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The Optimal Durability of Fixed Capital When Demand Is Uncertain

This paper considers the impact of demand uncertainty on the desired physical durability of fixed assets. We derive the theoretical relationship between the variability of price in a competitive industry and the desired durability of the industry's fixed capital. The theory implies that durability is a decreasing function of the variability of industry demand. We then test the theory on a cross-section of two-digit SIC code manufacturing industries.

I. Introduction

The purchase of fixed capital represents a commitment of resources long before they will be used. During the typical service life of plant and equipment, firms experience permanent shifts both in technology and in their economic environment, as well as repeated transient shocks. The durability of fixed capital makes it impossible to achieve perfect synchronization between the capital stock and the demand for the services of this stock. Periods of so-called excess capacity are inevitable. The losses which occur from periods of underutilized capacity constitute the cost of using durable resources. We would expect firms to take account of these costs in selecting production methods and designing its fixed plant.

The theory of durability has been elaborated extensively, although little attention has been paid to the interaction between it and demand variability.[1] Parks (1979) develops a general

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1. Such a model of durability abstracts entirely from maintenance. It is most "realistic" in the case of mass-produced equipment, e.g., automobiles, which are cheaper to replace than to renovate extensively.

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model of the service lives of assets as an extension of Terborgh's (1949) seminal work. Feldstein and Rothschild (1974) surveyed theoretical and empirical literature on realized depreciation rates. Durability, however, refers not to the realized service lives or depreciation rates of assets but to their ex ante planned economic lives. It has long been realized that asset durability is controlled by firms and industries. Haavelmo (1959) considers the link between the real interest rate and asset durability. More recently, Auerbach (1979) and Abel (1981) investigate the effects of tax treatment of depreciation on durability in the presence of inflation.

Section II of this paper is devoted to a theoretical analysis of the impact of uncertainty about the demand for capital services on the optimal durability of fixed assets. The general proposition that I shall prove is that when investment is irreversible, demand uncertainty lowers desired durability of assets. This proposition, which can be viewed as an extension of Arrow's (1967) analysis of the short-run dynamics of irreversible investment, leads to testable implications for the correlation between the uncertainty of capital contribution and the average durability of fixed assets. The remainder of the paper is devoted to an analysis of data on a cross-section of manufacturing industries, which provides some evidence in favor of my theoretical prediction.

II. Theory

The physical durability of fixed assets enters the planning and production decisions of the firm in two ways. First, the fact that physical assets are durable imposes a constraint on the firm's ability to disinvest; specifically, it hinders firms in making marginal adjustments to fluctuating demand conditions. This problem is not solved even when the firm can choose the durability of its fixed capital, since it must decide on an optimal durability before demand is known. This problem would be solved if resale markets for plant and equipment were perfect and frictionless and physical assets were general resources, since then capital services would be a perfectly variable input for the firm. In practice, however, durability represents a constraint on the firm to the extent that functional specialization of assets and transactions costs imposes a cost of liquidating fixed capital. This situation is reflected in the theoretical model by assuming that investment in fixed assets is irreversible.

Second, durability enters the firm's planning through the prices of physical assets. Buildings and equipment can be designed and constructed to achieve any degree of durability over a wide range. The more rugged the construction, the more expensive the asset is to produce. This cost increases less than proportionally with service life, however, because part of the cost of an asset is fixed regardless of its durability. The problem facing the firm is to find the optimal trade-off between reducing durability to gain short-run flexibility and raising durability to reduce the long-run average cost of a given level of capital services.

This trade-off can be derived from a simple model of the firm's investment problem. The firm produces a single homogeneous output by combining two homogeneous inputs, capital services and labor. I assume that labor, L, and the capital stock, k, enter a homothetic production function, f, which exhibits diminishing returns to scale. Factor prices, w, and p_K, respectively, are constant over time, although the price of capital goods is a function of durability. The firm is viewed as making a single once-and-for-all choice of the durability of fixed assets. (This means that the model is not a fully dynamic model, because changing structural adaptation is ruled out by assumption.) Product price, p, is a random variable. I assume that price shocks are purely transitory, in the sense that prices in different periods are independent and identically distributed.[2]

Fixed assets are assumed to have a service life of two periods. An asset which produces one unit of capital services in its first period produces $1 - \delta$ units in its second period.[3]

The firm's capital stock in period t is an aggregate of investments in periods t and $t - 1$. Specifically

$$k(t) = i(t) + (1 - \delta)i(t - 1). \tag{1}$$

The price of a new unit of capital, p_K, is a decreasing convex function of δ which is positive when $\delta = 0$.

