Michael L. Katz*
Princeton University

Firm-specific Differentiation and Competition among Multiproduct Firms

This paper examines a multifirm market where each firm produces several variants of a single good and each of the firm's customers chooses a single variant to purchase. When competition between firms drives down the price of one variant, the prices that a firm can charge for other variants in its product line may be lowered as well; the degree of competition in one variant submarket may affect the degree of competition in other submarkets. Consequently, firms may have incentives not to produce full product lines in order to maximize the profits that they earn from firm-specific differentiation advantages in specific variant submarkets.

I. Introduction

The analysis of competition among firms is central to the study of microeconomics and is the major theme of industrial organization theory. Almost all models of competition are constructed with the assumption that each firm produces a single product.[1] Looking at actual firms in our economy, it is clear that the overwhelming majority of businesses produce multiple products. Are models based on the single-product assumption adequate for the study of what are, in fact, multiproduct firms?

In many cases and for many questions, the answer is yes. This is, of course, why we continue to use such models. There are other situations, however, in which models of single-product firms are not appropriate, and in which such models fail to provide insight into the fundamental modes of competitive interaction. Such cases

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1. There have been models of competition among firms who produce goods in several markets each. Here, I am concerned with competition among firms each of which produces several goods in a *single* market.

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occur when there are important interrelationships among the set of products produced by a given firm, either on the demand side or in terms of the costs of production.

Markets in which the costs of production of two or more products are related have been discussed in the economics literature for some time, particularly the issue of the allocation of joint costs. More recently, Baumol, Panzar, Willig, and others have carefully examined the nature of competition in markets where products have interrelated cost functions. This work is surveyed in Baumol, Panzar, and Willig (1982).

Competition in markets where each firm's products are related on the demand side has received much less attention. Often a given firm produces a line of products where different members of this product line are at least partial substitutes for one another. For example, General Electric Corporation produces a variety of refrigerators that have different attributes but which share many features as well. A given consumer will choose from among the many models of refrigerator produced by General Electric and by the firms against whom GE competes in the refrigerator market. By lowering the price of a given model of refrigerator, General Electric can increase the sales of that model.

These additional sales will come from three sources. First, the lower price may attract new customers to the refrigerator market (in the present example, this effect is likely to be very small). Second, some consumers who would have purchased a competing brand of refrigerator will buy this General Electric model instead. Finally, the model for which the price has been lowered will gain sales at the expense of sales of other General Electric refrigerators. In setting the price of one member of a product line, a firm must consider the effects on the demands for its other products.

The profit-maximizing price policy for a monopolist manufacturing a line of products has been analyzed extensively, although far from completely (see Mussa and Rosen [1978] and O'Keeffe [1980] for excellent analyses of the monopoly case). The results of this literature show that the firm may use its product line as a means of sorting its customers and engaging in imperfect price discrimination. The monopolist may use a consumer's purchase decision from among the set of variants the firm produces as a signal of the buyer's underlying willingness to pay for the family of products.

The models on which these results are based all assume that there is a single producer. Thus, while these models provide good insights into the effect that setting the price of one product has on the sales of the firm's other products, the models ignore the effects of competition with other firms.

In this paper, I present a simple model of competition among multiproduct firms in markets where the products manufactured by any

given firm are related on the demand side. There are two notable exceptions to the general lack of attention paid to this area. First, Moorthy (1982) examines competition among multiproduct firms using rather different models from the one considered in the present analysis. Second, Oren, Smith, and Wilson (1983) examine the closely related problem of nonuniform pricing in multifirm markets.

The paper proceeds as follows. In Section II, the basic assumptions and notation are introduced. Section III presents some preliminary results. The nature of multiproduct competition is examined in Section IV through the use of some simple examples. It is shown that a firm with low costs of production may have zero output of a good, while a higher cost firm simultaneously has positive output. The paper closes with a brief conclusion.

II. A "General" Formulation of the Problem

A. Firms

The market to be analyzed consists of m firms indexed by j. Firm j produces n_j variants of the good. Each variant offered in the market is characterized by two attributes: its quality and its brand. Let q_{ij} denote the quality of the ith variant produced by the jth firm and let b_{ij} denote the variant's brand. It is assumed that any given firm must choose a single level of the brand attribute for all of its products: $b_{ij} = b_j$ for $i = 1, 2, \ldots, n_j$. The firm can set the quality levels for all products independently of one another.

The quality attribute should be interpreted as being any characteristic (or bundle of characteristics) such that, although they may differ in the intensity of their preferences, all consumers agree that more of the characteristic is better. There are several possible interpretations of the brand attribute and of the restriction that is be constant across all variants produced by a given firm. First, b_j may represent some physical attribute that is common to all of the firm's products and which cannot be duplicated by other firms. For example, before its patent expired, IBM was the exclusive manufacturer of "golf-ball" typewriters and produced several models, with varying "qualities." In other cases, the technology may be common to all firms but each firm specializes (i.e., chooses a single level of b_{ij} for all i) because of economies of joint production. If a firm must incur new fixed costs for each variant with a different value of b_{ij}, it may be unprofitable to produce all products with differing levels of the brand attribute.

Finally, b_j may represent a firm-specific reputation of the company, as opposed to a product-specific reputation. In markets where, before purchase, consumers have imperfect information about the attributes of the products, the overall reputation of the manufacturer may be an

important element in consumers' purchase decisions. Such effects are particularly likely to be strong when a given consumer makes repeated purchases (although not necessarily of the same variant of a given product or even of the same product) from a single firm over time. For example, a consumer's experience with a General Electric washing machine may influence his or her expectations of the attributes of a General Electric refrigerator (e.g., the quality of the repair service or the reliability of the refrigerator).[2]

The pattern of advertising that one observes in the media gives an indication of the importance of such brandwide effects. For example, many advertisements promote entire product lines rather than any single variant. Further, the recent introductions of Diet Coke and Budweiser Light signal that at least the marketers in these two firms believe that there can be important brand effects which carry over across variants of the product.

