Albert R. Wildt
University of Missouri

Russell S. Winer
Columbia University

Modeling and Estimation in Changing Market Environments*

Marketing research is often concerned with the modeling of market response. Over time, however, the nature of the relationships being modeled may change due to shifts in consumer tastes, changing marketing strategies, competition, and other reasons. Currently used constant-parameter models are not capable of adequately reflecting such changing market environments. Therefore, alternate modeling approaches should be considered. Recent work in statistics and econometrics suggests the appropriateness of variable-parameter models for such problems. This methodology permits model coefficients to change over time. In this paper various potentially useful variable-parameter models are discussed, reported marketing applications are examined, and issues pertaining to the application of this relatively new methodology to marketing problems are investigated.

In many marketing contexts, managers and researchers are concerned with modeling the response of performance variables (e.g., sales or market share) to one or more decision variables (e.g., advertising or price) or other predictor variables. Modeling efforts in these contexts may consider either the response of an individual entity—such as a single consumer or firm—or the aggregate response of a collection of entities. Usually they involve the specification of mathematical models where the unknown parameters are assumed a priori to be constant. These unknown parameters are then estimated by fitting the models to available data using some form of regression analysis. The rationale for expending effort in this area is the expectation that through activities of this type the decision maker gains understanding of the market process and therefore is able to make better decisions.

The response of a firm's market performance to decision/predictor variables may be moderated by numerous external factors, by competitive factors, and by the levels of the firm's decision variables. For example, the sensitivity of sales to price may vary depending on time of year, overall economic conditions, and/or levels of advertising or promotion; certain characteristics of

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the environment may result in sales' being more or less responsive to advertising, or lower price levels may result in an increase in the sales impact of advertising; or quality improvements in one firm's product may lead to reduced effectiveness of certain of the marketing mix variables of other firms. Complexities such as these might be characteristic of many consumer and industrial markets. Within such complex environments it is expected that the influence of decision/predictor variables on market performance will vary over time (and space where multiple cross-sections are considered). This argument holds regardless of whether individual or aggregate response is considered.

Recognition of the inherent complexities of market environments leads to two conclusions relative to the modeling of these phenomena. First, the typically used regression approaches are most likely unable to describe accurately such complex market environments, and second, alternate approaches should be examined and evaluated to ascertain their potential for improved description, prediction, and/or decision making.

One such approach, which permits the coefficients of response function variables to change over time (and cross-section), is variable-parameter models. This methodology would be particularly appropriate for modeling applications where (1) shocks in the market environment lead to permanent or temporary changes in model coefficients or (2) the coefficients themselves are the outcome of some process, either deterministic or stochastic in nature, which results in nonconstant values. Also, the methodology may be useful in cases where apparent coefficient variation exists because of changes in the aggregation weights of the underlying microcomponents where the data are aggregates, or when the underlying coefficients are constant but the model is misspecified.[1]

The intent of this paper is to focus attention on variable-parameter models as a potentially useful tool for modeling changing market structures. Specifically, this paper presents a brief summary of variable-parameter models, reviews reported marketing applications of variable-parameter models, and discusses issues relevant to the application and implementation of the variable-parameter methodology to marketing problems.

Overview of Variable-Parameter Models

Model Taxonomy

Variable-parameter models can be classified on the basis of the domain of variation, the type of variation, and/or the extent of variation. These

1. Some difference of opinion exists concerning this point; e.g., see Belsley and Kuh (1973) and Rosenberg (1973*a*).

classification bases are neither exhaustive nor mutually exclusive, but each is useful in providing insight into the topic area.

Considering the domain of variation, model coefficients may vary over time and/or space (cross-section). The emphasis in this paper is on the time domain. The type of variation exhibited by the coefficients may be either systematic or stochastic. Systematic variation is present when response coefficients are changing but the change is a function of observable variates. Stochastic-parameter variation arises when coefficient variation is the realization of some stochastic process which may be stationary (constant mean value) or nonstationary. Alternately, variable-parameter models can be viewed according to the extent of coefficient variation. In the general case, all model coefficients are subject to variation, whereas in the limiting case only the intercept varies. Other degrees of variation exist within these two extremes. These classification schemes are illustrated in table 1.

Model Types

From both the modeling and estimation perspectives, the type of variation exhibited by model coefficients is a useful classification base for viewing variable-parameter models. Using this classification base, a brief discussion of variable-parameter models is presented in this section. The models considered permit all model coefficients to vary and are limited to the time domain. A more detailed description of specific models discussed in the literature can be found in Maddala (1977) and Wildt and Winer (1979).

Most variable-parameter models are variants of the linear model

$$y_t = x_t'\beta_t + \mu_t, \qquad t = 1, \ldots, T, \tag{1}$$

where y_t = dependent variable for observation t, $x_t = p \times 1$ vector of fixed independent variables (including intercept) for observation t, $\beta_t = p \times 1$ vector of coefficients for observation t, and μ_t = disturbance term for observation t with $E(\mu_t) = 0$ (usually assumed serially uncorrelated with constant variance). The differences among the various variable-parameter models relate to the assumptions concerning the form or time path of the coefficient vector, β_t.

In general,

$$\beta_t = f(\beta_0, z_t, \beta_{t-1}, \ldots, \beta_{t-i}, \delta_t), \qquad t = 1, \ldots, T, \tag{2}$$

and if a linear relationship is assumed β_t may be expressed

$$\beta_t = \beta_0 + \Lambda z_t + \sum_{i=1}^{t-1} \Gamma_i \beta_{t-i} + \delta_t, \qquad t = 1, \ldots, T, \tag{3}$$

where $z_t = k \times 1$ vector of fixed known variates consisting of environmental, competitive, and/or decision variables (and may include elements of x_t or x_{t-i}), $\delta_t = p \times 1$ vector of disturbance terms with $E(\delta_t) =$

TABLE 1 **Types of Market Response Models Incorporating Response Parameter Variation**

	Type of Variation			
	Systematic		Stochastic	
Domain	Limited	General	Limited	General
Time	Intercept variation in response to an identifiable, time related source (e.g., seasonally changing intercept or discontinuity shifts)	All coefficients vary in response to an identifiable time-related source (e.g., seasonal variation in all coefficients or other systematic time variation)	Time-related random intercept (e.g., adaptive regression or variance components model)	Time-related regression and random coefficients models and sequential coefficient models
Space	Cross-sectional models with only intercept varying over cross section	General cross-sectional models with all coefficients varying over cross sections (or separate models for each cross section)	Cross-section-related random intercept (e.g., variance components model)	Cross-section-related random coefficients models

NOTE.—The category of mixed time and space domain has not been included as it simply represents a combination of the time and space categories.

