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An Empirical Investigation of the Possibility of Stochastic Systematic Risk in the Market Model

I. Introduction and Summary

In most empirical work on the market model, the parameters of that model are estimated by ordinary least squares, effectively assuming that the systematic risk of an asset, or portfolio, is constant through time. However, a plausible alternative hypothesis would allow the systematic risk of the stock of a company to vary through time. Such variation may arise through the influence of either microeconomic factors (such as operational changes in the company, or changes in the business environment peculiar to the company), or macroeconomic factors (such as the rate of inflation, general business conditions, and expectations about relevant future events). A detailed discussion of these points is provided by Rosenberg and Guy (1976*a*, 1976*b*). Support for the hypothesis that systematic risk varies through time is provided in the studies by Jacob (1971), Blume (1975), and Fabozzi and Francis (1978).

In this paper we allow the possibility that systematic risk of an asset is stochastic. In principle, an attempt to model this stochastic behavior could be made through allowing systematic risk to follow a member of the general ARIMA class of models of Box and Jenkins (1970). Ideally, the available data would be employed to suggest a specific model from this general class. However, this approach has to face very serious

We discuss the market model in which the possibility is allowed that beta is stochastic and obeys a first-order autoregressive process. Following a brief discussion of methodological issues in the estimation and testing of such models, results are reported on an empirical study of a large sample of monthly returns of common stock. We find strong evidence indicating stochastic systematic risk, but relatively little evidence against the random coefficient model.

methodological and computational difficulties, so that in this paper, as a modest first step, attention is restricted to the first-order autoregressive model. This includes, of course, as a special case, the random coefficient model in which systematic risk is taken to be white noise. The random coefficient model has been used in this context by Fabozzi and Francis (1978), Lee and Chen (1980), Alexander and Benson (1982), and Fabozzi, Francis, and Lee (1982). The use of the first-order autoregressive model has been considered by Sunder (1980) and Ohlson and Rosenberg (1982).

The purpose of the present paper is twofold. In the following section we briefly discuss some methodological issues arising in estimation and, more particularly, in hypothesis testing when the market model is generalized to allow the possibility that systematic risk follows a first-order autoregressive process. Our concern here is with the exposition of efficient procedures useful in the analysis of market data.

The third section of the paper reports the results of an extensive empirical study, in which the market model was fitted to monthly rates of returns for a large sample of stocks. Broadly speaking, we conclude that there is strong evidence for rejection of the fixed-parameter model against the alternative that systematic risk is stochastic. However, the case against the hypothesis that the autoregressive parameter, in the model for systematic risk, is zero is far less strong. Furthermore, based on the results of this particular study, we cannot make a strong case against the appropriateness of the random coefficient model.

II. Some Methodological Issues

Let R_t denote the rate of return, in the t th time period, on an asset and R_{mt} the corresponding market rate of return. Then we will write the market model

$$R_t = \alpha + \beta_t R_{mt} + \epsilon_t, \quad (1)$$

where ϵ_t is a random error term. The hypothesis of fixed systematic risk then implies, of course, that in (1) β_t is constant through time. As an alternative, we consider the possibility that β_t follows a first-order autoregressive process, with mean $\bar{\beta}$, so that

$$\beta_t - \bar{\beta} = \phi(\beta_{t-1} - \bar{\beta}) + a_t, \quad (2)$$

where a_t is white noise, assumed to be independent of the process ϵ_t . The special case of (2), where $\phi = 0$, is the random coefficient model.

The stochastic parameter model (1)–(2) can be estimated through numerical maximization of the exact log likelihood function, based on an assumption that the white noise error terms ϵ_t and a_t are normally

distributed. Pagan (1980) shows how the exact log likelihood function for such models can be constructed through use of the Kalman filter algorithm. Pagan also provides a convenient procedure for computing the information matrix of the parameters, so that asymptotic standard errors for the maximum likelihood estimators can be obtained. Some simplification of the computations necessary for the maximum likelihood estimation of the model (1)–(2), for the cases where ϕ is unrestricted and where $\phi = 0$, are available in Bos (1982).