In order to focus more sharply on the effects of demand-side interactions among a firm's products, the production technology is assumed to exhibit no economies of joint production or economies of scope. The restriction that b_{ij} must be the same for all variants produced by a given firm can be interpreted as arising from proprietary technology or firm-specific reputation. The production of each variant is assumed to be characterized by constant returns to scale. The jth firm's costs of producing x_{ij} units of a good with quality q_{ij} and brand b_j are $x_{ij} \cdot c(q_{ij}, b_j)$. Letting p_{ij} denote the price of the ijth product, firm j earns profits

$$\pi_j = \sum_{i=1}^{n_j} \{p_{ij} - c_j(q_{ij}, b_j)\}x_{ij}. \tag{1}$$

B. *Consumers*

Consumers have preferences over both qualities and brands. A consumer is characterized by three parameters: (1) basic willingness to pay for the product, w; (2) sensitivity to differences in product quality, s; and (3) the premium he or she is willing to pay to consume products of brand j, $\rho(b_j)$. A consumer with characteristics $[w, s, \rho(\cdot)]$ derives dollar benefits

$$w + sq_{ij} + \rho(b_j) \tag{2}$$

2. In his empirical analysis, Srinivasan (1979) demonstrates the importance of a "brand-specific effect," which he defines as "the component of a brand's overall performance that is not explained by the multi-attribute model" (p. 12), i.e., is not explained by the product's physical characteristics. Of the three interpretations of b_j that I consider here, Srinivasan's definition is closest to the interpretation of b_j as firm-specific reputation, where this reputation may be due to a firmwide marketing effort that is not measured by the multiattribute model.

from the consumption of one unit of a good with quality q_{ij} and brand b_j.

The brand premium may in turn be characterized by two parameters, $\rho(b_j) = -z|b_j - f|$, where z is the consumer's sensitivity to changes in the brand variable and f is a parameter whose interpretation varies with the nature of the firm-specific differentiation.

When b_j represents a physical characteristic shared by all of a given firm's products, f may be thought of as the location (in characteristic space) of the consumer's ideal brand. In this case, z will be greater than or equal to zero, as deviations between actual and ideal characteristics will lower the consumer's valuation of the product.

In markets where b_j represents reputation, f may be thought of as a stochastic noise term. When firm j has reputation b_j, this reputation will be perceived differently by various consumers, and f captures these differences.[3]

For both interpretations of b_j and f, I will assume that each consumer either buys one unit of the good or stays out of the market entirely. The consumer chooses the variant that yields the greatest level of surplus. Faced with the set of prices, qualities, and brands $\{(p_{ij}, q_{ij}, b_j)|i = 1, 2, \ldots, n_j, j = 1, 2, \ldots, m\}$, the consumer's preferred product is the one that solves

$$\max_{i,j} w + sq_{ij} - z|b_j - f| - p_{ij}. \tag{3}$$

If the maximum surplus from purchase is negative, the consumer makes no purchase and receives zero surplus.

The consumer's purchase decision can be broken down into two stages. First, for each brand (firm), the consumer choose his or her preferred variant of that brand. A consumer with characteristics (w, s, f, z) choosing among all variants of brand j will pick variant k, where for all $i \in \{1, 2, \ldots, n_j\}$, $w + sq_{kj} - z|b_j - f| - p_{kj} \geq w + sq_{ij} - z|b_j - f| - p_{ij}$ or, equivalently,

$$s\{q_{kj} - q_{ij}\} \geq p_{kj} - p_{ij}. \tag{4}$$

From equation (4), it is clear that a consumer's choice of variant from a given firm depends only on the consumer's sensitivity to changes in product quality, s. Let $i_j^*(s)$ denote the brand j variant that satisfies equation (4) for a consumer with quality sensitivity s.

Two properties of prices and preferred qualities follow almost immediately from equation (4). First, for a given brand, the prices and qualities of the variants must move together. If $p_{kj} > p_{ij}$ and $q_{kj} < q_{ij}$, no

3. One could allow a given consumer to have a different value of f with respect to each firm to capture differences in the consumer's exposure to various firms and their products.

consumer would choose the kjth product. Second, if two consumers, A and B, have tastes such that $s^A > s^B$, then $q_{i_j^*(s^A)j} \geq q_{i_j^*(s^B)j}$. To see why this relationship must hold, suppose to the contrary that consumer A prefers a variant with lower quality than the variant preferred by consumer B. By equation (4),

$$s^A \{q_{i_j^*(s^B)j} - q_{i_j^*(s^A)j}\} \leq p_{i_j^*(s^B)j} - p_{i_j^*(s^A)j}$$

and

$$s^B \{q_{i_j^*(s^B)j} - q_{i_j^*(s^A)j}\} \geq p_{i_j^*(s^B)j} - p_{i_j^*(s^A)j},$$

a contradiction since $s^A > s^B$. Intuitively, those consumers who value quality the most are the consumers who will purchase the greatest amount of it.

Having calculated the optimal variant of each brand, $\{i_j^*(s) \mid j = 1, 2, \ldots, m\}$, the consumer conducts the second stage of the purchase decision. The consumer chooses the brand whose optimal variant yields the highest level of surplus. Let $v_j^*(w, s, z, f)$ denote the surplus derived by a consumer with characteristics (w, s, z, f) from the consumption of his or her preferred variant from firm j:

$$v_j^* = w + sq_{i_j^*(s)j} - z|f - b_j| \quad - p_{ij} \text{ for } j = 1, 2, \ldots, n.$$

Define $v_0^* \equiv 0$, where "brand 0" is the decision to purchase none of the good.