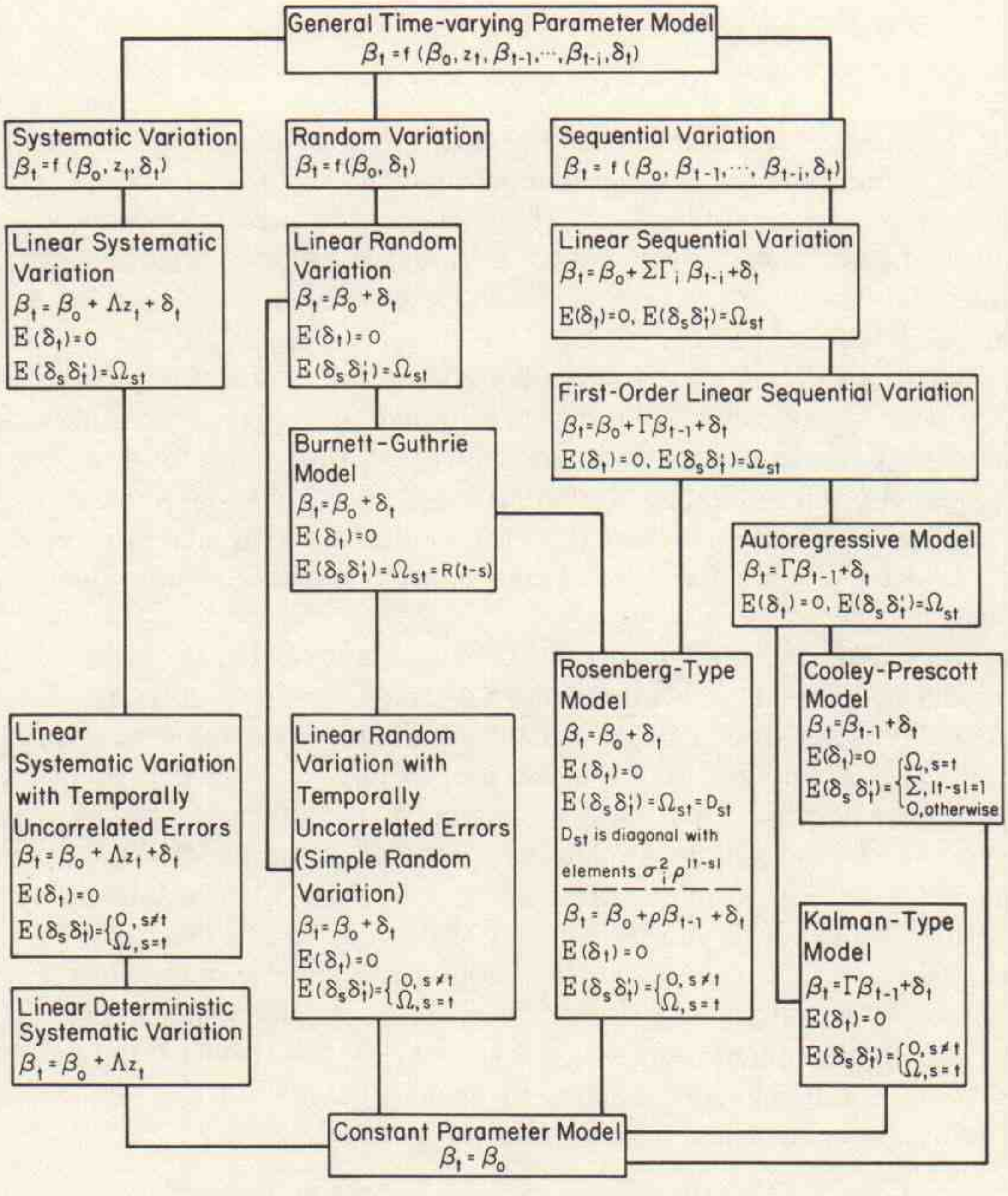

FIG. 1.—Overview of time-varying parameter models

0, $E(\delta_s\delta_t') = \Omega_{st}$, and $E(\mu_r\delta_s) = 0$ for all r and s, $\beta_0 = p \times 1$ vector of constants, $\Lambda = p \times k$ matrix of coefficients, and $\Gamma_i = p \times p$ matrix of coefficients. A flow chart summarizing selected variable-parameter models and the relationships among them is presented in figure 1.

Systematic parameter variation. Systematic variation occurs when model coefficients, β_t, are functionally related to one or more environmental, competitive, and/or decision variables. In addition to these variables, the specification of β_t may contain a constant and/or a disturbance term. In terms of equation (2), systematic variation occurs when

$$\beta_t = f(\beta_0, z_t, \delta_t), \qquad t = 1, \ldots, T, \tag{4}$$

or, under the assumption of a linear relationship,

$$\beta_t = \beta_0 + \Lambda z_t + \delta_t, \qquad t = 1, \ldots, T. \tag{5}$$

If the disturbance term, δ_t, is present, it may be correlated or uncorrelated temporally and/or contemporaneously. If the disturbance term is not present (or alternately if it has zero variance), the model is termed deterministic systematic.[2] Various systematic variation models with differing levels of complexity in the error structure are illustrated in the left panel of figure 1.

There are as many variants of the systematic variation process as there are specifications of β_t. And, although models incorporating systematic parameter variation may utilize any functional form, a linear model is often assumed. Marketing applications have employed time-varying specifications where β_t follows either a polynomial or an exponential curve as a function of time or is a function of the levels of specified decision variables.

Stochastic parameter variation. Stochastic variation arises when model coefficients are a realization of some stochastic process. Two major types of stochastic parameter variation are random and sequential. The distinction between the two is that sequential models hypothesize specific stochastic processes that coefficients follow over time, whereas random models hypothesize deviations from a mean vector resulting from strictly random variations. Unfortunately this distinction is not as sharp as it may seem.

1. Sequential variation models. Sequential variation models are characterized by a stochastic parameter process, usually an autoregressive or moving average process of a low order, which results in a random drift in coefficients over time. In terms of equation (2), sequential variation occurs when

$$\beta_t = f(\beta_0, \beta_{t-1}, \ldots, \beta_{t-i}, \delta_t), \qquad t = 1, \ldots, T. \tag{6}$$

As with other types of variation, a linear model is often used to represent sequential variation processes. Numerous linear sequential variation models are possible by simply placing restrictions on the coefficients and/or the covariance structure of the disturbance term. A number of these models are illustrated in the right panel of figure 1.

