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The Value of Corporate Debt with a Sinking-Fund Provision*

Most corporate debt issues contain a sinking-fund provision which provides for periodic retirement of a proportion of the issue prior to maturity. Retirement may be achieved either through a call at a specified price or through market purchases. This paper considers the value of a sinking-fund provision in the context of a contingent claims valuation model. It is concluded that the value of the sinking fund is determined by the risk of the underlying firm, the initial yield to maturity of the issue, and the sinking fund's redemption rate. The value of sinking-fund debt is calculated over a range of parameter values, and the conditions under which the sinking-fund provision is valueless are determined.

I. Introduction

A systematic investigation into the effects of sinking-fund provisions is important considering the preponderance of these provisions in corporate bond indentures. Approximately 80% of all publicly traded corporate issues contain a sinking-fund feature amortizing on average more than 50% of the principal amount of these issues. Thompson and Norgaard (1967) find these proportions to be stable over the 1960–67 period. McKeon (1980) reports 78.6% of all industrial bonds outstanding in 1979 contained a sinking-fund provision. Furthermore, sinking-fund provisions are a particularly common occurrence in medium- and low-grade issues but are less prevalent in high-grade issues. This relationship is most striking in utility issues. Thompson and Norgaard report that only 30% of Aa rated utilities issued in 1963 contained sinking-fund provisions whereas 100% of Baa rated new issues contained such a provision.

A sinking-fund provision obligates the firm to amortize a portion of the debt prior to maturity. It gives the firm the option to satisfy this amortization requirement by either purchasing the bonds to be redeemed from the market or calling the bonds by means of a sinking-fund call. Thus,

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the equity of a firm with an outstanding sinking-fund obligation can be viewed as a compound option on the value of the firm. On each date of the sinking-fund obligation, the stockholders have the option of purchasing the next option by satisfying the sinking-fund obligation or forfeiting the firm to the bondholders. The final stockholder option is to pay the principal due to the outstanding debt at maturity. That is, the first option can be exercised at either a predetermined exercise price (usually par) or a function of the value of the firm (the market value of the debt to be redeemed). The choice of exercise price is referred to in this paper as the delivery option associated with the sinking-fund obligation.

Studies of sinking-fund issues have emphasized the amortization feature and ignored the delivery option (Jen and Wert 1966; Geske 1977; Dyl and Joehnk 1979).[1] In particular they treat the sinking-fund provision as if the firm were required to retire the requisite proportion of the debt by a sinking-fund call. This makes a sinking-fund issue identical to a serial bond with the same amortization requirement, time to final maturity, and promised payment (Jen and Wert 1966). Hereafter this bond is called the *corresponding serial bond*.

The paper constitutes an extension of the literature on risky sinking-fund debt. Using contingent claims theory, the paper derives an analytical model of sinking-fund debt and the delivery option.[2] The value of a sinking-fund bond is found to be less than or equal to the value of the corresponding serial bond. This difference is the value of the delivery option. Furthermore, the value of the delivery option depends on the risk of the firm, the issue's amortization rate, and the initial yield to maturity. Simulation and empirical results indicate that the delivery option can have a substantial impact on the market value of sinking-fund debt.

Since the paper abstracts from interest rate uncertainty, there are conditions which render the delivery option valueless. These conditions can be determined at the time of issue. When they are satisfied, a sinking-fund bond always behaves as if it were a corresponding serial bond. The conditions for a valueless delivery option are more likely to be satisfied by issues which are of low quality at the time of issue. Debt which is high grade at the time of issue tends to carry a valuable delivery option. That is, the delivery option protects the issuer from default on originally high-quality issues which undergo unexpected deterioration in quality. However, the delivery option of issues which are low-quality at the time of issue offers no such protection.

1. A notable exception is Thompson and Norgaard (1967).

2. Consistent with recent contingent claims models of risky corporate debt, we assume the interest rate known with certainty over the life of the bond (Black and Scholes 1973; Merton 1974; Black and Cox 1976; Ingersoll 1977; Geske 1979; Ho and Singer 1982).

The investigation proceeds in the following sequence. Section II presents the model's underlying assumptions. Sinking-fund debt is valued as a contingent claim on the value of the firm and its comparative statics are presented. The value of sinking-fund debt is then compared to the value of a corresponding serial bond. Section III examines the delivery option. Analytic and simulation results are presented. Finally the conditions which render the delivery option valueless are derived. Section IV considers the impact of alternative financing rules on the value of a sinking-fund bond. The results are summarized and some overall conclusions are contained in Section V.

II. Bonds with a Sinking-Fund Provision

This section presents the basic assumptions underlying the model and the provisions of a sinking fund. A contingent claims valuation model for sinking-fund debt is derived, and the value and behavior of sinking-fund debt are analyzed and compared to the corresponding serial bond.

A. A Model of Sinking-Fund Debt

The assumptions employed are given below.

1. There are no transaction costs, taxes, or other frictions, and all participants have equal access to information.
2. The term structure of interest rates is flat and nonstochastic. The instantaneous compounding rate of interest is r.
3. The firm's capital structure consists of equity and a single debt instrument.
4. Throughout the life of the bond the firm's investment decision is known with certainty; the firm pays no dividends and makes no other distributions to stockholders.
5. Default occurs if the firm fails to satisfy the contractual obligations as set forth in the bond indenture. In default, ownership of the firm's assets is costlessly transferred to bondholders, and stockholders lose all claims to these assets.
6. The firm makes payments to bondholders only at maturity and at the time of a single sinking-fund obligation.
7. The sinking fund requires retirement of a given proportion of the principal amount of the debt at a specific time. The firm retains the right to redeem the debt either by way of a sinking-fund call or by purchasing the bonds from the market.
8. The sinking-fund payment is financed with new equity.

The perfect-market assumption 1 assures that the value of the firm is independent of its capital structure. Assumption 2 abstracts from term structure problems so that the firm's financial liabilities are affected by only one source of uncertainty, the evolution of the firm's asset value over time. Assumptions 3 and 4 are sufficient to assure against an unanticipated expropriation of wealth from bondholders to stockholders. Assumptions 5–7 describe the important elements of a bond indenture containing a sinking-fund provision. Assumption 8 preserves the market value of the firm at the time of the sinking fund.