The consumer's choice of brand, h, satisfies $v_h^* \geq v_j^*$ for all $j = 0, 1, 2, \ldots, m$.

Here we see the dual nature of the firm's problem. In the first stage of the consumer's optimization problem, the products of a given firm compete against one another. In the second stage, the products of the firm are competing against the products of the other firms in the market.

C. *Equilibrium*

In a fully general model, one would analyze the determination of the equilibrium values of prices, qualities, and brands. In this paper, I will treat the set of brands, $\{b_j \mid j = 1, 2, \ldots, m\}$ as exogenously given. Firms' decision variables are prices and qualities, with quantities determined by consumer demands.

Although each firm may not produce all possible variants, one can think of the firm as setting a price schedule for all possible quality levels, $p(q)$. This schedule can be taken to be infinite at all levels of q for which no output is produced. The equilibrium concept applied here is that of Nash-Bertrand equilibrium in prices. That is, each firm sets its price schedule on the assumption that other firms will keep their price schedules unchanged. An equilibrium obtains when no firm wants to alter its price schedule given the price schedules set by competing firms.

III. Two Preliminary Models

Before examining competition among multiproduct firms producing lines of products with related demands, it is useful to consider two related cases.

A. Monopoly

The following simple model of monopoly illustrates how a firm may use its customers' variant choices as a means of sorting these consumers. Consider the simplest such model, in which w and z are constant across all consumers, with $w \geq 0$ and $z = 0$. A consumer with quality sensitivity s derives gross benefits of $w + qs$ dollars from the consumption of one unit of the monopolist's output with quality q.[4] As s rises, the amount of money that a consumer is willing to pay for a unit of given quality rises as well.

The firm would like to charge all consumers their reservation prices for the units that they purchase, but the firm cannot observe the quality sensitivity of any given consumer directly. The firm can, however, set its prices in a way that induces consumers to reveal their quality sensitivity levels. The firm will use the fact that high-s consumers purchase high-quality variants as a basis on which to sort its customers and to price discriminate.

For the remainder of the paper, I will assume that the number of different quality sensitivity levels across consumers is finite and equal to t. The set of quality sensitivity levels is $\{s_1, s_2, \ldots, s_t\}$, where consumers are labeled so that $s_1 < s_2 < \ldots < s_t$. Let N_i denote the number of consumers for whom s is equal to s_i. As shown in Section II*B*, each consumer's choice of variant depends only on s_i. Thus, the firm will offer at most t variants. I will assume that the firm offers exactly t variants, $n_j = t$, although the firm may not produce positive quantities of all t variants. Label the variants so that q_i is the variant purchased by consumers with quality sensitivity s_i (I will suppress the firm index while considering a monopoly). As also shown in Section II*B*, if $i > k$ then $q_i \geq q_k$. Hence, q_1 is the bottom variant in the quality spectrum and q_t is the top variant.

In order to induce an s_i consumer to purchase variant i with quality q_i, the firm must set p_i so that the consumer prefers variant i to all other variants offered:

$$s_i q_i - p_i \geq s_i q_k - p_k \qquad \text{for all } k \in \{0, 1, \ldots, t\}, \tag{5}$$

where $q_0 \equiv 0 \equiv p_0$ denotes the possibility of consumers' exiting the market. Equation (5) states the "self-selection constraints," and they must hold for all $i = 1, 2, \ldots, t$.

4. This model is, essentially, one considered by Mussa and Rosen (1978) and by O'Keeffe (1980). These authors present more complete derivations of the monopoly analysis given here.

Given any set of prices and qualities, high-s consumers prefer variants with qualities at least as high as the qualities of the variants preferred by low-s consumers. Thus, if an s_i consumer prefers variant q_i to variant q_k for all k greater than i, then an s_{i-1} consumer also will prefer variant q_i to any higher-index variant. Similarly, if an s_i consumer prefers q_i to q_k for all k less than i, then so will an s_{i+1} consumer. Applying this argument repeatedly, the self-selection constraints may be reduced to

$$s_i q_i - p_i \geq s_i q_{i-1} - p_{i-1}, \quad \text{for all i,} \tag{6a}$$

and

$$s_i q_i - p_i \geq s_i q_{i+1} - p_{i+1}, \quad \text{for all } i < t. \tag{6b}$$

For a given set of quality levels, the firm will raise p_i until one of the two constraints with p_i on the left-hand side is binding. For the s_t consumers, this fact implies that

$$s_t q_t - p_t = s_t q_{t-1} - p_{t-1}, \tag{7}$$

since this is the only such constraint. s_t is greater than s_{t-1}. Hence, by equation (7), s_{t-1} consumers will strictly prefer variant q_{t-1} to variant q_t. The binding constraint for s_{t-1} consumers is that they not want to move down to the q_{t-2} variant, $s_{t-1}q_{t-1} - p_{t-1} = s_{t-1}q_{t-2} - p_{t-2}$. Continuing this process through backward induction, the binding constraint always is that s_i consumers will move down to variant q_{i-1} if p_i is raised further. All of the binding self-selection constraints are of the form given by equation (6a), and all of the constraints given by equation (6a) are binding.

Using this fact, the profit-maximizing prices can be calculated as functions of the set of variants produced. These prices are the maximal ones which satisfy equation (6a) with strict equality. Thus, profits may be expressed as a function of the set of qualities.

