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The Optimal Deductible for an Insurance Policy When Initial Wealth is Random*

The literature concerning the choice of optimal deductibles for insurance policies assumes that wealth is nonstochastic except for the value being insured. However, a decision maker's asset portfolio might easily contain other sources of risk, some of which might be uninsurable. This paper examines the choice of a deductible insurance contract in the presence of uninsurable background risk. Existing theorems concerning the optimality of full coverage are shown to hold only under restricted conditions concerning the correlation of insurable risk with other risky assets. More general propositions are established and the effects of risk aversion are considered.

I. Introduction

Individual wealth typically includes risky assets. Often some of these risks are insurable; but other risks, such as market valuation of stocks, inflation, and other general economic conditions, are usually not insurable. A series of recent papers by Kihlstrom, Romer, and Williams (1981), Ross (1981), Nachman (1982), and Pratt (1982) have shown that the introduction of background risk requires stronger definitions of risk aversion. We will use some of these results to examine the choice of the optimal deductible in an insurance contract when initial wealth is random.

An examination of insurance purchases within a broader portfolio is considered by Doherty (1981) and by Mayers and Smith (1983). These papers use a mean-variance framework to examine the separability of the insurance decision.

In general, portfolio relationships have nontrivial implications for insurance-buying strategies. In particular, correlation between insurable and noninsurable risk may affect the optimal level of insurance coverage. Deductible insurance is considered mainly because of its widespread use by many insurers. Furthermore, deductibles

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policies have received more attention in the literature in view of their apparently superior risk-reducing characteristics compared with other insurance arrangements (see, e.g., Arrow 1965).

A basic result in the insurance literature is the Bernoulli principle, which postulates that full coverage is optimal if insurance premiums are actuarially fair. Mossin (1968) extended this result to show that less than full coverage is optimal when the insurance premium includes a positive, proportional loading. In this paper, these results are examined for the case of random initial wealth and are seen to hold when initial wealth and insurable losses are independently distributed. The Bernoulli principle is also seen to hold under negative correlation between initial wealth and insurable losses but not under positive correlation.

The paper also makes use of some recent results on risk aversion with random initial wealth. In particular, results of Kihlstrom et al. (1981) and of Nachman (1982) are used to show that, ceteris paribus, a more risk-averse individual will obtain an insurance policy with a lower level of deductibility.

II. The Model

The basic model is analogous to those of Pashigian, Schkade, and Menefee (1966) and Gould (1969) for choosing the optimal level of deductible. Our approach differs in its consideration of the effects of random initial wealth. Consider a risk-averse individual with random initial wealth $\tilde{w}$. Let $\tilde{w} \in [\underline{w}, \overline{w}]$ with cumulative distribution function (c.d.f.) $G(w)$. In addition, suppose that this individual is exposed to some peril that results in loss x. Let x be the realization of a random variable $\tilde{x} \in [0, N]$ with c.d.f. $F(x)$. The individual's wealth prospect is the random variable $\tilde{Y} = \tilde{w} - \tilde{x}$.

Suppose that deductible insurance is available at any deductible level, $D \in [0, N]$. The (gross) premium for obtaining a policy with deductible level D is exogenously given by the function $P(D)$. This function is referred to as the premium schedule and is assumed to be strictly decreasing over its domain. On the purchase of an insurance contract with deductible level D, the individual's wealth prospect is

$$\tilde{Y} = Y(\tilde{x}, \tilde{w}, D) = \begin{cases} \tilde{Y}_1 = \tilde{w} - \tilde{x} - P(D) & \text{if } x < D \\ \tilde{Y}_2 = \tilde{w} - D - P(D) & \text{if } x \geqslant D. \end{cases} \tag{1}$$

It is convenient, notationally, to write (x, w) for the random pair $(\tilde{x}, \tilde{w})$; no confusion should arise. The distribution of Y depends upon the joint distribution of x and w. Let the joint c.d.f. be denoted by $H(x, w)$. The expected utility derived from this income prospect is[1]

