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Comments on "Gaussian Demand and Commodity Bundling"

Different commodities are often sold together as a "bundle" (e.g., complete dinners in a restaurant, stereo systems). In "mixed bundling" strategies, the individual components of the bundle are also offered for sale separately. In "block booking" or "pure bundling" strategies, they are not.

Via simple hypothetical examples of two-product firms, Stigler (1963) and Adams and Yellen (1976) have demonstrated that these selling practices can increase profit even if there is no cost advantage to bundling and the component products are independent in demand. It is obvious from these examples that heterogeneity in consumer tastes (especially in relative valuations of the firm's two products) is a necessary condition for profitable bundling. Unfortunately, more specific principles to describe concisely the necessary and/or sufficient conditions for profitable bundling are not so obvious. Since consumer demand in the examples is represented by a complete enumeration of consumer reservation prices, optimal prices and pricing strategies must be determined by trial and error. Marginal analysis is inappropriate or uninformative. More important, these numerical examples do not yield characterizations of profit for each strategy as functions of a small number of parameters that "summarize" the relevant features of the distribution of reservation prices in the consumer population.

By assuming that reservation prices (for the firm's two products) are distributed in the consumer population according to the bivariate normal probability law, Schmalensee (in this issue) has constructed a class of examples within which the profitability of bundling can be analyzed as a function of production costs, the mean and variance of the reservation price for each commodity, and the correlation between the two commodities' reservation prices. Although this analysis is sometimes mathematically complex, Schmalensee is quite thorough. Subject to the normality assumption, many of the questions suggested by the

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Stigler and Adams-Yellen examples (e.g., the role, necessity, and/or sufficiency of negatively correlated reservation prices) are largely resolved.

In this note I focus on the profitability of bundling in a slightly more general setting. The Stigler and Adams-Yellen assumptions about cost and demand (no cost advantage to bundling, bundle components independent in demand) are retained and, following Schmalensee, I assume that the distribution of consumer reservation prices has a continuous density. This makes profit a smooth function of prices and familiar marginal analysis is appropriate and informative. I do not assume, however, that the reservation-price distribution is restricted to any particular form (e.g., bivariate normal). Under these conditions, I show that

a) mixed bundling is equivalent to two-part pricing with no bundling (the bundle discount is like a fixed fee);

b) the profitability of bundling depends crucially on the behavior of Q/N in the absence of bundling, where Q is the total quantity of both commodities demanded and N is the number of consumers who demand a positive quantity of either commodity (at ordinary monopoly prices, introduction of a marginal bundle discount will increase profit if and only if Q/N is a decreasing function of prices at that point);

c) if an increase in prices (above the monopoly level) increases the number of consumers who buy only one of the two commodities, then bundling will increase profit;

d) if the reservation prices for the two commodities are not positively correlated, then bundling increases profit; and

e) whenever bundling does *not* increase profit, a form of promotional couponing *does* increase profit.

In concluding remarks, I argue that all of the foregoing results except *d* remain true if the firm's products are not independent in demand.

Mixed Bundling as Two-Part Pricing in the Adams-Yellen Model

Adams and Yellen (1976) consider a firm that produces two commodities, 1 and 2, with no fixed cost and with constant marginal costs C_1 and C_2. The demand facing the firm is such that

a) no consumer demands more than one unit of each commodity;

b) each consumer's reservation price (R_B) for a bundle containing one unit of each commodity is equal to the sum of his reservation prices (R_1, R_2) for the two individual commodities; and

c) it is impossible for consumers to resell the commodities.

Among the firm's pricing alternatives are

a) simple monopoly pricing, in which a single price is set for each commodity (P_1, P_2); and *b*) mixed bundling, in which a bundle (one unit of each commodity) is offered at a price P_B in addition to the individual commodities at prices P_1 and P_2.[1]

These demand conditions and alternative pricing schemes are illustrated in figures 1 and 2, respectively. In each figure, the marginal cost, selling price, and reservation prices of commodity 1 are measured on the horizontal axis. The same quantities for commodity 2 are measured on the vertical axis. At the illustrated prices, consumers with reservation prices (R_1, R_2) that plot in region A purchase only commodity 1. Consumers in region B purchase both commodities (the bundle in fig. 2). Consumers in region C purchase only commodity 2 and consumers in region D purchase nothing.

