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# Option Portfolio Strategies: Measurement and Evaluation

It is well known that options can increase the flexibility of returns available from investment strategies. However, actually describing the effect of option portfolio strategies and evaluating the performance of such strategies is a difficult task. In this paper we use an approximation algorithm to determine some of the important characteristics of option portfolio strategies. We also consider the measurement of performance for the strategies and discuss the pitfalls of applying several popular portfolio evaluation techniques to portfolios with option positions.

## I. Introduction

It is well known that options can increase the flexibility of returns available from investment strategies. Papers by Ross (1976), Breeden and Litzenberger (1978), and Arditti and John (1980) point toward spanning opportunities which increase the efficiency of the financial markets in meeting the investment objectives of investors. While these and other studies have demonstrated the potential of option portfolio strategies in molding the return distribution to better fit risk preferences, a number of other studies have used simulation techniques to give an empirical view of just what effect particular option stock and option bond strategies will have in altering risk return patterns.

A major study in this area is that of Merton, Scholes, and Gladstein (1978, 1982). Their study, presented in two papers, one on call option portfolio strategies (hereinafter MSGl) and the other on put option portfolio strategies (hereinafter MSG2), presents a broad and insightful discussion of many of the implications of combining options with stock portfolios. The MSG study employs a 12.5-year period for simulations in the first paper on portfolios using call options and a 14-year simulation period in the second paper on portfolios using put options. As they are careful to point out, their results depend on market trends which may not match ex ante expecta-

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tions of market performance at some later date (MSG1, pp. 184, 191, 210–11; MSG2, pp. 3, 29–30, 38–39, 41). Further, the variations in the market over the period make it difficult to separate the distributional characteristics that are inherent in the option and strategies from the characteristics that are idiosyncratic to the market during the period studied. This problem is complicated by the inability to keep the parameters constant during the sample period. With the riskless rate, the market rate of return, and the volatility of the stocks all varying, it is difficult to establish the contribution of the many factors to the observed return characteristics.

This paper uses a different simulation method to provide additional information concerning the effect of option portfolio strategies on return distributions. The results presented here are based on an approximation algorithm for determining the return distribution of optioned portfolios. This algorithm is described and tested in Bookstaber and Clarke (1983*a*, 1983*b*). The algorithm uses numerical methods to compute the return distribution of various option portfolio strategies.[1] It does so based on ex ante specifications, eliminating the problem faced by MSG in adapting their results, based on ex post market trends, to later ex ante expectations. It also allows the parameters, such as expected market return, option price, stock volatility, and riskless rate, to be specified. Also, we can trace out the entire return distribution, giving a fuller picture of the return and risk characteristics of the various option portfolios.

Ascertaining the sensitivity of this effect is critical, since all simulation studies to date depend on model-derived prices rather than the actual prices at which options are trading. In Section IV we will look in

1. Determining the characteristics of an optioned portfolio is not difficult if the option position can be revised continuously. In a continuous time model with constant revision of the option position, the effect of an option position on a portfolio can be determined easily by translating the option position into share equivalents through the use of the option hedge ratio. But in practice option positions are held over finite time intervals. And investors are interested in portfolio return characteristics over finite time horizons. In this case, the technique of translating the options into share equivalents is no longer straightforward. The difficulty in using the share equivalence approach for the determination of returns over more than an instant arises because the various stocks in the portfolios can take on any of a number of values over a finite time period and the share equivalent will differ according to the stock price that is realized. The computation of the share equivalent portfolio return will involve the enumeration of a number of conditional share equivalents across a large set of possible stock prices. For a portfolio of even a few stocks, the enumeration and the evaluation of the resulting probability statements becomes unwieldy. In the finite horizon case, the multivariant interaction between stocks necessitates considering the range of possible returns and related option payoffs for each stock conditional on the range of returns for all other stocks in the portfolio. The enumeration of these conditional returns becomes cumbersome even under the simplified assumptions of normality and independent residual errors. For example, with a moderate-sized portfolio of 20 optioned stocks, the determination of the portfolio return distribution involves the evaluation of over 1 million ($2^{20}$) conditional probability statements.

detail at the effect of option prices on the return distribution characteristics. As this section will show, it appears option mispricing does not cause the shape of the return distribution to change qualitatively. That is, the implications of the various option portfolio strategies we explore on the portfolio return distribution are not dependent on the option prices. This is a useful result, given the wide variety of option pricing models currently proposed.

In particular, we will show how the use of a number of popular portfolio performance measures that rely on the mean and variance of return can lead some option strategies to appear to outperform the more common stock-bond strategy, while other option strategies will appear to be inferior. The biases occur because call and put strategies each affect the distribution asymmetrically, while the usual mean-variance measures assume both tails of the distribution are affected equally by changes in variance.

For example, a strategy involving writing call options will result in a drop in the variance of return, but most of that decrease will occur precisely where variance is desirable, in the right tail of the distribution. It would seem reasonable that the return compensation for the variance reduction from covered call writing would then be greater than the return from the strategy of buying protective puts, a strategy which reduces variance where variance is undesirable, on the left tail of the distribution. The conventional mean-variance performance measures fail to take the effect of these strategies on the higher moments into account and will therefore mistakenly show the covered call strategy to have a more favorable risk-return trade-off—as measured in mean-variance terms—than the protective put strategy.

We will first summarize the algorithm we use in describing the return distribution for option portfolio strategies. We will then examine the four principal cases Merton et al. considered in their study—covered option writing, covered put option purchasing, option/paper buying, and naked put writing. We will next illustrate the effect varying options prices have on the return distributions, and finally we will consider the implication of option portfolio strategies for portfolio measurement and evaluation.

## II. The Simulation Method

This section summarizes the algorithm we apply to approximate the return distribution of the various strategies. The algorithm results approach the analytical solution as the number of stocks in the optioned portfolio becomes large. The algorithm has also been tested in Bookstaber and Clarke (1983*a*) by using Monte Carlo techniques to evaluate its accuracy for small portfolios. A more complete description of the algorithm is presented there.

The algorithm we use assumes that all stocks held by the investor have returns generated by the familiar single index model or market model:

$$\tilde{r}_i = r_f + \beta_i(\tilde{r}_m - r_f) + \tilde{u}_i,$$

where $\tilde{r}_i$ = the return on stock $i$, $r_f$ = the return on the riskless asset, $\beta_i$ = the systematic risk of firm $i$, $\tilde{r}_m$ = the return on the index, and $\tilde{u}_i$ = a random error term for firm $i$.