These assumptions describe the amount and timing of the payment to the bondholders. Notice that a bond with a single sinking-fund obligation can be viewed as a requirement to make payments to bondholders at two points in time: at maturity and at the time of the sinking-fund obligation. The promised payment at maturity is determined by the debt outstanding at the time, and it is known at the time of issue. On the other hand, the payment required to satisfy the sinking-fund obligation depends on the value of the firm at the time the obligation must be satisfied. This payment, in general, is not known ex ante.[3]

B. The Payments to the Sinking Fund

The payments to a sinking fund depend on the demand and supply prices of the bonds at the time of the sinking-fund obligation. This section derives demand and supply functions for the bonds to be redeemed and describes the payments to the sinking-fund obligation as a function of the firm's value at the time of the sinking-fund obligation. These results are used later to derive an analytical model of sinking-fund debt.

The following notation is employed:

$V(t)$ is the value of the firm at t, the time of the sinking-fund obligation.

F is the total face value of the bonds outstanding on the maturity date.

sF is the face value of the bonds to be redeemed by the sinking fund, where s is the redemption ratio, defined as the ratio of the face value of the debt to be redeemed by the sinking fund, to the face value of the remaining debt.[4]

τ is the time to maturity of the bond from the time of the sinking-fund obligation.

$B[V(t); F, \tau]$ is the value of the remaining debt once the sinking-fund obligation has been satisfied. It is a discount bond with face value F and time to maturity τ.[5]

c is the initial yield to maturity of the bond issue.[6]

3. It is assumed throughout that there is only one sinking-fund obligation. This is a simplifying assumption. The model could be extended to permit multiple sinking-fund obligations without affecting the results.

4. This terminology differs somewhat from that generally found in bond indentures. Indentures define the face value of the debt as the face value at the time of issue. In the notation of this paper, this face value is $(1 + s)F$.

5. See Merton (1974) for a study of the value and behavior of this bond.

6. It is assumed that the bonds are initially issued at a discount so that c is the solution to $B(0) = (1 + s)Fe^{-cT}$, where T is the time to maturity from the date of issue and $B(0)$ is its issue price. Thus par at t is $(1 + s)Fe^{-c(T-t)}$. In particular, if $\tau = T - t$, par is equal to $(1 + s)Fe^{-c\tau}$.

$sFe^{-c\tau}$ is the par value of the debt to be redeemed.[7]

$D(V(t); F, \tau) = V(t) - B(V(t); F, \tau)$ is the market value of the firm's equity (old plus new shares) just after the sinking-fund obligation is satisfied. It is the value of a call on the underlying value of the firm, with exercise price F and time to expiration τ (Merton 1974).

Define the bond's supply price as the minimum amount the firm must pay to satisfy the sinking-fund obligation. In competitive markets, bondholders offer their bonds to the firm for redemption only if the price for each bond is equal to (or greater than) the price that will obtain for the unredeemed bonds once the sinking-fund obligation is satisfied. Otherwise the firm redeems the bonds at par through a sinking-fund call. Since the value of the unredeemed bonds is $B[V(t); F, \tau]$, the supply price, as a function of the value of the firm at the time of the sinking-fund obligations $S[V(t); F, c, \tau, s]$ is given by[8]

$$S[V(t); F, c, \tau, s] - s \min \{Fe^{-c\tau}, B[V(t); F, \tau]\} = 0.$$

Define the demand price as the maximum amount the firm is willing to pay to satisfy the sinking-fund obligation. The firm satisfies the sinking-fund obligation only if the value of the equity (old plus new) once the sinking-fund obligation is satisfied is not less than the payment to the sinking fund. Therefore the demand price is the value of the equity, once the sinking fund is satisfied. That is,

$$D[V(t); F, \tau] = V(t) - B[V(t); F, \tau].$$

Given the parameters describing the bond indenture (F, τ, c, s), the supply and demand prices are increasing functions of the value of the firm at the time of the sinking-fund obligation. At that time, the firm has the option to default on the sinking-fund obligation, call the bonds for the sinking fund, or purchase the bonds in the open market. Default occurs if the demand price is less than the supply price. Therefore the point of indifference between defaulting or not, the bankruptcy point, is given by V_b, the solution to

$$D[V(t); F, \tau] - S[V(t); F, c, \tau, s] = 0. \tag{1}$$

If the issuer does not default on the sinking-fund obligation, it redeems the debt for the sinking fund in the most economical manner. When each bond's market value is equal to par it is indifferent between

7. Par is typically defined as the face value of the bond. This is because a coupon bond selling at par will earn the stated coupon rate if it is held to maturity and the firm does not default. In this paper, since no coupon is paid, par is defined so that a bond selling at par earns the initial yield to maturity if the firm does not default and the bond is held to maturity.

8. Let Z be the market value of the total issue just prior to the operation of the sinking fund. Since $\alpha = s/(1 + s)$ is the proportion of the debt which must be redeemed, the unredeemed debt is $(1 - \alpha)Z = B[V(t); F, \tau]$ and $Z = (1 - \alpha)^{-1}B(V(t); F, \tau)$. Thus, $\alpha Z = sB[V(t); F, \tau]$.

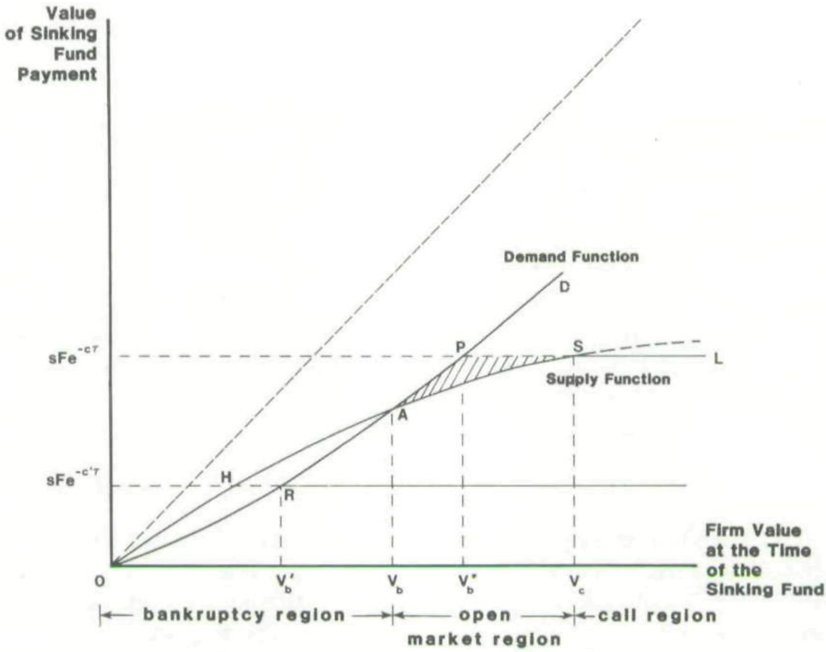

FIG. 1.—Supply and demand functions for bonds to be redeemed at the time of the sinking-fund obligation.