For the Adams-Yellen firm illustrated in figure 2, mixed bundling, with max $(P_1, P_2) \leq P_B < (P_1 + P_2)$, is equivalent to a simple form of two-part pricing. If the firm charges purchasers a fixed fee, F, plus marginal commodity prices V_1 and V_2, then the equivalent mixed bundling prices are $P_1 = F + V_1$ for commodity 1 alone, $P_2 = F + V_2$ for commodity 2 alone, and $P_B = F + V_1 + V_2$ for the bundle. Conversely, the two-part pricing scheme (F, V_1, V_2) that duplicates the mixed bundling prices (P_1, P_2, P_B) is given by a fixed fee $F = P_1 + P_2 - P_B$, and marginal prices $V_1 = P_B - P_2$ and $V_2 = P_B - P_1$. The two-part pricing analogue of figure 2 is illustrated in figure 3.

The fixed fee F in two-part pricing equals the "bundle discount" $(P_1 + P_2 - P_B)$ in mixed bundling. Thus, mixed bundling is more profitable than simple monopoly pricing if and only if, under two-part pricing, the optimal fixed fee (F) is positive.

I will illustrate first conditions for a positive optimal F in a simple symmetric case that is fairly easy to visualize. I will then add conditions that both characterize the general case and introduce a third pricing strategy which I will call "couponing."

The Symmetric Case

As in Schmalensee's paper in this issue it is assumed here that the distribution of consumers in reservation price space (R_1, R_2) can be described by a continuous density function $f(R_1, R_2)$. Thus, the fraction of the consumer population whose reservation prices fall in the rectangle $a < R_1 \leq b$ and $c < R_2 \leq d$ is given by:

$$\int_{R_1=a}^{b} \int_{R_2=c}^{d} f(R_1, R_2) dR_2 dR_1.$$

1. Pure bundling is not explicitly considered here but it can be thought of as the special case of mixed bundling in which $P_1 = P_2 = P_B$.

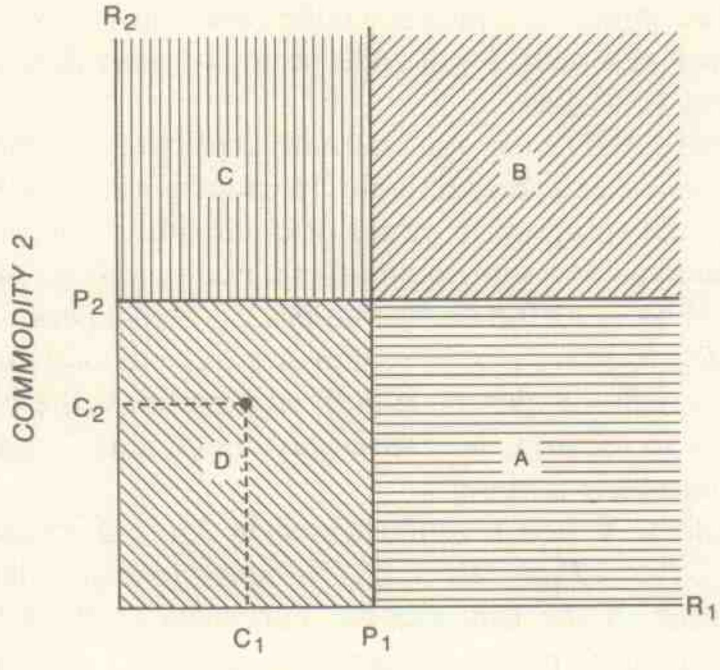

FIG. 1.—Simple monopoly pricing

This description of consumer heterogeneity will serve to make the total demand functions facing the firm differentiable and standard marginal analysis can be applied.