2. This assumption, which tends to minimize the degree of risk facing the firm, is not unreasonable. In general, a demand shock at the industry level is an aggregate of shocks of different durations. The average duration of the resulting price shock is shorter than that of the demand shock, however, because the industry long-run supply schedule is more elastic than the short-run supply schedule. This suggests that it is appropriate to model the price series as a low-order random process. There is a second reason for doing so. By treating the purchase price of capital goods as constant we have implicitly eliminated adjustment costs of the Lucas type (Lucas 1967). In the absence of adjustment costs, firms adjust fully to shocks whose duration is long relative to the economic life of assets. Given our model of depreciation, this fact also suggests using a low-order process.

3. This model is sufficient for the purposes of this paper because it provides for a carryover stock of fixed capital from one period to the next. The more traditional model, in which capital decays exponentially over a long (theoretically infinite) service life, does not yield any useful implications different from my model of depreciation, but it clutters the mathematical model. A more serious objection to my simple decay model is that it assumes that capital services which are not used are lost forever. That is to say, it assumes that capital goods decay whether or not they are being used. A more realistic view would be that the penalty for having excess capacity is that the firm must wait some time before it eventually recovers the services that it could otherwise have used immediately. While it would be interesting to work through the (more mathematically difficult) model based on this view, the one implication of immediate interest would not be changed.

The firm chooses at each time t all current variables—investment, employment, and output—to solve the following optimization problem:[4]

$$\max_{i(t),L(t)} EV(t) = E\left[\sum_{s=t}^{\infty} \theta^{s-t+1} \{p(s)f[k(s), L(s)] - wL(s) - p_K i(s)\} \,\middle|\, p(t); \right. \tag{2}$$

subject to $k(s) = i(s) + (1 - \delta)\, i(s - 1)$ for all $s \geqslant t$; $i(s) \geqslant 0$ for all $s \geqslant t$; $\{p(t), p(t + 1), \ldots\}$ i.i.d.; $p(s) \sim \Phi(p)$; $i(t - 1)$.

The first constraint is the same as equation (1) above, the second is the irreversibility constraint, and the third is the assumed structure of the price series. The final constraint, which fixes the initial carry-over capital stock, requires more explanation. At time t, investment levels in all previous periods are predetermined. They are not truly exogenous, however, because they are determined as solutions to the firm's optimization problem in previous periods. Since the decay rate, δ, affects the parameters of the period-by-period optimization problem in all periods, the carry-over capital stock for period t is a function of δ. Thus, the final constraint should be understood as asserting that $i(t - 1)$ solves the period $t - 1$ optimization problem.

The objective function which δ optimizes differs from $EV(t)$ in that it sums over the entire history of the firm, which is assumed to begin at time zero. It differs from $EV(t)$ also in that it cannot have an initial, predetermined carry-over capital stock; $i(0)$ must be an endogenous variable. These requirements are satisfied by the following optimization:[5]

$$\max_{\delta, i(0)} E\left(\sum_{t=1}^{\infty} \theta^t \{p(t)f[k^*(t), L^*(t)] - wL^*(t) - p_K i^*(t)\}\right) - C[i(0)], \tag{3}$$

subject to $[k^*(t), L^*(t), i^*(t)]$ is the solution to (3) for all $t \geqslant 1$, $0 \geqslant \delta \geqslant 1$, $i(0) \geqslant 0$.