satisfying the obligation by a call or by open-market purchases. This point of indifference, the call point, is given by V_c, the solution to

$$Fe^{-c\tau} - B[V(t); F, \tau] = 0. \tag{2}$$

The supply and demand functions are presented in figure 1. Curve segment *OHASL* represents the supply function and curve *ORAD* represents the demand function. Their intersection gives the bankruptcy point V_b. The call point V_c is the minimum firm value for which the supply price equals par. These two points give the limits to three distinct regions describing the firm's behavior. Firm values to the left of V_b represent the bankruptcy region, in which the firm defaults on the sinking-fund obligation. The region between V_b and V_c is the open-market region, where the sinking-fund obligation is satisfied with open-market purchases. Finally, the call region located to the right of V_c represents firm values where the sinking-fund obligation is satisfied by a sinking-fund call.[9]

C. The Payments to the Serial Bond and the Delivery Option

The behavior of the issuer of a corresponding serial bond is derived in a similar manner. The corresponding serial bond is identical to the sink-

9. The figures represents the case when $V_b < V_c$. The implications of a violation of this inequality are discussed in Sec. II*C*.

ing fund bond but without the delivery option. Both bonds have the same face value and maturity. Therefore the value of the unredeemed serial bond is $B[V(t); F, \tau]$ and the demand function is identical to the demand function for the sinking-fund bond, $D[V(t); F, \tau]$. However, since the serial bond has no delivery option its supply function is $sFe^{-c\tau}$ for all values of $V(t)$. It follows that the corresponding serial bond's bankruptcy point is V_b^*, the solution to equation (3).

$$D[V(t); F, \tau] = sFe^{-c\tau}. \tag{3}$$

In figure 1, the serial bond's supply function is represented by the horizontal line $sFe^{-c\tau}$. The serial bonds' bankruptcy point is at V_b^*. For firm values less than V_b^*, the issuer defaults on the serial bond obligation.

It is now possible to describe the delivery option. By virtue of the delivery option, the payments to the sinking-fund bond are less than or equal to the payments to the corresponding serial bond for all firm values. In the bankruptcy region the firm defaults on both the serial obligation and the sinking-fund obligation. Bondholders take over the firm and the payments to the two issues would be the same. In the call region the holders of either issue are paid par, $sFe^{-c\tau}$. However, in the open-market region, the sinking-fund bondholders receive less than the corresponding serial bondholders. For firm values between V_b and V_b^*, the firm defaults on the serial obligation and the maturing bonds are paid $D[V(t); F, \tau] = V(t) - B[V(t); F, \tau]$. In contrast, the sinking-fund bondholders are paid $sB[V(t); F, \tau]$, less than $D[V(t); F, \tau]$. For firm values between V_b^* and V_c the serial bondholders are paid par but the sinking-fund bondholders receive $sB[V(t); F, \tau]$, less than par. The difference between these payments, represented by the shaded area APS in figure 1, is due to the sinking-fund delivery option.

Note that the payment differential resulting from the delivery option depends on the location and curvature of the demand and supply functions. In fact it is possible for the sinking-fund obligation to contain no delivery option. This occurs when the demand function intersects the supply function at a firm value exceeding the call point—that is, when the market value of each bond exceeds par whenever the sinking-fund obligation is satisfied. In this case the firm never satisfies the sinking fund obligation with open-market purchases, there is no open-market region, and the payment to the sinking fund bond is identical to the corresponding serial bond for all firm values.[10] For example, if the par value of the debt to be redeemed were $sFe^{-c'\tau}$ in figure 1, the supply function would be represented by curve segments OHM. The bankruptcy point for both the serial bond and the sinking-fund bond would

10. The specific condition for a valueless delivery option is derived in Sec. III*B*. This depends on the redemption ratio, the risk of the firm, and the (nonstochastic) riskless rate of interest as well as the initial yield to maturity.

be at V_b' and the payments to the two issues would be the same for all values of $V(t)$.[11]

The existence of a valueless delivery option depends on the assumption of a nonstochastic risk-free interest rate and on the level of c, the initial yield to maturity of the debt. When the sinking-fund debt is rated low at the time of issue (i.e., c is high), the conditions for a valueless delivery option are likely to be satisfied. In this case the delivery option does not offer protection from default. On the other hand, debt which is high grade at the time of issue (i.e., when c is close to the riskless rate) will tend to carry a delivery option with some value.

This analysis leads to some interesting observations. First, the current value of the sinking-fund bond is less than or equal to that of the serial bond. In the open-market region the payment to the sinking-fund bond is strictly less than the payment to the serial bond. Outside of this region (and at maturity) the payments to the two bonds are the same. Thus the current value of a sinking-fund bond will differ from that of its corresponding serial bond by the present value of the delivery option (area ASP in fig. 1).

Second, some sinking-fund bonds do not have an open-market region. This occurs if the call point (determined by [2]) does not exceed the bankruptcy point (determined by [1]). That is, $V_c \leq V_b = V_b^*$. When this condition holds, the delivery option is valueless and the sinking-fund bond could be valued as if it were an equivalent serial bond. The payments to the two issues are the same for all possible values of the firm. Hence, the ex ante values of the two issues are always the same.

Third, when the delivery option has value, the probability of default on the sinking-fund obligation is less than that on the serial obligation. For firm values between V_b and V_b^*, the firm defaults on the serial bond but escapes default on the sinking-fund bond. Over this range, the par value of the sinking-fund debt exceeds the market value of the equity as well as the market value of the debt. The firm is not willing to satisfy the serial obligation. However, by exercising the delivery option the firm will satisfy the sinking-fund obligation through open-market purchases.

D. A Contingent Claims Model of Sinking-Fund Debt

The analysis so far provides some insight into the value of a sinking-fund obligation and the delivery option. In this section a contingent

11. Notice that the par value of the debt decreases as the coupon rate increases. However, the market values of the unredeemed bonds, $B[V(t); F, \tau]$, and thus the value of the equity once the sinking fund has been satisfied, are not affected by changes in the coupon rate. Therefore the supply function is affected only to the extent that the call region expands, increasing the probability of a sinking-fund call and reducing the probability of open-market purchases.

claims valuation equation is derived by imposing additional structure on the dynamics of firm value and the nature of capital markets. This permits explicit consideration of the determinants of the value and relative risk of a sinking-fund bond. In addition to the assumptions specified above, it is also assumed that trading can take place continuously and the firm's market value follows the Itô process given by $dV/V = \alpha dt + \sigma dw$, where α is the constant instantaneous expected rate of return of the firm, σ is its constant instantaneous standard deviation, and dw is the standard Gauss-Wiener process.