The tilde is used to denote a random variable. The return on the index is presumed to have a normal distribution with mean $\bar{r}_m$ and variance $\sigma_m^2$ while the error term has a mean of zero and variance of $\sigma_i^2$. The random error terms are assumed to be normally and independently distributed between firms.[2] Therefore, the conditional expected return on a stock given a return on the index is

$$E(\tilde{r}_i|r_m) = r_f + \beta_i(r_m - r_f) \equiv \bar{r}_i. \tag{1}$$

To maintain some comparability with the portfolio used in Merton et al., we have based our simulations on the parameter values for the Dow Jones Industrial Stocks. We have used a 6-month return horizon, as in the study of Merton et al. Only the 26 stocks with options listed as of August 1982 were included in the simulation. These stocks and the relevant parameter values are listed in the Appendix. The riskless rate, $r_f$, was set at a 10% annualized rate. Following Ibbotson and Sinquefield (1976), the annualized market return was assumed to have a mean of 16.4% and a standard deviation of 22.3%. The stocks are assumed to be equally weighted in the portfolio.

The conditional expected return on the combined stock option portfolio can be expressed as a function of the option strategy used in the portfolio. Consider the case of selling call options on $\alpha_i$ proportion of the shares of stock $i$ where the options have the same expiration date as the holding period for the stocks.[3] The conditional expected return on the shares of stock $i$ with the call option position given a return of $r_m$ on the index would be[4]

$$\begin{aligned} E(\tilde{r}_i'|r_m) &= E(\tilde{r}_i|r_m, r_i \leq r_i^e) \operatorname{prob}(r_i \leq r_i^e|r_m) \\ &+ [(1 - \alpha_i)E(\tilde{r}_i|r_m, r_i \geq r_i^e) + \alpha_i r_i^e] \operatorname{prob}(r_i \geq r_i^e|r_m), \end{aligned} \tag{2}$$

2. The assumption of normality is not critical to the development of the algorithm. We have used it here only for illustrative purposes. Other probability distributions could easily be used. The critical assumption is the independence of the error terms between the firms and with the index return.

3. The treatment for purchasing puts, buying calls, and other strategies is similar to that for writing call options. Only the inequalities and the signs of the expressions in eq. (2) will differ.

4. The notation $E(\tilde{r}_i|r_m, r_i \leq r_i^e)$ refers to the expected return on stock $i$, given a return of $r_m$ on the index and given that $r_i$ is less than or equal to the exercise return $r_i^e$ on the call option. Since $r_i = \bar{r}_i + u_i$, the probability that $r_i \leq r_i^e$ given $r_m$ is the same as the probability that $u_i \leq (r_i^e - \bar{r})$ given $r_m$.

where $r_i^e$ is the return on stock $i$ at which the stock price would equal the exercise price on the option.[5] This conditional expected return is the expected rate of return, conditional on the market return being $r_m$, that will be earned on a portfolio consisting of the shares of stock and call options on $\alpha_i$ proportion of the shares of stock, without regard to the purchase price of the options. We will adjust the returns for the option prices below. The first term in equation (2) is the expected return on the stock given that the return is less than the exercise return on the option. The second term allows for the fact that if the stock price is greater than the exercise price on the option, the shares of stock covered by the option position will only return $r_i^e$ instead of the full rise in the value of the stock.

Because the error term on each stock is assumed to be normally distributed, the probability statements can be evaluated giving

$$E(\tilde{r}_i'|r_m) = \bar{r}_i + \alpha_i\,[1 - \phi(y_i)](r_i^e - \bar{r}_i) - \alpha_i\bar{U}_i[1 - \phi(y_i)]. \quad (3)$$

In equation (3) $\phi(\cdot)$ is the standard normal cumulative distribution function, and $y_i = (r_i^e - \bar{r}_i)/\sigma_i$. $\bar{U}_i$ is the mean of the error term for stock $i$ given that $u_i \leqslant (r_i^e - \bar{r}_i)$. Using equation (3) for each separate stock gives the aggregate conditional expected return on the optioned stocks in the portfolio as

$$E(\tilde{r}_{op}|r_m) = \sum \theta_i E(\tilde{r}_i'|r_m) = \sum \theta_i \bar{r}_i + \sum \theta_i\, \alpha_i[1 - \phi(y_i)](r_i^e - \bar{r}_i - \bar{U}_i). \quad (4)$$

Here $\theta_i$ is the proportion of the initial portfolio held in stock $i$,

$$\theta_i = \frac{n_i S_i}{\sum n_i S_i},$$

where $n_i$ is the number of shares of stock $i$ and $S_i$ is the price of stock $i$. Note that $\theta_i$ is determined before any option proceeds or costs.

Equation (4) indicates that the conditional expected return on the combined portfolio is equal to the expected return on the stocks in the portfolio plus a weighted adjusted return on the stocks covered by an option position.

The conditional expected return on the total investment portfolio can be written as the gross return on the proceeds or cost of the option position plus the expected return on the combined option stock portfolio:

$$E(\tilde{r}|r_m) = \theta_c(1 + r_f) + E(\tilde{r}_{op}|r_m) \quad (5)$$

5. The conditional expected return here is given on the assumption the expiration date of the option coincides with the time horizon of interest for the distribution. Bookstaber and Clarke (1983*a*) explain a method for evaluating options at times before expiration or with differing times to expiration. The conditional expected return for the options can also be found analytically using a variation of the method presented by Rubinstein (1981).

where $\theta_c$ represents the ratio of the proceeds of the call option position to the total initial investment in the stocks. It is through the $\theta_c$ term that the cost or proceeds from the options enter into the calculations. The original development of the algorithm assumes that the proceeds from the options position were reinvested at the riskless rate as noted in equation (5).

We have adjusted the results in tables 1 and 2 to follow the same convention as Merton et al. in defining the investment base and the portfolio returns (e.g., MSG1, pp. 201, 206). The original development of the algorithm in Bookstaber and Clarke (1983*b*) did not reduce the investment base by the amount of the option premiums.

The expression in equation (5) calculates the expected return of the total investment portfolio conditional on the index having a return of $r_m$. As the return on the index changes, the expected return to the portfolio will also change. In order to calculate the return distribution of the portfolio, the expression for the conditional expected return must first be computed for each possible value of $r_m$. The unconditional distribution is then determined by weighting the conditional expected portfolio return with the probability that the conditional return $r_m$ will be realized.

Thus, for each value of $r_m$ for which the conditional expected return on the portfolio is computed, one point on the distribution function can be determined. As the conditional return on equation (5) is calculated for the entire range of possible values of $r_m$, the probability distribution can be fully developed.