A useful starting point for our analysis is to derive the optimal decay rate, δ, in the absence of uncertainty. If demand is constant over time, both investment and factor inputs are also constant. The relationship

4. This discount factor, θ, embodies the risk-adjusted cost of capital. Since the riskiness of investment in the firm depends on capital durability, θ is an endogenous variable. One need not rely on investor attitudes toward risk in order to understand the cost of the riskiness of capital durability. By treating θ as an exogenous constant, one implicitly assumes that the firm's cost of capital is the risk-free rate of interest, thus eliminating the effects of investor preferences.

5. The term $C[i(0)]$ in the objective function of this problem needs some explanation. I define $C[i(0)]$ as a cost function of $i(0)$, which leaves the firm indifferent between investing at time 0, before $p(1)$ is known, and investing at time 1. The presence of this term makes it unnecessary to impose an initial stock on the optimization problem. By making $i(0)$ endogenous, we avoid an undesirable transient part of the solution; the firm is in long-run equilibrium from the start.

between the capital stock and investment is then quite simple. An asset which provides k units of service in its first period provides a total of $(2 - \delta)k$ units of service over its lifetime. The service flow which a steady investment flow i yields is, therefore, $(2 - \delta) \cdot i$. Thus, the problem reduces to determining the single optimal investment rate and employment level, as functions of δ, and then to optimize over δ. The optimal investment level maintains the capital stock at the level where the value of the marginal product of capital equals the rental price of capital. When prices are constant over time, the rental price equals C_K, given by

$$C_K = \frac{p_K(\delta)}{1 + \theta(1 - \delta)}. \tag{4}$$

The decay rate enters δ in two ways: p_K depends on δ, and the present value of future replacement investment, $\theta\delta$, also depends on δ. The optimal decay rate is the one which minimizes C_K by trading off savings in initial cost, p_K, for higher replacement investment. The first-order condition for a minimum of C_K yields the following optimality condition for δ:

$$-\frac{dp_K}{d\delta} = \frac{\theta p_K(\delta)}{1 + \theta(1 - \delta)} = \theta C_K(\delta). \tag{5}$$

The second-order condition requires the p_K be convex, as we have assumed. The left-hand side of the equation (5) equals the marginal savings per period, in the form of lower asset prices, from a unit increase in δ. An increase in that component of p_K raises p_K, while leaving the derivative unchanged as a function of δ. Equilibrium is restored at a lower decay rate. Thus, the larger the share of asset prices which does not depend on durability, the greater is the optimal durability of assets. This implication is entirely consistent with the usual rule for inventory policy, which dictates that the reorder quantity is an increasing function of the lumpy reorder cost.

We are now prepared to introduce demand uncertainty into the analysis. The inequality constraint on gross investment introduces shadow prices into the solution. The value of the shadow price in period t, which is denoted $Z(t)$, is the cost of the irreversibility constraint for period t. The first-order conditions for optimal investment and labor input in period t, derived for problem (2), are the following (subscripts of f denote partial derivatives):

$$w = p(t)f_L[k(t), L(t)] \tag{6a}$$

$$\begin{aligned} p_K = {} & p(t)f_K[k(t), L(t)] \\ & + \theta(1 - \delta)E\{p(t + 1)f_K[k(t + 1), L(t + 1)] \mid p(t)\} \\ & + Z(t). \end{aligned} \tag{6b}$$

The sum of the first two right-hand-side terms of equation (6b) equals the demand price of investment at time t. Clearly, when $Z(t)$ is positive, investment is not justified, because then the cost of new assets exceeds their demand price. Positive gross investment occurs when Z is zero.

The capital equation can be converted into a flow equilibrium condition, which relates the value of the marginal product of capital in equilibrium to the rental price of capital. Some simple manipulations, moreover, permit us to express the expectation on the right-hand side of (6b) in terms of expected values of future Z's. We obtain in this way the equilibrium condition

$$p(t)f_K[k(t)], L(t)] = \frac{p_K}{1 + \theta(1 - \delta)} - Z(t) - \sum_{s=t+1}^{\infty} [-\theta(1 - \delta)]^{s-t} E[Z(s)|i(t)]. \tag{7}$$

The first term is the same as the rental price of capital in the static model. The third term is a risk premium on current investment, which is simply the additional income which capital must earn in normal times in order to offset the occasional periods of depressed income. Since on average over time, assets simply earn back at the margin their initial cost, the premium is positive. That is, the infinite summation is negative.[6]