The two sequential variation models proposed to date are autoregressive linear models, consider only the time domain, and assume

2. By the inclusion of the error term it may appear that parameter variation is both systematic and stochastic. In fact, Rosenberg (1973*a*) indicates that stochastic parameter variation arises when the variation includes a component that is the realization of some stochastic process, in addition to whatever component is related to observable variables. Belsley (1973) considers a model of the exact same form to be systematic and nonstochastic. For purposes of this paper we will refer to parameter variation that is directly related, at least in part, to the variation in observable variables as systematic variation, even though a stochastic element may be present.

$$\beta_t = \Gamma\beta_{t-1} + \delta_t, \qquad t = 1, \ldots, T, \tag{7}$$

where Γ is a $p \times p$ matrix of coefficients. One was developed by Kalman (1960) and is often called a "Kalman filter." Cooper (1973) and Sarris (1973), among others, consider the Kalman filter model and assume a simple error structure where the disturbance vector, δ_t, is uncorrelated over time. The other was proposed by Cooley and Prescott (1973) and considers the coefficient vector to consist of a permanent time-varying component and an error term. The model assumes

$$\beta_t = \beta_t^* + \mu_t,$$

and

$$\beta_t^* = \beta_{t-1}^* + v_t, \qquad t = 1, \ldots, T, \tag{8}$$

where β_t^* is the permanent component of the coefficient and μ_t and v_t are error terms. The Cooley-Prescott model is a special case of (7) with $\Gamma = I$ and a complex error structure. Also, in the Cooley and Prescott model, δ_t is correlated with β_{t-1}. The relationships among these and other sequential variation models are illustrated in figure 1.

2. Random variation models. Random variation models hypothesize a time-varying coefficient vector which is a function of a constant term and a disturbance term. Assuming additivity, the random coefficient vector (see center panel of fig. 1) is expressed

$$\beta_t = \beta_0 + \delta_t, \qquad t = 1, \ldots, T, \tag{9}$$

which is equivalent to equation (3) with $\Lambda = 0$ and $\Gamma_i = 0$ for all i. Different forms of the random variation model may be obtained by placing restrictions on the covariance structure of δ_t in equation (9). While some type of restriction is necessary if model parameters are to be estimated, complex disturbance structures permitting coefficients to be correlated over time are possible. In the simpler, more restrictive case, coefficient vectors are assumed temporally uncorrelated.

Burnett and Guthrie (1970) and Rosenberg (1973*b*) both consider random variation models with correlated coefficient vectors. The Burnett-Guthrie model, the more general of the two, is a time-series cross-section model with coefficient variation occurring only over time. For this model the covariance of β_s and β_t, $E(\delta_s\delta_t')$, is determined by the difference in the time periods of the two vectors. Rosenberg's model, also a time-series cross-section model, is a special case of the Burnett-Guthrie model with the stochastic elements of the coefficient vector following a first-order autocorrelated process.[3] More restrictive ran-

3. Rosenberg specifies his model as $b_t = (1 - p)b + pb_{t-1} + v_t$, where v_t is random error. It can be shown that an equivalent specification is $b_t = b + w_t$, where $w_t = pw_{t-1} + v_t$. Given Rosenberg's specification it appears appropriate to classify the model as a sequential variation model, whereas given the alternate, equivalent specification it ap-

dom parameter models have been considered by Rao (1965) and Hildreth and Houck (1968). Rao's model is a time-series, cross-section model, whereas Hildreth-Houck consider only a single cross-section with coefficients uncorrelated with each other, $E(\delta_t \delta_t') = D$, a diagonal matrix, and no model error term, $E(\mu_s \mu_t) = 0$, for all s and t.

The various random variation models discussed above and the interrelationships among them are illustrated in the center panel of figure 1. Examination of figure 1 reveals that the random variation models are special cases of the systematic variation models when Λ (of eq. [5]) equals zero and also special cases of the sequential variation models when all Γ_i are zero. In addition, the constant parameter model is nested within (a special case of) all the variable parameter models illustrated in figure 1.

Marketing Applications of Time-varying Parameter Models

A small number of applications of time-varying parameter models have appeared in the marketing literature. These applications, all exploratory studies involving single-equation models, are summarized in table 2 and are briefly discussed below.

Systematic Variation Applications

Most time-varying parameter marketing studies assume coefficients change over time in a systematic fashion.[4] Beckwith (1972) analyzed the Lydia Pinkham data (Palda 1964) by modeling coefficients as polynomials in time and using dummy variables allowing discrete shifts for specified time periods. He found increasing advertising effectiveness and decreasing influence of lagged sales over the years analyzed. Parsons (1975) assumed the sales effects of lagged sales and advertising for Sapolio, a well-known soap product, followed separate exponential curves over the product's life cycle. He concluded that the advertising elasticity for Sapolio decreased while lagged sales effect increased over time. In another study using the Sapolio data, Erickson (1977), using a polynomial in time for the advertising effect and a constant lagged sales effect, found that advertising's impact on sales did not decrease monotonically over time, as Parsons had found, but rose to a peak late in the life of the product. Arora (1979) used an exponential model to analyze how promotional variables affect sales of ethical drugs. He concluded

pears appropriate to classify it as a random variation model. One notes that the distinction between sequential models and random models with serially correlated disturbances is not a clear one.

4. Numerous studies have used models incorporating systematic seasonal variation, usually limited to the intercept term. These are not considered here.

that the advertising elasticity decreased and the lagged sales elasticity increased over time, while the elasticities of direct mail and retailing were relatively time invariant. Erickson and Montgomery (1979) considered two pharmaceutical products, one with declining sales and another with increasing sales, and modeled the relationship between market share and three communication variables, sampling and literature, direct mail, and journal advertising sales, using time polynomials. They concluded that advertising and sampling effectiveness decreased while direct mail effectiveness increased for the declining brand and that the effectiveness of all variables increased for the growing brand over the time period studied. The estimated variation of all coefficients in this study was relatively small. Simon (1979) studied three German product categories: pharmaceuticals, detergents, and household cleaners. Using various models with price and lagged sales effects as a function of time, he concluded that price elasticities decreased to a minimum at the maturity stage and then increased during the decline stage of the product life cycle. Parsons and Abeele (1981) estimated the sales response of a Belgian ethical drug where sales call elasticity was assumed to vary as a function of the number of samples and handouts used. They concluded that a positive relationship existed between sales call elasticity and the use of collateral material.

Sequential and Correlated Random Variation Applications

A number of studies utilize stochastic sequential or correlated random models in modeling parameter variation over time. Winer (1979) used the aforementioned Cooley-Prescott (1973) model, a sequential stochastic approach, to analyze the Lydia Pinkham data. He found results similar to Beckwith's with regard to advertising and lag sales. An application of Rosenberg's stochastically convergent regression, a correlated random stochastic model, is described by Johansson (1978). The context is that of estimating the demand for gasoline over time and across individuals, but no empirical results are provided. Erickson (1981) used a sequential model (Sarris 1973) as an exploratory technique in examining the ethical pharmaceutical considered earlier by Erickson and Montgomery (1979). This exploratory analysis led to the estimation of a discrete shift (dummy variable) model which indicated decreased effectiveness of advertising and sampling but no change in direct mail effectiveness. Pekelman and Tse (1980) modeled market share as a function of lag market share and advertising share, where the coefficient on advertising share follows a first-order sequential time-varying pattern, in an illustration of their adaptive control approach to the advertising budgeting decision. Liu and Hanssens (1981) used a Bayesian approach to estimate a Kalman-filter-type model relating sales and price for an inexpensive branded gift item. This study considered data from multiple cross-sections but only time-domain parameter