The index algorithm computes equation (5) conditional on a range of values for $r_m$, and assigns a probability weighting to each resulting expected total portfolio return according to the probability of the corresponding index return. The index algorithm may be summarized as follows:[6]

1. Compute the expected return on the total portfolio conditional on $r_m$ using equation (5).
2. Assign a cumulative probability to the resulting total portfolio return equal to the probability of the corresponding value of $r_m$.
3. Integrate step 1 and step 2 for all admissable values of $r_m$ to trace out the cumulative probability distribution for the total portfolio

6. The algorithm is flexible in allowing varying option positions with varying exercise prices on different stocks in the portfolio. The portfolio may also include stocks with no option position taken by setting $\alpha_i = 0$. The algorithm determines the distribution for a finite (noninstantaneous) investment horizon and does not rely on assumptions of continuous portfolio revision. For simplicity we have made the assumption in the presentation of the algorithm that the investor's time horizon equals the time horizon of the option contracts. However, this assumption can be relaxed, and the algorithm can be modified to use in the case that the investment horizon is less than the time to expiration of the options. Doing so requires the use of an option valuation formula and a change in the partitioning of eq. (2).

**TABLE 1 Probability of Returns for Writing Call Options Exercise Return is 0%**

| Return Interval (Semiannual) | Proportion of the Portfolio Covered ($\alpha$) 0 | .25 | .50 | .75 | 1.0 |
|---|---|---|---|---|---|
| Below −40 | .07 | .05 | .03 | .02 | .01 |
| −30 to −40 | .49 | .33 | .24 | .16 | .11 |
| −20 to −30 | 2.51 | 1.86 | 1.34 | .94 | .65 |
| −10 to −20 | 8.37 | 6.83 | 5.34 | 4.01 | 2.89 |
| 0 to −10 | 18.14 | 17.19 | 15.45 | 13.05 | 10.31 |
| 10 to 0 | 25.62 | 29.14 | 33.35 | 38.26 | 44.51 |
| 20 to 10 | 23.55 | 27.91 | 34.39 | 42.25 | 41.52 |
| 30 to 20 | 14.10 | 13.26 | 9.29 | 1.31 | .0 |
| 40 to 30 | 5.50 | 3.05 | .55 | .0 | .0 |
| 40+ | 1.65 | .36 | .01 | .0 | .0 |
| Mean | 8.04 | 7.70 | 7.34 | 6.96 | 6.56 |
| S.D. | 14.98 | 12.91 | 10.78 | 8.67 | 6.72 |
| Skewness | .0 | −.25 | −.63 | −1.22 | −2.12 |
| $\beta$ | .95 | .82 | .68 | .53 | .37 |

**TABLE 2 Probability of Returns for Purchasing Put Options Exercise Return is 0%**

| Return Interval (Semiannual) | Proportion of the Portfolio Covered ($\alpha$) 0 | .25 | .50 | .75 | 1.0 |
|---|---|---|---|---|---|
| Below 40 | .07 | .0 | .0 | .0 | .0 |
| −40 to −30 | .49 | .09 | .0 | .0 | .0 |
| −30 to −20 | 2.51 | 1.23 | .18 | .0 | .0 |
| −20 to −10 | 8.37 | 7.58 | 5.32 | 1.28 | .0 |
| −10 to 0 | 18.14 | 20.90 | 24.58 | 29.22 | 31.10 |
| 0 to 10 | 25.62 | 28.43 | 31.42 | 34.48 | 37.46 |
| 10 to 20 | 23.55 | 23.53 | 23.06 | 22.15 | 20.85 |
| 20 to 30 | 14.10 | 12.65 | 11.13 | 9.60 | 8.15 |
| 30 to 40 | 5.50 | 4.42 | 3.50 | 2.71 | 2.07 |
| 40+ | 1.65 | 1.16 | .80 | .55 | .36 |
| Mean | 8.04 | 7.68 | 7.34 | 7.00 | 6.67 |
| S.D. | 14.98 | 13.50 | 12.11 | 10.81 | 9.64 |
| Skewness | .0 | .22 | .47 | .74 | 1.11 |
| $\beta$ | .95 | .86 | .76 | .67 | .59 |

return. Using the estimated points in the cumulative distribution, calculate the height of the density function at each point consistent with the cumulative probability distribution.

## III. The Characteristics of Portfolio Returns for Alternative Option Portfolio Strategies

We will illustrate the algorithm described in Section II with four popular option portfolio strategies: writing covered calls, protective put buying, call option/paper buying, and writing uncovered puts. Needless to say, these four can only begin to cover the many strategies that are possible. But they provide a representative look at how options alter the return characteristics of a stock portfolio. These are also the four principal option portfolio strategies studied by Merton et al. We employ the same holding period and definition of portfolio returns as Merton et al., so this section also provides an alternative examination of the strategies studied in their historical simulations. The stocks we use in this situation are listed in the Appendix, along with their betas, dividend yields, and residual standard errors. To conform with Merton et al. these stocks were selected from those stocks in the Dow Jones 30 Industrials which had listed options. These stocks are equally weighted in the portfolio.

*Writing Covered Call Options*

Writing covered call options is a popular option strategy among portfolio managers. Writing call options seems to offer the best of both worlds: If the stock appreciates and is called away, the portfolio still gains the option premium and also receives dividends for the period before exercise. If the option is out of the money, the portfolio will gain the stock appreciation up to the exercise price. If the stock declines in price, the loss is buffered by the option premium, so the total loss from the position will be less than if no option were written.

Figures 1 and 2 present the return distribution resulting from writing call options on the underlying stock portfolio. Figure 1*A* is the distribution of the underlying stock portfolio when no options are written. As is shown in table 1, this reference portfolio has an expected semiannual return of 8.04% and a standard deviation of return of 14.98%. Since the return is assumed to be normally distributed, the mean and variance are sufficient statistics for the distribution.

However, the distribution becomes more complex as an increasing proportion of call options is written on the stocks in the portfolio. Figures 1*B*, 1*C*, and 1*D* show the portfolio return distribution when the proportion of the portfolio covered by call options, which we denote by $\alpha$, rises from zero to 25%, 50%, and 75%, respectively. The fully covered portfolio, where $\alpha = 1.0$, is shown in figure 2*B*. (Since the high peak of the fully covered position requires a different scaling, it is presented separately.) The exercise price of all the options in this strategy is assumed to equal the current stock price. That is, the exercise return is 0%. With this and all other strategies, the time to expiration is 6 months.

The two apparent characteristics of this strategy on the portfolio returns are, first, the truncation of the right-hand portion of the distribution, and second, the rightward shift in the distribution. Essentially, writing a call option involves selling the desirable right-hand tail of the distribution, with the payment reflected in the option premium re-

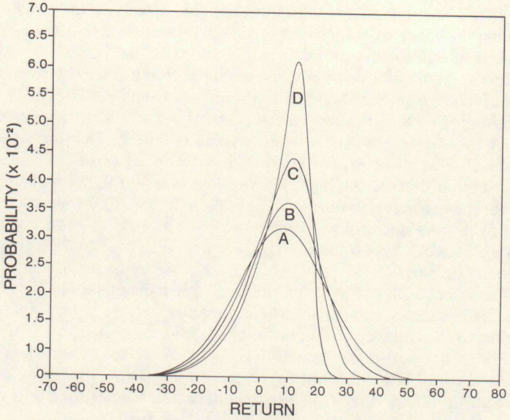


FIG. 1.—Return distribution of a portfolio with call options written on 0% (*A*), 25% (*B*), 50% (*C*), and 75% (*D*) of the stock portfolio. The exercise price of the options is equal to the current stock price.