The term "symmetric" will imply two things: (i) $f(R_1, R_2) = f(R_2, R_1)$ for all (R_1, R_2); and (ii) equal marginal production costs ($C_1 = C_2 = C$). Thus, in the symmetric case, loci of constant consumer density (level curves of f) are symmetric about the 45-degree line in the reser-

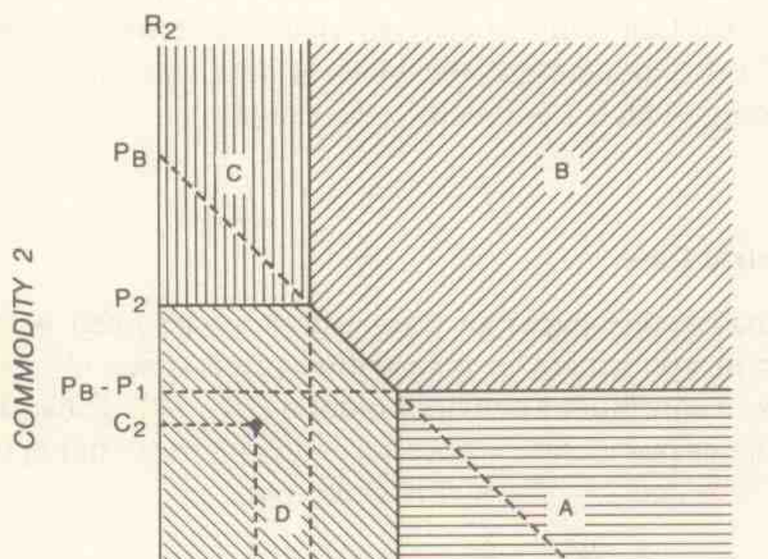

FIG. 2.—Mixed bundling

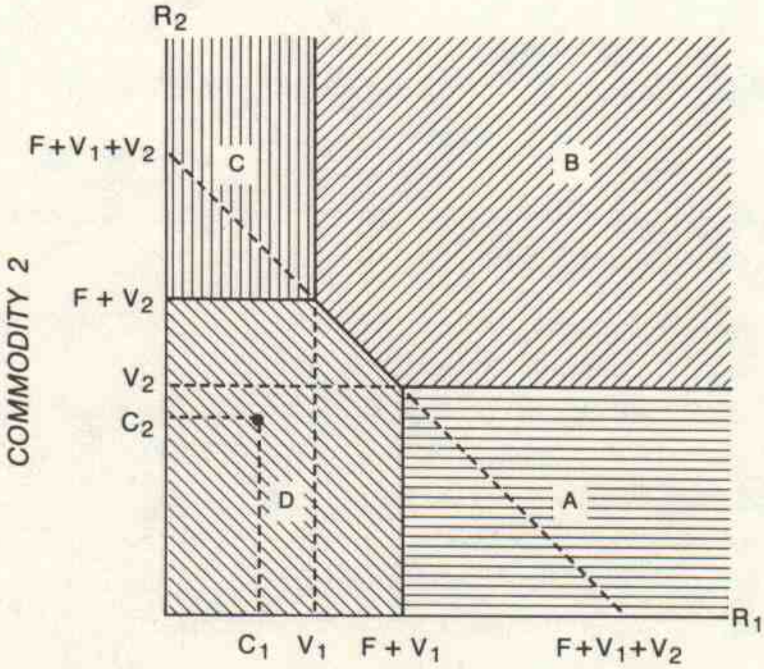

FIG. 3.—Two-part pricing

vation price plane and marginal costs (C_1, C_2) can be depicted as a point on the 45-degree line. This is illustrated by an example in figure 4.

In the symmetric case, the firm can restrict itself (without loss of profits) to mixed bundling with equal individual prices ($P_1 = P_2 = P$) or, equivalently, to two-part pricing with equal marginal prices ($V_1 = V_2 = V$). At the two-part prices (F, V) the firm's profit, $\pi(F, V)$, is given by