What quality levels does the monopolist set for its output? In particular, how do these levels compare with the socially efficient levels? The efficient level of quality for consumer i, q_i^e, is the quality level that maximizes $s_i q - c(q)$. Hence $s_i = c'(q_i^e)$.[5] It turns out that under monopoly

$$N_i\{s_i - c'(q_i)\} = \left\{\sum_{k=i}^{t} N_i\right\}\{s_{i+1} - s_i\} \geq 0, \tag{8}$$

with strict inequality holding for all i less than t.[6] Equation (8) says that the monopolist sets q_i inefficiently low for all consumers except those with the strongest quality sensitivity.

5. I will assume throughout the paper that $\partial^2 c_f / \partial q_{ij}^2 > 0$; marginal quality improvements become increasingly costly.

6. See Mussa and Rosen (1978) or O'Keeffe (1980) for details of this derivation.

The reason for this distortion is that as q_i is increased, p_i can be increased by at most s_i times that change in q_i without inducing s_i consumers to switch variants. But the incremental quality is worth s_{i+1} times the change in q_i to s_{i+1} consumers. Recalling that s_{i+1} consumers were just indifferent between q_i and q_{i+1}, the firm must lower p_{i+1} by $(s_{i+1} - s_i) \times$ (the change in q_i) in order to prevent s_{i+1} consumers from switching downward. Once p_{i+1} is lowered, it is necessary to lower p_{i+2}, p_{i+3}, on up to p_t. When q_i is increased, the surplus derived by all higher-index consumers rises. The firm counts this increase in surplus as a loss, and it sets q_i at an inefficiently low level.

From the fact that equation (6a) holds with strict equality, equation (8) implies that $p_{i+1} - p_i = s_{i+1}(q_{i+1} - q_i) > c(q_{i+1}) - c(q_i)$. Given the distribution of consumer demand functions assumed here, the price-cost margin rises with the quality of the variant. The firm is able to discriminate against high-s consumers by using their variant choices as signals of their willingness to pay for the firm's output.

B. Variants with Unrelated Demands

Now, suppose that there are several multiproduct firms in the market but that the self-selection constraints for consumers' choices among the variants produced by a given firm are not binding. Equivalently, assume that each firm can prevent a type i consumer from purchasing any variant produced by the firm other than variant i through some exogenous mechanism. This market can be thought of and modeled as a set of independent submarkets, one for each value of s_i in the consumer population. Restricting the analysis to symmetric equilibria (i.e., $p_{ij} = p_{ik}$ and $q_{ij} = q_{ik}$ for all i, j, and k), one can examine the effects of firm-specific differentiation in each submarket. The degree of competition in the ith submarket can be measured by the equilibrium price-cost margin in that submarket, $p_{ij} - c(q_{ij})$.

Consider a market in which there are two firms, firm 1 and firm 2, with $b_1 = 0$ and $b_2 = 1$. Consumers are assumed to satisfy:

ASSUMPTION A: Consumers may be indexed by i, where (a) w is the same for all consumers; (b) all type i consumers have quality sensitivity s_i and brand sensitivity z_i; (c) $s_i > s_{i-1}$; (d) $z_i > z_{i-1}$; and (e) there are N_i type i consumers with preferred brands distributed uniformly over the interval [0, 1].

A consumer with characteristics (w, s_i, z_i, f) derives gross dollar benefits of $w + s_i q_{i1} - z_i f$ from the consumption of the ith variant produced by firm 1 and benefits of $w + s_i q_{i2} - z_i(1-f)$ from the consumption of the ith variant produced by firm 2. Interpreting the brand component as arising from physical product differences, $z_i \geq 0$ for all i. A consumer with characteristics (z_i, f) is willing to pay a premium of up to $(1 - 2f)z_i$ to consume a variant of given quality from firm 1 instead

of from firm 2. Hence, firm 1 has market power with respect to consumers for whom f is near 0, while firm 2 has market power with respect to consumers whose ideal brands are near 1.

Each firm chooses the price, p_{ij}, and quality, q_{ij}, of the variant that it sells in the ith submarket. Given the (assumed) absence of binding self-selection constraints across variants produced for different submarkets, each firm will set q_{ij} at the efficient level, q_i^e. The market shares in each submarket will be determined by the two prices set in that submarket. All consumers whose brand 1 premiums exceed the difference between the prices of brands 1 and 2 will purchase variant q_{i1}. The remaining consumers will purchase variant q_{i2}. This condition is

$$(1 - 2f)\, z_i \geqslant (p_{i1} - p_{i2}) \tag{9a}$$

or

$$f \leqslant \frac{p_{i2} - p_{i1}}{2\, z_i} + \frac{1}{2}. \tag{9b}$$

Taking the demand function derived from equation (9b), the profit level of firm 1 is

$$\pi^1 (p_{i1}, p_{i2}) = N_i \left\{ \frac{p_{i2} - p_{i1}}{2\, z_i} + \frac{1}{2} \right\} \{p_{i1} - c(q_i^e)\}.$$

Firm 1 will set p_{i1} at the level for which

$$0 = \pi^1_{p_{i1}} (p_{i1}, p_{i2}) = \frac{N_i}{2} \left\{ \frac{p_{i2} - p_{i1}}{z_i} + 1 - \frac{p_{i1} - c(q_i^e)}{z_i} \right\}.$$

A similar condition holds for firm 2's choice of p_{i2}. The condition for a symmetric equilibrium is $p_{ij} - c(q_i^e) = z_i$. Thus, the higher the brand sensitivity of consumers in the ith submarket, the greater is each firm's market power and the higher is the equilibrium price-cost margin.

IV. Multiproduct Competition

The effects of competition on the price-quality schedule are examined in this section. It turns out that many of the basic features of the monopolistic solution hold in the multifirm case as well when firm-specific differentiation is present. Some features of the multifirm equilibrium do not arise in the monopoly case, however. In particular, questions concerning the efficient allocation of output across firms arise when there are multiple firms.

