \theta)^b]\right\} S \tag{12}$$

and the equilibrium spot price is

$$P(S) = \left\{\frac{-r}{b} - \frac{\lambda}{b}[1 - (1 + \theta)^b]\right\}^b S^b. \tag{13}$$

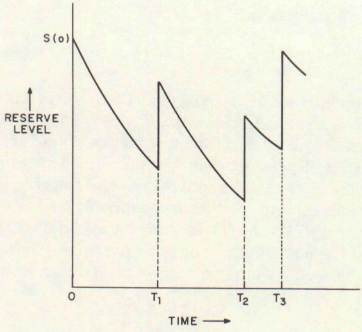

FIG. 1

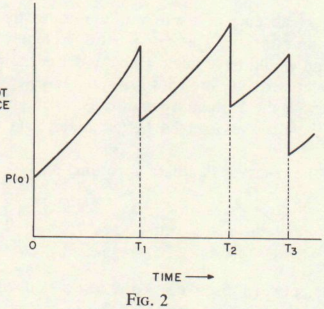

FIG. 2

The proof of this theorem is in Appendix B. Q.E.D.

Before proceeding to analyze the results reported in this theorem, I shall discuss the nature of the competitive environment assumed. The notion of competitiveness used here is that the arrival of additional stocks affects adversely the spot price at which the competitors can sell in the common spot market. No benefits are conferred to the existing competitive producers as a consequence of increased stocks. In this sense the environment is markedly different from the previous setting in which all the increased stock came under the control of the monopolist.[11]

11. Several points deserve to be made at this juncture. The implication that competitors suffer a price decline without any benefit of increased stocks under their control is

For notational convenience, define

$$\Delta_c \equiv \left\{ \frac{-r}{b} - \frac{\lambda}{b} [1 - (1 + \theta)^b] \right\} > 0. \tag{13a}$$

Note that the optimal extraction policy is a linear increasing function of the reserves. It is also an increasing function of the market interest rate r and the mean arrival rate λ of the discoveries. These are intuitively reasonable results. The equilibrium spot price is a decreasing function of the level of the reserves. The spot price is also a decreasing function of the market interest rate r and the mean arrival rate λ of discoveries. Further, one can use (12) and (13) to study the evolution of the reserves over time and the equilibrium spot price over time. Substituting the optimal extraction policy into the reserve level dynamics yields

$$dS(t) = -\Delta_c S dt + S dq(t). \tag{14}$$

Similarly, one can obtain the expected spot price:

$$E(dP) = -\Delta_c bP dt + \lambda\{P[S(1 + \theta)] - P(S)\}dt. \tag{15}$$

Notice that in the absence of discoveries the stocks are depleting at the rate of Δ_c per unit of time. This rate is strictly greater than the rate at which depletion would have taken place under certainty (with no discoveries). Under certainty the spot price increases at the rate of $dP/P = r dt$, confirming the Hotelling hypothesis.[12] Under uncertainty the evolution of the reserves can be better understood with the aid of figure 1.

Recall that the reserves fluctuate according to

$$ds(t) = -\Delta_c S dt + \begin{cases} \xrightarrow{\lambda dt} S\theta \\ \xrightarrow[1 - \lambda dt]{} 0. \end{cases} \tag{16}$$

In the periods between discoveries, the reserves decline at a rate exceeding the certainty rate $(-r/b)$. Upon the "arrival" of additional reserves, the reserves level rises to a higher level. A sample path of the reserve level shown in figure 1 illustrates the pattern of the evolution of reserves. A similar behavior is exhibited by the spot prices. Note that the spot prices evolve according to (15). Between periods of discov-

implicit in the arbitrage arguments which appear in Dasgupta and Stiglitz (1981). This may be justified by imagining the "new stocks" are held by new competitive firms which enter the spot market thereby increasing the overall reserve level and bringing down the spot price. In (12) and (13) I assume the needed restrictions to ensure $\Delta_c > 0$.