$$\pi(F, V) = F \cdot N(F, V) + (V - C) \cdot Q(F, V),$$

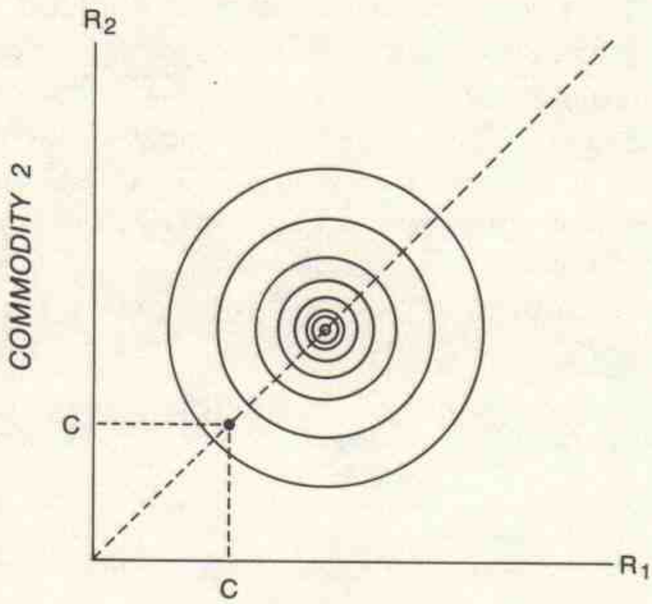

FIG. 4.—Symmetric costs and reservations prices

where

$$N(F, V) \equiv \text{number of purchasers at prices } (F, V)$$

and

$$Q(F, V) \equiv \text{total number of units (of both individual products) demanded at prices } (F, V).$$

This profit function is identical in form to the profit function of a two-product firm that faces interrelated demand and uses simple monopoly pricing. In this analogy, F and V become the prices of two different products; N and Q are the respective quantities demanded; and 0 and C are their respective marginal costs.

If N and Q are regarded as demands for two different products, then the products are complementary in demand.[2] Thus, it is not possible to conclude immediately that the profit-maximizing value of F must be positive.[3] The following condition, however, implies that the optimal value of F will be positive if N and Q are not "too complementary":

> If the profit maximizing simple monopoly price is $\hat{V}$; that is, $\max_V \pi(0, V) = \pi(0, \hat{V})$, then $\partial\pi/\partial F$ is positive at $(0, \hat{V})$ if and only if $\partial/\partial V\,(Q/N)$ is negative at $(0, \hat{V})$.[4]

The ratio Q/N is the average unit purchase per customer (a "customer" being a consumer who buys at least one unit). Since each customer buys either one or two units, Q/N can also be interpreted as one plus the fraction of customers who buy both component products. Mixed bundling (or two-part pricing) is more profitable than simple monopoly pricing if an increase in the simple monopoly price, $\hat{V}$, would decrease the fraction of customers who buy both products. This occurs when the *number* of consumers who buy both products is more elastic with respect to V than is the number who buy only one product.[5] If, for example, a marginal increase in price above $\hat{V}$ actually *increases* the number of consumers who buy only one product, then introducing a bundle discount (or a fixed fee) will always increase profit. (This case typically occurs when the monopoly price is below the average consumer reservation price, as in fig. 5.)

Figure 5 is an illustration of simple monopoly pricing in the sym-

2. All first-order price derivatives (N_F, N_V, Q_F, Q_V) of N and Q are negative and the cross-price derivatives N_V and Q_F are equal. The matrix of first-order price derivatives is negative definite.

3. Complementarity is a necessary but not sufficient condition for one of the products to be priced as a "loss leader" (see, e.g., Allen 1938, pp. 359–62). If two products are gross substitutes, then the two-product monopoly prices must be above marginal costs.

4. At $(0, \hat{V})$, $\pi_V = Q + (\hat{V} - C)Q_V = 0$ and $\pi_F = N + (\hat{V} - C)Q_F$. The condition for $\pi_F > 0$ then follows from the fact that $Q_F = N_V$.