A. The Importance of Firm-specific Differentiation

To see the importance of firm-specific, or brand, differentiation (as opposed to product differentiation, which is measured by q), consider the equilibrium in a market where there is no brand differentiation and all firms have the same costs of production, $c(q)$. The following proposition states that, in the absence of brand differentiation, competition

among firms will drive the price of each variant to its marginal cost and all individuals will consume variants with efficient quality levels.

PROPOSITION 1: If $z \equiv 0$ for all consumers, or if $b_j = b_k$ for all j and k, then $p_{ij} = c(q_{ij})$ and $c'(q_{ij}) = s_i$ for all i and j such that $x_{ij} > 0$.

A formal proof of proposition 1 is presented in the Appendix. Intuitively, one may explain the result in the following way. In the absence of firm-specific differentiation, competition among firms drives p_{ij} down to $c(q_{ij})$ for all i and j. The firm earns zero profits on all variants; the firm is indifferent among consumers' choices of variants. Thus, the self-selection constraints have no effect on profits, and the firm will set q_{ij} as if there were no self-selection constraints. In the absence of such constraints, there is no reason for the firm to set q_{ij} at an inefficient level. This proposition says that the perfectly competitive outcome will obtain when there is no brand differentiation.

When brand differentiation is present in the market, firms will have market power with respect to consumers for whom z is large, and the equilibrium may deviate from the perfectly competitive one. There are many possible cases of parameter values. To simplify the study of markets with brand differentiation, I will assume that given any two consumers, A and B, if consumer A is more quality conscious than consumer B (i.e., $s^A > s^B$), then consumer A also is more brand conscious than consumer B (i.e., $z^A > z^B$). Casual empiricism suggests that consumers tend to be more brand conscious at the high end of the quality spectrum. For example, the existence of generic food products may indicate that brand effects are not important at the low end of the quality spectrum for these goods, while brands appear to be important as one moves up the quality spectrum.

When brand differentiation arises from physical differences in the products across firms, a consumer who is sensitive to physical quality differences is likely to be sensitive to brand differences as well. For example, consumers purchasing bottom-of-the-line bicycles may not care about the particular brand that they choose, while consumers purchasing top-of-the-line, lightweight touring bicycles may be more sensitive to differences across brands. A similar pattern may hold in the market for automobiles.

In markets where brand differentiation stems from reputation effects, this positive correlation may arise when high-quality characteristics (such as reliability) are difficult to observe before purchase. For example, with major home appliances, where durability is a key component of quality, the reputation of the producer may become increasingly important as one moves up the product line.

B. Symmetric Equilibria

It is useful to study symmetric and asymmetric equilibria separately. In this part, I consider symmetric equilibria, where the set of variant

quality levels offered is the same for each firm. Asymmetric equilibria, in which firms offer different sets of variant qualities, are discussed in Section IV*C*.

Given the positive correlation between consumers' brand and quality sensitivities, firms will have greater market power with respect to high-s, high-z consumers than with respect to low-s, low-z consumers. As shown in Section III, product differentiation is stronger and competition is less intense in the submarket serving high-z consumers. In the absence of self-selection constraints, the variants sold to high-z consumers would have larger price-cost margins than would the variants sold to low-z consumers. It also was shown in Section III that, in the absence of competition, one would expect the price-cost margin to rise with s.

Due to these two factors, one would expect that per unit sales profitability generally is greater at the high end of the quality spectrum in a given firm's product line. Because of this pattern of profitability, the firm will be concerned with the possibility of high-z, high-s, consumers shifting their variant choices down the quality spectrum. As in the monopoly case considered in the previous section, a firm may set quality inefficiently low for some of its variants to prevent this downward switching.

PROPOSITION 2: If in equilibrium the price-cost margin rises as one moves up the quality spectrum, then $c'(q_{ij}) \leq s_i$ for all i and j.

Note that a firm may set q_{kj} at the efficient level for some k less than t if the pattern of competition is such that, in equilibrium, the self-selection constraints are not binding. This result never obtains the simple monopoly model of Section III*A*. There, the firm always raises its prices until at least one self-selection constraint is binding for each level of s_i.

A formal proof of proposition 2 is given in the Appendix. The intuition behind this result is the following. Given the form of the self-selection constraints, the only reason a firm would set q_{ij} above q_i^e would be to prevent type $(i - 1)$ consumers from switching to variant i. But if the price-cost margin is constant or rising, it is not in the firm's interest to engage in inefficient behavior to prevent such upward switching.

Proposition 2 is not entirely satisfactory in that it *assumes* that the price-cost margin rises as one moves up the quality spectrum. The price-cost margin may be observable in a particular market, and thus one could determine whether the theorem was valid for that market. It is helpful, however, not to have to rely on this assumption. The following result is based on the underlying values of the demand and brand parameters. Like proposition 2, it shows that profit-maximizing firms may set variant quality levels inefficiently low.

PROPOSITION 3: Suppose that (*a*) there are two firms, 1 and 2, with $b_1 = 0$ and $b_2 = 1$; and (*b*) consumers satisfy assumption A. Then, in any symmetric equilibrium in which all consumers are served, q_i is less than or equal to the first-best level of quality, q_i^e.

A proof of proposition 3 is given in the Appendix.

The possibility of consumers' switching variants can have two general effects on the pattern of competition across variants. First, competition at the low end of the quality spectrum may be inhibited by strong brand differentiation at the high end of the spectrum. Suppose, for example, that (*a*) there are two firms, with $b_1 = 0$ and $b_2 = 1$, and (*b*) consumers satisfy assumption A, with $t = 2$.