The combination of uncertainty and irreversibility lessens the value of replacement investment. The cost of asset deterioration is equal to the value of the capital services which are lost. In the static case—or when investment is reversible—the value of the marginal product is always equal to C_K. It follows that C_K is the marginal per period benefit from reducing the decay rate. When the irreversibility constraint is binding, however, the lost value of capital services is equal to the marginal product of capital and is less than C_K. The effect of the irreversibility of investment is therefore to lessen the cost of decay and to raise the optimal decay rate, δ. This proposition can be proved formally by differentiating the firm's objective function with respect to δ.

Equation (8) is an equilibrium condition derived by directly differentiating equation (3) with respect to δ. Primes denote derivatives with respect to δ.

6. The right-hand side of equation (7) is an increasing function of investment, $i(t)$, and can be interpreted as the marginal cost of capital services in period t. The "fixedness" of fixed assets depends on the slope of this schedule; the steeper it is the more fixed is capital. Interestingly, the inelastic part of this function corresponds not to rising out-of-pocket production expenses but to a rising marginal risk premium for investment in specialized assets. As a result, it would never appear in accounting records of production "costs."

$$\frac{dW}{d\delta} = E\Bigl(\sum_{t+1}^{\infty} \theta^t\{p(t)\, f_K[k(t),\, L(t)]k'(t) \\ -\, p_K' i(t) \,-\, p_K i'\,(t)\}\Bigr) \,-\, C'i'(0) \,=\, 0. \tag{8}$$

Since W is the expected value of net cash flows, its derivative equals the expectation of the derivative of the future flows. Terms involving the derivative of $L(t)$ drop out, because $L(t)$ is an optimum. The derivatives $k'(t)$ and $i'(t)$ are related by equation (2) and its derivative.

$$k'(t) \,=\, i'(t) \,+\, (1 \,-\, \delta)i'(t \,-\, 1) \,-\, i(t \,-\, 1). \tag{9}$$

Equation (7) contains a risk premium, which we will denote by $R(t)$; (7) then becomes

$$p(t)f_K[k(t)L(t)] \,=\, C_K \,-\, Z(t) \,+\, R(t). \tag{10}$$

Substituting into (8) yields the necessary condition

$$0 \,=\, E\left\{\sum_{t=1}^{\infty} \theta^t[-p_K' i(t) \,-\, \theta C_K i(t) \,+\, R(t)i(t)]\right\}. \tag{11}$$

Solving for p_K', we obtain

$$-p_K' \,=\, \theta C_K \,-\, \frac{1}{\Sigma\theta^t E[i(t)]} \cdot \Sigma\theta^t E[R(t)i(t)]. \tag{12}$$

$R(t)$, which is defined as a conditional expectation given $i(t)$, is an increasing function of $i(t)$. Since the distribution of product prices is stationary over time, the derived random variables $R(t)$ and $i(t)$ are also stationary. It follows that the unconditioned expectations on the right-hand side of (12) are independent of t. Factoring them out of the infinite series leaves only the binomial series in θ in both numerator and denominator. The θ's therefore cancel from the second term, leaving a somewhat simpler expression:

$$-\, p_K' \,=\, \theta C_K \,-\, \frac{E[R(t)I(t)]}{E[i(t)]}\,. \tag{13}$$

The proposition follows by comparing equations (5) and (13). The interpretation of (13) is that the benefits of durability in terms of reduced replacement investment equal θC_K. This return must be discounted by the marginal risk associated with investment when investment is irreversible. $R(t)$ equals the marginal risk in period t associated with investment at time t. The product of R and i, therefore, equals the total cost of the riskiness of current investment. Dividing by the expected value of $i(t)$ transforms this into a cost per unit of fixed capital and thus restates this cost in units which are comparable to C_K and p_K. The link between price—that is, demand—uncertainty and durability exists because the riskiness of investment is greater, the greater is the probability that future investment will be zero.