1. Stieltjes integration is used to allow for discontinuities in H.

$$EU(Y) = \int_{\underline{w}}^{\overline{w}} \int_0^D U(Y_1)dH(x, w) + \int_{\underline{w}}^{\overline{w}} \int_D^N U(Y_2)dH(x, w), \qquad (2)$$

where $U(\cdot)$ represents the individual's von Neumann-Morgenstern utility of final wealth and E denotes the mathematical expectations operator. Risk aversion implies that U is monotone increasing and strictly concave.

The level of D that maximizes (2) will satisfy[2]

$$\frac{dEU}{dD} = |P'(D)| \left[\int_{\underline{w}}^{\overline{w}} \int_0^D U'(Y_1)dH(x, w) + \int_{\underline{w}}^{\overline{w}} \int_D^N U'(Y_2)dH(x, w) \right] - \int_{\underline{w}}^{\overline{w}} \int_D^N U'(Y_2)dH(x, w) = 0. \qquad (3)$$

To interpret (3), note that $|P'(D)|$ is the marginal price of increasing coverage (i.e., decreasing D), whereas the bracketed term is simply $EU'(Y)$. Hence, the first term on the right-hand side of (3) represents the marginal cost, in terms of expected utility, of the increased insurance premium for additional coverage. The second term represents the marginal increase in expected utility for covering previously uncovered losses, slightly below D. Thus, (3) has a standard marginal benefit equals marginal cost interpretation.

To facilitate the analysis, consider a conventional form of the premium schedule which entails an actuarially fair price with or without proportional loading. In particular, consider

$$P(D) = (1 + \gamma) \int_D^N (x - D)dF(x), \qquad (4)$$

where $\gamma \geqslant 0$. Note that

$$P'(D) = -(1 + \gamma) \int_D^N dF(x) = (1 + \gamma)\,[F(D) - 1]. \qquad (5)$$

A fundamental theorem of deductible insurance is that full coverage, $D = 0$, is optimal to the individual when the price is actuarially fair. This is the well-known Bernoulli principle. Furthermore, less than full coverage, $D > 0$, is optimal when the loading is strictly positive, as was shown by Mossin (1968). These results continue to hold when initial wealth is random and wealth and losses are independently distributed. However, these results may not hold if x and w are not independent.

2. The second-order condition for (3) is not trivially satisfied. However, this paper does not concern itself with this condition and assumes it to hold. For further discussion on the second-order condition, see Mossin (1968) and Schlesinger (1981).

III. Independence of Loss and Initial Wealth

Under the condition that x and w are independently distributed, the Bernoulli principle continues to hold as is shown in the following proposition.

PROPOSITION 1. Let $P(D)$ be defined as in (4). Then if D^* denotes the expected-utility-maximizing level of deductibility,

$$D^* = 0 \quad \text{if } \gamma = 0 \tag{i}$$

and

$$D^* > 0 \quad \text{if } \gamma > 0. \tag{ii}$$

PROOF. Consider the optimality condition, (3). Since x and w are independent, it follows from (5) that

$$\begin{aligned}(1 + \gamma)\,[1 - F(D)]\Big[\int_{\underline{w}}^{\overline{w}} \int_0^D U'(Y_1)dF(x)dG(w) + [1 - F(D)] \\ \int_{\underline{w}}^{\overline{w}} U'(Y_2)dG(w)\Big] + [F(D) - 1] \int_{\underline{w}}^{\overline{w}} U'(Y_2)dG(w) = 0.\end{aligned} \tag{6}$$

On simplifying (6), some manipulation yields

$$\begin{aligned}\int_{\underline{w}}^{\overline{w}} \int_0^D [U'(Y_1) - U'(Y_2)]dF(x)dG(w) \\ + \frac{\gamma}{1 + \gamma} \int_{\underline{w}}^{\overline{w}} U'(Y_2)dG(w) = 0.\end{aligned} \tag{7}$$

By assumptions made on the utility function, $U'(Y_1) - U'(Y_2)$ is strictly negative for $Y_1 > Y_2$. In the case where $\gamma = 0$, (7) cannot hold unless $D = 0$. In the case where $\gamma > 0$, the left-hand side of (7) will be strictly positive when $D = 0$. Hence it must be the case that $D > 0$ if (7) holds. Q.E.D.