12. One of the very early investigations of exhaustible resources was a paper by Hotelling (1931). In this work he showed that in an efficient exhaustible resources market the spot price of the resource should increase over time at the rate of interest. This is true under monopolistic regimes as well as under certainty, with constant price elasticity of demand.

eries the spot price rises at a rate exceeding the risk-free rate. The actual rate at which the spot price increases is given by $-b\Delta_c dt$. Substituting for Δ_c, this rate is found to be $r + \lambda[1 - (1 + \theta)^b]$. Even though the producer is risk neutral, he sets the extraction rate such that the prices rise at a rate in excess of the sure rate of interest. The economic intuition for this has been recognized in the literature:[13] with uncertain arrival of additions to reserves, the producer faces the possibility of capital losses on the stocks held currently. This induces him to demand a return in excess of the risk-free return in periods between the discovery dates. The excess return demanded depends on the demand elasticity parameter b, the mean arrival rate λ, and the fractional addition θ to the reserves. In figure 2, a sample path of spot prices is provided.

Compare the extraction policy and the spot prices under the monopolistic market structure with those under the competitive market structure: clearly the monopolist extracts at each instant a lower rate than the competitive producers. Consequently the spot prices under the monopolistic structure are generally higher than the corresponding prices in the competitive regime. As the monopolist is more conservative, the average reserve level is higher than the reserves under the competitive regime.

Although the pattern of evolution of spot price is similar to the one shown in figure 2 under both regimes, in periods between discoveries the rate of increase in spot price differs sharply across the two regimes. The Hotelling result (namely, price rising at the rate of interest) serves as the dividing line: in the monopolistic regime prices rise at a rate less than the risk-free rate and in the competitive regime prices rise at a rate in excess of the risk-free rate.

In this sense, the model predicts that the market structure of the exhaustible resource places empirically refutable restrictions on the intertemporal price path. By observing the spot price behavior between successive discoveries, it is possible to draw some inferences concerning market structure of the resource.

My model in this section also has some implications for the valuation of natural resources under uncertainty. Miller and Upton (1983) point out that, under certainty, the market value of the reserve is simply the current spot value. They call this the Hotelling valuation principle. The value function of the monopolist is $J = \beta^b S^{1+b} \equiv PS$. Thus the value to the monopolistic producer is also the current spot value of the resource.

13. This result is a more general one than our own derivation might suggest. See Dasgupta and Stiglitz (1981), for instance. However, this result must be viewed within the context of the comments made in 11 above.

III. Optimal Extraction Policy and Equilibrium Spot Price under Alternative Market Structures

In the previous section both monopolistic and competitive market structures were analyzed with uncertain discoveries. I shall now examine the extraction problem separately under a monopolistic market structure as well as a competitive market structure with no discoveries. Under certainty with isoelastic demand function and zero extraction costs, it has been shown that the optimal extraction policy pursued by a pure monopolist is identical to the intertemporal competitive equilibrium so long as the competitive industry also faces the same isoelastic demand. The economic intuition behind this result was pointed out earlier.

In this section I demonstrate that even in the presence of uncertainty, the optimal extraction rate of the pure monopolist and the (aggregate) optimal extraction rate of the competitive industry are identical. In fact, this result turns out to be robust with respect to the constant relative risk aversion class of preference. Furthermore, I explicitly derive the optimal extraction policy and the equilibrium spot prices under both market structures. These results provide the foundation for pricing claims on nonrenewable resources later in this paper. Throughout the rest of this section I specialize to isoelastic demand functions and to uncertain reserve levels with no exploration so that the nonrenewable nature of the resource is captured in the analysis.

A. Monopolistic Market Structure

The monopolist owns the reserves which fluctuate over time according to the stochastic differential equation

$$dS(t) = -R(S, t)dt + \sigma S dz. \tag{17}$$

Equation (17) is a special case of (1) studied in Section II. The next theorem characterizes the optimal extraction policy of the monopolist and the equilibrium spot price.

