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Comments on "Firm-specific Differentiation and Competition among Multiproduct Firms"

Michael Katz's (1984) paper is interesting both because it strengthens an important link between marketing and economics and because it extends the analysis of an insightful paper by Mussa and Rosen (1978) in a useful way.[1] Katz generalizes Mussa and Rosen's model by including brand preference as well as quality preferences in the specification of the (linear) utility function of consumers. By this device, he is able to consider product differentiation in a market containing two or more firms. The significance for marketing of this extension is substantial, because the product strategies of firms often explicitly recognize the product lines of rival firms.

I. Applications

The type of model studied in this paper has many implications for further theoretical and empirical research in marketing. Consider, for example, the simplest monopolistic version of the model in which customers fall into only two quality-sensitivity groups. The first-order conditions on q_1 and q_2 are[2]

$$N_2[s_2 - c'(q_2)] = 0 \tag{1}$$

and

$$N_1[s_1 - c'(q_1)] + N_2[s_1 - s_2] = 0. \tag{2}$$

1. K. Sridhar Moorthy (1982) has also developed some valuable generalizations and extensions of the Mussa-Rosen model.

2. In these expressions N_i is the number of consumers in group i, s_i is the quality sensitivity level of consumers in group i, and q_i is the quality variant for consumers in group i where $i = 1, 2$. The notational conventions are described in Secs. I, II, and III of Katz's paper. In particular $s_1 < s_2$ as in Katz's case.

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Solving,

$$c'(q_2) = s_2 \tag{3}$$

$$c'(q_1) = s_1 - \frac{N_2}{N_1}(s_2 - s_1). \tag{4}$$

Equation (3) reflects the well-known result that the consumer group with the highest quality sensitivity obtains the level of quality at which the marginal utility of an increment of quality (measured in units of the numeraire) equals its marginal cost. This is the level of quality that would be supplied to this group of consumers in a competitive market. Consumers with lower sensitivity to quality will be offered a level of quality less than that provided in a competitive market because the quality provided to this group affects the price that can be charged to the customers with high quality sensitivity. The second term on the right side of (4) reflects this upstream externality.

The simple form of these equations makes it easy to study the effect of parameter changes on the firm's product-line decision. For example, as N_1 increases, ceteris paribus, the upstream externality gets less weight in equation (4) and the right side of this equation approaches s_1. If N_1 is very large compared with N_2, the monopolist will choose a value of q_1 that is close to the competitive solution. On the other hand, if N_2 is large relative to N_1, the quality produced for consumers with low quality sensitivity declines. Similarly, the larger the gap in quality sensitivity (i.e., the larger is $s_2 - s_1$), the lower the quality offered to the low sensitivity group.

In the extreme, large differences in the number of consumers in each group or large differences in their sensitivity to quality can lead the firm to abandon production of the lower-quality item. When the low-sensitivity consumers are not served, the firm will set $P_2 = s_2 q_2$ and choose q_2 so that $c'(q_2) = s_2$ (i.e., the quality provided to the high-sensitivity group does not change). In this case, profit is $V_2 = N_2[s_2 q_2 - c'(q_2)]$. When both groups are served, profit is

$$\begin{aligned} V_{12} &= N_2[s_2 q_2 - (s_2 - s_1)\, q_1] + N_1[s_1 q_1 - c(q_1)] \\ &= V_2 - N_2\,[(s_2 - s_1)q_1] + N_1[s_1 q_1 - c(q_1)]. \end{aligned}$$

Insofar as the optimal value of q_2 is the same regardless of whether the low-quality product is sold, V_2 is a constant in the last equation. Thus, the decision to produce the lower-quality good depends on which value of q_1 maximizes

$$V_{12} - V_2 = -\,N_2[(s_2 - s_1)q_1] + N_1[s_1 q_1 - c(q_1)].$$

Assuming that $c'(q_1) > 0$, $c''(q_2) > 0$ and $c(0) = 0$, it will be optimal to produce to the lower-quality good only if

$$s_1 - \frac{N_2}{N_1}(s_2 - s_1) > 0, \tag{5}$$

because, otherwise, the first-order condition for the maximum of $(V_{12} - V_2)$ with respect to q_1 would require a negative value for q_1. Obviously, if N_2 gets very large relative to N_1, the inequality in (5) will not be satisfied. Thus, for sufficiently large N_2, the firm may choose not to sell any of the product to the low-sensitivity group. Similarly, if $(s_2 - s_1)$ gets large enough, the firm may decide not to sell to the consumers with low sensitivity to quality.

It is never true that the monopolist will sell only to the group with low sensitivity to quality. To see why, suppose the firm is producing $\hat{q}_1$ and profitably selling it at the price $\hat{p}_1$ to the low-sensitivity group. Then, $N_1[\hat{p}_1 - c(\hat{q}_1)] > 0$ and $(s_1\hat{q}_1 - \hat{p}_1) \geqslant 0$. But at this price, $(s_2q_2 - \hat{p}_1) > 0$ and all of the higher-quality-sensitive customers would also want the product. Even if the firm did not charge the higher-quality-sensitive consumers a higher price, its profit would rise to $(N_1 + N_2)[\hat{p}_1 - c(q_1)]$ if it sold to them. This argument holds in general; if the firm's optimal product strategy involves selling to consumers of type s_k, it will also sell to consumers with quality sensitivity greater than s_k. Thus we have a result of some empirical interest.

