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The Demand for Beer, Wine, and Spirits: A Systemwide Analysis*

This paper illustrates how the systemwide approach to consumer demand can be extended so that it can be applied to narrowly defined commodity groups. We use the consumption of beer, wine, and spirits to show how the approach can be applied to estimate income and price elasticities of demand. When the consumer's utility function is appropriately separable in alcoholic beverages and all other goods, it is possible to confine our attention to the three beverages and ignore all other goods. We use the demand model for a number of simulations designed to analyze the rapid growth of wine consumption.

I. Introduction

The systemwide approach to the analysis of consumer demand considers the multivariate structure of the problem in which the consumer allocates his income to all goods simultaneously. This approach combines the theory of the consumer with empirical analysis and has enjoyed much popularity over the past decade. For surveys of these developments, see Brown and Deaton (1972), Phlips (1974), Powell (1974), Theil (1975–76, 1980), Barten (1977), and Theil and Clements (1980).

Most previous applications of the systemwide approach have used national accounts commodity groups (food, clothing, housing, and so on). For many business and government policy purposes, however, these groups are much too broad. For example, to analyze the effects on consumption of all but the simplest changes in indirect taxes, we would need considerably more disaggregation. Similarly, market researchers need to analyze demand at the individual product level for purposes of forecasting and formulating pricing and other policies. The objective of this paper is to use the consumption of beer, wine, and spirits to illustrate how the approach can be ap-

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plied to give insights into the structure of demand for more narrowly defined commodity groups. When the consumer's utility function is appropriately separable in alcoholic beverages and all other goods, it is possible to confine our attention to the three beverages and ignore all other goods. In Section II of the paper we set out the so-called differential version of the systemwide approach. In Section III we use the alcohol data to estimate demand equations for beer, wine, and spirits. We then use the demand model in Sections IV and V to (1) explain the rapid growth of wine consumption and (2) measure the welfare cost of alcohol taxes. Section VI contains concluding comments.

II. Differential Demand Equations

In this section we first formulate the consumer's demand equations for all n goods in terms of differentials. These are unconditional demand equations as they depend on all prices and total expenditure. The coefficients of these equations are not necessarily taken to be constant, so the model is general. We then block the n goods such that they form groups that are separable in the consumer's utility function. This leads to a composite demand equation for each group, as well as conditional demand equations within each group. The variables of the conditional equation are exclusively concerned with the group to which the good belongs, allowing us to focus attention on demand within the group. Finally, for estimation we set out a Rotterdam parameterization. This section is based on Theil (1975–76, 1980).

Unconditional Demand Equations

We write p_i, q_i for the price and quantity demanded for good i ($i = 1, \ldots, n$), $M = \Sigma_{i=1}^{n} p_i q_i$ for total expenditure ("income" for short) and $w_i = p_i q_i / M$ for the ith budget share. Under general conditions, the demand for good i can be written as (see App.)

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij} d\left(\log \frac{p_j}{P'}\right), \tag{1}$$

where $\theta_i = \partial(p_i q_i)/\partial M$ is the ith marginal share; $d(\log Q) = \Sigma_{i=1}^{n} w_i d(\log q_i)$ is the Divisia volume index of the change in the consumer's real income; $d(\log p_j/P')$ is to be interpreted as the change in the deflated price of $j d(\log p_j) - d(\log P')$, where $d(\log P') = \Sigma_{i=1}^{n} \theta_i d(\log p_i)$ is the Frisch price index; and

$$\nu_{ij} = \lambda p_i u^{ij} p_j / M \tag{2}$$

is the (i, j)th price coefficient, where λ is the marginal utility of income and u^{ij} is the (i, j)th element of the inverse of the Hessian of the utility function $[\partial^2 u / \partial q_i \partial q_j]^{-1}$. In view of (2), the symmetry of this Hessian means that the matrix of price coefficients $[\nu_{ij}]$ is also symmetric. A sufficient second-order condition for a budget-constrained maximum is that the Hessian be negative definite; (2) thus implies that $[\nu_{ij}]$ is negative definite. The price coefficients are subject to the constraint that the row sums of $[\nu_{ij}]$ are proportional to the corresponding marginal shares (see App.),

$$\sum_{j=1}^{n} \nu_{ij} = \phi\theta_i \qquad i = 1, \ldots, n, \tag{3}$$

where $\phi = (\partial \log \lambda / \partial \log M)^{-1}$ is the reciprocal of the income elasticity of the marginal utility of income. We shall refer to ϕ as the income flexibility. For further details of (1), see Theil (1975–76).