THEOREM 2: Assume that

a) The monopolist is risk averse and his preferences can be represented by $u(\pi)$ which satisfies $u' > 0$; $u'' < 0$, and

$$\frac{-u''(\pi)\pi}{u'(\pi)} = (1 - \gamma); \tag{18}$$

and

b) The monopolist faces a demand function which is isoelastic and can be represented by

$$P(R) = R^b \tag{19}$$

where $-1 < b < 0$. Then, the optimal extraction policy is

$$R = \beta_1 S \tag{20}$$

where

$$\beta_1 \equiv \frac{\rho + \frac{1}{2}\sigma^2[1 - (1 + b)\gamma]\,(1 + b)\gamma}{1 - \gamma(1 + b)}. \tag{21}$$

The optimal indirect value function is given by

$$J(S) = \frac{\beta_1^{(1+b)\gamma - 1}}{\gamma} S^{(1+b)\gamma} \equiv \frac{\beta_1^{\gamma-1}}{\gamma}(PS)^{\gamma}. \tag{22}$$

The proof is by substitution of (18)–(22) into (5), (6a), and (6b). Under the assumptions of theorem 2, it may be verified that they are satisfied. Q.E.D.

We find that the optimal extraction policy pursued by the monopolist is linear in the reserves. The actual amount extracted depends on the rate of impatience of the monopolist, the degree of relative risk aversion, the level of uncertainty about the fluctuations in the reserves, and, of course, the price elasticity of demand for the resource. The rate of extraction is a decreasing (increasing) function of σ^2 if $\gamma < 0$ (> 0). Regardless of the value of γ, the rate of extraction is an increasing function of the impatience factor of the monopolist. These are intuitively plausible conclusions. The key feature of the extraction policy which I shall exploit in later sections is that it is linear in the reserves. The equilibrium spot price of the resource at any instant is found by substituting (20) into the spot price function (19) to get

$$P(S) \equiv \beta_1^b S^b \equiv aS^b,\ a > 0. \tag{23}$$

Equation (23) describes precisely how the spot price of the resource inherits the preference attributes of the monopolist, the parameters of the technology, and the amount of reserves at each point in time. From (22), note that the utility value of the reserves to a monopolist following the optimal policy is not PS but a nonlinear function of PS. The nonlinearity depends on the risk aversion parameter γ. As $\gamma \to 1$, the Hotelling valuation becomes the limiting utility value J. Thus the monopolist values his reserve according to the Hotelling valuation principle. I shall now turn to a competitive market structure.

B. Competitive Market Structure

Consider a number, N, of atomistic identical firms owning identical reserves. This setting is to be contrasted with a setting where firms extract from a common pool. In this paper I do not address this "common access" problem.[14] It is assumed that the demand schedule is given by (19). Taking this schedule as given and acting as a price taker, each firm must decide on the optimal extraction policy for the resource. Each firm maximizes its expected discounted lifetime utility of profits obtained by selling the extracted resource in the spot market. Each firm is endowed with an identical field from which the reserves fluctuate as given by

$$dS_i(t) = -R_i dt + \sigma S_i dz, \ i = 1, 2, \ldots, N, \tag{24}$$

where $S_i(t)$ is the reserve level for the ith firm and R_i is the rate of extraction by the ith firm. $S = \Sigma S_i$ is the total reserve level in the economy. Note that fluctuations in reserves in each field are perfectly correlated. The aggregate rate of extraction is denoted by R. The aggregate reserve level in the economy fluctuates as per $dS = -Rdt + \sigma Sdz$. The profits of the ith firm are given by $\pi_i = PR_i$. Each firm has the following objective function

$$\max_{\{R_i\}} E_0 \int^{\infty} e^{-\rho t} u(\pi) dt$$

subject to the dynamics (24). The Bellman equation of optimality for this problem is

$$\begin{aligned} 0 = \max_{\{R_i\}} \ & u(\pi_i) - \rho J + J_{S_i}(-R_i) + J_{S_i S_i}\sigma^2(S_1^2) - J_S R \\ & + \tfrac{1}{2} J_{SS}\sigma^2 S^2 + J_{S_i S}\sigma^2 S(S_i). \end{aligned} \tag{25}$$