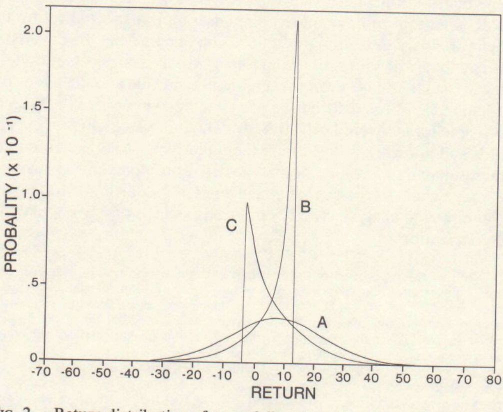


FIG. 2.—Return distribution of a portfolio with call options written on 100% of the stock portfolio (*B*), and with put options purchased on 100% of the stock portfolio (*C*). The exercise price of both options is equal to the current stock price. Figure 2*A* is the reference stock portfolio return distribution.

ceived. The truncation results from the sale of part of the tail, from having the stock called away if it rises above the exercise price. The shift results from the premium value added to the portfolio, which reduces the initial investment base required to purchase the stocks.

The change in the shape of the return distributions is evident by referring to table 1. For the underlying portfolio with $\alpha = 0$, the probability of receiving a return below $-20\%$ is 3.07%. This probability diminishes as the portfolio is covered with option positions and the distribution shifts to the right. For the case of $\alpha = 1.0$, the probability of receiving a return below $-20\%$ drops to 0.77%. The truncation on the right-hand side is more pronounced—the probability of receiving a return above 20% is 0 for $\alpha = 1.0$, compared to over 21.25% for the underlying portfolio.[7]

The effect of this strategy on the first three moments of the distribution and on the overall portfolio betas is shown in table 1. The expected return for the fully covered portfolio is 6.56%, compared to 8.04% for the reference portfolio of underlying stocks. While the expected return drops by 18%, the standard deviation of the portfolio drops by over 50%, from 14.98% to 6.72%. The overall beta of the portfolio drops by over half, from 0.95 to 0.37.

Covered call option writing significantly reduces exposure. However, the steep reduction in exposure as measured by standard deviation can be misleading. Most of the drop in variance occurs in that part of the distribution where high variance is desirable—in the right-hand tail. It is natural to have more compensation in expected return for a drop in standard deviation when that drop occurs from the covered call strategy than when it occurs from a strict stock-riskless asset trade-off, since in the latter case the desirable right-hand side and the undesirable left-hand side of the distribution are affected symmetrically. Indeed, it is possible for a covered call strategy actually to give a higher expected return and a lower variance than the underlying stock portfolio.[8] This brings out an important point that we will return to later: the changes in the return characteristics that arise from option portfolio strategies are too complex to analyze strictly on the basis of the first two moments of the distribution.

7. While $\alpha$, the proportion of the portfolio with the option position, determines the amount of truncation, the exercise return of the option position determines the location of that truncation. As would be expected, the higher exercise return moves the truncation more to the right, with the degree of truncation less pronounced the higher the exercise return. The rightward shift in the distribution is also smaller, reflecting the smaller premium for call options with a higher exercise price.

8. This will be the case if the exercise return is high and the option premium is large due to a high volatility. This is in contradiction to MSG1, p. 205. For example, in a portfolio where all stocks have $\beta = 1.0$ and $\sigma = .30$, the stock portfolio will have a mean return of 7.891 and standard deviation of return of 15.77%. If all stocks are covered by call options with an exercise return of 30%, the mean return will increase to 8.71% while the standard deviation will decline to 13.20%.

### *Buying Put Options*

In two obvious respects, buying put options is the mirror image of writing covered call options. While writing covered calls truncates the right-hand side of the distribution and shifts the distribution to the right, buying protective puts truncates the left-hand side of the distribution and shifts the distribution to the left. The truncatation results as the put option acts to offset the decline in the stock price. The shift results since the cost of the put premium increases the investment base of the portfolio.

Figure 3 shows the returns for the put option strategy with $\alpha = .25$, .50, and .75 (figs. 3*B*, 3*C*, and 3*D*, respectively) and the return distribution of the reference stock portfolio (fig. 3*A*). The values are tabulated in table 2. The distribution when put options are purchased on all the stocks in the portfolio is presented in figure 2*C*. These figures are all generated for options purchased at the money, that is, for an exercise return of 0%.

The most obvious characteristic of the put option strategy is the reduction of downside risk and the maintenance of upside potential. The probability of incurring a severe loss can be reduced and even brought to zero with the appropriate option strategy. Indeed, in some cases it is even possible to insure against receiving any negative return.[9] This is as would be expected, since analytically put options can be characterized as an insurance contract to protect the purchaser from a decline in the price of the underlying stock.

As table 2 indicates, both the expected return and standard deviation drop as the proportion of the portfolio with put options held increases. For $\alpha = 1.0$, the expected return is 6.67% compared to 8.04% for the reference portfolio, and the standard deviation is 9.64% compared to 14.98%. A more interesting comparison is between this put option strategy and the covered call writing strategy. The two strategies have much the same effect on expected return. For $\alpha = 1.0$, the expected return for the put strategy is virtually identical to that of the call strategy, 6.67%, compared to 6.56%. But the standard deviation for the put strategy is actually higher, 9.64% compared to the 6.72% for covered call writing. If variance is used as a proxy for risk, covered call writing will be preferred to buying puts. But variance is not a suitable proxy for risk, since the option strategies reduce variance asymmetrically. The call truncates the right-hand side of the distribution and thereby reduces the desirable upside variance. The put, on the other hand, elimi-

9. This will be the case if the exercise return is high and the option premium is low due to low volatility. This is in contradiction to MSG2, p. 33. For example, in a portfolio where all stocks have $\beta = 1.0$ and $\sigma = .05$, the stock portfolio will have a mean return of 8.71% and a standardized deviation of return of 13.20%. If protective puts are purchased on all stocks, the mean return drops to 5.14% with a standard deviation of 4.43%, and the minimum possible return is positive at 3.31%.