Block-independent Preferences

Let the n goods now be divided into G groups, $S_1, \ldots, S_G$, such that each good belongs to only one group. Further, let the consumer's preferences be such that the utility function is the sum of G subutility functions, each involving the quantities of only one group,

$$u(\mathbf{q}) = \sum_{g=1}^{G} u_g(\mathbf{q_g}), \tag{4}$$

where $\mathbf{q} = [q_1, \ldots, q_n]'$ and $\mathbf{q_g}$ is the vector of the q_i's that fall under S_g. Then, when the goods are numbered appropriately, the Hessian $[\partial^2 u / \partial q_i \partial q_j]$ and its inverse become block-diagonal. Accordingly, specification (4) is known as block-independent preferences (Theil 1975–76).

Under block independence, $\nu_{ij} = 0$ for i and j in different groups (see [2]) and (3) for $i \in S_g$ becomes

$$\sum_{j \in S_g} \nu_{ij} = \phi\theta_i \qquad g = 1, \ldots, G. \tag{5}$$

The demand equation (1) for $i \in S_g$ becomes

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j \in S_g} \nu_{ij} d\left(\log \frac{p_j}{P'}\right), \tag{6}$$

so that the only deflated prices which appear are those of goods belonging to the same group as i.

Composite Demand Equations

We write $W_g = \Sigma_{i \in S_g} w_i$ and $\Theta_g = \Sigma_{i \in S_g} \theta_i$ for the average and marginal shares of group g, and define the group volume and Frisch price indexes as $d(\log Q_g) = \Sigma_{i \in S_g} (w_i/W_g) d(\log q_i)$, $d(\log P'_g) = \Sigma_{i \in S_g} (\theta_i/\Theta_g) d(\log p_i)$. If we then add (6) over $i \in S_g$ and use (5) and the symmetry of $[\nu_{ij}]$, we obtain the composite demand equation for S_g as a group (see App.),

$$W_g d(\log Q_g) = \Theta_g d(\log Q) + \phi\Theta_g d\left(\log \frac{P'_g}{P'}\right). \tag{7}$$

Thus only the deflated price of the group $d(\log P'_g/P') = d(\log P'_g) - d(\log P')$ and income affects the demand for the group as a whole. The income and own-price elasticities for the group are Θ_g/W_g and $\phi\Theta_g/W_g$, respectively.

Conditional Demand Equations

Combining (6) and (7) we obtain (see App.)

$$w_i d(\log q_i) = \frac{\theta_i}{\Theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} \nu_{ij} d\left(\log \frac{p_j}{P'_g}\right). \tag{8}$$

This is the demand equation for $i \in S_g$, given the demand for the group as a whole $W_g d(\log Q_g)$. As the variables on the right of this equation are exclusively concerned with the group S_g to which the ith commodity belongs, it is known as a conditional demand equation. The term θ_i/Θ_g is the conditional marginal share of i within the group S_g, with $\Sigma_{i \in S_g} \theta_i/\Theta_g = 1$.

Equation (8) can be formulated in terms of absolute (undeflated) prices by using the definition of $d(\log P'_g)$ (see above [7]) to write the price term as

$$\begin{aligned}
&\sum_{j \in S_g} \nu_{ij}\left[d(\log p_j) - \sum_{k \in S_g} \frac{\theta_k}{\Theta_g} d(\log p_k)\right] \\
&= \sum_{j \in S_g} \nu_{ij} d(\log p_j) - \phi\theta_i \sum_{k \in S_g} \frac{\theta_k}{\Theta_g} d(\log p_k) \qquad (9) \\
&= \sum_{j \in S_g} (\nu_{ij} - \phi\Theta_g \theta'_i \theta'_j) d(\log p_j) = \sum_{j \in S_g} \pi^g_{ij} d(\log p_j),
\end{aligned}$$

where the first step is based on (5), $\theta'_i = \theta_i/\Theta_g$ is the conditional marginal share of i, and

$$\pi^g_{ij} = \nu_{ij} - \phi\Theta_g \theta'_i \theta'_j \qquad i, j \in S_g \tag{10}$$

is the (i, j)th conditional Slutsky coefficient. Substitution of the fourth member of (9) in (8) gives the absolute price version of the conditional demand equation for $i \in S_g$,

$$w_i d(\log q_i) = \theta'_i W_g d(\log Q_g) + \sum_{j \in S_g} \pi^g_{ij} d(\log p_j). \tag{11}$$