As was shown in Section III, if the self-selection constraints did not exist, the equilibrium would entail each firm offering two variants, with $c'(q_{ij}) = s_i$ and $p_{ij} = c(q_{ij}) + z_i$ for $i = 1, 2$ and $j = 1, 2$. This outcome may violate the self-selection constraint that s_2 consumers not prefer the lower-quality variant. In order to prevent switching, each firm must raise the level of surplus s_2 consumers derive from purchase of the high-quality variant, which reduces the profits earned by that variant. Or, each firm must reduce the level of surplus that s_2 consumers derive from the low-quality variant.

By setting a low price in the low-quality submarket, a firm may lower its profits higher up in the product line. Thus, full-line firms are less likely to compete vigorously at the lower end of the quality spectrum than are single-product firms. The price-cost margin for the low-quality product, $p_{1j} - c(q_{1j})$, may be greater than z_1, the level at which it would be set if there were no self-selection constraints. Thus, the presence of buyers for the higher-quality variant may lead to a reduction in the surplus derived by purchasers of low-quality variants. The presence of high-z, high-s consumers may be a negative externality for low-z, low-s consumers. This pattern of competition is illustrated in figure 1.

On the other hand, the existence of low-z, low-s consumers may

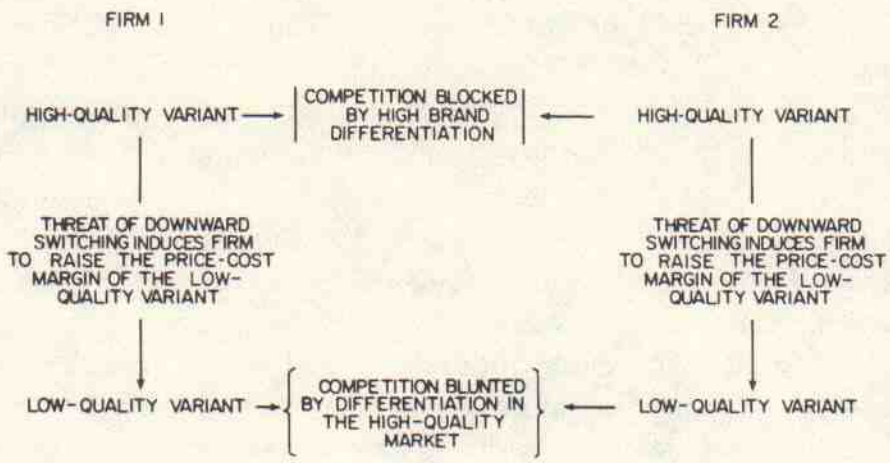

FIG. 1

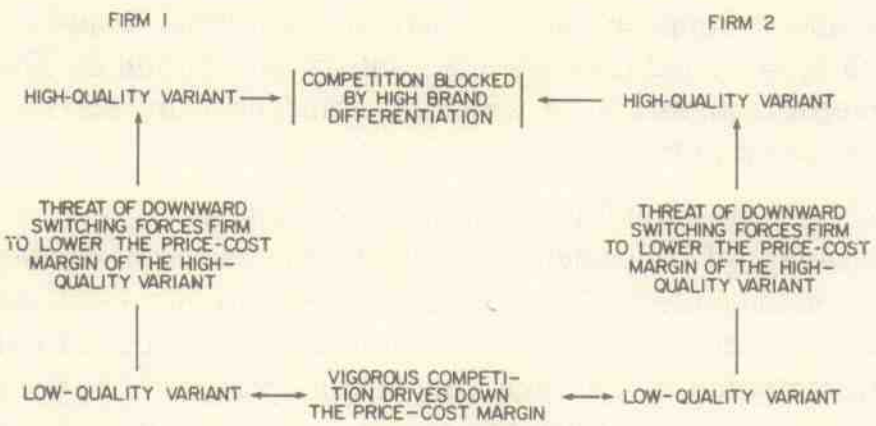

FIG. 2

confer a positive externality upon high-z, high-s consumers. If the low-quality market is sufficiently large, firms will compete for sales in this market even though this competition may force the firms to reduce prices at the high end of the quality spectrum (which would otherwise be isolated from competition by the presence of strong brand differentiation effects). Figure 2 illustrates this pattern of competition.

C. *Asymmetric Equilibria*

The possibility of consumers' switching among the variants of a given firm also can affect firms' decisions whether to offer complete product lines or to compete only in particular market segments. Asymmetric equilibria can arise in which some firms decide to offer only partial product lines even though it is socially efficient for these firms to offer complete lines.

A firm with a strong brand attribute may not offer products at the low end of the quality spectrum in order to maximize the profits that it earns at the high end of the quality spectrum. As a result, a firm with an inferior brand may be left to serve consumers at the low end of the spectrum.

Consider the following example. There are two consumer types of equal size. For type 1 consumers, $s_1 = 2$ and $z_1 = \frac{1}{2}$. For type 2 consumers, $s_2 = 3$ and $z_2 = 4$. For all consumers, $w = 0$ and the consumer's preferred level of the brand attribute is $f = 0$. Firm 1 has a brand attribute, $b_1 = 0$, that is closer to all consumers' preferred locations than is the brand attribute of firm 2, $b_2 = 1$. A consumer with brand preference intensity z is willing to pay a premium of $z|b_2 - b_1|$ to consume a unit of given quality from firm 1 rather than from firm 2. The firms have identical unit cost functions, $c(q) = 1 + q^2/2$.