With these assumptions, standard arbitrage arguments lead to a valuation equation in terms of the value of the firm. Adapting the Cox-Ross-Rubinstein (1979) methodology, the value of the debt can be expressed as the present value, in a risk-neutral world, of the payments to the debt.[12] This gives the value of the debt, conditional on the current value of the firm and the stochastic process generating firm value as

$$D = VN(x_b - \sigma\sqrt{t}) + e^{-rt}\int_{x_b}^{\infty} f(x)B(x)dx + se^{-rt}\int_{x_b}^{x_c} f(x)B(x)dx + N(-x_c)sFe^{-c\tau-rt}. \tag{4}$$

The parameters x_b and x_c are the solutions to (5) and (6), respectively.

$$x_b = \frac{ln\left[\dfrac{(1+s)B(x_b)}{Ve^{rt}}\right]}{\sigma\sqrt{t}} + \tfrac{1}{2}\sigma\sqrt{t}, \tag{5}$$

$$B(x_c) = Fe^{-c\tau}. \tag{6}$$

$B(x)$ is a function defined as

$$B(x) = N(y - \sigma\sqrt{\tau})Ve^{-1/2\sigma^2 t + \sigma\sqrt{t}x + rt} + N(-y)Fe^{-r\tau}, \tag{7}$$

where

$$y = \left(\ln\left[\frac{Fe^{-rT}}{V}\right] + \tfrac{1}{2}\sigma^2 T - x\sigma\sqrt{t}\right)/\sigma\sqrt{\tau}, \tag{8}$$

t is the time to the operation of the sinking fund, T is the time to maturity of the debt issue, and $\tau = T - t$, the time to maturity from the time of operation of the sinking fund. The standard normal probability density function is $f(s) = (1/\sqrt{2}\pi)\, e^{-1/2s^2}$; $N(z) = \int_{-\infty}^{z} f(s)ds$ is the standard normal distribution function, r is the instantaneous riskless rate, and $x = \{\ln[V(t)/V] - (r - \tfrac{1}{2}\sigma^2)t\}/\sigma\sqrt{t}$ is a standardized normal state variable representing the (stochastic) value of the firm at t, the time of the sinking-fund obligation.

12. See the Appendix for a derivation.

Equation (4) describes the value of the debt as a function of the provisions of the bond indenture as well as the value and risk of the underlying firm. The parameters, x_b determined by equation (5), and y determined by equation (8), give the bankruptcy points at t and T, respectively. $N(x_b)$ and $N(y)$ would be the respective probabilities of bankruptcy in a risk-neutral world. The parameter x_c is determined by (6) and gives the call point for the sinking-fund provision. Finally, $B(x)$ is the market value of the unredeemed portion of the debt issue just after the operation of the sinking fund (assuming the firm successfully satisfied the sinking-fund obligation). $B(x)$ is determined by (7) and is equal to the market value of a discount bond of maturity τ with contractual obligation F at maturity, conditional on x, the state variable at t.

Given the assumed dynamics of the value of the firm over time, the parameters x_b and x_c and the state variable x are related to the state variable $V(t)$ and the parameters V_b and V_c in the previous section. Specifically, the state variable representing the bankruptcy point x_b can be expressed as

$$x_b = [\ln(V_b/V) - (r - \tfrac{1}{2}\sigma^2)t]/\sigma\sqrt{t}.$$

Similarly, the call point x_c can be expressed as

$$x_c = [\ln(V_c/V) - (r - \tfrac{1}{2}\sigma^2)t]/\sigma\sqrt{t}.$$

The terms in equation (4) lend themselves to a straightforward intuitive interpretation. The current value of the debt is the present value of four claims associated with the debt issue. The first term is the present value of the bondholder's claim if the firm defaults on the sinking-fund obligation. The second term is the present value of the claims of the unredeemed bondholders if the firm satisfies the sinking-fund obligation. The last two terms are the present values of the redeemed bondholders' claims if the firm satisfies the sinking fund through open-market purchases and through a call, respectively.

The comparative statics of the sinking-fund issue are presented below.[13]

$$D_V = N(x_b - \sigma\sqrt{t}) + e^{-rt}\left[\int_{x_b}^{\infty} f(x)\,\frac{\partial B(x)}{\partial V}\,dx + s\int_{x_b}^{x_c} f(x)\,\frac{\partial B(x)}{\partial V}\,dx\right] > 0,$$

$$D_F = e^{-rt}\left[\int_{x_b}^{\infty} f(x)\,\frac{\partial B(x)}{\partial F}\,dx + s\int_{x_b}^{x_c} f(x)\,\frac{\partial B(x)}{\partial F}\,dx + N(-x_c)\,se^{-c\tau}\right] > 0,$$

13. The bankruptcy point x_b and the call point x_c are determined by the firm. Since the firm chooses these points to minimize the value of the debt, $\partial D/\partial x = 0$ for $x = x_b$ and $x = x_c$, the first-order condition for a minimum.

$$D_r = -TFe^{-rT}\left[\int_{x_b}^{\infty} f(x)N(-y)dx + s\int_{x_b}^{x_c} f(x)N(-y)dx\right] < 0,$$

$$D_T = -rFe^{-rT}\left[\int_{x_b}^{\infty} f(x)N(-y)dx + s\int_{x_b}^{x_c} f(x)N(-y)dx\right] < 0,$$

$$D_s = e^{-rt}\left[\int_{x_b}^{x_c} f(x)B(x)dx + Fe^{-c\tau}N(-x_c)\right] > 0,$$

$$D_c = -\tau N(-x_c)sFe^{-(c\tau+rt)} < 0.$$

As one would expect, the value of the debt increases with V and F and decreases with σ, T, and r. These results are standard in the risky bond valuation literature and are explained in Merton (1974) for the case of a discount bond. The comparative statics with respect to s and c are unique to sinking-fund debt. An increase in the redemption ratio s increases the expected payments to the redeemed debt. The current value of the debt rises by the marginal increase in the present value of these expected payments. An increase in the initial yield to maturity c decreases the call price of the debt. The expected payment to the sinking-fund decreases and therefore the current value of the debt declines.