By dividing both sides of this equation by w_i, we find that $(\theta_i/\Theta_g)(W_g/w_i) = (\theta_i/w_i)/(\Theta_g/W_g)$ is the ratio of the income elasticity of the good to that of the group to which it belongs. We shall refer to this ratio as the conditional income elasticity of i. We also find that π^g_{ij}/w_i is the conditional price elasticity, that is, the elasticity of q_i with respect to the absolute price p_j $(i, j \in S_g)$.

It follows from (5) and (10) and the fact that $\Sigma_{j\in S_g}\theta'_j = 1$ that

$$\sum_{j\in S_g} \pi^g_{ij} = 0 \qquad i \in S_g. \tag{12}$$

This reflects the homogeneity proposition that a proportionate change in all prices in the group, total consumption of the group remaining unchanged, does not affect the demand for any good in the group. We shall refer to (12) as demand homogeneity.

If S_g consists of n_g commodities, the $n_g \times n_g$ matrix of price coefficients referring to S_g is a principal submatrix of the $n \times n$ price coefficient matrix $[\nu_{ij}]$. As the latter matrix is symmetric, so is the former. It then follows from (10) that the conditional Slutsky matrix $[\pi^g_{ij}]$ is symmetric,

$$\pi^g_{ij} = \pi^g_{ji} \qquad i, j = 1, \ldots, n_g. \tag{13}$$

We shall refer to this as Slutsky symmetry. The negative definiteness of $[\nu_{ij}]$ together with (10) and (12) imply that $[\pi^g_{ij}]$ is negative semidefinite with rank $n_g - 1$.

A Parameterization of the Conditional Demand Equations

To apply (11) to finite-change data, we replace w_i with its arithmetic average over the periods $t - 1$ and t, $\overline{w}_{it} = (w_{it} + w_{i,t-1})/2$, and $d(\log x)$ with $Dx_t = \log x_t - \log x_{t-1}$, the log change in x. We also use the Rotterdam parameterization of treating the coefficients of (11) as constants (Theil 1975–76), so that the estimating equations are

$$\overline{w}_{it}Dq_{it} = \theta'_i \overline{W}_{gt} DQ_{gt} + \sum_{j=1}^{n_g} \pi^g_{ij} Dp_{jt} + \epsilon_{it}, \qquad i = 1, \ldots, n_g, \tag{14}$$

where $\overline{W}_{gt} = \Sigma_{i=1}^{n_g}\overline{w}_{it}$; $DQ_{gt} = \Sigma_{i=1}^{n_g}(\overline{w}_{it}/\overline{W}_{gt})Dq_{it}$; and the ϵ_{it}'s are serially independent, normally distributed disturbances with zero means and a constant contemporaneous covariance matrix.

III. Demand Equations for Alcoholic Beverages

In this section we first present price-quantity data for the consumption of beer, wine, and spirits. We then use these data to estimate (14) under two conditions, that total consumption of alcohol is a predetermined variable and that it is endogenously determined. Finally, we give the unconditional demand responses.