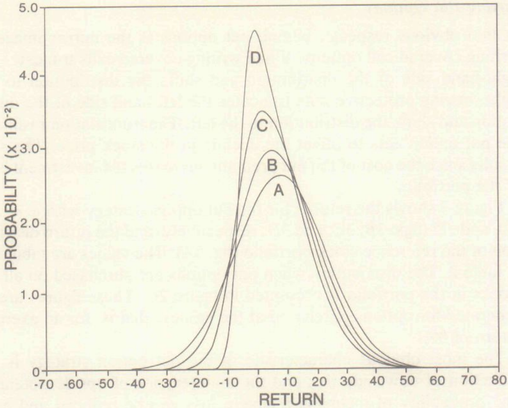


FIG. 3.—Return distribution of a portfolio with put options purchased on 0% (*A*), 25% (*B*), 50% (*C*), and 75% (*D*) of the stock portfolio. The exercise price of the options is equal to the current stock price.

nates the variance more on the undesirable left-hand portion of the return distribution. It is clear that more than variance is needed in assessing the risk of the option portfolio strategies. The third moment gives more information—call writing is negatively skewed while put buying is positively skewed—but even that will not be enough to fully describe and delineate the strategies. It is possible to find two strategies which generate the same values for both variance *and* skewness but which have different mean returns.

This suggests a shortcoming of using simulation methods which rely on summary statistics to describe option portfolio strategies, but more important, it underscores the limitations suggested earlier in applying conventional evaluation methods, which rely heavily on first- and second-moment measures, to describe and evaluate the risk and performance of option portfolio strategies. We will return to this point in the next section.

The insurance characteristics of buying put options are apparent from figures 2 and 3. But while the probability of having a large loss can be made arbitrarily small, the protection is not costless. One obvious cost is that of the option, and this is reflected in the left-hand shift of the distribution. Another, less obvious cost is reflected in the increase in probability mass just above the point of truncation. That is to say, while the probability of a very low return declines, the probability mass associated with the very low returns is just pushed up slightly, and consequently the probability of a moderately low return increases. As is shown in table 2, the probability of receiving a return below $-20\%$ declines from 3.07% for $\alpha = 0$ to 0.18% for $\alpha = 0.5$, and to 0.0 for $\alpha = 0.75$ and for $\alpha = 1.0$. But the probability of receiving a return between $-10\%$ and 0% increases from 18.14% for $\alpha = 0$ to 24.58% for $\alpha = 0.5$, and to 31.10% for $\alpha = 1$. Clearly the mechanism by which the put option insurance policy is paid for is far more complex than that simply denoted by the option premium.

*Call Option/Paper Buying*

Call options are conventionally viewed as highly levered, speculative securities. But it is now widely recognized that the leverage and risk of options can be reduced to convert the strategy of buying call options into a prudent, low-risk strategy. Indeed, by holding a sufficiently small proportion of the portfolio in call options, with the remainder held in a low-risk money market instrument, portfolio risk can be reduced substantially below that of the underlying stock portfolio.

Figure 4*B* shows the return distribution for a call/paper strategy where 10% of the portfolio funds are held in call options, with the remainder held in money market instruments.[10] Given the price of the options, with our data set, this strategy leads to $\alpha = 1.15$, where the options are held in place of the underlying stocks. Figure 4*C* gives the return distribution of the share-equivalent portfolio, the option portfolio which matches the price movement of the underlying stocks. Since the share equivalent, derived as the inverse of the hedge ratio, is an instantaneous measure, the actual share equivalent will vary over time, and this static option portfolio will not exactly match the stock price movement over any finite time horizon.[11]

The option/paper strategy dramatically truncates the left-hand side of the distribution, since the maximum loss is limited to the fraction of the portfolio in calls less the interest on the paper, and still allows for some appreciation should the underlying stocks increase in value. As $\alpha$ increases, the distribution spreads out, the truncated left tail shifts downward, and the right tail extends further to the right. Table 3 also shows the result of holding all the portfolio in options, the extreme case of this strategy. Here, 11.5 options are held in place of each 100 shares of stock of the underlying portfolio. In this case, the distribution truncates at $-100\%$ return, and the right tail extends well beyond that of the underlying portfolio. Roughly a third of the probability mass rests below $-40\%$, a third between $-40\%$ and 40%, and a third above 40%.

10. This corresponds to the option/paper strategy used in the Merton et al. study.

11. The need to use the share equivalent in simulation tests which directly compare the portfolio of optioned stocks with the underlying stock portfolio is discussed by Gastineau and Madansky (1979).

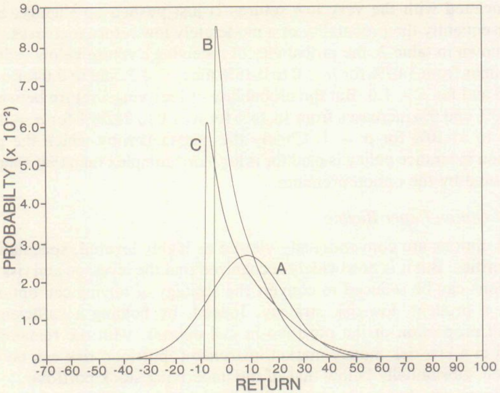


FIG. 4.—Return distribution of a portfolio of call options, with 1.15 options purchased for every 100 shares of stock in the reference portfolio (*B*), and with 1.685 options purchased for every 100 shares of stock in the reference portfolio (*C*). Figure 4*A* is the reference portfolio.

**TABLE 3 Probability of Returns for Call/Paper Buying**

| Return Interval (Semiannual) | 0 | Option Position Relative to Stock Position in Reference Portfolio ($\alpha$) 1.0 | 1.15 | 1.685 | 11.5 |
|---|---|---|---|---|---|
| Below −40 | .07 | .0 | .0 | .0 | 37.38 |
| −40 to −30 | .49 | .0 | .0 | .0 | 4.41 |
| −30 to −20 | 2.51 | .0 | .0 | .0 | 4.06 |
| −20 to −10 | 8.37 | .0 | .0 | .0 | 3.77 |
| −10 to 0 | 18.14 | 29.23 | 33.11 | 40.37 | 3.54 |
| 0 to 10 | 25.62 | 43.42 | 37.73 | 26.36 | 3.32 |
| 10 to 20 | 23.55 | 19.32 | 18.85 | 16.50 | 3.14 |
| 20 to 30 | 14.10 | 6.44 | 7.60 | 9.41 | 2.97 |
| 30 to 40 | 5.50 | 1.38 | 2.20 | 4.60 | 2.80 |
| 40+ | 1.65 | .20 | .51 | 2.76 | 34.58 |
| Mean | 8.04 | 6.19 | 6.37 | 6.97 | 21.25 |
| S.D. | 14.98 | 8.49 | 9.65 | 13.56 | 106.12 |
| Skewness | .0 | 1.23 | 1.23 | 1.23 | 1.23 |
| $\beta$ | .95 | .51 | .58 | .81 | 6.34 |

As with buying put options on the stocks in the portfolio, the call/paper strategy concentrates much of the mass near the point of truncation. While there is no risk of a return below $-10\%$, there is over a .40 probability of a return between $-10\%$ and 0% for the portfolio with $\alpha$ 1.685.[12]

## *Uncovered Put Option Writing*

Just as purchasing put options leads to a mirror image of the returns generated in writing calls, so writing uncovered put options mirrors the call/paper strategy. While the call/paper strategy increasingly truncates the left-hand side of the distribution as the $\alpha$ of the portfolio is increased, writing puts results in a truncation that moves more to the right as $\alpha$ increases. And rather than the right-hand tail extending out as with the call/paper strategy, the left-hand tail becomes more pronounced.