Under the socially efficient outcome, firm 1 would offer a complete product line and firm 2 would have no sales. Simple calculations demonstrate that the following outcome is an equilibrium, however: (i) firm 1 offers a single variant with quality equal to 3 and price equal to $9 = w$

$+ s_2 3 - z_2|b_1 - f|$; (ii) firm 2 offers a single variant with quality equal to 2 and price equal to $7/2 = w + s_1 2 - z_1|b_2 - f|$; (iii) all type 1 consumers make purchases from firm 2; (iv) all type 2 consumers make purchases from firm 1.

The following intuition lies behind this example. Firm 1 has a valuable reputation on which it would like to earn rents. The firm sells its output only to those customers with respect to whom it has greatest market power. If firm 1 were to enter the low-quality submarket, this product would draw customers away from the high-quality variant on which firm 1 earns a greater per unit return. Thus, firm 1 stays out of the low end of the market entirely in order to maximize the rents it earns on its reputation through sales to brand-sensitive consumers. Firm 2, however, does not have a good reputation. Brand-sensitive consumers will not purchase units from firm 2. Instead, this firm makes sales to brand-insensitive consumers. Given that all consumers prefer brand 1 to brand 2 and production costs are identical, this pattern of sales is inefficient.

The pattern of specialization may be inefficient in terms of direct production costs as well. The forces identified here could lead to the same pattern of specialization even in a market where firm 1 has lower costs of providing the low-quality variant than does firm 2. This misallocation of output across firms may hold in some segments of the airline industry, for example, where the low-quality variants are budget flights on relatively new carriers and the high-quality variants are regularly scheduled flights on the major trunk airlines who have established reputations.

When there are more than two firms in the market, specialization is particularly likely when some of the firms are viewed by low-z, low-s consumers as producing brands that are near-perfect substitutes. Competition among these firms will drive prices down toward costs. If $z = 0$ for the low-end consumers, then prices will be equal to costs.

Since $z = 0$ for consumers in the low-end submarket, a firm with a good brand image will find that this reputation does not help it sell its low-quality variant; the firm will have to set price equal to unit costs for its variant in the low-quality submarket. The high-reputation brand will earn zero profits in the low-quality submarket and may be forced to lower prices to its brand-sensitive customers at the high end of the quality spectrum. It will not be profitable for a firm with a strong brand image to produce low-quality variants. Similar effects would be present in markets where consumers at the low end of the quality sensitivity spectrum had small but positive levels of brand sensitivity.

One case in which firms are likely to have asymmetric reputations is in markets where one or more firms are new entrants. The examples above suggest that entry should occur at the low end of the product quality spectrum and that it may not be profitable for an existing, high-

reputation firm to continue producing variants at the low end of the spectrum.[7]

V. Conclusion

The examples and results presented here demonstrate that there may be important interactions across variants in a product line. There may be spillovers between the submarkets for high- and low-quality variants, where the degree of competition in one submarket affects the degree of competition in another. As a result of these spillovers, some firms may choose not to offer full product lines even in cases where such specialization is inefficient from a social perspective. This specialization is particularly likely to occur when there is intense competition at the low end of the quality spectrum.

The analysis presented here only begins to develop our understanding of competition among multiproduct firms. The problem being analyzed is a difficult and complex one. There is a great deal of room for further work in this area, and there are many possible cases to consider. At this stage in our knowledge of multiproduct competition, it may be most useful to examine actual market data in order to develop a sense of the likely parameter values. If certain relationships appear regularly (e.g., a positive correlation between brand and quality sensitivity across consumers), then the number of theoretical cases that must be considered can be reduced substantially.

Once the empirically important cases are identified, one can develop propositions for these particular parameter relationships. For example, if it were found that brand and quality sensitivity do move together, and if the cost and demand data were available, then one could test the hypothesis that consumers making purchases at the low end of the quality spectrum would be willing to pay more than the marginal cost of quality improvement. One also could attempt to find evidence of inefficient specialization by firms.

Appendix

This Appendix presents the proofs of propositions 1–3. The statements of these propositions are given in the text.

PROOF OF PROPOSITION 1: In the absence of brand differentiation, a consumer with quality sensitivity s_i will choose the firm for which $s_i q_{ij} - p_{ij}$ is greatest. Suppose that, in equilibrium, for some h and l, $p_{hl} > c(q_{hl})$, and consider the behavior of a competing firm, k. There are two possibilities: x_{hk} may be positive or equal to zero. If $x_{hk} > 0$, then it must be the case that $s_h q_{hl} - p_{hl} = s_h q_{hk} -$

7. In markets where b_j represents a physical attribute of the brand, the entrant may well have a superior brand and these results would not apply.

p_{hk}. Firm k can lower all of its prices, $\{p_{ik} | i = 1, 2, \ldots, n_k\}$, by an infinitesimal amount, ϵ, and generate new profits of at least

$$x_{hl}\{p_{hl} - \epsilon - c(q_{hl})\} - \epsilon \sum_{i=1}^{n_k} x_{ik}. \tag{A1}$$

For ϵ sufficiently small, equation (A1) will be positive. Similar arguments can be made in the other case to show that $p_{ij} = c(q_{ij})$.

Suppose that for some h firm l sets $p_{hl} = c(q_{hl})$ and sets q_{hl} such that $c'(q_{hl}) \neq s_h$. In particular, suppose that $c'(q_{hl}) < s_h$. Then firm k could set the level of q_{hk} such that $q_{hk} > q_{hl}$ and $c'(q_{hk}) < s_h$. Firm k must set p_{hk} to satisfy $s_h q_{hl} - p_{hl} \leq s_h q_{hk} - p_{hk}$ and will choose

$$p_{hk} = p_{hl} + s_h(q_{hk} - q_{hl}). \tag{A2}$$

But $p_{hl} = c(q_{hl})$ and $c'(q_{hk}) \leq s_h$. Hence, equation (A2) implies $p_{hk} > c(q_{hl}) + c'(q_{hk})(q_{hk} - q_{hl}) > c(q_{hk})$. Shifting by consumers with other values of s_i does not affect profits since $p_{ij} = c(q_{ij})$ for all other variants. Firm k would earn profits from the sale of units to consumers with quality sensitivity s_h, a contradiction of the fact that in equilibrium $p_{ij} = c(q_{ij})$ for all i and j. A similar argument may be used to show that $c'(q_{hl})$ cannot be greater than s_h. Q.E.D.