No-Jump Theorem: If a monopolistic firm has a product strategy that includes sales to consumers with s_k quality sensitivity, then it will also sell to all consumers with quality sensitivity greater than s_k. In particular, if the firm sells to any consumers at all, it will surely sell to the group with the highest quality sensitivity.[3]

The empirical usefulness of the no-jump theorem is limited by the assumptions on which it rests. In particular, as Katz shows later in the paper, this theorem need not hold when there are two or more competitive multiproduct brand-differentiated firms. He provides an example of a two-firm industry in which one firm serves the customers with high quality sensitivity and the other firm serves customers with low quality sensitivity.

II. Bunching

The simple monopoly case discussed above indicates how these models might be used in theoretical and empirical analyses of marketing practices. Unfortunately, the model becomes more difficult to apply when it is extended to more general situations. Consider the case of a monopolist facing three or more groups with different sensitivities to product quality. The no-jump theorem continues to hold, but beyond that it is much more difficult to obtain clean analytical results. This is because there is an important qualification concerning the first-order conditions, which Katz does not mention in his paper. Consider the

3. Mussa and Rosen (1978) point out this result in their paper.

application of Katz's first-order equations when there are t consumer groups with different quality sensitivities (where $t > 2$):

$$c(q_t) = s_t$$
$$c'(q_i) = s_i - H_i(s_{i+1} - s_i) \quad i \in \{1,2, \ldots, t-1\},$$

where

$$H_i = \left(\sum_{k=i}^{t} N_k\right) \Big/ N_i.$$

It is tempting to apply the earlier (i.e., $t = 2$) results here (e.g., it would be nice to show that if H_i increases q_i is reduced as the above conditions seem to suggest). Unfortunately, at this level of generality, we cannot tell whether the first order condition will hold for any group other than s_t. This is because the right-hand side of the inequalities above need not be monotonically increasing and hence, since $c''(\cdot) > 0$, the q_i sequence need not be monotonically increasing. This is not a problem when $t = 2$, because monotonicity can be assured with two groups if $s_2 > s_1$.

When monotonicity fails (and it is easy to find values of N_i that result in nonmonotonicity), the monopolist will produce less than t different qualities of the product and some of the different consumer groups will be bunched together and sold the same quality. This bunching phenomenon limits the usefulness of the first-order conditions for finding general propositions about product-line planning. The effort to generalize beyond the simple case makes it more difficult to find unambiguous general principles that describe the behavior of firms. The complexity is compounded, as Katz notes, when the model is extended to multiproduct rivals. On the other hand, the multiproduct case is analytically very interesting, because it illustrates how varied firm and industry behavior can be in more general versions of the model.

III. Assumptions

The diversity of solutions generated by this class of models suggests that these models are quite general. In fact, however, the assumptions of the models are remarkably restrictive. The utility function is linear in the quality-sensitivity parameter and two-piece linear in brand preferences. Moreover, each consumer is assumed to demand only one unit of the good, whatever its price, and there are no income effects. On the supply side, firms are assumed to have constant marginal cost that is an increasing and convex function of quality. The cost of producing any given quality is independent of the firm's level of production of other qualities. Mussa and Rosen introduced some of these assumptions for reasons of tractability and argued that they can be thought of as a linearization of more general functions in the neighborhood of the equilibrium. It is understandable and perhaps even desirable that restrictive assumptions are used at the outset of a new line of inquiry. It is disconcerting, however, that these assumptions become standardized in the research that follows from the initial work. The danger is that the results will be treated as more general than is appropriate. Thus, for example, the self-selection constraints do not take into account the quantity effects of price changes that would be present in a more general model of demand. These quantity effects would make it more difficult to characterize the optimal solution but are likely to have important theoretical and empirical implications. When demand is not perfectly inelastic, it is more difficult to use price as a device to discriminate among consumers, and the bunching problem becomes more severe.

Most of the models treat the number of firms in the market as exogenous, suggesting that the equilibrium concept is not general enough to deal with all of the issues that are likely to arise in planning marketing strategy. It would be interesting to see if an extension of the models shows whether firms can use product-line planning as a strategy to discourage entry to their industry.

IV. Summary

Katz's paper extends the work of Mussa and Rosen in a way that provides many useful insights for both economics and marketing. His analysis indicates that even with the fairly restrictive assumptions in these models, it is difficult to establish general results, so that even more restrictions may be required if the models are used in empirical research. The lack of general results does not diminish the substantial value of this approach for analyzing product-line strategies in the context of traditional economic theory. That these models offer a means of applying economic analysis to a complicated problem in marketing makes them very interesting and suggests that continued research along the lines suggested by Katz's paper will be fruitful.

References

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