Figures 5*B* and 5*C* present the distribution for writing uncovered put options for $\alpha = 1.0$ and $\alpha = 2.46$, respectively. The intervals for figure 5 are summarized in table 4. This table also presents the intervals of the distribution for $\alpha = 13.8$, the extreme case where a put option position is taken with proceeds equal to the funds available for investment.

A comparison of this strategy with the call/paper strategy underscores the difficulty in describing and evaluating the various strategies by using summary statistical measures. For $\alpha = 1.0$, the expected return of both strategies is virtually the same—6.17 for puts compared to 6.19 for calls. The standard deviation for the put strategy is lower, however, 7.57 compared to 8.49 for calls. The reason for this is the same as it is for the discrepancy between call writing and protective put buying. For call/paper buying, the variance is taken from the undesirable left-hand side of the distribution while for put writing it is taken from the desirable right-hand side. As a consequence, the put strategy gives a higher expected return relative to standard deviation in compensation. As with the other two strategies, here variance is not an adequate proxy for risk.

12. In MSG1, the call/paper strategy greatly outperformed the underlying stock portfolio over a number of sample periods when the stock portfolio had substantial gain. The call/paper return was roughly double that of the stocks. The results of this section show that the concern of Merton et al. that the returns and the stock performance over the periods were atypical, not representative of the performance that would be expected ex ante of the call/paper strategy, is well placed (MSG1, pp. 222–26). In contrast to their simulation results, the call/paper strategy with $\alpha = 1.15$ will generally move less than the market, as indicated by its average $\beta$ of .58. The potential for high returns with this strategy is noticeably lower than for the reference stock portfolio. The idiosyncrasies evident in the simulations of the call/paper strategy are not surprising, since this strategy can be very sensitive to slight variations in the returns of the underlying stocks. This is particularly true as the $\alpha$ increases, since then large variations in returns may result from slight variations in the stock portfolio.

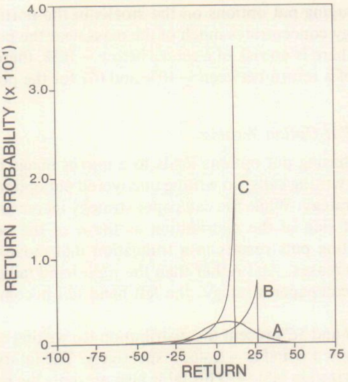


FIG. 5.—Return distribution of a portfolio of uncovered puts, with 1.0 options sold for every 100 shares of stock in the reference portfolio ($B$), and with 2.46 options sold for every 100 shares of stock in the reference portfolio ($C$). Figure 5$A$ is the reference portfolio.

**TABLE 4 Probability of Returns for Writing Uncovered Put Options**

| Return Interval (Semiannual) | Option Position Relative to Stock Position in Reference Portfolio ($\alpha$) 0 | 1.0 | 1.38 | 2.46 | 13.8 |
|---|---|---|---|---|---|
| Below − 40 | .07 | .02 | .32 | 4.07 | 20.28 |
| −40 to −30 | .49 | .15 | .80 | 2.61 | 1.90 |
| −30 to −20 | 2.51 | .89 | 2.24 | 3.93 | 2.06 |
| −20 to −10 | 8.37 | 3.82 | 5.42 | 5.76 | 2.23 |
| −10 to 0 | 18.14 | 12.44 | 11.68 | 8.39 | 2.47 |
| 0 to 10 | 25.62 | 40.37 | 26.17 | 12.69 | 2.69 |
| 10 to 20 | 23.55 | 42.32 | 53.37 | 23.12 | 3.01 |
| 20 to 30 | 14.10 | .0 | .0 | 39.43 | 3.36 |
| 30 to 40 | 5.50 | .0 | .0 | .0 | 3.81 |
| 40+ | 1.65 | .0 | .0 | .0 | 58.20 |
| Mean | 8.04 | 6.17 | 6.71 | 8.45 | 21.32 |
| S.D. | 14.98 | 7.58 | 10.77 | 21.04 | 96.89 |
| Skewness | .0 | −1.79 | −1.79 | −1.79 | −1.79 |
| $\beta$ | .95 | .43 | .61 | 1.20 | 5.51 |

## IV. The Effect of Premium Levels on Returns

Option portfolio strategies are often instituted more to take advantage of the return potential of options than consciously to alter portfolio return characteristics. That is, the alteration of portfolio returns is generally a side product of option strategies, not the primary objective. But just as it is useful for the portfolio manager to know what the aggregate effect of the many options positions is on returns, and whether that effect is consistent with the strategic objectives of the portfolio, it is also important to know how sensitive the distributional effects will be to the mispricing of options entered into the portfolio. While it is unlikely that option positions will all uniformly have identical mispricing, or be mispriced to a high degree, tests which assume such mispricing are useful to see if systematic mispricing has any effect on option portfolio return characteristics.

Figures 6 and 7 present the returns of the call option writing and the put option buying strategies, respectively. In both cases, the exercise return is 0% and $\alpha = .5$. The premiums are set 20% below the theoretical value (figs. 6*B*, 7*B*), at the theoretical value (figs. 6*C*, 7*C*), and 20% above the theoretical value (figs. 6*D*, 7*D*). We use the dividend-adjusted Black-Scholes model to establish the theoretical values. Figures 6*A* and 7*A* present the reference stock portfolio.

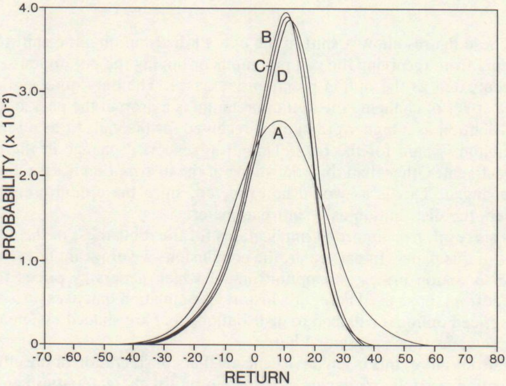


FIG. 6.—Effect of premium value on the return distribution of covered call option writing. The option premium is 20% below the fair value (*B*), equal to the fair value (*C*), and 20% abvove the fair value (*D*). Figure 6*A* is the reference portfolio. The exercise price is equal to the current stock price, and $\alpha = .5$.