III. An Empirical Model of Durability and Demand Uncertainty

The foregoing abstract argument establishes that capital asset durability is endogenous and that among other things it depends on demand uncertainty. Only empirical research can establish whether this effect is quantitatively significant. The empirical model which I shall derive in this section represents one approach to this issue. In the process, it must come to grips with a basic identification problem which was not evident previously. By exploiting fully the structure of the model, we are able to derive a single-equation empirical model which solves the problem of simultaneity. Out of necessity, since the necessary data exists only for industries and not for individual firms, it is based on an industry rather than a firm model. The shift of focus from firm to industry simplifies the model in some ways without obscuring the ultimate objective.

For the firm, optimal durability is a function of the variability of product price. Being a competitor, it takes the distribution of price as exogenously given. The stochastic distribution of price is endogenous to the industry, however. By lessening the durability of its fixed plant, the firm secures a flatter marginal cost schedule. The motive for reducing durability is to convert sunk costs into avoidable costs.[7] If all firms in an industry choose a less durable fixed plant, the industry supply schedule becomes more elastic, and price variability is reduced. This fact causes a problem of simultaneity in an industry cross-section.

Optimal durability and price variability for an industry are functions of two exogenous factors: variability of industry demand and the cost function of durability as an attribute of fixed assets. If we consider two industries which differ in the uncertainty of demand, but which face equal cost functions of assets, the industry whose demand is more uncertain will have more variable prices and less durable capital. This, of course, is the essential implication of my theory, and it is the proposition we wish to test. If the two industries faced different cost functions of assets, however, the direction of the association between price variability and durability cannot be predicted. The industry which finds durability of assets to be a relatively cheap attribute to purchase may adopt highly durable fixed capital, even though it must then live with the greater price uncertainty which results. If the exogenous factors could be quantified and measured, this system could be modeled and

7. The adaptation of capital durability is only one part of the broader phenomenon. Stigler (1939) argued that firms adopt production methods which confer output flexibility, depending on the variability of their demand. Fuss and McFadden (1978) model the implications for the choice of a production function; Orr (1967) considers the implications for inventory policy and for a "base" vs. "peak load" mix of capacity. Sharkey (1977) and Sheshinski and Drèze (1976) have derived further results for the optimal mix of production facilities.

estimated by standard simultaneous equations techniques, but data on the shapes of asset price functions are not available.

The association between the durability of fixed plant and the variability of capital contribution overcomes this handicap, although the simple correlation between durability and variability of capital contribution suffers from the same identification problem.[8] The difference arises from an additional identifying restriction on the derived demand for fixed assets: the derived demand is inelastic.[9]

Consider a competitive industry which consists of firms like the one modeled in the preceding section. The individual short-run derived demands aggregate to an industry-wide derived demand for fixed capital. Industry capital stock in period t, $K(t)$, is a function of the rental price of assets, $\rho(t)$, the wages of other factors, and the level of industry demand. $\rho(t)$ entered the theoretical model of the firm as the right-hand side of equation (7). Assume that, in the aggregate, the derived demand for capital can be expressed in the constant elasticity form,

$$K(t) = K^D[\rho(t)] = A\rho(t)^{-\epsilon^D}e^{\eta(t)}. \tag{14}$$

The shock series $[\eta(t)]$ is a stationary white noise process which is derived from the industry demand shocks. Denote the variance of $\eta(t)$ by σ^2. ϵ^D, the own-price elasticity of industry demand for capital, is a function of the within-industry distribution of firm-level production functions and of the wage rate of labor. ϵ^D is, however, assumed to be independent of K.

Equation (7) states that the marginal cost of capital at time t, which the firm equates to the rental price of capital, contains a risk premium which depends on the size of the capital stock. This condition can be thought of as being a short-run supply schedule of capital services to the firm. For the present we are only interested in the fact that these firm-level supply schedules can be aggregated across the industry to yield an aggregate supply schedule of capital to the industry. Assume that the aggregate schedule takes the constant-elasticity form

$$K(t) = K^S[\rho(t)] = B\rho(t)^{\epsilon^S}. \tag{15}$$

The elasticity of short-run supply, ϵ^S, is a function of the durability of fixed assets. The faster assets decay, the flatter (more elastic) the individual supply schedules are. In the aggregate, then, ϵ^S is an increasing function of the decay rate of fixed assets. In terms of the model, we can represent ϵ^S as a function,

$$\epsilon^S = \epsilon(\delta), \qquad \epsilon' > 0. \tag{16}$$

8. Capital contribution by definition equals the residual income of the owners of capital after all hired factors and materials have been paid.