It is instructive to compare the value of the sinking-fund debt to that of its corresponding serial bond. The value of the serial bond differs from that of the sinking-fund bond as a result of a different bankruptcy point at the time of the serial redemption, and the payment to the debt if the serial obligation is satisfied. The value of the corresponding serial bond is given by

$$G = VN(x_b' - \sigma\sqrt{t}) + e^{-rt}\int_{x_b'}^{\infty} f(x)B(x)dx + N(-x_b')sFe^{-c\tau-rt}, \quad (9)$$

$$x_b' = \frac{\ln\left[\dfrac{sFe^{-c\tau} + B(x_b')}{Ve^{rt}}\right]}{\sigma\sqrt{t}} + \tfrac{1}{2}\,\sigma\sqrt{t}, \quad (10)$$

where $B(x)$ and y are defined in equations (7) and (8).

It can be shown that equations (9), (10), (7), and (8) are equivalent to the compound option formula for a serial bond as given by Geske (1977). The bankruptcy point at the time the serial redemption must be satisfied is given by x_b' and determined by (10).

Notice that when the sinking-fund bond's bankruptcy point equals its call point the firm never satisfies the sinking-fund obligation through open-market purchases. By our previous argument, sinking-fund debt is identical to an equivalent serial bond. This is easily verified by letting $x_b = x_c$ in equations (4)–(8). The third term in (4) vanishes and equation (4) becomes equation (9). Using equation (6), $B(x_b) = Fe^{-c\tau}$. Then $(1 + s)B(x_b) = B(x_b) + sFe^{-c\tau}$ and equation (5) can be written as

$$x_b = \frac{\ln\left[\dfrac{sFe^{-c\tau} + B(x_b)}{Ve^{rt}}\right]}{\sigma\sqrt{t}} + \tfrac{1}{2}\sigma\sqrt{t}. \tag{11}$$

Equation (11) is the same as equation (10) determining x_b'. Thus $x_c = x_b = x_b'$, and equations (4) and (5) become identical to equations (9) and (10).

III. The Delivery Option

So far the analysis has concentrated on a comparison of a sinking-fund bond with its corresponding serial bond. Recall that the difference between these two is due to the sinking fund's unique provision permitting its issuer the choice of satisfying the sinking-fund provision by a sinking-fund call or by purchasing the requisite number of bonds from the market. In this paper this choice is called the delivery option. This section studies the value and behavior of the delivery option.

A. The Value of the Delivery Option

Since the sinking-fund bond differs from the serial bond only by virtue of its delivery option, the value of the delivery option Ω can be obtained by subtracting the value of the sinking-fund bond from its corresponding serial bond. Where

$$\begin{aligned}\Omega = G - D = {} & V[N(x_b' - \sigma\sqrt{t}) - N(x_b - \sigma\sqrt{t})] \\ & - (1 + s)e^{-rt}\int_{x_b}^{x_c} B(x)f(x)dx \\ & + sFe^{-rt-c\tau}[N(-x_b') - N(-x_c)] + e^{-rt}\int_{x_b'}^{x_c} B(x)f(x)dx.\end{aligned} \tag{12}$$

The value of the delivery option is positive when $x_b < x_b' < x_c$. It depends on the specific conditions of the bond indenture—the promised payment at maturity (F), the initial yield to maturity (c), and the redemption ratio (s)—as well as the riskless rate of interest and the risk and value of the firm. These parameters can be easily determined at the time of issue.

The comparative statics for the delivery option with respect to c, s, and V are presented below.

$$\Omega_c = -\tau sFe^{-rt-c\tau}[N(-x_b') - N(-x_c)] \tag{13}$$

$$\Omega_s = e^{-rt}\int_{x_b'}^{x_c}[Fe^{-c\tau} - B(x)]f(x)dx - e^{-rt}\int_{x_b}^{x_b'} B(x)f(x)dx \tag{14}$$

$$\begin{aligned}\Omega_V = {} & \int_{x_b}^{x_b'} f(x - \sigma\sqrt{t})dx - \int_{x_b}^{x_b'} f(x)\frac{\partial B(x)}{\partial V}dx \\ & + s\int_{x_b}^{x_c} f(x)\frac{\partial B(x)}{\partial V}dx.\end{aligned} \tag{15}$$

The value of the delivery option is inversely related to the initial yield to maturity. An increase in initial yield reduces the par value of the debt. This has the effect of decreasing the call point so that open-market purchases are less likely and the delivery option is less valuable.

Increasing the redemption ratio can increase or decrease the value of the delivery option. An increase in the redemption ratio is equivalent to an increase in the bonds to be redeemed at t while keeping the promised payment at maturity the same. This has the effect of increasing the probability of default at t (for both the sinking-fund bond and the serial bond), while leaving the call point unchanged.[14] Thus the open-market region decreases, reducing the value of the delivery option. On the other hand, an increased proportion of the sinking-fund debt is subject to open-market purchases. This has the effect of increasing the value of the delivery option.

The value of the delivery option can increase or decrease with the firm's value, depending on the current value of the firm. When firm value is low, there is a positive relationship between the value of the firm and the delivery option. In this case the probability of default is high so that the delivery option will have a relatively low value. A marginal increase in firm value reduces the probability of default, increasing the probability that the delivery option will be utilized and raising its value. On the other hand, when the value of the firm is sufficiently high, there is a negative relationship between the value of the firm and the delivery option. In this case there again is a relatively small probability that the delivery option will be utilized, since the call price is likely to be less than the market price of the debt. However, a further rise in firm value increases the probability of a call, thereby reducing the value of the delivery option.

The simulation results presented in table 1 give the value of the delivery option relative to the value of the total debt (Ω/D), for selected leverage ratios (D/V), initial yields to maturity, redemption ratios, and levels of firm risk. Notice that the impact of the delivery option can be substantial, amounting to over 10% of total debt value. Ceteris paribus, the value of the delivery option, as a percentage of the debt value, first increases with leverage than decreases. Furthermore the ratio Ω/D, increases with the standard deviation of the firm. This results from the nature of an option which gains value as the standard deviation of the underlying asset increases. Over the parameter values of the table, Ω/D decreases as the redemption ratio increases. This is because the market value of the debt increases sufficiently to offset any increase in the value of the delivery option as s increases.