TABLE 2 Marketing Applications of Time-varying Parameter Models

Basic Model Type and Reference	Product	Model Used	Dependent Variable	Independent Variables	Major Findings
Systemic variation:					
Beckwith (1972)	Lydia Pinkham	Polynomial in time	Annual dollar sales	Advertising lag sales	Advertising effectiveness increased and lag sales influence decreased over time
Parsons (1975)	Sapolio	Exponential in time	Annual unit sales	Advertising lag sales	Advertising elasticity decreased and lag sales elasticity increased
Erickson (1977)	Sapolio	Polynomial in time	Annual unit sales	Advertising* lag sales	Advertising effect increased to peak late in product life then declined
Arora (1979)	Ethical drugs	Exponential in time	Monthly dollar sales	Advertising direct mail detailing	Advertising elasticity decreased, lag sales elasticity increased, direct mail and detailing elasticity time invariant
Erickson and Montgomery (1979)	Ethical pharmaceutical	Polynomial in time	Monthly market share	Sampling direct mail advertising	Advertising and sampling elasticities decreased and direct mail elasticities increased
Simon (1979)	Pharmaceuticals, detergents, and cleaners	Hyperbolic cosine†	Quarterly bimonthly unit sales	Price lag sales	Price elasticities decreased to minimum at maturity then increased
Parsons and Abeele (1981)	Ethical pharmaceutical	Multiplicative interactive	Unit sales	Sales calls‡ mailings lag sales	Sales call elasticity increases with the use of samplers and handouts
Stochastic, pure random variation:					
None on time-series data	N.A.	N.A.	N.A.	N.A.	N.A.
Stochastic, sequential variation:					
Little (1966)	Simulation	Rosenberg (1973*b*)	N.A.	N.A.	N.A.

Stochastic, random variation with correlated coefficients:					
Johansson (1978)	Gasoline	Rosenberg (1973*b*)	N.A.	N.A.	N.A.
Winer (1979)	Lydia Pinkham	Cooley-Prescott (1973)	Annual dollar sales	Advertising lag sales	Advertising effectiveness increased and lag sales influence decreased over time
Erickson (1981)	Ethical pharmaceutical	Sarris (1973)	Monthly market share	Sampling direct mail advertising	Increasing then decreasing sampling elasticity; inconclusive shifts in direct mail elasticity, slight decrease in advertising elasticity
Pekelman and Tse (1980)	Cereal	Kalman filter	Market share	Advertising share* lag market share	N.A.
Liu and Hanssens (1981)	Inexpensive gift item	Kalman filter§	Biweekly sales	Price	Price elasticity increased slightly over time period
Other:					
Wildt (1976)	Consumer product	Moving window	Bimonthly unit sales	Advertising promotion lag sales	Lag sales elasticity increased, promotion elasticity decreased, no change in advertising elasticity over time
Mahajan et al. (1980)	Lydia Pinkham	Feedback	Annual dollar sales	Advertising lag sales	Oscillation in lag sales and possible oscillation in advertising effects
Mahajan et al. (1980)	Lydia Pinkham	Moving window	Annual dollar sales	Advertising lag sales	Advertising effectiveness increased and lag sales influence decreased over time

* Only advertising coefficient was time varying.
† On price variable.
‡ Only sales call coefficient varied (with amount of samples and handouts).
§ Cross-sectional, time-series model with parameters varying only over time.

variation was considered. Two other studies have utilized sequential time-varying parameter models but did not estimate them with actual data. Little (1966) was the first published marketing study to incorporate time-varying coefficients. The context was that of developing a model firms could use to allocate promotional expenditures optimally over time. Although the study was largely expository, one of the promotional parameters in the sales response function was hypothesized to follow the sequential stochastic scheme later postulated by Rosenberg (1973*b*). Erickson (1979) performed a Monte Carlo analysis in the context of estimating model coefficients over the product life cycle. He attempted to find which of five procedures—OLS, moving window OLS, OLS on a time polynomial, ridge regression on the time polynomial, or Sarris's (1973) sequential stochastic method—best estimated a four-parameter model with each parameter varying in a systematic but different time path. The results indicate that performance depends on sample size, degree of independent variable covariation, and shape of parameter variation.

Simple Random Variation Applications

Simple random variation models yield only a mean coefficient vector and do not track the effect of marketing policies over time. These models are not as valuable to the decision maker as other models which permit parameter tracking over time. We are unaware of any time-varying simple random coefficient models in the marketing literature, but random coefficients models have been applied in a cross-section time-series context (e.g., Bass and Wittink 1975).

Other Reported Applications

Two studies have utilized procedures which cannot be classified by the typology of table 1, since they do not specify a model for the time path of the coefficients. Such approaches are probably more useful at an exploratory stage of analysis. A study by Wildt (1976) utilized moving window regression (Belsley 1973) to analyze the effect of advertising, promotion, and several other variables on industry unit sales. He found that advertising effectiveness did not appreciably vary over time, but the influence of promotion decreased and lagged sales increased. Mahajan, Bretschneider, and Bradford (1980) applied both moving window regression and an adaptive estimation approach (Carbone and Longini 1977) to obtain time-varying coefficients for advertising and lagged sales for the Lydia Pinkham data. The moving window analysis yielded results consistent with both Beckwith (1972) and Winer (1979), but markedly different from the adaptive approach.

Summary

The major purpose of many marketing applications of time-varying parameter models was to illustrate an estimation technique, although some did attempt to address substantive issues. Independent of their objective, a number of the applications did consider the same data base or addressed the same marketing question. Three applications (Beckwith 1972; Winer 1979; Mahajan et al. 1980) used the Lydia Pinkham data. All assumed similar linear models with sales a function of advertising expenditures and lagged sales. The results of four of the five time-varying estimation procedures used—systematic discrete-shift, polynomial in time, sequential-stochastic, and moving window models—consistently indicated increasing advertising effectiveness and decreasing influence of lagged sales over time. The adaptive approach (Mahajan et al. 1980) gave results inconsistent with the other four models—an inconsistency unexplained by the authors.

Two applications estimated time-varying models for the Sapolio data. Parsons (1975) estimated a log-linear model with unit sales a function of advertising and lagged sales, no intercept term, and the coefficients varying exponentially with time. Erickson (1977) considered the same data and a similar model form but specified a constant intercept and constant lagged sales coefficient and modeled the advertising coefficient as a third-degree polynomial in time. Parsons concluded that advertising elasticity (the coefficient of advertising in the log-linear model) decreased (exponentially) over time and lagged sales elasticity increased (exponentially) over time. Erickson concluded that advertising elasticity followed a second-degree function of time, rising to a peak and then falling in the latter half of the data period. The inconsistent results from these two studies could be expected due to the restrictive nature of the models employed. The Parsons model forced monotonicity on the elasticities and, because of the form of the model and the relative magnitude and correlation patterns of the variables, virtually assured that advertising and lagged sales elasticities would vary inversely. Erickson's model forced the lagged sales coefficient to be constant but allowed freedom of time movement for the advertising coefficient. But the data characteristics together with the selected model formulation all but precluded the large variation in advertising elasticity noted by Parsons. Both models were overly rigid, especially given the exploratory nature of the investigations, and, in fact, each model formulation essentially precluded the possibility of obtaining results consistent with the other study.