9. The fact that capital services are a relatively small share of total cost implies that their derived demand is inelastic. Numerous econometric studies (e.g., Berndt and Wood 1975; Griffin and Gregory 1976) find this to be the case.

The model of industry short-run behavior consists of the preceding three equations—the demand and supply of fixed assets in the short run and the technical relationship which determines ϵ^S in terms of δ—and a fourth relationship which determines (optimal) durability. The optimal service life of fixed assets is a function of exogenous endowments which determine the shape of the cost function $p_K(\delta)$ and of the variance of demand shocks, σ^2.

$$\delta = \delta[p_K(\delta), \sigma].^{10} \tag{17}$$

The behavior of capital contribution, as a function of ϵ^D, σ, and functions (16) and (17), can be derived from this system of equations. Let σ_c denote the standard deviation of the first differences of logarithms of contribution.

The total derivative of σ_c with respect to δ, allowing both σ and ϵ^S to vary, is ambiguous in sign. Differentiating (21) with respect to δ yields

$$\begin{aligned}\frac{d\sigma_c}{d\delta} = &-\frac{(1-\epsilon^D)}{(\epsilon^D+\epsilon^S)^2}\cdot\sigma\cdot\frac{\partial\epsilon^2}{\partial\delta}\\ &+\frac{(1+\epsilon^S)}{(\epsilon^D+\epsilon^S)}\cdot\frac{1}{d\delta/d\sigma}\lesseqgtr 0.\end{aligned} \tag{18}$$

The first term is negative, because the partial derivative, $\partial\epsilon^S/\partial\delta$, is positive and $1 - \epsilon^D$ is positive. The second term is positive because the derivative of δ with respect to σ is positive.

While this derivative is ambiguous, the second derivative of σ_C is unambiguously positive, because the second term increasingly dominates the first term as δ gets larger. This claim is easily proved from (18). Let δ rise. The elasticity of supply of capital approaches infinity; capital becomes increasingly a variable factor of product. Then the effect of endowed durability becomes less significant, because the fraction $(1 - \epsilon^D)/(\epsilon^D + \epsilon^S)$ approaches zero. The second term approaches the reciprocal of $d\delta/d\sigma$, which is positive. Thus, as δ increases, the right-hand side of (18) is increasingly dominated by its second, positive term.

10. The theoretical model implies that the durability of each kind of fixed asset is determined by a relation like eq. (17). Aggregating over a heterogeneous universe of capital goods modifies the relationship because the firm can substitute one kind of asset for another. Given several kinds of assets, each with its own price structure, the firm will prefer a production method which utilizes durable assets if demand is stable. The more its demand fluctuates, the more attractive is a production method which uses short-term fixed assets. Thus, the average sensitivity of durability of an aggregate capital stock to demand uncertainty is greater than that of the "average" subspecies of fixed assets. My rather rough, macroscopic empirical tests are not able to separate the effects of shifting the durability of specific kinds of assets from the effects of shifting the average durability of the capital stock by capital/capital substitution.

The first and second derivatives of σ_c with respect to δ can be modeled econometrically by a simple quadratic polynomial in δ:

$$\sigma_c = \beta_0 + \beta_1\delta + \beta_2\delta^2; \qquad \beta_2 > 0. \tag{19}$$

As $2\beta_2$ equals the second derivative of σ_c with respect to δ, it is predicted to be positive. The first derivative is not a constant and therefore is not unambiguously signed. Differentiating equation (19) with respect to δ yields

$$\frac{d\sigma_c}{d\delta} = \beta_1 + 2\beta_2\delta \gtrless 0. \tag{20}$$

Either sign for β_1 is consistent with the theory. The theory predicts that ϵ^S is an increasing function of δ. If the derivative of ϵ^S with respect to δ is large enough, β_1 will be positive.