14. An increase in s can be represented by an increase in the par value of the debt and a rotation of the supply curve in fig. 1. The call point, defined by eq. (2), and the demand function are independent of s. Thus the call point remains unchanged while the bankruptcy points increase.

TABLE 1 The Value of the Delivery Option as a Percentage of the Value of Sinking-Fund Debt

Leverage (D/V) (%)	$\sigma = .3$ $s = 2$ $c = .106$ (1)	$\sigma = .4$ $s = 2$ $c = .104$ (2)	$\sigma = .4$ $s = 2$ $c = .106$ (3)	$\sigma = .4$ $s = 2$ $c = .108$ (4)	$\sigma = .4$ $s = 4$ $c = .106$ (5)	$\sigma = .4$ $s = 6$ $c = .106$ (6)	$\sigma = .5$ $s = 2$ $c = .104$ (7)	$\sigma = .5$ $s = 2$ $c = .106$ (8)
34	.26	3.90	3.23	2.65	.84	.15	10.38†	9.51‡
37	.27	3.95	3.29	2.71	.87	.15	10.33†	9.40
39	.29	3.96	3.31	2.74	.51	.16	10.09	9.33
42	.30	3.97*	3.33*	2.75	.89	.16	9.87	9.14
44	.31	3.96	3.33	2.77*	.89	.16	9.73	9.02
46	.32	3.94	3.32	2.77	.89*	.16	9.53	8.86
48	.32	3.91	3.30	2.76	.89	.16	9.35	8.74
50	.33	3.87	3.27	2.74	.88	.16*	9.16	8.53
52	.33	3.83	3.25	2.72	.88	.16	8.99	8.38
54	.34	3.78	3.21	2.70	.87	.16	8.73	8.18
56	.34	3.73	3.17	2.69	.86	.16	8.53	7.96
58	.34	3.65	3.12	2.63	.86	.16	8.29	7.75
60	.34*	3.58	3.06	2.58	.84	.16	8.03	7.52
62	.34	3.50	3.00	2.54	.83	.15	7.76	7.27
64	.34	3.39	2.93	2.47	.81	.15	7.50	7.03
68	.34	3.22	2.77	2.34	.78	.15	6.94	6.50
70	.33	3.10	2.67	2.28	.75	.14	6.62	6.22
72	.32	2.97	2.57	2.19	.73	.14	6.30	5.93
74	.32	2.85	2.47	2.11	.71	.14	5.98	5.65
76	.31	2.85	2.47	2.11	.71	.14	5.65	5.33
78	.30	2.57	2.23	1.91	.64	.12	4.93	4.66
80	.29	2.40	2.12	1.80	.61	.12	4.93	4.66
82	.27	2.25	1.97	1.70	.57	.11	4.56	4.31

NOTE.—All values are calculated for a bond with time to maturity of 20 years and time to the sinking-fund payment of 10 years. A more complete table is available from the authors. σ = standard deviation; s = redemption ratio; c = coupon rate.

*This is the maximum value for the column.

†The maximum value for this column is 10.42%, occurring at a leverage ratio of .28.

‡The maximum value of this column is 9.75%, occurring at a leverage ratio of .30.

It is again instructive to consider the values of the delivery option of issues with a low initial yield to maturity compared with issues with a high initial yield. Notice that, ceteris paribus, the delivery options of the low-yield issues have a greater relative value than those of the high-yield issues. This is due, in part, to the greater importance of the delivery option in avoiding default for bonds carrying a low initial yield when they suffer a deterioration in quality, as noted in Section II. However, low-yield bonds also contain delivery options which are substantially more valuable than those of high-yield bonds at all leverage ratios. Thus the delivery option is also more valuable for high-quality (low-yield) issues which remain high-quality than for low-quality (high-yield) issues which undergo a substantial increase in quality after issue. This is because, ceteris paribus, the call point for low-grade issues is less than it is for high-grade issues. As a result, for a given current leverage ratio, bonds which were issued with a high yield possess a more valuable delivery option than those issued with a low yield. This higher call point also explains why the delivery options associated with low-yield debt achieve a maximum value (relative to that of the debt) at higher leverage ratios.

B. The Conditions for a Valueless Delivery Option

The above results hold when the delivery option has some value. Section II*B* showed that the delivery option could be valueless. Given a nonstochastic interest rate this occurs when the sinking-fund debt's par value is sufficiently low, so that the supply price equals the demand price in the call region. In this case, the delivery option is valueless because the firm never satisfies the sinking-fund obligation with open-market purchases. This section examines the conditions for a valueless delivery option.

From Section II*C* it is clear that the conditions for a valueless delivery option are satisfied as long as the bankruptcy point x_b (given by eq. [5]) is not less than the call point x_c (given by eq. [6]). Thus the conditions for a valueless delivery option can be derived in terms of the determinants of x_b and x_c. In particular, let x_b equal x_c and substitute equation (6) into equation (5). Simplifying gives the condition for a valueless delivery option as

$$k = N(-d)(1 + s)\mathrm{k} + N(d - \sigma\sqrt{\tau}), \tag{16}$$

where $k = e^{-(c^* - r)\tau}$ and $d = \ln[(1 + s)\mathrm{k}]/\sigma\sqrt{\tau} + \frac{1}{2}\sigma\sqrt{\tau}$.

Notice that this condition depends on four parameters describing the sinking-fund obligation and firm value: the initial yield premium of the debt $(c - r)$, the redemption ratio s, the time to maturity from the time of the sinking-fund obligation τ, and the risk of the firm σ.

Define the critical risk premium $(c^* - r)$ as the minimum yield premium which renders the delivery option valueless. Its value depends on the remaining three parameters given in equation (16). Table 2 and figure 2 give simulation results describing the critical risk premium for selective values of s and σ. Notice that the critical risk premium can be as low as 54 basis points. Thus, it is not unreasonable to expect a substantial proportion of sinking-fund debt to carry a valueless delivery option.

The concave curves in figure 2 represent the locus of critical risk premiums and redemption ratios for various levels of firm risk. Given σ, any point on or above the relevant curve is a combination of initial risk premium and redemption ratio which renders the delivery option valueless. The critical risk premium is inversely related to the redemption ratio for a given level of firm risk. This is because a decrease in the redemption ratio reduces the bankruptcy point. In order for the bankruptcy point and the call point to remain the same, the par value of the debt must also decline. That is, the risk premium must increase.