TABLE 1 **Alcohol Quantity and Price Log-Changes, Australia, 1955–56/1976–77**

	Beer		Wine		Spirits	
Year	Quantity (Dq_1)	Price (Dp_1)	Quantity (Dq_2)	Price (Dp_2)	Quantity (Dq_3)	Price (Dp_3)
1956–57	−5.359	14.205	−2.447	5.982	−10.427	0
1957–58	.209	1.230	−.730	4.879	1.220	0
1958–59	.476	.366	.684	6.108	4.853	0
1959–60	1.735	.849	.901	7.514	8.429	0
1960–61	−.292	1.795	−2.960	7.363	−.390	8.406
1961–62	−.410	.709	.479	1.361	.289	1.315
1962–63	1.235	.587	3.039	1.584	−2.588	5.656
1963–64	3.355	1.971	4.608	1.082	7.639	.894
1964–65	3.083	1.368	.708	5.809	7.271	1.982
1965–66	.045	8.152	8.737	7.395	−11.977	13.734
1966–67	2.452	4.291	11.501	4.709	−.712	−5.069
1967–68	3.304	4.114	10.430	6.953	11.586	0
1968–69	3.032	3.025	8.657	12.275	−1.068	6.766
1969–70	2.612	3.477	8.094	6.004	10.476	2.856
1970–71	1.699	6.274	−3.016	9.198	.939	3.041
1971–72	−.135	5.104	1.904	5.850	5.653	.264
1972–73	3.010	5.238	10.084	−.873	12.249	.875
1973–74	7.028	7.623	11.430	2.924	.643	20.202
1974–75	1.060	13.327	11.200	14.993	−4.268	25.660
1975–76	−2.130	22.672	6.158	18.523	−3.534	12.946
1976–77	−.965	8.943	4.463	7.748	9.773	.386
Mean	1.193	5.491	4.473	6.542	2.193	4.758

NOTE.—The quantities are per capita. The year 1956–57, e.g., refers to the transition from 1955–56 to 1956–57. All entries are to be divided by 100.

The Data

Our data refer to the consumption of beer, wine, and spirits in Australia over the period 1955–56/1976–77. Table 1 gives the price-quantity data and table 2 the budget shares. As can be seen, on average, beer consumption per capita increased by about 1.2% per annum, wine by 4.5%, and spirits by 2.2%. The budget share for total alcohol has remained more or less constant over this period at about 6% (table 2). Expenditure on beer, expressed as a percentage of total alcohol expenditure, has fallen by about 10 percentage points to 66% in 1976–77 (see the third to last col. of table 2). This relative decline in beer expenditure mirrors the dramatic growth of the share of wine in total alcohol, which increased from 8.6% in 1956–57 to 18.6% in 1976–77. One of our objectives is to explain this rapid growth in the wine share. The spirits share in total alcohol has been relatively stable at about 16%. Further details of the data and sources are given in a separate appendix, available on request.

Estimates with Total Alcohol Consumption Predetermined

We assume that the total consumption of alcohol ($\overline{W}_{gt}DQ_{gt}$) and prices (Dp_{jt}) are predetermined variables and use maximum likelihood to esti-

TABLE 2 Arithmetic Averages of Unconditional and Conditional Budget Shares for Alcohol: Australia, 1955–56/1976–77

	Unconditional				Conditional		
Year	Beer ($\overline{w}_1$)	Wine ($\overline{w}_2$)	Spirits ($\overline{w}_3$)	Total alcohol ($\overline{W}_g$)	Beer ($\overline{w}_1/\overline{W}_g$)	Wine ($\overline{w}_2/\overline{W}_g$)	Spirits ($\overline{w}_3/\overline{W}_g$)
1956–57	4.6325	.5256	.9587	6.1168	75.734	8.593	15.673
1957–58	4.7115	.5274	.8823	6.1211	76.970	8.616	14.414
1958–59	4.6336	.5415	.8847	6.0597	76.465	8.936	14.599
1959–60	4.4809	.5552	.8991	5.9351	75.497	9.354	15.149
1960–61	4.3314	.5609	.9247	5.8170	74.462	9.643	15.896
1961–62	4.2709	.5661	.9477	5.7846	73.831	9.785	16.383
1962–63	4.1877	.5673	.9408	5.6957	73.524	9.959	16.517
1963–64	4.1115	.5656	.9446	5.6217	73.137	10.061	16.802
1964–65	4.0759	.5676	.9747	5.6181	72.549	10.102	17.349
1965–66	4.1326	.6059	.9802	5.7185	72.266	10.594	17.140
1966–67	4.2397	.6778	.9164	5.8339	72.674	11.618	15.707
1967–68	4.2709	.7525	.8839	5.9073	72.299	12.738	14.963
1968–69	4.2805	.8550	.9028	6.0383	70.889	14.160	14.951
1969–70	4.2479	.9492	.9278	6.1248	69.355	15.497	15.148
1970–71	4.2308	.9749	.9387	6.1444	68.857	15.866	15.277
1971–72	4.1829	.9686	.9139	6.0654	68.964	15.969	15.067
1972–73	4.0888	.9646	.9200	5.9734	68.451	16.148	15.401
1973–74	4.0500	.9588	.9632	5.9720	67.817	16.054	16.129
1974–75	3.9724	.9969	1.0088	5.9781	66.449	16.676	16.875
1975–76	3.9885	1.0839	.9927	6.0651	65.761	17.872	16.367
1976–77	3.9918	1.1303	.9496	6.0716	65.745	18.616	15.639
Mean	4.2434	.7569	.9359	5.9363	71.509	12.708	15.783