It is to be noted that each competitive firm takes price as given. Hence the total resources S are taken as given. This accounts for (S, S_i) being the state variables. $J(S_i, S)$ is the indirect utility function which represents the expected utility derived by the ith firm when it proceeds optimally into the future and when the state of the economy is characterized by (S). Assuming interior solutions, the necessary and sufficient first-order condition is (with respect to R_i)

$$u'(\pi_i)P = J_{S_i}. \tag{26}$$

Each firm extracts up to a point when its marginal utility of profits in dollar value is equal to the marginal utility in the reserve level. It will be of interest to ask whether the aggregate rate of extraction under a competitive market structure differs from the rate of extraction found

14. The common access problem raises thorny and important questions about the nature of property rights as well as legislation and implementation of laws governing property rights. Fishing rights and bidding for oil fields are examples of legislation which attempts to internalize the externality associated with the common property resource. Loury (1978) derives consumption policies in a competitive regime under uncertainty. But he does not address the issue of alternative market structures. He focuses on how the possibility of exhaustion affects optimal consumption behavior and (to a lesser extent) on learning about the distribution of resources over time.

under a monopolistic market structure. The next theorem addresses this issue.

THEOREM 3: Assume that each competitive producer is risk averse and his preference can be represented by $u(\pi) > 0$; $u'' < 0$, and

$$\frac{-u''(\pi)\pi}{u'(\pi)} = (1 - \gamma).$$

Furthermore, let the demand schedule be given by (19). Then (*a*) aggregate extraction rate under the competitive market structure is identical to the extraction rate followed by the monopolist. (*b*) Equilibrium spot prices are identical in both market structures. (*c*) The extraction policy for firm i is

$$R_i = \beta_1 S_i \tag{27}$$

where

$$\beta_1 \equiv \frac{\rho + \frac{1}{2}\sigma^2\{1 - (1 + b)\gamma\}(1 + b)\gamma}{1 - \gamma(1 + b)}. \tag{28a}$$

The indirect value function is

$$J(S_i, S) = \alpha P^{\gamma} S_i^{\gamma} \equiv \alpha(PS_i)^{\gamma} \tag{28b}$$

where

$$\alpha \equiv \frac{1}{\gamma}\left[\frac{1 - \gamma(1 + b)}{\rho + \frac{1}{2}\sigma^2\{1 - (1 + b)\gamma\}(1 + b)\gamma}\right]^{1-\gamma}. \tag{28c}$$

The proof is once again by substitution. Substitute (27)–(28c) into (5), (6a), and (6b). Under the assumption of theorem 3, they are satisfied. Q.E.D.

Theorem 3 generalizes the findings of Weinstein and Zeckhauser (1975), Stiglitz (1976), and Dasgupta and Heal (1979) to an uncertain environment where the participants are risk averse. Our analysis confirms that the power of the monopolist is considerably weakened in a nonrenewable resources market. The monopolist, just like the competitive firms, will eventually expect to exhaust the natural resource. Theorem 3 suggests that his ability to rearrange the patterns of extraction to maximize the present value of discounted expected utility profits is severely limited. The setting used in this section as well as in the previous section required the producers to set their extraction rates at each instant in time. This was achieved through the optimal extraction policy function which was fixed but state contingent. I precluded credible precommitments.[15] Thus producers pursued decision rule

15. I am grateful to Douglas Diamond for pointing this out to me. The strategy space used in this paper admits only decision rule strategies. See Reinganum and Stokey (1981) for an elaboration of this point.

strategies. Relaxing this assumption and allowing the producers to make commitments might well alter the conclusions. At least in the context of common property resource extraction, Reinganum and Stokey (1981) have shown that the period of commitment in dynamic setting can make a nontrivial difference to the outcome in imperfectly competitive markets.