IV. Empirical Test

Equation (19) can be interpreted as a regression model, with the addition of a residual term which accounts for other factors—the price elasticity of industry demand, for example—which have an impact on the variability of capital contribution. This section presents regression estimates of the parameters of equations (19) and (20) obtained from a cross-section of manufacturing industries. The parameter estimates are summarized in table 1. In this section I shall discuss first the data on which the regressions are based and then interpret the parameter estimates.

The theoretical model only calls for two variables—one which measures capital durability and the other which measures the variability of capital contribution. My measure of δ was constructed from Census of Manufactures and Annual Survey of Manufactures data, using the published investment and gross book value series. As a result, it is as free as possible of accounting practices, and of the IRS guidelines in particular. As nearly as possible, it reflects the actual scrappage practices of firms over a 13-year period, 1967–74. This yields a variable, denoted N, which is a measure of the mean service life of fixed assets employed by each industry in the sample. The reciprocal of N, denoted DPN, is a proxy for the depreciation rate. Details of the construction of this variable and its stochastic properties appear in Gibbons (1984).

This estimation procedure was carried out for 19 industries: SIC 20 to SIC 36, SIC 38, and SIC 371. The unevenness of the available data made it impossible to estimate N at a more disaggregated level. Aggregation to the two-digit SIC code level severely limits the number of observations, and it forces a high degree of aggregation on the notion of the industry capital stock. Our data set has some important advantages

TABLE 1 Regressions of *SC* and *CVC* on Durability

Variable	1	2	3	4
Constant	63.9	78.7	55.2	48.8
	(3.14)	(3.53)	(2.82)	(2.98)
DPN	−21.6	. . .	−17.6	. . .
	(2.96)**		(2.51)*	
DPN^2	2.05	. . .	1.43	. . .
	(3.19)**		(2.32)*	
N	. . .	−7.03	. . .	−5.32
		(−3.04)**		(3.13)**
N^2	. . .	.171	. . .	.148
		(2.93)**		(3.46)**
$\bar{R}^2$	.4700	.3841	.3534	.5615

NOTE.—*t*-ratios in parentheses; 16 degrees of freedom.
* Significant at the 5% level.
** Significant at the 1% level.

over three-digit or four-digit SIC code industry, however. First, our sample comprises nearly the entire manufacturing sector. Data availability at a three- or four-digit SIC code level would limit us to subsample of manufacturing. Second, the census data appear to contain a great deal of random measurement error, which would almost surely be greater for more disaggregated data.[11]

My proxy for the variability of capital contribution is based on annual time series of industry value added and payrolls for the period 1958–72. I first defined a measure of capital contribution in year t. As reported in the Annual Survey of Manufactures, C_{it} equals value added minus payroll of industry i in year t. My measure of the variability of capital contribution for industry i, SC_i, equals the standard deviation of first differences of logarithms of the C_{it} over the period 1958–72. Since the survey data are based on costs at manufacturing plants, payroll does not include wages of employees who are not assigned to manufacturing plants. Thus C_{it} contains some wages of managerial employees. In addition, it contains income on other assets—inventory and patent rights, for example—as well as income on investment in fixed assets. If these components of income are highly correlated, as one would ex-

11. A major source of noise is year-to-year changes in the assignments of plants to four-digit industries. When a plant "moves," gross book value of the losing industry is reduced and gross book value of the recipient is raised, without any corresponding adjustment of their (previously reported) gross investment series. The amount of noise which is present in even the two-digit level data is disturbing. On occasion, the gross book value of a two-digit industry increases in 1 year by more than reported gross investment. This paradoxical event could occur because of changes in the composition of the industry, caused by moving firms from one two-digit industry to another. It sometimes occurs, moreover, in noncensus years, when the definitions of industries remain unchanged. A further difficulty is that the census data are incomplete at the four-digit level. Existing data permit one to estimate asset horizon for only about 30 four-digit industries.

pect since they are all incomes on industry, specialized capital, SC_i, is a good proxy from the variability of each one. I also experimented with a modified form of the variable SC_i, called CVC_i. The modified variable CVC_i equals SC_i divided by the growth rate of C_{it}. This procedure controls for the part of the variability of contribution which is related to the growth rate.