Intuitively, the results of proposition 1 are not surprising. Given a deductible that is optimal at every level of initial wealth, one would expect this deductible to be optimal for expectations on w when w and x are independent.

Turning attention toward risk aversion, recall the Arrow-Pratt measure of absolute risk aversion,[3]

$$r(Y) = \frac{-U''(Y)}{U'(Y)}. \tag{8}$$

One individual is said to be uniformly more risk averse than another individual if the Arrow-Pratt measure of risk aversion for the first individual is at least as great as that of the second individual and is strictly greater somewhere in every arbitrary wealth interval. A funda-

3. See Pratt (1964) and Arrow (1965).

mental theorem of insurance is that if U_1 is uniformly more risk averse than U_2, a lower level of deductibility (i.e., more coverage) is optimal under U_1.[4]

Kihlstrom et al. (1981) showed that the Arrow-Pratt measure of risk aversion might not be appropriate when initial wealth is random. This is because the Arrow-Pratt measure of "uniformly more risk averse" is not necessarily order preserving under expectations on w, as was shown by Nachman (1982). Examples, where the Arrow-Pratt measure does not work appropriately, are given by Kihlstrom et al. (1981) and Ross (1981) and will not be repeated here.

As an alternative, Kihlstrom et al. propose that U_1 be considered uniformly more risk averse than U_2 if

$$\frac{-\int_{\underline{w}}^{\overline{w}} U_1''(Y)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_1'(Y)dG(w)} \geqslant \frac{-\int_{\underline{w}}^{\overline{w}} U_2''(Y)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_2'(Y)dG(w)}, \tag{9}$$

for all realizations of the random variable x. The following theorem of Kihlstrom et al. provides a sufficient condition for (9) to hold.

THEOREM. If U_1 is uniformly more risk averse than U_2 in the Arrow-Pratt sense, and if $-U_i''(Y)/U_i'(Y)$ is a nonincreasing function of Y for either $i = 1$ or $i = 2$, then (9) holds.

An assumption of nonincreasing Arrow-Pratt risk aversion is not too restrictive given the fairly wide acceptance of this assumption throughout the literature. This result will be used following proposition 2.

Before we proceed further, the following lemma is needed.

LEMMA. If U_1 is uniformly more risk averse than U_2 in Kihlstrom et al.'s sense, then for any $D \in (0, N)$,

$$\frac{\int_{\underline{w}}^{\overline{w}} U_1'(Y_2)dG(w)}{\int_{\underline{w}}^{\overline{w}} \int_0^D U_1'(Y_1)dF(x)dG(w)} > \frac{\int_{\underline{w}}^{\overline{w}} U_2'(Y_2)dG(w)}{\int_{\underline{w}}^{\overline{w}} \int_0^D U_2'(Y_1)dF(x)dG(w)}. \tag{10}$$

PROOF. By hypothesis, (9) holds. This implies that, for $z \in [-N, 0]$,

$$\frac{d}{dz} \log \frac{\int_{\underline{w}}^{\overline{w}} U_1'(w + z)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_2'(w + z)dG(w)} < 0. \tag{11}$$

4. This was shown by Schlesinger (1981). If $D \neq 0$, then a strictly lower deductible is optimal. For expositional ease, the phrases "lower deductible," "more coverage," etc., will not be taken to imply necessarily strict inequalities. Strict-inequality versions of the results are straightforward, though tedious, and are left to the reader.