The utility value of the reserves to a risk-averse, price-taking producer is given by the indirect value function J in (28b). As $\gamma \rightarrow 1$, the limiting value of J is the Hotelling value PS_i. In general the value is a nonlinear function of the current spot value PS_i. Equations (22) and (28b) indicate that the monopolist and the competitive producer(s) will use the same value assessments. In computing their utility value, both the monopolist and the competitive producer(s) use the Hotelling valuation principle.

I conclude this section by examining the distributional properties of the reserves and the spot price at equilibrium. Substituting the optimal extraction policy in (17) yields the following reserve-level dynamics:

$$\frac{dS}{S} = -\beta_1 dt + \sigma dz.$$

The dynamics given above are those of a geometric Brownian motion. The state space is the interval $(0, \infty)$. Consider the economy which starts with a reserve level of S_0. Then the expected reserves at time t are given by

$$E[S(t) \mid S_0] = S_o \exp\{(-\beta_1 t)\}.$$

The variance of reserve levels can be expressed as

$$\mathrm{var}[S(t) \mid S_0] = S_0^2 \exp[-2\beta_1 t](\exp[t\sigma^2] - 1).$$

From the equations above it can be seen that the reserves are expected to decline over time (depending on the rate of extraction through β_1), but there is also variability in the reserve level through time.

In a similar manner it is possible to write down the dynamics of spot price on the nonrenewable resource as

$$\frac{dP}{P} = \left[-b\beta_1 + \frac{b\,(b-1)\,\sigma^2}{2}\right]dt + b\sigma dz.$$

The spot price follows a geometric Brownian motion whose state space is again the interval $(0, \infty)$. The spot price has a positive drift with stochastic variations around it. Specifically,

$$E[P(t) \mid P_0] = P_0 \exp\left\{-b\beta_1 + \frac{b\,(b-1)\,\sigma^2}{2}\right\}t.$$

The positive drift in the spot price dynamics is an increasing function of β_1 and σ^2.

If I assume that the participants are risk neutral and substitute β_1 under this assumption from (21) in the equation above, I get

$$E[P(t) \mid P_0] = P_0 \exp\{ r - b\sigma^2 \} t.$$

Thus the spot price is expected to rise at a rate in excess of the risk-free return. The excess amount depends on the price elasticity of demand and σ^2. The variance of the spot price is given by

$$V[P(t) \mid P_0] = P_0^2 e^{2\,(r-b\sigma^2)t}[e^{b^2\sigma^2 t} - 1\,].$$

IV. Arbitrage Relationships in Spot and Future Markets

In this section I derive and discuss the properties of futures prices on the nonrenewable resource. The optimal extraction policy and the equilibrium spot prices derived previously provide the basis for the analysis contained here. First I derive the futures prices with stochastic exogenous discoveries. Subsequently, I derive futures prices under pure extraction with no discoveries.

Consider a forward contract on the nonrenewable resource which matures at T. The forward contract value (after the date it was written) is denoted by F. I retain the assumptions made in Section IA. The forward contracts are assumed to be price such that $E\,(dF/F\,) \triangleq rdt$. This condition is not as restrictive as it may seem at first glance. The specification used assumes that the local expectations hypothesis (LEH) prevails in the forward market. As Cox et al. (1981) have shown, this hypothesis is fully consistent with equilibrium models with no arbitrage. Furthermore, the model can be easily extended to accommodate a factor risk premium that is proportional to the level of the short-term riskless rate of interest. The assumption of LEH permits me to illustrate the issues involved without modeling preferences explicitly, and it involves no significant loss of generality. The expected change in the forward contract value (by generalized Itô's lemma) is

$$E\,(dF) = (-F_\tau - F_S\Delta_c S + \lambda\,\{\,F[S(1 + \theta)] - F(S)\})dt.$$

Combining these two equations and simplifying yields

$$-F_\tau - F_S\Delta_c S + \lambda\{F[S(1 + \theta)] - F(S)\} - rF = 0. \tag{29}$$

I denote $\tau \equiv T - s$ to represent the time to maturity of the forward contract for any time $s \in (t,\, T)$. At time t of writing the forward contract, the forward price $\phi(t,\, T)$ will be so chosen as to make the value of the forward contract equal to zero. At maturity the value of the forward contract will be

$$F(S,\, 0) = P[S(T) \mid\; - \phi(t,\, T). \tag{30}$$

PROPOSITION 1: The equilibrium value of a forward contract is given by

$$F(S, \tau) = P(S) - \phi(t, T) \exp\{-r\tau\}, \tag{31}$$

where $P(S)$ is given by equation (13) and $\tau \equiv T - t$.