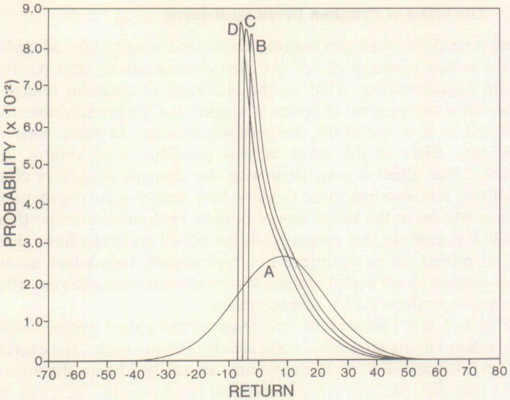


FIG. 7.—Effect of premium value on the return distribution of purchased put options. The option premium is 20% below the fair value (*B*), equal to the fair value (*C*), and 20% above the fair value (*D*). Figure 7*A* is the reference portfolio. The exercise price is equal to the current stock price, and $\alpha = .5$.

These figures show a shift in the overall distribution—the shift that occurs from receiving the call premiums or buying the put options—is accentuated as the option premium increases. The only other noticeable effect of changing the option premium is a drop in the peak of the distribution as a higher premium is received for the calls or as a lower premium is paid for the puts. This drop reflects a change in the net investment. Other than this, the shape of the distribution is essentially unchanged. This is as would be expected, since the option premium enters the distribution as a shift parameter.

This result has important implications for the robustness of the analysis of this paper. In particular, the conclusions we draw are not sensitive to option prices. An option model which generates prices that differ from those used here, or a historical simulation that uses possibly mispriced options, will lead to distributions that are shaped essentially the same as those presented here.

Furthermore, this insensitivity means that consideration of the effect of option portfolio strategies on portfolio return characteristics can be kept largely separate from any determination of success in finding mispriced options.

## V. The Evaluation of Portfolios with Option Positions: The Pitfalls of Some Common Evaluation Methods

The strategies discussed in this paper are representative of a number of option portfolio strategies, although it goes without saying that they can only give a general sense of the alterations these strategies will bring to portfolio returns in practice. The exact return distribution will differ for every set of options and stocks, according to the time of maturity and exercise return of the options, the standard error of the residuals of the portfolio, and a number of other factors. However, this paper does provide a useful benchmark in understanding the effect of options on portfolio returns, and perhaps more important, it points out some pitfalls in describing option portfolio return characteristics and in evaluating option portfolio performance.

### *The Capital Market Line for Portfolios with Option Positions*

We have mentioned repeatedly that summary statistical measures will give an incomplete picture of the option portfolio return distribution. The use of expected return and variance will give a distorted view of the relative performance of alternative strategies for the simple reason that these strategies result in return distributions that depend on more than mean and variance.

This point is illustrated in figure 8. Here we have plotted the annualized expected return and standard deviation of portfolios with put or call option positions taken on the underlying stocks in the portfolio. These values are taken from tables 1 and 2. It appears that the portfolio's return can be made to deviate from the capital market line. The use of call options appears to generate a risk return trade-off more favorable than that available in the market while the use of put options appears to generate returns less favorable. This is very deceptive, however. The standard deviation ignores the effects of the higher moments on the desirability of the return distribution. It is not a good descriptive measure of the risk of the portfolio.

This result of the use of options in a portfolio has important implications for portfolio performance evaluations. The performance of a particular portfolio is often compared to the performance of a well-diversified portfolio of stocks which has been levered appropriately to give the same standard deviation and by implication the same risk as the particular portfolio. The return on the reference portfolio is then compared with the return on the specific portfolio to see if performance has been better or worse.

When options are used in the particular portfolio, however, the returns of that portfolio should not be compared with a reference portfolio of the same standard deviation. The two return distributions

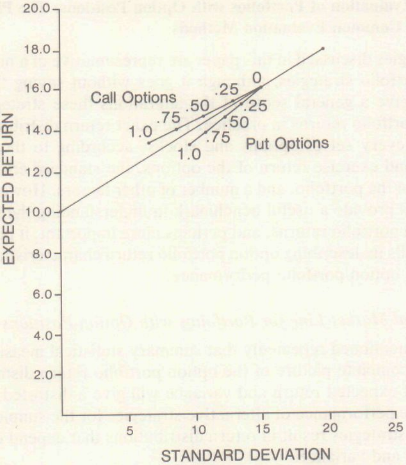


FIG. 8.—Annualized expected returns as a function of the standard deviation of the optioned portfolios. The longest line is the capital market line for stock portfolios. The points correspond to the proportion of the stocks covered by an option position.

would not generally be similar at all. Options can alter the shape of the return distribution substantially and so even though the two distributions may have similar standard deviations of return, the risk characteristics of the two may be quite different. In the case of the examples presented in figure 8, the portfolio manager who wrote call options would tend to have better performance evaluations relative to the reference portfolio while a manager who purchased put options would show relatively poor performance on average. In reality, it would not be possible to tell whether one or the other should be praised or penalized using that comparison.

The distortions in the use of the capital market line for portfolio evaluation appear in the security market line as well. A plot of expected return on the portfolio beta will lead to the same biases observed in figure 8: writing call options will tend to lead to apparently extraordinary performance, while buying some options will lead to apparently suboptimal performance. As with the use of the standard deviation of returns, care must be taken in measuring the beta of the portfolio and in its interpretation. As a second-moment measure, the beta will have the same limitations as have already been mentioned for the standard deviation. Furthermore, the beta is not constant over the range of stock returns when dealing with option portfolios; the betas reported in tables 1 and 2 are the average betas for each portfolio across the full range of the probability distribution.

### *Distortions in Two Popular Evaluation Measures*

The difficulties in using the capital market line and security market line for evaluating portfolios with option positions carry over into the use of two popular portfolio performance measures: the Sharpe ratio and the Treynor index. The Sharpe ratio, also called the reward-to-variability ratio, takes the difference between the expected return of the portfolio, $R_p$, and the risk-free rate, $R_f$, and divides this by the standard deviation of portfolio return, $\sigma$, to account for risk. The resulting measure is $S_p = (R_p - R_f)/\sigma$. The Treynor index, also called the reward-to-volatility ratio, is similar to the Sharpe ratio but uses the beta rather than the standard deviation of return, yielding the measure $T_p = (R_p - R_f)/\beta$.

Since both of these measures rely on a second moment term, they will face the same distortions in measuring the performance of portfolios with option positions that were discussed above. Table 5 shows the values of the Sharpe and Treynor measures for the underlying stock portfolios ($\alpha = 0$), and for the stock portfolios with increasingly higher call and put positions ($\alpha = .25, .50, .75,$ and $1.0$). The Sharpe and Treynor measures for the underlying portfolio are .20 and 3.20, respectively. Since the stock returns in this example have been generated using the CAPM, these measures will reflect returns that are fair and consistent with that model. A higher measure will suggest a higher than fair return and therefore superior performance. Conversely, a lower value for the measures will suggest below average performance.