5. This is not equivalent to a comparison of the demand elasticities of the two consumer groups.

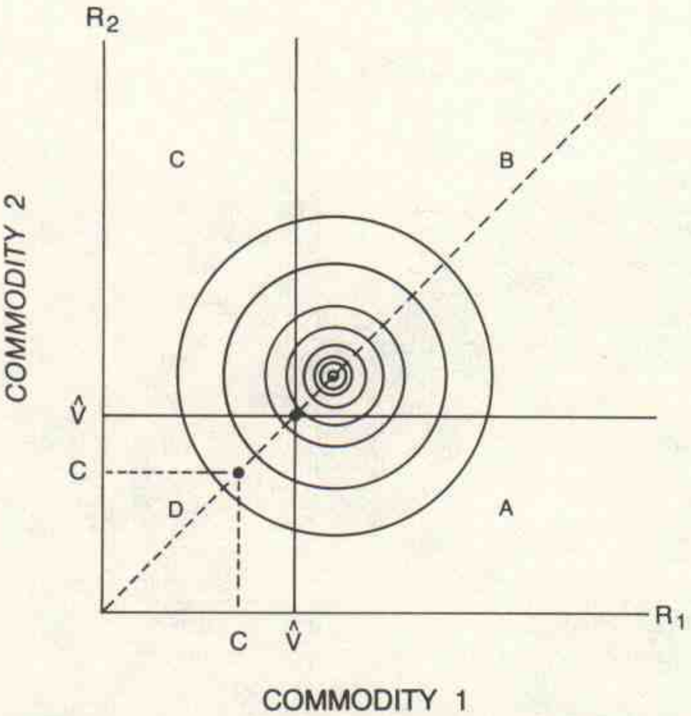

FIG. 5.—Simple monopoly pricing with independently distributed reservation prices.

metric case. In terms of the (symmetric) bivariate distribution of consumer reservation prices, the ratio Q/N is an increasing function of $\text{prob}\{R_1 > V \mid R_2 > V\}$, the probability that a randomly chosen consumer will buy commodity 1 given that he buys commodity 2. This conditional probability is equal to the number of consumers in region B of figure 5 divided by the combined number of consumers in regions B and C. If R_1 and R_2 are distributed independently in the consumer population (as in fig. 5), then $\text{prob}\{R_1 > V \mid R_2 > V\} = \text{prob}\{R_1 > V\}$, which is decreasing with respect to V. Thus, if R_1 and R_2 are distributed independently, $\partial/\partial V\ (Q/N) < 0$ and hence mixed bundling is more profitable than simple monopoly pricing.

If R_1 and R_2 are negatively correlated in the consumer population, then (a fortiori) $\text{prob}\{R_1 > V \mid R_2 > V\}$ and hence Q/N are decreasing functions of V.[6] Thus, negative correlation between R_1 and R_2 also implies that mixed bundling is more profitable than simple monopoly pricing. An example of this case is illustrated in figure 6.

Only in the case of positive correlation between R_1 and R_2 is it possible for the derivative $\partial/\partial V\ (Q/N)$ to be positive for some values of V (with $F = 0$).[7] In this case, the relative profitability of mixed bun-

6. I am using the term "negatively correlated" somewhat loosely here. The exact meaning I intend is that $\text{prob}\{R_1 > X \mid R_2 > Y\}$ be nonincreasing in Y for all (X, Y) and strictly decreasing in Y for some values of (X, Y).

7. A necessary condition is that $\text{prob}\{R_1 > X \mid R_2 > Y\}$ be strictly increasing in Y for some values of (X, Y).