NOTE.—The year 1956–57, e.g., refers to arithmetic averages of budget shares in 1955–56 and 1956–57. All entries are to be divided by 100.

mate (14) for $i = 1, 2, 3$, subject to the homogeneity and symmetry restrictions (12) and (13). The estimates are given in table 3.[1] Constant terms have been added to each equation to take account of trendlike changes in tastes and other factors. A preliminary analysis indicated that two constants are needed in each equation, one for the first nine observations (α_i) and one for the remaining 12 (β_i).[2] As can be seen, there is a significant trend into wine and out of spirits in the second part of the period. The estimate of β_i for wine implies that per capita consumption is growing autonomously at an exponential rate of $.53/(\overline{w}_{it} \times 1000) = .53 \times 100/1.13 \times 1000 = 4.7\%$ per annum in 1976–77. The conditional marginal shares are estimated quite precisely and they indicate that when alcohol expenditure increases by \$1.00, expenditure on beer rises by 54 cents, wine by 10 cents, with the remaining 37 cents being spent on spirits. All the diagonal elements of the conditional Slutsky matrix are negative, as they should be, and significant.[3] The

1. We use Wymer's (1977) estimation program RESIMUL.

2. This analysis consisted of plotting the residuals from the model with one constant in each equation. For wine they are distinctly positive for the last 12 observations. With the two constants, the residuals are more random.

3. That is, these coefficients have t-values greater than 2.

TABLE 3 **Conditional Demand Equations for Alcoholic Beverages, Australia, 1955–56/1976–77**

$$\bar{w}_{it}Dq_{it} = \alpha_i D_t + \beta_i(1 - D_t) + \theta_i' \bar{W}_{gt} DQ_{gt} + \sum_{j=1}^{3} \pi_{ij}^{g} Dp_{jt} + \epsilon_{it}$$

	Constants		Conditional Marginal Share	Conditional Slutsky Coefficients				
Beverage	$\alpha_i \times 1{,}000$	$\beta_i \times 1{,}000$	(θ_i')	$\pi_{i1}^{g} \times 100$	$\pi_{i2}^{g} \times 100$	$\pi_{i3}^{g} \times 100$	R^2	DW
Beer	−.039	−.107	.539	−.464	.135	.329	.92	2.20
	(.103)	(.115)	(.046)	(.153)	(.117)	(.101)		
Wine	.066	.525	.095	...	−.300	.165	.74	1.49
	(.078)	(.084)	(.033)		(.122)	(.074)		
Spirits	−.027	−.419	.366	...	...	−.494	.78	1.66
	(.100)	(.112)	(.044)			(.105)		

NOTE.—D_t = 1 for 1956–57/1964–65, 0 otherwise. R^2 is the squared correlation coefficient between actual and predicted values of dependent variable and thus lies in the range [0, 1]. DW (with the usual single-equation definition) tests for autocorrelation of residuals from restricted model. Asymptotic SEs in parentheses.