The portfolio with call options looks more impressive as the option position increases; with a fully covered position ($\alpha = 1.0$), the Sharpe ratio has risen over 10% to .23 and the Treynor index has risen from 3.20 to 4.21. For put options, the opposite is the case. The Sharpe ratio drops to .17 and the Treynor index drops to 2.83. This is the case even though the option prices, like those of the stocks, are fair. These results are consistent with the observations of figure 8. And the error in using these measures for portfolio evaluation is the same as well. Both measures ignore the asymmetric effect the standard deviation has on the return distributions of both the put and the call positions, and simply do not present the whole picture. The application of the Sharpe ratio or the Treynor index to portfolios with option positions will lead

TABLE 5 **Sharpe and Treynor Measures for Various Option Positions**

| | Proportion of Each Stock with an Option Position | | | | |
|---|---|---|---|---|---|
| | 0 | .25 | .50 | .75 | 1.0 |
| Calls: | | | | | |
| Mean | 8.04 | 7.70 | 7.34 | 6.96 | 6.56 |
| S.D. | 14.98 | 12.91 | 10.78 | 8.67 | 6.72 |
| $\beta$ | .95 | .82 | .68 | .53 | .37 |
| Sharpe | .20 | .21 | .22 | .23 | .23 |
| Treynor | 3.20 | 3.29 | 3.44 | 3.69 | 4.21 |
| Puts: | | | | | |
| Mean | 8.04 | 7.68 | 7.34 | 7.00 | 6.67 |
| S.D. | 14.98 | 13.50 | 12.11 | 10.81 | 9.64 |
| $\beta$ | .95 | .86 | .76 | .67 | .59 |
| Sharpe | .20 | .20 | .19 | .19 | .17 |
| Treynor | 3.20 | 3.12 | 3.08 | 2.99 | 2.83 |

to inevitable distortions. The performance of call writing will be overestimated, and the performance of put buying will be underestimated. Needless to say, other option portfolio strategies will face similar evaluation difficulties.

The result that a number of mean-variance-related performance measures are inappropriate for evaluating portfolios with option positions is not to say these measures are generally invalid. Rather, it is that in this case they are being misapplied. They are being used in a world other than that for which they were designed, a world where more than mean and variance matters. These measures are CAPM based, whereas the option prices, and certainly the complex return structure of the total option-stock portfolio, cannot in general be priced within the CAPM framework.

The two ways around the difficulties with evaluation methods are either to use an option pricing method that is consistent with the CAPM assumptions or to develop an evaluation method that fits into the merged world of option and stock returns.

The inconsistency of option pricing, which under the Black-Scholes model is based on a lognormal distribution, and stock pricing, which in our analysis follows the conventional assumption of a normal distribution, is not central to the distortions in the evaluation methods. Employing the model developed by Rubinstein (1976), we have regenerated the portfolios with covered written calls and with purchased puts under the assumption that stocks follow a lognormal distribution. This makes the underlying assumptions for the stock and option prices consistent. It does not, however, reduce the distortions we have reported. The performance of covered call writing continues to dominate that of buying protective puts even when the inconsistencies of the distributions being used are eliminated.

## Conclusion

The step from recognizing the theoretical value of using options in portfolio management to understanding empirically just what effect option portfolio strategies have on portfolio return characteristics is of critical practical importance. Historical simulation studies are useful in providing a background against which later performance can be measured. However, there are well-recognized limitations inherent in such studies, limitations which may be compounded in the case of short time periods and by the complexity of the return characteristics of portfolios with option positions. The objective of this paper is to apply a second technique, one which computes the return distribution through an algorithm rather than through simulations which use past data, in answering this empirical question. Since the algorithm can describe the entire return distribution and depends on specified expectational assumptions, it fills in some of the weaknesses inherent in historical simulations.

In addition to providing a description of the return characteristics of the various option portfolio strategies, this paper leads to some useful conclusions for the effect of option mispricing on the nature of portfolio returns and for the use of conventional evaluation methods to measure the performance of portfolios with option positions. We have shown that the return characteristics of optioned portfolios are not very sensitive to the pricing of the options used. Deviations from the correct price of as much as 20% do not lead to significant changes in the shape of the return distribution. The principal effect of such mispricing is only to shift the distribution.

Perhaps the most valuable observation of the paper is the distortions in applying mean-variance measures to the evaluation of option portfolio strategies. Generally, the use of these measures—the capital market line, security market line, Sharpe ratio, or Treynor measure, for example—will lead to significant biases.[13] For example, call writing will appear to outperform the market, while buying protective puts will appear to result in suboptimal performance.

The complex nature of the return distributions generated by these strategies requires more than the first two moments for either descriptive or evaluation purposes.

13. Although we have not treated it explicitly, the Jensen measure, the use of the statistical significance of the intercept in the capital asset pricing model, will lead to the same distortions, for it too is based on the mean-variance criterion and is closely related to the concepts behind the capital market line and security market line.

## Appendix

| Stock | Beta | Dividend Yield* | Residual Standard Error* |
|---|---|---|---|
| International Paper | 1.10 | .063 | .086 |
| Allied Chemical | 1.20 | .077 | .193 |
| Aluminum Co. of America | 1.05 | .075 | .260 |
| U.S. Steel | 1.05 | .056 | .273 |
| Bethlehem Steel | 1.25 | .050 | .243 |
| General Electric | .95 | .050 | .250 |
| Esmark | .95 | .046 | .210 |
| Owens-Illinois | .90 | .080 | .112 |
| Union Carbide | 1.00 | .079 | .056 |
| AT&T | .65 | .104 | .133 |
| Exxon | .85 | .111 | .147 |
| Du Pont | 1.15 | .089 | .145 |
| General Foods | .75 | .059 | .143 |
| Standard Oil of Cal. | 1.05 | .089 | .188 |
| Proctor and Gamble | .70 | .051 | .155 |
| United Technologies | 1.15 | .057 | .250 |
| Texaco | .95 | .107 | .199 |
| Woolworth | .90 | .095 | .220 |
| Kodak | .95 | .045 | .059 |
| Sears | .85 | .068 | .192 |
| American Can | .85 | .104 | .258 |
| American Brands | .65 | .093 | .308 |
| Johns Manville | 1.00 | .073 | .270 |
| Westinghouse | 1.20 | .062 | .256 |
| General Motors | .85 | .052 | .258 |
| Goodyear | .75 | .056 | .249 |
| Average | .95 | .073 | .219 |

*Values are listed in annualized terms.

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