Figure 2 shows that for a given redemption ratio, the critical risk premium increases as the risk of the firm increases. This results from the effect of firm risk on the market value of the unredeemed debt. An increase in firm risk reduces the value of the unredeemed debt (at each level of V). Thus the bankruptcy point decreases as σ increases, so that the par value of the debt must decrease for the delivery option to remain valueless.

Of particular interest are the parameters which do not affect the critical risk premium. It is independent of x, the state variable at the time of operation of the sinking fund, and V, the current value of the firm. That is, these conditions are observable at the time the bonds are issued. It follows that if the delivery option is valueless at the time of issue, then it remains valueless throughout the life of the bond. Conversely, a bond possessing a delivery option having some value at the time of issue retains some value until the time of the sinking-fund obligation. These results hold regardless of the evolution of firm value over time. Furthermore, the critical risk premium is independent of the promised payment at maturity. Thus it is independent of the initial market value of the debt and the leverage ratio.

IV. Financing Restrictions

Throughout the analysis it has been assumed that the sinking-fund obligation is financed externally through the sale of junior securities. This assumption is a severe restriction on the financing ability of the firm. Although bond indentures generally afford substantial protection from issuing senior securities, the ability to finance a debt obligation through the sale of real assets or through the generation of cash flow is less limited.

TABLE 2 Relationship between the Redemption Ratio(s) and the Critical Risk Premium ($c^* - r$) for Different Values of the Standard Deviation (σ)

$\sigma = .20$		$\sigma = .30$		$\sigma = .45$		$\sigma = .55$	
s	$c^* - r$ (%)	s	$c^* - r$ (%)	s	$c^* - r$ (%)	s	$c^* - r$ (%)
.0123	12.16	.0845	12.85	.3291	14.88	.5954	16.71
.0637	7.55	.2058	8.80	.5579	11.36	.9101	13.40
.1516	4.98	.3458	6.54	.7780	9.32	.1198	11.44
.2657	3.41	.4962	5.08	.9921	7.95	1.4695	10.09
.3985	2.40	.6536	4.08	1.2019	6.94	1.7297	9.09
.5451	1.72	.8150	3.34	1.4085	6.17	1.9815	8.31
.7023	1.27	.9824	2.78	1.6127	5.55	2.2270	7.66
.8676	.94	1.1152	2.36	1.8150	5.04	2.4674	7.13
1.0395	.71	1.3244	2.02	2.0158	4.62	2.7035	6.67
1.2164	.54	1.4992	1.74	2.2154	4.26	2.9362	6.28

NOTE.—The table represents values for a bond with a time to maturity of 20 years and time to the sinking-fund payment of 10 years. A more complete table is available from the authors.

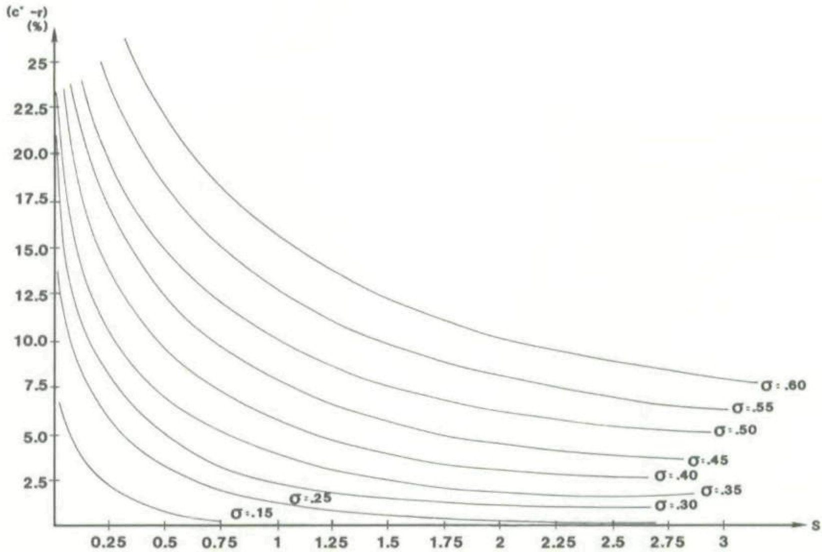

FIG. 2.—The critical risk premium $(c^* - r)$ as a function of firm risk (σ) and the redemption ratio (s).

An alternative financing arrangement would permit the firm to finance the sinking-fund obligation from operating cash flow or from the sale of assets. Under this alternative, the delivery option remains a valuable feature. In fact, the firm never defaults on the sinking-fund obligation, and the delivery option is always valuable. Thus the sinking-fund bond is strictly less valuable than the corresponding serial bond.

To prove this, we first show that the value of the sinking-fund bond is less than or equal to the value of the corresponding serial bond at the time of the sinking-fund obligation. Then, since the sinking-fund obligation can always be satisfied by open-market operations, the sinking-fund bond is shown to be strictly less valuable than the corresponding serial bond.

Define X as the value of the assets sold to satisfy the sinking-fund obligation. Then the value of the unredeemed debt, once the assets have been sold is $B(V - X; F, \tau)$, where $X = s \min [Fe^{-c\tau}, B(V - X; F, \tau]$ and V is the value of the firm prior to the sale of assets. Thus $X \leqslant sFe^{-c\tau}$ for all V. It is necessary to show that

$$sFe^{-c\tau} + B(V - sFe^{-c\tau}; F, \tau) \geqslant X + B(V - X; F, \tau), \quad (17)$$

where the right-hand side of expression (17) is the value of the sinking-fund bond and its left-hand side is the value of the corresponding serial bond at the time of the sinking-fund obligation. Expression (17) can be written as

$$(V - X) - (V - sFe^{-c\tau}) \geqslant B\,(V - X; F, \tau) - B\,(V - sFe^{-c\tau}; F, \tau).$$

Since by the nature of a discount bond, $dB(y)/dy < 1$ for all y and $X \leq sFe^{-c\tau}$, the mean value theorem assures that (17) must hold.

Furthermore, the firm never defaults on the sinking-fund obligation. This can be established by showing that

$$V - X - B(V - X) > 0 \tag{18}$$

for all V. Expression (18) can be written as

$$V - X > B(V - X). \tag{19}$$

Now X must be less than V. Otherwise the unredeemed bonds $B(V - X)$ would have no value (since all the assets would be sold and paid to the redeemed bonds). But if the redeemed bonds were purchased in the market,

$$X = sB(V - X). \tag{20}$$

Clearly, if $X > V$, (20) cannot hold since the left-hand side is positive and the right-hand side is zero. Hence $X < V$ for all V. Therefore (19) must hold since $B(y) < y$ for all positive values of y. That is, a discount bond is always less valuable than the underlying assets. In our case, $y = V - X$, and $B(y)$ is the value of the unredeemed debt, $B(V - X)$. Thus with this financing arrangement the firm never defaults on the sinking-fund obligation. There is no critical risk premium, and the delivery option is always valuable.