TABLE 4 Conditional Demand Elasticities for Alcoholic Beverages, Australia, 1955–56/1976–77

Beverage	Conditional Income Elasticity $(\theta_i' \overline{W}_{gt}/\overline{w}_{it})$	Conditional Price Elasticities		
		$\pi_{i1}^g/\overline{w}_{it}$	$\pi_{i2}^g/\overline{w}_{it}$	$\pi_{i3}^g/\overline{w}_{it}$
1956–57:				
Beer	.71	−.10	.03	.07
Wine	1.11	.26	−.57	.31
Spirits	2.34	.34	.17	−.52
1976–77:				
Beer	.82	−.12	.03	.08
Wine	.51	.12	−.27	.15
Spirits	2.34	.35	.17	−.52
Sample means:				
Beer	.75	−.11	.03	.08
Wine	.75	.18	−.40	.22
Spirits	2.32	.35	.18	−.53

SOURCE.—Based on table 3 estimates and table 2 data.

off-diagonal π_{ij}^g's are positive, indicating that the three beverages are pairwise substitutes, and only one is insignificantly different from zero.[4] The fit of the equations is satisfactory given that they are in first-difference form.

The compatibility of the homogeneity and symmetry restrictions with the data can be verified by means of a likelihood ratio test. The unrestricted version of the model is (14), including constant terms, without the constraints (12) and (13). Asymptotically, minus twice the difference between the log-likelihood values for the restricted and unrestricted models is distributed as $X^2(3)$.[5] The observed value of the test statistic is 4.05, less than the critical value at the 5% level of 7.81. Thus we are unable to reject homogeneity and symmetry.

Table 4 gives the conditional demand elasticities evaluated at the beginning and end of the sample period, as well as at sample means. At sample means, the conditional income elasticities are .8, .8, and 2.3, for beer, wine, and spirits, respectively. This indicates that, within alcohol, beer and wine are necessities and spirits is a strong luxury. Note, however, that at the beginning of the period wine has a conditional income elasticity greater than one. All the conditional price elasticities are less than one in absolute value, with the own-price elasticities being −.1, −.4, and −.5 (at sample means). As expected, there is only a moderate amount of substitutability between the three beverages.

4. The characteristic roots of the estimated conditional Slutsky matrix are 0, −.448, −.810 (all × 100), which verifies that this matrix is negative semidefinite.

5. As one of three estimating equations is redundant, the homogeneity constraint (12) involves two restrictions while symmetry (13) involves one.

Estimates with Total Alcohol Consumption Endogenous

We now let total alcohol consumption $\overline{W}_{gt}DQ_{gt}$ become endogenously determined by adding the composite demand equation to the three conditional equations. The composite equation in terms of infinitesimal changes is (7). As before, we replace budget shares with their arithmetic averages and infinitesimal logarithmic changes by finite log changes; and treat the coefficients as constants. Numbering goods such that the alcoholic beverages are the first three, the estimating equation is thus

$$\overline{W}_{gt}DQ_{gt} = \Theta_g DQ_t + \phi\Theta_g\left(\sum_{i=1}^{3} \theta_i' Dp_{it} - \sum_{j=1}^{n} \theta_j Dp_{jt}\right) + E_{gt}, \quad (15)$$

where $DQ_t = \Sigma_{i=1}^{n}\overline{w}_{it}Dq_{it}$ and E_{gt} is a serially independent, normally distributed disturbance with a zero mean and a constant variance.

Equations (14) and (15) form a system of four simultaneous equations with endogenous variables $\overline{w}_{it}Dq_{it}$ $(i = 1, 2, 3)$ and $\overline{W}_{gt}DQ_{gt}$. The variables taken to be predetermined are the prices and DQ_t. Note that (15) implies that the only random component of $\overline{W}_{gt}DQ_{gt}$ is the disturbance E_{gt}. Hence, we can treat $\overline{W}_{gt}DQ_{gt}$ as predetermined in (14) if E_g is independent of the disturbance in (14) ϵ_{it}. Remarkably, Theil's (1975–76, 1980) theory of rational random behavior implies that these disturbances are independent. Thus, our previous estimates of (14) by itself are consistent under the theory of rational random behavior.

In this subsection we do not rely on this theory and estimate (14) and (15) simultaneously, allowing the disturbances E_{gt} and ϵ_{it} to be correlated. A comparison of the two sets of estimates can be interpreted as an informal test of the theory.[6] In addition, the estimation of (15) allows us to obtain the unconditional demand responses.