Five of the applications included in table 2 consider pharmaceutical products, but no generalization can be drawn from these studies. This is understandable because different products, in different stages of development, were considered and different variables were used in the model formulations. Also, a number of the marketing studies have explicitly examined the issue of changing elasticities over the product life cycle, while others implicitly investigate the same issue by reason of the extended time duration of the data used. Six studies included in table 2 estimated time-varying elasticities of advertising and lagged sales for lengthy data series. Three of these studies (Arora 1979; Parsons 1975; Wildt 1976), using data for different products in different industries, concluded that the impact of lagged sales increased over the life cycle and, except for Wildt, that advertising elasticity decreased. Three other studies (Beckwith 1972; Mahajan et al. 1980; Winer 1979), each using the same Lydia Pinkham data, reached the opposite conclusions.

Although the results to date are far from conclusive, some generalizations are probably warranted. First, too little consideration seems to be given to alternative varying parameter specifications. Models are often selected on convenience rather than theoretical grounds. Thus, it would seem that a more systematic approach to modeling and estimating in a changing market environment is needed. Second, evidence suggests that market response to policy variables does change over time. The implication of this is that time-varying models appear to have legitimate application to estimating market structures.

Issues relating to the Application of Variable Parameter Models in Marketing

The application of the variable parameter methodology to marketing problems is largely unexplored. The position taken in this paper is that variable parameter models have yet to be proven but offer sufficient promise to warrant consideration. In this section we first consider the potential benefits to be derived from using variable parameter models. This is followed by discussion of the implications of model misspecification and the application of variable parameter models to marketing problems where the form of coefficient variation may be unknown. These topics have not received sufficient attention in the early stages of research in this area. At present there is no prescribed procedure available for selecting an appropriate model when uncertainty exists concerning the type or form of variation, and the effects of model misspecification are largely unknown. Summary comments on the ease and cost of implementing the computational procedures are provided. Further issues relating to variable parameter models are outlined in the final part of this section, which addresses areas for future research.

Potential Contributions to Marketing Theory and Practice

The concept of temporal change in market response has considerable intuitive appeal, but marketing theory in this area is virtually nonexistent. An exception is the speculation by Mickwitz (1959), popularized by Kotler (1971), concerning the changing impact of marketing mix variables over the product life cycle.

In the absence of accepted theory regarding the nature of changing market response, it may be argued that it is inappropriate to press forward with the development and application of variable parameter models to marketing problems. We do not subscribe to this view. Instead, we foresee two possible contributions of this methodology relative to the accumulation of marketing knowledge. First, the observation of marketing phenomena is an essential element in the development of marketing knowledge, and exploratory research activities employing variable parameter methods in conjunction with a variety of different data bases may provide insights which will aid in that development. Second, once hypotheses are generated, through either inductive or deductive means, the variable parameter techniques represent a methodology for testing these hypotheses. For these reasons, we feel that appropriate methodological and empirical research on variable parameter techniques can contribute to the development of marketing theory.

In addition to a contribution to marketing knowledge, there is the issue of marketing practice. From a managerial perspective, the detection of a changing market environment, even by means of a misspecified model, may provide insight which will lead to large payoffs. Given that currently used constant parameter models are not capable of adequately reflecting the changing environment characteristic of many market situations, alternative modeling approaches should be considered. The potential advantages of models which permit coefficients of response function variables to change over time are many. First, the decision maker can estimate current values of response function parameters and is not limited to estimating averages across historical data. For example, it is important to know the response to advertising expenditures or the price elasticity for the most recent time period if decisions are to incorporate up-to-date information. Second, improved forecasting capabilities can be realized with these models (Winer 1979). This is important if forecasts are used to plan production runs or determine quotas. Third, if the decision maker wishes to determine optimal levels of policy variables, the use of coefficient estimates from more typical models can yield results which vary considerably from the true optimal levels, whereas better approximations may be provided by variable parameter estimates. Fourth, if the model incorporates competitive policy variables, a manager could determine if patterns of competition have changed over time. Finally, if parameters do not change over time, the variable parameter models will typically indicate a stationary environment and give results comparable with more commonly used constant parameter models.

Implications of Model Misspecification

There are certain costs associated with the use of variable parameter models and the accompanying estimation procedures. Also, there are costs associated with not employing the appropriate model in the presence of parameter variation. To a large extent, the overall usefulness of variable parameter models will be determined by the magnitude of these costs. In this section we consider these costs in terms of misspecification error. Three types of misspecification error are considered. The first is the error resulting from the use of constant parameter models in the presence of parameter variation. If this error is small, the added effort of using variable parameter models may not be justified. However, the greater the magnitude of this error, the greater the incentive for using variable parameter models. The second type of error is that resulting from the use of a variable parameter model if no parameter variation is present. A large error here would indicate that serious consequences may result from the use of variable parameter models. The third type of error is that incurred in using an inappropriate variable parameter model in the presence of parameter variation. If this error is large relative to the error associated with using constant parameter models in the presence of parameter variation, it would suggest that it may be inadvisable, in many instances, to employ variable parameter models.

The problem of model misspecification relative to variable parameter models has not been extensively investigated. Two recent studies have considered this issue. Erickson (1979) employed a Monte Carlo approach with a deterministic systematic variation (single equation, four parameter, polynomial in time) model and five estimation procedures: OLS, moving window regression, a systematic polynomial in time model estimated with OLS, the same polynomial in time model estimated with ridge regression, and Sarris's random sequential model. He evaluated the five procedures on mean absolute and mean squared differences between estimated and true coefficient values and mean squared prediction error on a holdout sample but reached no general conclusions. His results indicate that, for the particular model considered, relative performance of the estimation procedures varies with the shape of coefficient time path, sample size, and covariation among variables. Wildt and Winer (1979) undertook an analytical examination of four single-equation two-parameter models: an OLS model, a simple random (Hildreth-Houck) model, a random sequential (modified Rosenberg) model, and a systematic variation (linear in time) model. The study investigated the consequences of misspecifying each of the four models when using each of the remaining models as the estimation model. Tentative conclusions based on the Wildt-Winer results are presented in figure 2 and discussed below.