The proof of this theorem is by actual substitution of (31) into (29). The boundary condition (30) is satisfied as well. Q.E.D.

COROLLARY: The futures price on the nonrenewable resource is given by[16]

$$\phi(t, T) = P(S)\, e^{r\tau}. \tag{32}$$

The futures price is obtained by setting (31) to zero and solving for $\phi(t, T)$. In other words, the futures price is found by setting the value of forward contract equal to zero. Note that this result comes about from standard arbitrage arguments and does not depend on the local expectation hypothesis. Q.E.D.

Clearly the futures prices and forward contracts inherit the behavior of spot prices. Since I have discussed the spot price behavior in detail already, the properties of the futures prices can be readily inferred. In particular, futures prices also follow a pattern similar to the one in figure 2.

I now turn to pricing futures contracts when there is no discovery. The dynamics of the reserve levels are assumed to be given by (17). Note that the optimal extraction policy may be represented as $R = \beta_1 S$ and the equilibrium spot price as $P(S) = aS^b$. Let us once again consider a forward contract denoted by $F(S, \tau)$. The dynamics of F can be found by applying Itô's lemma:

$$\frac{dF}{F} = \left(- \frac{F_\tau}{F} - \frac{F_S R}{F} + \tfrac{1}{2} \frac{F_{SS}\sigma^2 S^2}{F} \right) dt + \frac{F_S \sigma S}{F}\, dz. \tag{33}$$

Naturally when the contract matures the value of the forward contract is the spot price at maturity minus the contracted forward price set at the date t when the contract originated.

It should be recalled that the resource is not storable. Under certainty with zero cost of extraction this poses no problem. In an uncertain environment, however, this precludes a direct application of arbitrage arguments which rely on storing the good in a riskless manner. It may aid one's intuition to think of reserve fluctuations as the outcome of a "risky storage" technology.

In order to obtain the equilibrium price of a forward contract I con-

16. Since I have assumed the interest rate to be nonstochastic, forward prices are identical to futures prices. See Cox et al. (1981) for instance. This result is valid if the only feature distinguishing the forward contract from the futures contract is the "marking-to-the-market" feature.

tinue to assume that the equilibrium expected over an instantaneous holding period is the risk-free return. Applying this equilibrium condition to (29) and simplifying by using the optimal extraction policy yields

$$-F\tau - F_S\beta_1 S + \tfrac{1}{2}F_{SS}\sigma^2 S^2 - rF = 0. \tag{34}$$

This valuation equation can be solved to determine the value of contracts at equilibrium at any time $s \in (t, T)$, subject to the boundary condition stated in (30).

PROPOSITION 2: The equilibrium value of a forward contract is given by

$$F(S, \tau) = P(S)\exp\{[b\beta_1 + \tfrac{1}{2}b\,(b - 1)\sigma^2] - r\}\tau - \phi\,(t, T)\exp(-r\tau). \tag{35}$$

The proof of this proposition is by actual substitution of (35) into (34). The boundary condition is satisfied as well. Q.E.D.