Since log $(\cdot)$ is an increasing function, it follows that

$$\frac{\int_{\underline{w}}^{\overline{w}} U_1'(w + z_1)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_2'(w + z_1)dG(w)} < \frac{\int_{\underline{w}}^{\overline{w}} U_1'(w + z_2)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_2'(w + z_2)dG(w)} \tag{12}$$

for all $z_1 > z_2$.

Rearranging yields

$$\frac{\int_{\underline{w}}^{\overline{w}} U_1'(w + z_1)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_1'(w + z_2)dG(w)} < \frac{\int_{\underline{w}}^{\overline{w}} U_2'(w + z_1)dG(w)}{\int_{\underline{w}}^{\overline{w}} U_2'(w + z_2)dG(w)}. \tag{13}$$

Let $z_1 = -x - P(D)$ and $z_2 = -D - P(D)$. Since $z_1 > z_2$ for $x < D$,

$$\frac{\int_0^D \int_{\underline{w}}^{\overline{w}} U_1'(w + z_1)dG(w)dF(x)}{\int_{\underline{w}}^{\overline{w}} U_1'(w + z_2)dG(w)} < \frac{\int_0^D \int_{\underline{w}}^{\overline{w}} U_2'(w + z_1)dG(w)dF(x)}{\int_{\underline{w}}^{\overline{w}} U_2'(w + z_2)dG(w)}. \tag{14}$$

Inverting (14) completes the proof. Q.E.D.

This lemma can be used to show that, ceteris paribus, an individual who is uniformly more risk averse, in Kihlstrom et al.'s sense, will purchase more insurance.

PROPOSITION 2. If U_1 is uniformly more risk averse than U_2 in Kihlstrom et al.'s sense, then the optimal insurance contract under U_1 includes a lower deductible.

PROOF. Let D^* denote the optimal deductible for U_2. Using the optimality condition, (3), for x and w independent,

$$\frac{1}{|P'(D^*)|} = \frac{\int_{\underline{w}}^{\overline{w}} \int_0^{D^*} U_2'(Y_1)dG(w)dF(x)}{(1 - F(D^*)) \int_{\underline{w}}^{\overline{w}} U_2'(Y_2)dG(w)} + 1. \tag{15}$$

By the lemma above, the right-hand side of (15) will be lower if U_2 is replaced by U_1. It follows that

$$\begin{aligned} \frac{\partial EU_1(Y)}{\partial D}\Big|_{D^*} = |P'(D^*)| \Big[& \int_{\underline{w}}^{\overline{w}} \int_0^{D^*} U_1'(Y_1)dG(w)dF(x) \\ & + (1 - F(D^*)) \int_{\underline{w}}^{\overline{w}} U_1'(Y_2)dG(w)\Big] \\ & - (1 - F(D^*)) \int_{\underline{w}}^{\overline{w}} U_1'(Y_2)dG(w) < 0. \end{aligned} \tag{16}$$

Consequently, D^* is too high for U_1 and a lower deductible should be purchased. Q.E.D.

An immediate consequence of proposition 2 and Kihlstrom et al.'s theorem is the following:

Corollary. If U_1 is more risk averse than U_2 in the Arrow-Pratt sense, and if $-U_i''(Y)/U_i'(Y)$ is a nonincreasing function of Y for either $i = 1$ or $i = 2$, then the optimal insurance contract under U_1 includes a lower deductible.

IV. Nonindependence of Loss and Initial Wealth

The optimal insurance decision behaves conventionally when x and w are independently distributed. Only slight modifications were needed, in the preceding section, to obtain results that parallel those in the literature for nonrandom initial wealth models. However, these results fall suspect when the assumption of independence between x and w is discarded.