The first issue considered is the consequence of using a constant parameter model in the presence of parameter variation. As noted in the first row of figure 2, constant parameter models are likely to yield coefficient estimates very near to the average coefficient for the time period considered. Therefore, the consequences of using a constant

<table>
<tr><th rowspan="2">ESTIMATED MODEL</th><th colspan="4">TRUE MODEL</th></tr>
<tr><th>OLS</th><th>PURE RANDOM</th><th>SEQUENTIAL</th><th>SYSTEMATIC</th></tr>
<tr><td>OLS</td><td></td><td colspan="3">$\hat{B}$ LIKELY TO BE NEAR SAMPLE MEAN OF B_T
CONSEQUENCES WILL VARY GIVEN CONTEXT</td></tr>
<tr><td>RANDOM</td><td rowspan="3">NO MAJOR DIFFICULTY SO LONG AS OLS IS NESTED IN ESTIMATED MODEL</td><td></td><td colspan="2">$\hat{B}$ LIKELY TO BE NEAR SAMPLE MEAN OF B_T</td></tr>
<tr><td>SEQUENTIAL</td><td>NO PROBLEM IF MODELS NESTED</td><td></td><td>POTENTIAL PROBLEMS</td></tr>
<tr><td>SYSTEMATIC</td><td>NO PROBLEM IF MODELS NESTED</td><td>POTENTIAL PROBLEMS</td><td></td></tr>
</table>

FIG. 2.—Consequences of misspecification based on Wildt-Winer results (sequential includes correlated random variation models, such as Rosenberg's [1973*b*]).

parameter model when the underlying coefficients do vary is minimal when the deviation of the time-varying coefficients from their average is small. But this may be of little consolation when these deviations are large, as may be the case when the underlying process is characterized by systematic or sequential variation. It should be noted that a careful examination of the OLS residuals will, in many instances, lead to the rejection of the constant parameter model when coefficients are time varying.

The second issue of concern is the consequence of using variable parameter models when no parameter variation is present. This situation is considered in the first column of figure 2 and represents no serious problem so long as the constant parameter model is nested within (a special case of) the variable parameter model used for estimation. (For the case of nested models, standard statistical testing procedures are available.)

The third issue is the consequence of using an inappropriate variable parameter specification. This situation is considered in the remaining cells of figure 2. As is noted in the second column of figure 2, if the underlying variation is simple random the consequences of using an inappropriate variable parameter estimation model will likely be minimal. This type of model misspecification may be detected using standard statistical testing procedures whenever the true random model is nested within the misspecified variable parameter estimation model. As noted by the second row of figure 2, when the estimation model is a simple random variation model the estimated mean coefficients are likely to be near the sample means of the time-varying coefficients. As when using a constant parameter estimation model, the consequences of misspecification in this context will vary considerably given the degree of variation in the time path of the true coefficients. It is noted that the careful examination of residuals from the estimated random variation model may suggest the inappropriateness of the specification. The potentially most serious misspecification problems relate to cases involving the systematic and stochastic sequential models. Unfortunately, given the research conducted to date, no general conclusions concerning the implications of this type of misspecification can be drawn.

Other types of misspecification not considered in figure 2 are those relating to misspecification within a class of models, for example, the use of inappropriate systematic variation models to estimate parameters which are subject to systematic variation. We are not aware of published research dealing with this type of problem.

In conclusion, the consequences of employing variable parameter models when no parameter variation is present appear to be minimal, especially if appropriate testing is performed. In contrast, the consequences of using a constant parameter model is the presence of parameter variation can be severe. These results provide support for the use of variable parameter models. This support would be stronger if mixed results were not obtained when considering the consequence of inappropriate variable parameter specifications in time-varying environments.

Applying Variable Parameter Models in Marketing Contexts

The application of the variable parameter models previously discussed would seem to require a priori knowledge of the exact type of coefficient variation present so that the appropriate model could be used. However, in marketing applications there may be considerable uncertainty concerning the exact form of coefficient variation, and one might question the appropriateness of these models. The issue here is whether, in the presence of a suspected changing market environment, it is better to use a constant coefficient model which does not acknowledge the suspected change or to employ a possibly misspecified variable parameter model. In a large part this is a question which can be approached empirically. But, at least from an intuitive perspective, the use of variable parameter models, which attempt to address the prob-

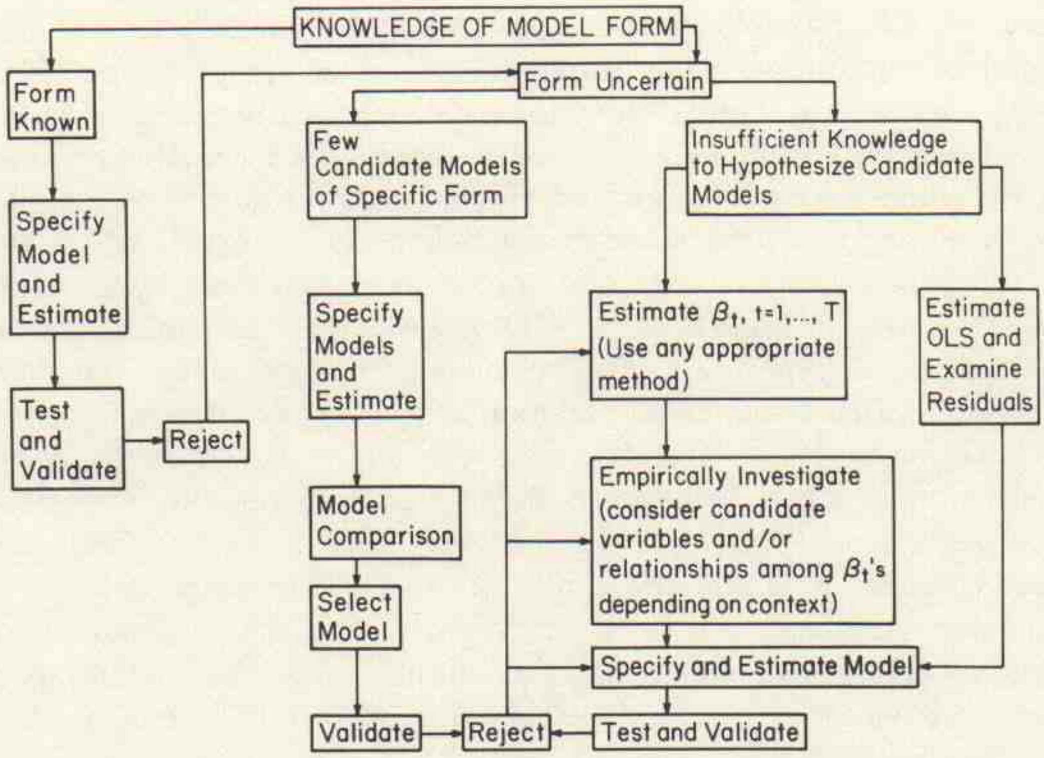

FIG. 3.—Approach to the estimation of time-varying market structures

lem of time-varying market structure, possesses considerable appeal. Also, it is noted that the constant parameter model is a special case of most variable parameter models, and therefore if coefficient variation is not present it is likely that the variable parameter models will so indicate.