The specification for the equilibrium value for forward contracts is intuitively appealing. As the reserve levels increase, the forward contract prices tend to fall. As the uncertainty about the reserve level increases, the forward contract prices tend to increase. This is a reasonable conclusion. As the reserve level uncertainty increases, forward contracts that deliver one unit of the reserve in future are valued highly. In (35) it is seen that the extraction rate directly influences the term structure of forward contract prices. In the previous section I endogenously determined β_1 for both competitive and monopolistic market structures. Under the assumption of risk neutrality,

$$\beta_1 = \frac{r - \frac{b}{2}(1 + b)\,\sigma^2}{-b}.$$

Using this in (35) and simplifying yields

$$F(S, \tau) = P(S)\exp(-b\sigma^2)\tau - \phi(t, T)\exp(-r\tau). \tag{36}$$

This expression is useful in identifying the following properties of the term structure of forward contract prices. $\partial F/\partial R > 0$, indicating that the value of the forward contracts of all maturities tend to increase as the interest rates rise. The term structure of forward contract prices is positively sloped.

From (36) one can determine the equilibrium forward prices or futures prices by picking that value of $\phi(t, T)$ which causes at time t the value of the forward contract to go to zero. This ensures that at the equilibrium forward prices, the market for forward contracts clears.

PROPOSITION 3: The equilibrium futures or forward price is given by

$$\phi(t, T) = P(S)\exp[(r - b\sigma^2)\tau]. \tag{37}$$

There are some interesting properties that the equilibrium futures prices have in this model. As σ^2 increases, the futures prices tend to increase. This once again reflects that the usefulness of futures as a hedge against any unanticipated shortage of the exhaustible resource. The term structure of the futures prices is positively sloped. Frequently the relationship between spot price and futures price is expressed in terms of the basis which is defined as $\phi(t, T) - P[S(t)]$.

The specification in equation (37) demonstrates that the basis in the futures markets depends on the price elasticity of demand of the resource, spot price, variability in the reserve level fluctuations, and the time to maturity of the contract.

The approaches presented in this section permit a rich intertemporal evolution for the futures prices and spot prices. However, the relationship between the futures price and the spot price obtained in this paper is not new and is well known. Indeed the relationship (32) could be obtained under more general conditions. The advantage of the models presented in this paper lies in their predictions about the distributional properties of spot prices, futures prices, and their evolution over time.

I have employed a competitive spot market assumption so that standard arbitrage transactions are possible. If the underlying spot market is monopolistic there is some question whether a futures market could be sustained if the monopolist also were to trade in the futures market. The results of Anderson and Sundaresan (1984) suggest that if the competitive participants have rational expectations, then at equilibrium there will be no futures trading in the absence of hedging motives on the part of the monopolist.

V. Empirical Implications and Conclusions

The results of this paper have interesting empirical implications. The pioneering work of Hotelling (1931) suggested that, under certainty in a frictionless world, the price of a nonrenewable resource should rise at the market rate of interest to preclude arbitrage. His work has led economists to look for association between interest rate movements and price of nonrenewable resources. My model suggests that in the presence of uncertain exogenous discoveries the spot price of a nonrenewable resource should evolve as depicted in figure 2. Specifically, in a competitive market structure my model places the following restrictions on the spot price evolution: (1) the spot price is expected to increase at a rate exceeding the risk-free rate in periods between discoveries and (2) the amount by which the rate of increase of spot price exceeds the risk-free rate depends on the nature of the nonrenewable resource. The key determinants of this excess expected returns are (*a*) the price elasticity of demand of the resource (assumed to be a constant), (*b*) the mean arrival rate of discoveries, and (*c*) the magnitude of additions to the available reserves. Finally, my model predicts that the spot price evolution is punctuated by discontinuous drops in periods during which successful discoveries are made. According to my model the difference between the rate of spot price appreciation and the risk-free rate is unique to the natural resource market.

If the market structure is monopolistic, then my model with stochastic exogenous discoveries predicts that the rate of increase in spot price in periods between discoveries should be less than the riskless rate. The determinants of the difference between the rate of appreciation of the spot price and the riskless rate are the same variables as before. My model with discoveries has interesting implications for the futures price behavior as well. The futures prices given by (32) imply that the rate of change of futures prices in periods between discoveries is positive and given by $\lambda[1 - (1 + \theta)^b]$.