V. Summary and Conclusions

This paper investigates the implications of sinking-fund provisions on the pricing and behavior of corporate bonds. Special attention is focused on the delivery option feature of these provisions—that is, the option to satisfy the sinking-fund provision either through market purchases of the bonds or through a sinking-fund call.

The analysis shows that the value of a delivery option depends on the risk of the firm, the amortization rate of the issue, and the initial yield to maturity. Simulation results indicate that the value of the delivery option can be substantial compared to the market value of the debt. The delivery option is particularly important for sinking-fund issues which have suffered an unexpected deterioration in quality subsequent to the time of issue. Under these circumstances the delivery option serves to substantially reduce the probability of default on, and the expected payment to, the sinking-fund obligation (compared to the equivalent serial bond).

The results derived here have important empirical implications. A sinking-fund provision has two distinct features affecting bond prices. First, the amortization feature shortens a bond issue's average maturity (Jen and Wert 1966). Second, the delivery option can reduce the expected payments to bondholders. These features have opposing impacts on bond prices. The amortization feature tends to reduce bond risk (Ho and Singer 1982), and thus yield premiums should be inversely related to the sinking fund. On the other hand, this analysis indicates that the delivery option tends to reduce bond prices, increasing nominal bond yields. Empirical studies of corporate bond yields have either ignored the sinking-fund provision (Fisher 1959; Johnson 1967; Weinstein 1981), or treated only the amortization feature (Dyl and Joehnk 1979). Allowing for the separate impact of the amortization and delivery option features in empirical studies of the determinants of bond yields may improve our understanding of the pricing of corporate bonds.[15]

Appendix

The methodology derived in Cox-Ross-Rubinstein (C-R-R) is adapted to derive a valuation model for the debt. Consider a sinking-fund bond described by assumptions 1–8 with n periods to a sinking-fund obligation and $n + m$ periods to maturity. Assume that the value of the firm follows a binomial process, with a risk-neutral probability p that the value of the firm is Vu, and $(1 - p)$ that the value of the firm is Vd at the end of the first period. The current value of a sinking-fund bond, conditional on the current value of the firm, can be written as the risk-neutral present value of the payments to the debt given by

$$\begin{aligned} D = {} & \frac{1}{\hat{r}^n} \sum_{k=0}^{\beta} b(k, n, p) Vu^k d^{n-k} + \frac{(1+s)}{\hat{r}^n} \sum_{k=\beta+1}^{\gamma} b(k, n, p)\Delta(V_k) \\ & + \frac{sF\hat{c}^{-m}}{\hat{r}^n} \sum_{k=\gamma+1}^{n} b(k, n, p) + \frac{1}{\hat{r}^n} \sum_{k=\gamma+1}^{n} b(k, n, p)\Delta(V_k), \end{aligned} \tag{A1}$$

where $V_j = Vu^j d^{n-j}$ $b(k, n, p)$ is the binomial probability density function of k success in n trials, where p is the probability of success in one trial,
$\hat{r}$ is one plus the single-period riskless rate of interest,
n is the number of periods to the operation of the sinking fund, and
$m + n$ is the number of periods to maturity of the debt.
$\Delta(V_k)$ is the present value of the unredeemed bonds at the time of operation of the sinking fund, conditional on the value of the firm at that time, V_k.
β is the maximum number of up moves of the value of the firm over the n periods consistent with default on the sinking fund. It is given by

$$Vu^{\beta} d^{n-\beta} < (1 + s)\Delta(V_{\beta}) \leq Vu^{\beta+1} d^{n-\beta-1}. \tag{A2}$$

The minimum number of up moves permitted for the firm to redeem the sinking-fund obligation through a call $\gamma + 1$, is given by

$$Vu^{\gamma} d^{n+\gamma} \leq \Delta(V_{\gamma}) + sF\hat{c}^{-m} < Vu^{\gamma+1} d^{n-\gamma-1}. \tag{A3}$$

15. It is of interest to note that Zwick (1980) documents wide variations over time in the yield differential between private placements and comparable publicly distributed bonds. The study attributes this to slow adjustments of private rates to changes in public markets. However, a plausible alternative explanation is the variation in the value of the delivery option associated with public issues since private placements do not contain sinking-fund delivery options.

The value of the unredeemed bonds at the time of sinking-fund obligation is satisfied, conditional on the value of the firm V_j at that time, is given by

$$\Delta(V_j) = \frac{1}{\hat{r}^m}\left\{\sum_{k=0}^{\alpha_j} b(k, m, p)V_j u^k d^{m-k} + \sum_{k=\alpha_j+1}^{m} b(k, m, p)F\right\}, \quad \text{(A4)}$$

where α_j is the maximum number of up moves permitted, from the time of the sinking-fund obligation, for the firm to default at maturity. α_j is given by

$$Vu^{\alpha_j - j}d^{n-j+m-\alpha_j} < F \leq Vu^{\alpha_j + j + 1}d^{n-j-m-\alpha_j+1}. \quad \text{(A5)}$$

The continuous-time version of (A1)–(A5) is derived by considering the limits as n and m become arbitrarily large while the time span of each period becomes arbitrarily small. The following substitutions to the discrete-time model follow C-R-R.

$$r = (n/T)\ln \hat{r}, \quad p = \tfrac{1}{2} + \tfrac{1}{2}\frac{(r - \tfrac{1}{2}\sigma^2)}{\sigma}\sqrt{T/n},$$

$$u = e^{\sigma\sqrt{T/n}}, \quad c = (m/T)\ln \hat{c},$$

$$d = e^{-\sigma\sqrt{T/n}}$$

The random variables j and α_j, defined in (A1)–(A5), may be standardized to yield

$$x = \frac{j - np}{\sqrt{np(1 - p)}}, \quad y = \frac{\alpha_j - \ell np}{\sqrt{\ell np(1 - p)}},$$

where $\ell n = m$, the number of periods between the sinking-fund obligation and maturity.

Substituting x and y into (A1)–(A5), and letting $m = \ell n$, results in a system of four equations depending on the number of periods n in the first time interval. By letting $n \to \infty$, the system of equations (A1)–(A5) converges to equations (4)–(8) in the body of the paper.

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