The approach taken in the estimation of time-varying market structures will depend on one's knowledge and expectation concerning the type or form of variation. A suggested procedure for this estimation problem is diagrammed in figure 3 and discussed in the remainder of this section. In some instances the appropriate model may be known a priori (see the left panel of fig. 3), in which case the estimation and testing procedures are relatively straightforward. However, in many contexts varying degrees of uncertainty exist concerning the type and/or specification of coefficient variation, and estimation and model selection may be more difficult.

When the form of parameter variation is not specified a priori a number of options exist, two of which we feel are worthy of discussion. One option is to specify a number of possible models, estimate each, and employ model comparison procedures to select the appropriate model. A second option is to perform exploratory analyses to gather empirical information that may assist in the specification of possibly appropriate models. In either case the validation of the selected model with a holdout sample is strongly suggested.

The first option (see the center panel of fig. 3) is fairly straightforward, with the possible exception of difficulties in model selection, and would be appropriate when the initial choice of model can be effectively narrowed to a few candidate model specifications. To aid in the model selection problem in this case various tests of variable parameter structures have been proposed. Cooley and Prescott (1976) describe a procedure for testing whether coefficients are constant where the alternate hypothesis is sequential variation as described by equation (8). LaMotte and McWhorter (1978) describe an exact procedure for testing the null hypothesis of no parameter variation against the alternate hypothesis of sequential variation described by equation (7) and $\Gamma = I$. Garbade (1977) also presents a procedure, based on the same model employed by LaMotte and McWhorter, for examining the stability of regression coefficients. Further tests of parameter variation can be constructed by noting the implications of restricting parameters of the various models. For instance, when testing for parameter time variation involves a comparison of functional forms, classical (Ramsey 1974) or Bayesian (Gaver and Geisel 1974; Leamer 1978) model selection procedures may be used.

In other cases we recommend the second option (see the right panel of fig. 3), conducting an exploratory analysis. The specific exploratory analysis employed may differ depending upon the context and the predisposition of the researcher. A common feature of the exploratory approaches is estimation with a model not hypothesized to be representative of the process under investigation and through the examination of this estimated model hypothesizing a specific model or models which are subsequently estimated and tested.

One type of exploratory analysis involves the estimation of model coefficients for various time periods (i.e., estimation of β_t, $t = 1, \ldots, T$) of equation (1). The resulting coefficient estimates are then examined empirically to provide insight into the form and/or variables relating to the time-varying process. The result of this investigation will be a hypothesized model which is estimated and then tested (presumably employing data not utilized in the investigative procedure). The method the researcher employs to obtain the estimates will depend on the particular context. One method is to utilize the moving window regression procedure (Belsley 1973). The moving window method involves the estimation of separate OLS regressions on selected overlapping subsets of observations and results in the determination of a time path of estimated model coefficients. A second method, which may be useful when parameter variation is hypothesized to follow a smooth time path, is to estimate a systematic variation model characterized by a k-degree polynomial in time. A third method would be to employ one of the sequential variation models (Cooley and Prescott 1973; Sarris 1973) to yield a time path of estimated coefficients. It is noted that any of these three methods involves the use of a misspecified model. Other options would include the application of ad hoc estimation procedures such as a feedback modeling approach (Carbone and Longini 1977; Mahajan et al. 1980).

A second type of exploratory analysis is to compute the OLS (constant parameter) model and examine the resulting residuals. Brown, Durbin, and Evans (1975) present a technique for testing the consistency of regression relationships over time using OLS results. These procedures are incorporated into a computer program, TIMVAR. (For comments relating to limitations of the Brown-Durbin-Evans procedure, see Cox et al. 1975.) Other tests have also been proposed. For example, Garbade (1977) proposed the use of a procedure to test for parameter variation on the basis of OLS results, even though the model on which the test is based is not considered to be the sole alternative hypothesis. Garbade, in a Monte Carlo study, investigates the power of the Brown-Durbin-Evans procedure for specified structures and concludes that his test is more powerful than the Brown-Durbin-Evans test in rejecting the false null hypothesis of stability. Also, these results indicatetd that neither methodology gave misleading results when the coefficients were in fact stable.

Cost and Ease of Implementation

For many of the time-varying coefficient models discussed in this paper, computational routines are available or may be developed with minimal effort and cost. For the systematic variation models, the hypothesized functional expression for parameter variation is substituted into the basic model and estimated by whatever procedures are appropriate. This often results in a nonlinear model which may be estimated using a number of available computer packages with nonlinear regression procedures. Both simple and correlated random coefficient models may be restated as ordinary regression models with complex disturbance terms. These models can all be estimated using generalized least-squares procedures. Typically, this will require the researcher to develop the appropriate computational routine. Computational procedures for the two sequential parameter models mentioned in this paper are available. Cooley and Prescott (1973) indicate that the program and write-up for their model may be obtained from either of the authors, and a program for the Kalman filter model is available on the TROLL system. The moving window regression procedure, which has been suggested in cases where systematic variation is suspected, is a cumbersome procedure but may be implemented with relative ease. The development of a program to do this is relatively straightforward and can employ existing regression analysis subroutines. Also, ESP and

TSP both contain procedures capable of generating moving window results.

Areas for Future Research

The position that substantial benefits can be derived from the *accurate* estimation of time-varying coefficients is easily supported and probably consistent with the views of most marketers. At this juncture the two major issues appear to be: (1) Is the presently available variable parameter methodology capable of delivering these accurate estimates? (2) If it is, what is the most effective implementation strategy, that is, for any given situation what estimation procedure or method or what sequence of procedures or methods is best? The investigation of these two issues represents a logical next step for research in this area. It is expected that this investigation will use a combination of analytical and simulation approaches in conjunction with a variety of known time-varying and constant coefficient processes and will be directed toward the evaluation of existing estimation methods and testing procedures. This evaluation should ascertain the ability of the methods and procedures to identify correctly the underlying model structure and yield accurate coefficient estimates, and, in the event of incorrect specifications and/or inaccurate coefficient estimates, the robustness of the resulting estimates and implied decision strategies. The outcome of this initial suggested investigation should provide insight concerning the advisability of further investigation, for example, the development of additional testing and/or estimation procedures.

Summary

Many marketing situations are characterized by changing market environments. In a changing environment, the application of constant parameter models seems to yield estimates which approximate the "average" coefficient. However, the use of these average estimates in decision making may result in seriously nonoptimal decisions because they do not represent the current situation. A methodology which allows the accurate estimation of time-varying coefficients would appear to be of considerable value. Recently developed variable parameter models may represent such a methodology. We have briefly summarized some variable parameter models which have appeared in literature, described marketing applications of these models, and discussed a number of issues relevant to the application of this methodology. Evidence relating to the efficacy of these models is far from conclusive, but the intuitive appeal of the approach and the limited research evidence available lead us to believe the approach is worthy of consideration and future investigation.

References

Arora, Rajinder. 1979. How promotion elasticities change. *Journal of Advertising Research* 19 (June): 57–62.