Although the correlation between x and w is insufficient in repesenting the joint effects of x and w on the riskiness of the prospect $w - x$, this goes beyond the scope of the present paper.[5] Some of the effects of positive and negative correlation are discussed below. If negative correlation exists between x and w, higher losses have a greater likelihood of occurrence at lower wealth levels, whereas lower losses are more likely to occur when initial wealth is high. The opposite is true under positive correlation.

Intuitively, negative correlation is "bad news" to a risk-averse individual. High losses are most likely to occur when the individual is poorest and can least afford to absorb the loss. ("When it rains, it pours.") Thus, bad luck compounds itself with high x and low w occurring simultaneously. Although the opposite is also true—low x and high w occur together—risk averseness implies that the individual's prospect is less desirable than with independence between x and w.

Since high losses are likely to be exacerbated by the simultaneous occurrence of low wealth, insurance against x would seem even more attractive when negative correlation exists than when x and w are independent. The results are not as robust under negative correlation, partly because of the insufficiency of the correlation measure. A constrained version of the Bernoulli principle holds if we assume the distribution of x and w is bivariate normal.

Proposition 3. Suppose negative correlation exists between x and w, and deductible insurance is available at an actuarially fair price. Then

5. This insufficiency is apparent if one considers that a zero correlation coefficient does not necessarily imply independence. An alternative measure is presented in Doherty and Schlesinger (1982).

full coverage, $D = 0$, is the expected-utility-maximizing level of deductibility.

Proof. This proposition implicitly assumes that insurance is a contract of indemnity; thus contractual conditions preclude negative deductibles. Under these circumstances, it is sufficient to show that $dEU(Y)/dD$ is negative for all $D \in (0, N)$. Since insurance premiums are actuarially fair, it follows from (5) that $|P'(D)| = 1 - F(D)$. Hence, differentiating (2),

$$\begin{aligned} \frac{dEU}{dD} = [1 - F(D)] \int_0^D \int_{\overline{w}}^{\overline{w}} U'[w - x - P(D)]dG(w|x)dF(x) \\ - F(D) \int_D^N \int_{\overline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w|x)dF(x) \end{aligned} \tag{17}$$

for all D.

Since $U'' < 0$, for $D \in (0, N)$, $U'[w - x - P(D)] < U'[w - D - P(D)]$. Hence

$$\begin{aligned} \frac{dEU}{dD} < \int_0^D \int_{\overline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w|x)dF(x) \\ - F(D) \int_0^N \int_{\overline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w|x)dF(x). \end{aligned} \tag{18}$$

Note that

$$\begin{aligned} &\int_0^N \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w|x)dF(x) \\ &= \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)] \int_0^N dG(w|x)dF(x) \\ &= \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w), \end{aligned} \tag{19}$$

where $G(w)$ is the marginal c.d.f. of w.

Now if (x, w) has a bivariate normal distribution with negative correlation between x and w, $\rho < 0$, then

$$\begin{aligned} &\int_0^D \int_{\underline{w}}^{\overline{w}} wdG(w|x)dF(x) = \int_0^D E(w|x)dF(x) \\ &= \int_0^D [Ew + (\rho\sigma_w/\sigma_x)(x - Ex)]dF(x) \\ &> \int_0^D EwdF(x) = \int_0^D \int_{\underline{w}}^{\overline{w}} wdG(w)dF(x). \end{aligned} \tag{20}$$

Since U' is monotonic decreasing, (20) implies that

$$\int_0^D \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w|x)dF(x) < \int_0^D \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w)dF(x). \tag{21}$$

Using (19) and (21), in comparison with (18), it follows that

$$\begin{aligned} \frac{dEU}{dD} &< \int_0^D \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w)dF(x) \\ &\quad - F(D) \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w) \\ &= \int_0^D dF(x) \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w) \\ &\quad - F(D) \int_{\underline{w}}^{\overline{w}} U'[w - D - P(D)]dG(w) = 0. \end{aligned} \tag{22}$$

Consequently, dEU/dD is negative for all $D \in (0, N)$ and $D = 0$ is optimal. Q.E.D.