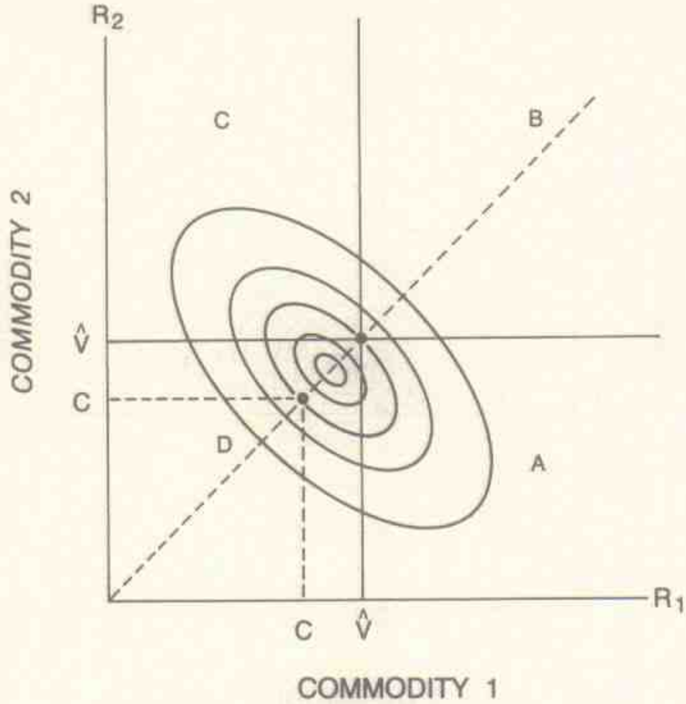

FIG. 6.—Simple monopoly pricing with negatively correlated reservation prices.

dling cannot be determined from demand conditions alone. If demand *and* cost conditions are such that the monopoly price $\hat{V}$ falls in a range where Q/N is increasing, then mixed bundling will be *less* profitable than simple monopoly pricing.

Examples of relatively unprofitable mixed bundling necessarily have the property that, as V is increased above $\hat{V}$, most of the reduction in total quantity demanded (Q) is due to customers who "leave the store" (customers whose quantity demanded goes to zero).[8] If the fraction of customers who buy both products, $(Q/N) - 1$, is sufficiently large in this situation, then introduction of a bundle discount is roughly equivalent to reducing price to the most inelastic demanders and profit falls. An exaggerated example of this case is illustrated in figure 7.

Mixed Bundling in the General (Asymmetric) Case

In the general case, mixed bundling and the equivalent two-part pricing are illustrated, respectively, in figures 2 and 3. As a function of prices, the firm's profit is given by:

$$\pi(F, V_1, V_2) = F \cdot N(F, V_1, V_2) + (V_1 - C_1) \cdot Q^1(F, V_1, V_2) + (V_2 - C_2) \cdot Q^2(F, V_1, V_2),$$

8. $(Q + \Delta Q)/(N + \Delta N) > Q/N$ implies (since ΔN and ΔQ are negative) that $\Delta N/\Delta Q > N/Q$. The ratio N/Q, in turn, is at least equal to ½.

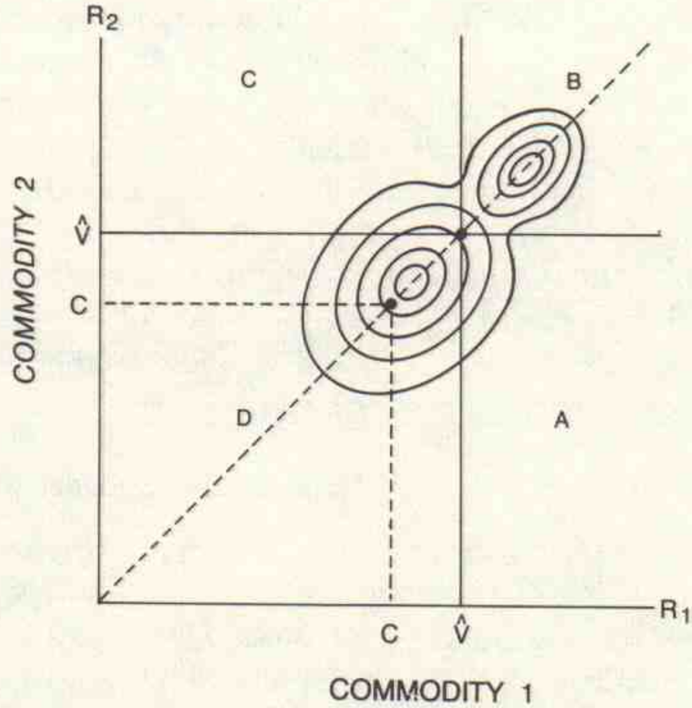

FIG. 7.—Simple monopoly pricing with positively correlated reservation prices.

where

$N(F, V_1, V_2) \equiv$ number of customers at prices (F, V_1, V_2),
$Q^1(F, V_1, V_2) \equiv$ quantity of commodity 1 demanded, and
$Q^2(F, V_1, V_2) \equiv$ quantity of commodity 2 demanded.