The results in the absence of discoveries also place refutable restrictions on the evolution of spot prices and future prices. For instance, in Section III I showed that along the optimal extraction path equilibrium spot prices are lognormally distributed with a positive expected return in excess of the risk-free return. The amount by which the rate of increase of spot price exceeds the risk-free rate depends on the price elasticity demand b and the variance σ^2 of the reserve-level fluctuations. The futures pricing specification (37), however, implies that the expected rate of change in future prices is zero, in sharp contrast to the implication of the specification (32).

In Sections II and III I extended the Hotelling valuation principle to a setting which accommodated risk aversion and uncertainty.

Future work along these lines can examine the impact of costly extraction efforts on spot, forward, and future prices. For instance, if the unit cost of extraction is an increasing function of the cumulative amount extracted, then the spot and futures prices will systematically reflect this dependence. Dasgupta and Heal (1979) and Miller and Upton (1983) have examined the implications of a nonzero cost of extraction on the intertemporal spot price behavior. In the context of this paper, the introduction of extraction costs will cause the optimal depletion policy of the monopolist to differ from that of the price-taking competitive industry. In a competitive regime, if the extraction costs were to rise with cumulative production, then the time path of net spot prices would rise at a rate below the riskless rate of interest. As a consequence, the first two results stated in the abstract of this paper would be altered once positive extraction costs were introduced to this model. The modifications to the Hotelling valuation principle with positive extraction costs are not easy to obtain. Furthermore, the role of active trading in the futures markets and its effect on optimal extraction and exploration policies is another line of inquiry. If futures markets could serve to allow credible precommitments, then the outcome in such a setting could be used to examine the role of futures markets on the resource allocation features in the spot market. A general equilibrium extension of this work would require that the demand for the nonrenewable good be determined endogenously. Furthermore, the market structure in the spot market can be oligopolistic. For instance, instead of acting as price takers and taking the spot price as given, the producers may elect to form a cartel (as in oil) and set either the amount extracted each time or price to be charged each time. Equilibrium models of oligopoly can be constructed to study cooperative or noncooperative extraction policies and their impact on the prices of claims on the resource. Finally, the empirical implications of the model developed in this paper suggest directions for future research. The spot price data and the futures price data on goods such as oil and metals can be used in conjunction with appropriate econometric models to test the implications of the model.

The framework used in this paper can be suitably modified to study the behavior of spot and futures prices of harvestable and storable commodities. The pertinent literature in this area is too vast to be surveyed here, and any attempt to link the results of this paper with those of Cootner (1967) and Samuelson (1957), among others, must be postponed to a future study.[17]

Appendix A

The monopolist's problem is

$$0 = \max_{\{R\}}\{R^{1+b} - \rho J - J_S R + \lambda[J(S + \theta S) + J(S)]\}. \tag{A1}$$

The first-order condition is

$$(1 + b)R^b = J_S. \tag{A2}$$

It may be verified that

$$R = \beta S,$$

$$J = \beta^b S^{1+b},$$

and

$$\beta = \frac{\rho - \lambda[(1 + \theta)^{1+b} - 1]}{-b}.$$

Solve (A1) and (A2). Since the monopolist is assumed to be risk neutral, $\rho = r$. The proof is complete. Q.E.D.

17. The intraharvest and interharvest behavior of spot and futures prices has received extensive investigation in Samuelson (1957) and Cootner (1967). Recently Goldman and Sosin (1981) have also addressed this issue.

Appendix B

Equation (11) can be written as

$$R = \frac{P + \lambda\{P(S) - P[S(1 + \theta)]\}}{-P_S}.$$

With isoelastic demand $P = R^b$ the above equation can be written as

$$-P_S P^{1/b} = P + \lambda\{P(S) - P[S(1 + \theta)]\}.$$

It is easy to verify that $P = \Delta^b S^b$ solves the above equation, where

$$\Delta = \frac{-r}{b} + \frac{\lambda}{-b}[1 - (1 + \theta)^b] > 0.$$

Clearly $R = \Delta s$. Q.E.D.

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