If positive correlation exists between x and w, the Bernoulli principle may be violated. Since x and w vary together, a natural hedge is provided. A risk-averse individual who is offered the option of insuring against loss x may wish to retain some of the risk inherent in x as an offset to the riskiness in w.

As an example, consider $\tilde{w} = \tilde{x} + k$ where k is some positive constant.[6] In this example, there is perfect positive correlation between $\tilde{x}$ and $\tilde{w}$. Without insurance, an individual's prospect is the nonrandom amount $\tilde{w} - \tilde{x} = k$. If insurance (for x) is available at actuarially fair prices, the individual would not choose to fully insure, thus violating the Bernoulli principle. In fact, it follows from Jensen's inequality that, for this example,

$$EU(w - x) = U(Ew - Ex) > EU(w - Ex). \tag{23}$$

Since the last expression in (23) represents expected utility when full coverage against x is purchased at an actuarially fair price, (23) shows that no coverage is strictly preferred to full coverage. Thus, the Bernoulli principle is violated in the above example.

It can be shown that no coverage is, in fact, the optimal level of insurance in this example. Letting $k = 0$ for notational ease,

$$\tilde{Y} = \begin{cases} Y_1 = -P(D) & \text{if } x < \mathrm{D} \\ Y_2 = x - D - P(D) & \text{if } x \geq D. \end{cases} \tag{24}$$

6. Kihlstrom et al. provide a similar example in an unpublished version of their 1981 article.

Since prices are actuarially fair,

$$\frac{dEU}{dD} = [1 - F(D)]F(D)U'[-P(D)] - F(D)\int_D^N U'[x - D - P(D)]dF(x). \tag{25}$$

Since U is concave, for $D \in (0, N)$

$$\int_D^N U'[x - D - P(D)]dF(x) < \int_D^N U'[-P(D)]dF(x) = [1 - F(D)]U'[-P(D)]. \tag{26}$$

Applying (26) in (25), dEU/dD is positive on $(0, N)$ when $F(D) > 0$. It follows that no coverage, $D = N$, must be optimal.

As a caveat, it must be pointed out that this is only an example. Positive correlation, even perfect positive correlation, does not always imply that no coverage is optimal. For example, if w is bounded on a relatively small interval, such as [1000, 1001], while x is allowed to vary on a fairly large interval, such as [0, 1000], it becomes apparent that the degree of correlation does not significantly affect the level of insurance purchased. Even with perfect positive correlation, the variability in w in the above example is likely to be negligible. Consequently, insurance purchases on x will be nearly identical to those made when w is nonrandom.

V. Summary and Conclusion

This paper has extended some basic results on deductible insurance purchases to the case where initial wealth is random. If random initial wealth is stochastically independent of the insurable loss, three basic results were shown to hold:

i) the Bernoulli principle: full coverage is optimal when the price of insurance is actuarially fair;
ii) less than full coverage is optimal when insurance prices include a positive, proportional loading; and
iii) ceteris paribus, a more risk-averse individual purchases a policy with a lower deductible.

In the case of iii, it was necessary either to use an alternative to the Arrow-Pratt measure of risk aversion or to make additional assumptions concerning the nature of individual preferences. Unfortunately, this result is not easily extendable to the case of nonindependence of loss and initial wealth. This is primarily because of the lack of an appropriate measure of "more risk averse" in this situation.[7]

7. Pratt (1982) introduces some restrictions on conditional certainty equivalents that may be useful in future constructions of appropriate measures of risk aversion.

The extension of ii to the case of nonindependence is not trivial. This is partly due to the inadequacy of correlation as a measure of the interrelationship of the riskiness of x and w. This precludes an extension of ii without some alternative measure of joint riskiness.

The Bernoulli principle, i, holds in the case of negative correlation between x and w but may be violated if there is positive correlation.

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