Bass, Frank M., and Wittink, Dick R. 1975. Pooling issues and methods in regression analysis with examples in marketing research. *Journal of Marketing Research* 12 (November): 414–25.

Beckwith, Neil E. 1972. Regression estimation of the time varying effectiveness of advertising. Unpublished paper, Columbia University.

Belsley, David A. 1973. On the determination of systematic parameter variation in the linear regression model. *Annals of Economic and Social Measurement* 2 (October): 487–94.

Belsley, David A., and Kuh, Edwin. 1973. Time-varying parameter structures: an overview. *Annals of Economic and Social Measurement* 2 (October): 375–80.

Brown, R. L.; Durbin, J.; and Evans, J. M. 1975. Techniques for testing the constancy of regression relationships over time. *Journal of the Royal Statistical Society, Series B* 37:149–63.

Burnett, Thomas D., and Guthrie, Donald. 1970. Estimation of stationary stochastic regression parameters. *Journal of the American Statistical Association* 65 (December): 1547–53.

Carbone, Robert, and Longini, Richard L. 1977. A feedback model for automated real estate assessment. *Management Science* 24 (November): 241–48.

Cooley, Thomas F., and Prescott, Edward C. 1973. Varying parameter regression: a theory and some applications. *Annals of Economic and Social Measurement* 2 (October): 463–73.

Cooley, Thomas F., and Prescott, Edward C. 1976. Estimation in the presence of stochastic parameter variation. *Econometrica* 44:167–84.

Cooper, J. Phillip. 1973. Time-varying regression coefficients: a mixed estimation approach and operational limitations of the general Markov structure. *Annals of Economic and Social Measurement* 2 (October): 525–30.

Cox, D. R., and others. 1975. Discussion of the paper by Dr. Brown, Professor Durbin and Mr. Evans. *Journal of the Royal Statistical Society, Series B* 37:163–92.

Erickson, Gary M. 1977. The time-varying effectiveness of advertising. In Barnett A. Greenberg and Danny N. Bellenger (eds.), *Contemporary Marketing Thought*. Chicago: American Marketing Association.

Erickson, Gary M. 1979. Systematic time-variation of market response: the search for an effective estimation methodology. Marketing Department Working Paper no. 79-996, University of Pennsylvania, Wharton School.

Erickson, Gary M. 1981. Time-varying parameter estimation as exploration. *Decision Sciences* 12 (July): 428–40.

Erickson, Gary M., and Montgomery, David B. 1979. Measuring the time-varying response to market communication instruments. Paper presented at ORSA/TIMS Conference on Market Measurement and Analysis, Stanford, Calif., March.

Garbade, Kenneth. 1977. Two methods for examining the stability of regression coefficients. *Journal of the American Statistical Association* 72 (March): 54–63.

Gaver, Kenneth M., and Geisel, Martin S. 1974. Discriminating among alternative models: Bayesian and non-Bayesian methods. In P. Zarembka (ed.), *Frontiers in Econometrics*. New York: Academic Press.

Hildreth, Clifford, and Houck, James. 1968. Some estimators for a linear model with random coefficients. *Journal of the American Statistical Association* 63:584–95.

Johansson, Johnny K. 1978. Unobserved variables, stochastic parameter regression, and demand analysis: the case of gasoline hoarding. In Subhash C. Jain (ed.), *Research Frontiers in Marketing: Dialogues and Directions*. Chicago: American Marketing Association.

Kalman, R. E. 1960. A new approach to linear filtering and prediction problems. *Transactions of ASME, Series D, Journal of Basic Engineering* 83 (March): 95–108.

Kotler, Philip. 1971. *Marketing Decision Making: A Model Building Approach*. New York: Holt, Rinehart & Winston.

La Motte, Lynn Roy, and McWhorter, Archer Jr. 1978. An exact test for the presence of random walk coefficients in a linear regression model. *Journal of the American Statistical Association* 73 (December): 816–20.

Leamer, Edward E. 1978. *Specification Searches: Ad Hoc Inference with Nonexperimental Data*. New York: Wiley.

Little, John D. C. 1966. A model of adaptive control of promotional spending. *Operations Research* 6 (November–December): 1075–97.

Liu, Lon-Mu, and Hanssens, Dominique M. 1981. A Bayesian approach to time-varying cross-sectional regression models. *Journal of Econometrics* 15:341–56.

Maddala, G. S. 1977. *Econometrics*. New York: McGraw-Hill.

Mahajan, Vijay; Bretschneider, Stuart; and Bradford, John. 1980. The feedback approaches to modeling structural shifts in market response. *Journal of Marketing* 44 (Winter): 71–80.

Mickwitz, Gosta. 1959. *Marketing and Competition*. Helsingfors: Centratryckerick.

Palda, Kristian S. 1964. *The Measurement of Cumulative Advertising Effects*. Englewood Cliffs, N.J.: Prentice-Hall.

Parsons, Leonard J. 1975. The product cycle and time-varying advertising elasticities. *Journal of Marketing Research* 12 (November): 476–80.

Parsons, Leonard J., and Abeele, Piet Vanden. 1981. Analysis of sales call effectiveness. *Journal of Marketing Research* 18 (February): 107–13.

Pekelman, Dov, and Tse, Edison. 1980. Experimentation and budgeting in advertising: an adaptive control approach. *Operations Research* 28 (March–April): 321–47.

Ramsey, James B. 1974. Classical model selection through specification error tests. In P. Zarembka (ed.), *Frontiers in Econometrics*. New York: Academic Press.

Rao, C. R. 1965. The theory of least squares when the parameters are stochastic. *Biometrika* 52:447–58.

Rosenberg, Barr. 1973*a*. A survey of stochastic parameter regression. *Annals of Economic and Social Measurement* 2 (October): 381–97.

Rosenberg, Barr. 1973*b*. The analysis of a cross-section of time series by stochastically convergent parameter regression. *Annals of Economic and Social Measurement* 2 (October): 399–450.

Sarris, Alexander H. 1973. A Bayesian approach to estimation of time-varying regression coefficients. *Annals of Economic and Social Measurement* 2 (October): 501–23.

Simon, Hermann. 1979. Dynamics of price elasticity and the product life cycle: an empirical study. *Journal of Marketing Research* 16 (November): 439–52.

Wildt, Albert R. 1976. The empirical investigation of time dependent parameter variation in marketing models. In Kenneth L. Bernhardt (ed.), *Marketing: 1776–1976 and Beyond*. Chicago: American Marketing Association.

Wildt, Albert R., and Winer, Russell S. 1979. The specification and estimation of dynamic market structures. Working Paper no. 79-057, University of Georgia, College of Business Administration. Presented at TIMS/ORSA Joint National Meeting, Washington D.C., May 1980.

Winer, Russell S. 1979. An analysis of the time varying effects of advertising: the case of Lydia Pinkham. *Journal of Business* 52 (October): 563–76.