The preceding variables are the only ones required by my highly abstract theoretical model. The dependent variable is a function of many other variables as well, however. It would be desirable to estimate a more complete model which controls for other effects, in order to eliminate possible sources of spurious correlation. The small number of degrees of freedom, however, discourages any major attempt to introduce other variables.[12]

The results of several alternative formulations of the regressions model are presented in the table. Columns 1 and 2 are models which have SC_i, as the dependent variable, columns 3 and 4 have CVC_i as their dependent variable. Column 1 exactly corresponds to equation (19) with a residual term added:

$$SC_i = \beta_0 + \beta_1 DPN_i + \beta_2 DPN_i^2 + V_i. \tag{21}$$

The theory predicts that β_2 is positive. Column 2 corresponds to an alternative functional form, in which DPN_i is replaced by its reciprocal, N_i, which is in units of a service life of assets. The regressions take the form

$$SC_i = \gamma_0 + \gamma_1 N_i + \gamma_2 N_i^2 + U_i. \tag{22}$$

The prediction that β_2 is positive corresponds, under this change of variables, to the prediction that γ_1 is negative. Because of the nature of

12. Besides the regressions reported in the table, I estimated other regressions which include other variables. In no case were these regressions appreciably different from those reported. The two additional variables they contained were (1) the share of value added in industry revenues, which is a proxy for the elasticity of derived demand for capital, and (2) the growth rate of industry gross investment. The growth rate of investment may be correlated with measurement error in measured durability: if the survivor curve of asset has a large variance, *DPN* may have a spurious correlation with the growth rate of investment, although the sign of the correlation depends on the exact shape of the investment path. Neither of these variables changes the coefficients of durability or their significance level, however. The following are typical of these regressions: G equals the growth rate of the capital expenditures over the period 1958–73. When we add it to the regression in col. 1 of the table, we obtain

$$SC = \text{constant} \underset{(-2.85)}{-\ 21.9\, DPN} \underset{(3.08)}{+\ 2.07\, DPN^2} \underset{(0.2)}{+\ .071 G} + U.$$

SHM equals one minus the ratio of value added to sales. When we add *SHM* to the col. 1 regression we obtain

$$SC = \text{constant} \underset{(-2.85)}{-\ 20.5\, DPN} \underset{(3.08)}{+\ 1.96\, DPN^2} \underset{(0.2)}{+\ .115\, SHM} + U,$$

with the t-ratios in parentheses.

the transformation, γ_2 and β_1 should also have opposite signs. These predictions are all strongly supported by my regression estimates. Despite the small number of degrees of freedom, every coefficient is significant at the 5% level, and most are significant at the 1% level; *t*-ratios are reported in parentheses. The quadratic coefficient of *DPN* and the linear coefficient of *N* are significant at the 1% level in three of the four regressions. The signs of the linear coefficient of *DPN* and the quadratic coefficient of *N* also tend to reinforce my interpretation of the data. Although the theory does not predict a sign for β_1, it does predict that β_1 will be negative only if the induced effect—which runs from demand uncertainty to durability—dominates the cross-industry variation in durability. The observed pattern of signs is the one which would occur, for example, if all industries face the same cost function of durability. Without arguing for this extreme view of durability, it seems reasonable to conclude that demand uncertainty is not a minor factor in the choice of optimal durability.

V. Conclusion

The evidence considered here does not yield anything like conclusive proof of the proposition that firms adapt the durability of the fixed capital to their uncertainty about demand. The main deficiency of the evidence, as I have already mentioned, is that the census data afford too few observations to permit estimation of a more complete structural model, controlling for spurious relationships. A larger model, perhaps based on individual firm data, would have another, more important advantage. Many aspects of the firm's activities which are ordinarily taken to be fixed parameters are, like asset durability, endogenous to demand uncertainty. A larger model (i.e., more data) would make it possible to investigate several such factors simultaneously. Such a study, implementing Stigler's thesis about induced flexibility, has not been attempted despite the importance of the subject and the length of time which has passed since his original theoretical paper appeared.

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