Mixed bundling yields higher profits than simple monopoly pricing if $\partial\pi/\partial F$ is positive at $(0, \hat{V}_1, \hat{V}_2)$, where $\hat{V}_1$ and $\hat{V}_2$ are the optimal simple monopoly prices of commodities 1 and 2. As in the symmetric case, the sign of $\partial\pi/\partial F$ depends on the behavior of Q/N (where Q is the total quantity of both products).

> If $\hat{V}_1$ and $\hat{V}_2$ are the optimal simple monopoly prices, then $\partial\pi/\partial F > 0$ at $(0, \hat{V}_1, V_2)$ if and only if the ratio $(Q^1 + Q^2)/N$ is decreasing at $(0, \hat{V}_1, \hat{V}_2)$ with respect to simultaneous equal percentage increases in both monopoly markups, $\hat{V}_1 - C_1$ and $\hat{V}_2 - C_2$.[9]

A useful corollary of this condition is that mixed bundling is always more profitable than simple monopoly pricing if, when $F = 0$, the ratio $(Q^1 + Q^2)/N$ is everywhere decreasing with respect to both prices, V_1

9. At $(0, \hat{V}_1, \hat{V}_2)$, $\pi_F = N + (\hat{V}_1 - C_1) \cdot Q^1_F + (\hat{V}_2 - C_2) \cdot Q^2_F = N + (\hat{V}_1 - C_1) \cdot N_{V_1} + (\hat{V}_2 - C_2) \cdot N_{V_2}$ (since $Q^1_F = N_{V_1}$ and $Q^2_F = N_{V_2}$). Thus $\pi_F > 0$ if and only if the elasticity of N with respect to simultaneous equal percentage increases in both markups is greater than -1. At $(0, \hat{V}_1, \hat{V}_2)$, however, the elasticity of each of the quantities, Q^1 and Q^2, with respect to its own markup is equal to -1. Since $Q^1_{V_2} = Q^2_{V_1} = 0$ when $F = 0$, the elasticity of the combined quantity, $Q^1 + Q^2$, with respect to simultaneous equal percentage increases in both markups, is also equal to -1.

and V_2. As argued below, this demand condition is satisfied when the correlation between reservation prices R_1 and R_2 is nonpositive.

The ratio $(Q^1 + Q^2)/N$ is one plus the fraction of customers who buy both products. When $F = 0$, this fraction is uniquely determined as a strictly increasing function of the following two conditional probabilities: (i) prob$\{R_1 > V_1 \mid R_2 > V_2\}$; and (ii) prob$\{R_2 > V_2 \mid R_1 > V_1\}$. If probability i is not increasing with respect to V_2 and probability ii is not increasing with respect to V_1, then the ratio $(Q^1 + Q^2)/N$ must be decreasing with respect to both V_1 and V_2 and mixed bundling is more profitable than simple monopoly pricing.[10]

If Mixed Bundling Does Not Pay, Then Couponing Does

Suppose a firm deviates from simple monopoly pricing by offering *simultaneously* a bundle discount of $\$X$ and "one per customer" coupons worth $\$X$ toward any purchase. Customers who buy only commodity 1 then pay $\hat{V}_1 - X$. Customers who buy only commodity 2 pay $\hat{V}_2 - X$, and customers who buy both commodities pay $\hat{V}_1 + \hat{V}_2 - 2X$. Thus, this deviation from monopoly pricing is equivalent simply to reducing both component prices by $\$X$.

The profit-maximizing simple monopoly prices are $\hat{V}_1$ and $\hat{V}_2$. If $X = 0$, $V_1 = \hat{V}_1$, and $V_2 = \hat{V}_2$, the derivative of profit with respect to X is equal to zero. Thus, the marginal effects of mixed bundling and couponing are equal in absolute magnitude but opposite in sign. If the (marginal) introduction of a bundle discount lowers profit, then the alternative strategy of introducing coupons must increase profit.

Figure 8 illustrates consumer response to an $\$X$ one per customer coupon. As in the previous figures, consumers with reservation prices in region A buy only commodity 1. Consumers in region B buy both commodities and consumers in region C buy only commodity 2. The coupon is equivalent to a negative fixed fee under two-part pricing.