In practical applications, a number of hypotheses about the parameters of the model (1)–(2) may be of interest. In particular, in our empirical investigation of the following section we will want to consider tests of three hypotheses:

i) For a process with stochastic parameters, so that the variance, σ_a^2 , of a_t of (2) is taken to be bigger than zero, we can test the null hypothesis of a random coefficients model against the alternative that the parameters β_t obey a first-order autoregressive process with nonzero parameter ϕ . Thus, the null hypothesis to be tested is that ϕ in (2) is zero. Two tests of this hypothesis immediately suggest themselves. First, the ratio of the estimate of ϕ to its estimated standard error can be used, based on the asymptotic normality of the corresponding random variable. Alternatively, the likelihood function can be maximized subject to the constraint $\phi = 0$ and a likelihood ratio test employed.

ii) We may wish to test the null hypothesis of a fixed-parameter model against the alternative of a random coefficient model. Thus the null hypothesis is that σ_a^2 is zero, while the alternative is that this variance is positive, with the autoregressive parameter ϕ taken to be zero. An asymptotically valid procedure can be obtained through the Lagrange multiplier test of Silvey (1959), since, as noted by Chant (1974), the asymptotic null distribution of the test statistic remains χ^2 when the null hypothesis forces parameters to be on the open boundary of the parameter space implied by the alternative hypothesis. In fact, Breusch and Pagan (1979) have derived the form of the Lagrange multiplier test statistic for general problems of this nature, though it should be emphasized that for this particular problem the appropriate alternative hypothesis is one-sided.

iii) Finally, we would like to test the null hypothesis of a fixed-parameter model against the alternative that β_t is stochastic and obeys a first-order autoregressive process but, by contrast with ii, with the autoregressive parameter ϕ unspecified. Since the nuisance parameter ϕ is unidentified under the null hypothesis, this problem differs from the usual hypothesis-testing framework. However, Davies (1977) has shown in general how the Lagrange multiplier test can be extended to deal with this type of problem. The details of Davies's test have been worked out for this particular application by Watson (1980).

III. Stochastic Parameters and the Market Model: An Empirical Investigation

We fitted the model (1)–(2) to monthly data on a sample of 464 stocks traded on the New York Stock Exchange. These data, covering a period of 10 years from January 1970 to December 1979, were taken from a monthly COMPUSTAT computer tape. The market rate of return, R_{mt} of (1), is the return on the value-weighted market portfolio taken from a monthly computer tape from the Center for Research in Security Prices (CRSP) at the University of Chicago. The stochastic parameter market model was estimated by numerical maximization of the exact log likelihood function. Table 1 shows a summary of the results of some of the hypothesis tests described in the previous section.

The first column of table 1 shows the results of the Lagrange multiplier test of the null hypothesis of a fixed-parameter model, that is,

$$H_0: \sigma_{\delta_i}^2 = 0,$$

against the random coefficient alternative,

$$H_1: \sigma_{\delta_i}^2 > 0, \quad \phi = 0.$$

We see that, at the 5% significance level, the null hypothesis is rejected for a clear majority of our series, while the fixed-parameter model is rejected at the 1% level for very nearly one-half of these series. The second column of the table gives results for the Davies-Watson test of

TABLE 1 Summary of Some Test Results for the Market Model (464 Stocks)

	1	2	3	4	5	6
Five-percent-level tests:						
Number	270	272	37	34	75	63
Percentage	58.2	58.6	8.0	6.7	16.2	13.6
One-percent-level tests:						
Number	234	233	12	8	51	40
Percentage	49.8	50.2	2.6	1.7	11.0	8.6

Notes:

1. Number of rejections of $H_0: \sigma_{\delta_i}^2 = 0$ against $H_1: \sigma_{\delta_i}^2 > 0, \phi = 0$.
2. Number of rejections of $H_0: \sigma_{\delta_i}^2 = 0$ against $H_1: \sigma_{\delta_i}^2 > 0, \phi$ unspecified.
3. Number of series for which $H_0: \sigma_{\delta_i}^2 = 0$ is rejected against $H_1: \sigma_{\delta_i}^2 > 0, \phi$ unspecified, and $H_0: \phi = 0$ is rejected against $H_1: \phi \neq 0$, using the likelihood ratio test.
4. Number of series for which $H_0: \sigma_{\delta_i}^2 = 0$ is rejected against $H_1: \sigma_{\delta_i}^2 > 0, \phi > 0$, and $H_0: \phi = 0$ is rejected against $H_1: \phi > 0$, using the likelihood ratio test.
5. Number of series for which $H_0: \sigma_{\delta_i}^2 = 0$ is rejected against $H_1: \sigma_{\delta_i}^2 > 0, \phi$ unspecified, and $H_0: \phi = 0$ is rejected against $H_1: \phi \neq 0$, using the t -test based on $\hat{\phi}$.
6. Number of series for which $H_0: \sigma_{\delta_i}^2 = 0$ is rejected against $H_1: \sigma_{\delta_i}^2 > 0, \phi = 0$, and $H_0: \phi = 0$ is rejected against $H_1: \phi > 0$, using the z -test based on $\hat{\phi}$.

the null hypothesis of a fixed slope parameter in the market model, against the alternative that this parameter is stochastic, obeying a first-order autoregressive process, that is, $H_1: \sigma_a^2 > 0$, ϕ unspecified. In aggregate, the results for these two tests are very similar. Taken together, our evidence casts very serious doubt on the hypothesis that the fixed-parameter market model for assets is generally appropriate.

When there is evidence suggesting that systematic risk is stochastic rather than fixed, it becomes relevant to ask whether the risk parameter is purely random or autocorrelated, obeying, perhaps, a first-order autoregressive process with nonzero parameter ϕ . Thus, in columns 3–6 of table 1, we show results for tests of the null hypothesis, $H_0: \phi = 0$, against the alternative, $H_1: \phi \neq 0$, for those series for which evidence of stochastic parameters was found. Both the likelihood ratio test and a t -test based on the ratio of the maximum likelihood estimate, $\hat{\phi}$, of the autoregressive parameter to its estimated standard error were employed. Though the latter test indicated more significant results than the former, the overall impression remains that the evidence of autoregressive rather than purely random behavior in systematic risk is not very strong for the great majority of assets in our sample. For example, at the 5% significance level, the null hypothesis of fixed parameters was rejected by the Davies-Watson test for 272 of the 464 stocks. For the 272 stocks, the hypothesis of a random coefficient model was rejected at the 5% level for only 37 and 75 stocks, respectively, using the two tests.

It is pertinent to ask, given these observations, whether we have found strong evidence *in favor* of the random coefficient specification, or simply failed to find strong evidence *against* it. In fact, we believe that the latter interpretation is the more appropriate, as evidenced by the results in table 2. This table summarizes the values found for the

TABLE 2 Maximum Likelihood Estimates, $\hat{\phi}$, for the Autoregressive Parameter

	1	2		1	2
-.1 to -.9	15	8	0 to .1	25	22
-.9 to -.8	11	7	.1 to .2	33	25
-.8 to -.7	20	14	.2 to .3	28	24
-.7 to -.6	29	17	.3 to .4	18	11
-.6 to -.5	28	15	.4 to .5	17	7
-.5 to -.4	25	18	.5 to .6	13	6
-.4 to -.3	18	12	.6 to .7	18	10
-.3 to -.2	27	19	.7 to .8	8	6
-.2 to -.1	27	18	.8 to .9	13	8
-.1 to 0	24	18	.9 to 1	8	6

NOTES:

1. Numbers for all 405 series.

2. Numbers for those series for which $H_0: \sigma_a^2 = 0$ is rejected at the 5% level by the Davies-Watson test.

maximum likelihood estimate of the autoregressive parameter ϕ . In fact, for 59 of our 464 series the stochastic parameter model could not be estimated. In these cases, the likelihood function attained its maximum at $\sigma_\phi^2 = 0$, so that ϕ was unidentified, suggesting strongly, of course, the appropriateness of the fixed-parameter specification for these series. For the remaining 405 stocks, table 2 shows the values obtained for the maximum likelihood estimates $\hat{\phi}$, both for all series and for those for which the null hypothesis of fixed parameters (or time-invariant systematic risk) was rejected at the 5% level by the Davies-Watson test.