PROOF OF PROPOSITION 2: Suppose, to the contrary, that $c'(q_{ij}) > s_i$. Consider the effects of lowering q_{ij} by dq and lowering p_{ij} by $s_i dq$. In the absence of any shifting, the change in profits is

$$x_{ij}\{dp - c'(q_{ij})(-dq)\} = x_{ij}\{-s_i + c'(q_{ij})\}dq > 0. \tag{A3}$$

Now, consider the effects of the price and quality change on consumers' variant choices. The surplus of s_i consumers is unaffected. All higher-index consumers (i.e., consumers for whom quality sensitivity is greater than s_i) will derive less surplus from the ijth variant after this shift. Hence, no consumer will be induced to switch downward to the ijth variant. Consumers with quality sensitivity less than s_i will derive greater surplus from the ijth variant after the changes in q_{ij} and p_{ij}. Any such upward shifts would increase the firm's profits, since the price-cost margin rises with quality (recall that high-s_i consumers purchase high-quality units).

Therefore, by equation (A3), it is profitable for the firm to change q_{ij} and p_{ij}, a contradiction of the optimality of the firm's price and quality levels. Q.E.D.

PROOF OF PROPOSITION 3: Consider the set of variants offered by one of the firms. As long as the only binding self-selection constraints are that type i consumers are indifferent between the ith and $i - 1$st variants (eq. [6a]), variant quality levels will be set less than or equal to their efficient levels for the reason given in the discussion of monopoly. Hence, to establish the present result, one must show only that consumers are not indifferent between their variant and the variant of next higher quality.

Suppose that some consumer type, k, is indifferent between variants k and $k + 1$. Define $\bar{k}$ as the smallest index such that $\bar{k} > k$ and type $\bar{k}$ consumers strictly prefer variant $\bar{k}$ to variant $\bar{k} + 1$. Define $\underline{k}$ as the largest index such that $\underline{k} < k$ and type $(\underline{k} - 1)$ consumers strictly prefer variant $\underline{k} - 1$ to variant $\underline{k}$. If $\bar{k} = t$, then these consumers have no higher-quality variant to which to switch, and if $\underline{k} = 1$, then there is no lower index consumer type who is indifferent between variants $\underline{k} - 1$ and $\underline{k}$.

From the definitions of $\underline{k}$ and $\bar{k}$, for all $i \in \{\underline{k}, \underline{k} + 1, \ldots, \bar{k}\}$, type i consumers are indifferent between variants i and $i + 1$. Given this indifference, it must be the case that the price-cost margin does not rise as one moves up the quality spectrum, or else the firm would induce each consumer type to switch upward;

$$p_{i+1} - c(q_{i+1}) \leq p_i - c(q_i) \quad \text{for all } i \in \{\underline{k}, \underline{k} + 1, \ldots, \bar{k}\}. \tag{A4}$$

Now, consider the price-cost margins of variants $\underline{k}$ and $\bar{k}$. The fact that type $(\bar{k} - 1)$ consumers are indifferent between variants $\bar{k} - 1$ and $\bar{k}$ implies that type $\bar{k}$ consumers strictly prefer variants $\bar{k}$ to variant $\bar{k} - 1$; $s_{\bar{k}}(q_{\bar{k}} - q_{\bar{k}-1}) > s_{\bar{k}-1}(q_{\bar{k}} - q_{\bar{k}-1}) = p_{\bar{k}} - p_{\bar{k}-1}$.[8] By definition, type k consumers strictly prefer variant $\bar{k}$ to variant $\bar{k} + 1$. Therefore, there are no binding self-election constraints involving switching by type $\bar{k}$ consumers. The only binding self-selection constraints in which p_k possibly appears are those involving switching by other consumer types to variant $\bar{k}$. Therefore, the price-cost margin for this variant is greater than or equal to what it would be in the absence of self-selection constraints. Using the results from Section III*B*,

$$p_{\bar{k}} - c(q_{\bar{k}}) \geq z_{\bar{k}}. \tag{A5}$$

Similarly, one can show that for type $\underline{k}$ consumers the only self-selection constraints containing $p_{\underline{k}}$ that possibly are binding involve switching by type $\underline{k}$ consumers to other variants. Thus, the price-cost margin for variant $\underline{k}$ is less than or equal to what it would be in the absence of self-selection constraints;

$$z_{\underline{k}} \geq p_{\underline{k}} - c(q_{\underline{k}}). \tag{A6}$$

Combining equations (A4) through (A6), one obtains $z_{\underline{k}} \geq z_{\bar{k}}$, a contradiction of assumption A*d*. Therefore, for all $i \in \{1, 2, \ldots, t\}$, type i consumers strictly prefer variant i to variant $i + 1$. The only self-selection constraints that may be binding in equilibrium involve downward shifting, and each firm sets the quality level of its variants at or below their efficient levels. Q.E.D.

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8. Here, I am assuming that $q_{\bar{k}}$ is strictly larger than $q_{\bar{k}-1}$. The proof can be modified to treat the possibility that $q_{\bar{k}}$ is equal to $q_{\bar{k}-1}$ using techniques similar to those employed in Katz (1983).