In contrast with a bundle discount, the effect of coupons on quantities demanded is entirely due to the attraction of new customers. (A bundle discount primarily increases the quantity demanded by existing customers.) This is an essential distinction between coupons and bundle discounts. When a bundle discount (or positive fixed fee) reduces profit (as in fig. 7), there must be significantly more consumers on the zero/one unit margin (the upper and right-hand borders of region D in the figs.) than on the one/two margin (the lower and left-hand borders of region B).[11] This, in turn, is the condition that allows coupons to increase profit.

10. In general, this condition is not mathematically equivalent to "nonpositive correlation" between R_1 and R_2. I believe, however, that the condition captures the general connotation of nonpositive correlation and thus I have used that term.

11. The introduction of coupons increases profit if the ratio $(Q^1 + Q^2)/N$ increases as the markups $\hat{V}_1 - C_1$ and $\hat{V}_2 - C_2$ are increased by equal percentage amounts.

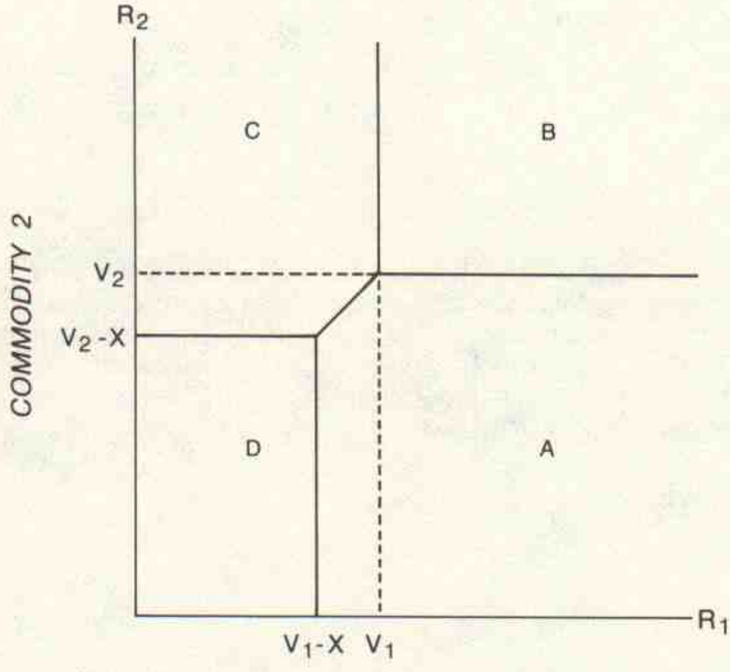

FIG. 8.—One per customer coupons worth $X toward any purchase

Concluding Remarks

In many actual examples of bundling, the bundle components appear to be related in demand. The assumption of independence in demand (that each consumer's reservation price for the bundle is equal to the sum of his reservation prices for the components) is analytically convenient, but it was allegedly made only to emphasize the point that bundling *can* increase profit even if the bundle components are independent in demand.[12] This logical proposition has very limited empirical relevance unless one believes that independence in demand is the "worst case" for the profitability of bundling. But surely it is not.

Fortunately, the incremental cost of modeling the case of interrelated demand seems confined largely to the loss of the simple Adams-Yellen diagrams. Most of the conclusions of this note, for example, do not depend on the independence assumption. The analogy between bundling and two-part pricing, the formulation of profit as a function of two-part prices, the relation between bundling profitability and the behavior of Q/N, the duality between bundling and couponing—all of these remain valid in the case of interrelated demand. Only the propositions concerning the distribution of component reservation prices depend on the independence in demand assumption.

Based on the behavior of Q/N (or equivalently, the fraction of customers that buy both components) it would seem that, ceteris paribus, the most favorable case for bundling as a price discrimination device is

12. See, e.g., Adams and Yellen (1976), p. 476.

the case where the bundle components are substitutes in demand (as, e.g., in "variety packs"). On price discrimination grounds alone, the bundling of complementary components is harder to explain.

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