The results in table 2 do not indicate a terribly tight clustering of the autoregressive parameter estimates around zero, even when attention is restricted to those cases in which the fixed-parameter model is rejected at the 5% level. On the contrary, a great many of the parameter estimates differ substantially from zero. This observation suggests that our tests of the hypothesis $\phi = 0$ may not be very powerful. We confirmed the impression by noticing that very frequently the asymptotic standard errors associated with the parameter estimate $\hat{\phi}$ were large.

On the basis of our analysis of 120 monthly observations on each of 464 stocks, we have found strong evidence for randomness of systematic risk in the market model. We have not, however, found a great deal of evidence that risk is autocorrelated rather than purely random. Longer series of observations would, of course, have led to more powerful tests of this last hypothesis. However, this consideration must be balanced against the possibility of model instability through time. Our results are also restricted to monthly data. It is likely that any autoregressive behavior in systematic risk would be more manifest in data observed at shorter time intervals, though we have no empirical evidence to offer on this question.

References

- Alexander, G. J., and Benson, P. G. 1982. More on beta as a random coefficient. *Journal of Financial and Quantitative Analysis* 17:27-36.
- Blume, M. 1975. Betas and their regression tendencies. *Journal of Finance* 30:785-95.
- Bos, T. 1982. Exact maximum likelihood estimation of the Kalman filter model. Ph.D. dissertation, University of Illinois at Urbana-Champaign.
- Box, G. E. P., and Jenkins, G. M. 1970. *Time Series Analysis, Forecasting and Control*. San Francisco: Holden Day.
- Breusch, T. S., and Pagan, A. R. 1979. A simple test for heteroscedasticity and random coefficient variation. *Econometrica* 47:1287-94.
- Chant, D. 1974. On asymptotic tests of composite hypotheses on non-standard conditions. *Biometrika* 61:291-98.
- Davies, R. B. 1977. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* 64:247-54.
- Fabozzi, F. J., and Francis, J. C. 1978. Beta as a random coefficient. *Journal of Financial and Quantitative Analysis* 13:101-16.

- Fabozzi, F. J.; Francis, J. C.; and Lee, C. F. 1982. Specification error, random coefficient, and the risk return relationship. *Quarterly Review of Economics and Business* 22 (1): 23-31.
- Jacob, N. L. 1971. The measurement of systematic risk for securities and portfolios: Some empirical results. *Journal of Financial and Quantitative Analysis* 6:815-34.
- Lee, C. F., and Chen, S. N. 1980. A random coefficient model for re-examining risk-decomposition method and risk-return relationship test. *Quarterly Review of Economics and Business* 20 (4): 58-69.
- Ohlson, J., and Rosenberg, B. 1982. Systematic risk of the CRSP equal-weighted common stock index: A history estimated by stochastic parameter regression. *Journal of Business* 55:121-45.
- Pagan, A. 1980. Some identification and estimation results for regression models with stochastically varying coefficients. *Journal of Econometrics* 13:341-63.
- Rosenberg, B., and Guy, J. 1976a. Prediction of beta from investment fundamentals. Part 1. *Financial Analysts Journal* 32 (3): 60-72.
- Rosenberg, B., and Guy, J. 1976b. Prediction of beta from investment fundamentals. Part 2. *Financial Analysts Journal* 32 (4): 62-70.
- Silvey, S. D. 1959. The Lagrangian multiplier test. *Annals of Mathematical Statistics* 30:389-407.
- Sunder, S. 1980. Stationarity of market risk: Random coefficients tests for individual stocks. *Journal of Finance* 35:883-96.
- Watson, M. W. 1980. Applications of Kalman filter models in econometrics. Ph.D. dissertation. University of California, San Diego.

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