The overall Frisch price index in (15) can be written as

$$\sum_{j=1}^{n} \theta_j Dp_{jt} = \Theta_g \sum_{j=1}^{3} \theta_j' Dp_{jt} + (1 - \Theta_g) \sum_{k=4}^{n} \frac{\theta_k}{\Sigma_{l=4}^{n}\theta_l} Dp_{kt}, \quad (16)$$

as $\theta_j' = \theta_j/\Theta_g$ and $\Theta_g = \Sigma_{i=1}^{3}\theta_i = 1 - \Sigma_{l=4}^{n}\theta_l$. In words, the overall Frisch price index is a weighted average of the Frisch indexes for alcohol and all other, the weights being the group marginal shares. The estimation procedure can be simplified by eliminating the marginal shares not involving alcohol. We do this by approximating the Frisch index of all other in (16), $\Sigma_{k=4}^{n} (\theta_k/\Sigma_{l=4}^{n}\theta_l)Dp_{kt}$, by the change in the consumer price index excluding alcohol (see App.) DP_{0t}, so that

$$\sum_{j=1}^{n} \theta_j Dp_{jt} \approx \Theta_g \sum_{j=1}^{3} \theta_j' Dp_{jt} + (1 - \Theta_g)DP_{0t}. \quad (17)$$

6. As an alternative, Hausman's (1978) specification test could be adapted to test for simultaneity in this context.

For estimation we substitute in (15) the right side of (17) for $\Sigma_{j=1}^{n} \theta_j Dp_{jt}$. We also approximate the change in real income DQ_t by the difference between the log change in per capita total consumption expenditure and that of the consumer price index.

The maximum likelihood estimates are given in table 5. All the estimates for the conditional demand equations are highly consistent with those of table 3. This result gives support to the theory of rational random behavior, which allows $\overline{W}_{gt}DQ_{gt}$ to be treated as predetermined. Looking at the estimates for the composite equation, there is a significant autonomous trend out of alcohol in the first part of the period. The marginal share for the group is highly significant and indicates that a \$1.00 rise in income leads to a 6 cent increase in alcohol expenditure. Finally, the value of the income flexibility is quite close to previous estimates.[7] Testing the homogeneity and symmetry restrictions, as before, gives an observed X^2 value of 5.66, again less than the critical value of $X^2(3)$ at the 5% level.[8]

Table 6 gives the conditional demand elasticities and the income and own-price elasticities for the group. As is to be expected, the conditional elasticities are similar to those of table 4. The income elasticity of demand for alcoholic beverages as a whole is 1.0, while the own-price elasticity is $-.6$.

The Unconditional Demand Responses

The conditional demand equation (14) depends on total alcohol consumption and the prices of the three beverages. Accordingly, the conditional Slutsky coefficient π_{ij}^{g} measures the effect of a change in the price of alcoholic beverage j on the consumption of beverage i with total alcohol consumption held constant. From equation (15), this total also depends on the price of j. Thus a change in p_j has a direct effect on i, via the conditional demand equation, and an indirect effect via the composite demand equation. These two effects can be combined to give the total effect by substituting the right side of (15) for $\overline{W}_{gt}DQ_{gt}$ in (14). This yields

$$\overline{w}_{it}Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{3} \pi_{ij}Dp_{jt} + \pi_{i0}DP_{0t} + \eta_{it}, \tag{18}$$

where

$$\theta_i = \Theta_g \theta_i' \tag{19}$$

is the unconditional marginal share of i;

$$\pi_{ij} = \pi_{ij}^{g} + \phi\Theta_g(1 - \Theta_g)\theta_i'\theta_j' \qquad \pi_{i0} = -\phi\Theta_g(1 - \Theta_g)\theta_i' \tag{20}$$

7. See, e.g., Theil (1975–76, 1980), Clements and Theil (1979), Clements and Nguyen (1980), Clements (1981), and Theil and Suhm (1981).

8. The characteristic roots of the estimated conditional Slutsky matrix are 0, $-.440$, $-.627$ (all $\times$ 100), which verifies that this matrix is negative semidefinite.

TABLE 5 **Conditional and Composite Demand Equations for Alcoholic Beverages, Australia, 1955–56/1976–77**

$$\bar{w}_{it}Dq_{it} = \alpha_i D_t + \beta_i(1 - D_t) + \theta_i' \overline{W}_{gt} DQ_{gt} + \sum_{j=1}^{3} \pi_{ij}^{g} Dp_{jt} + \epsilon_{it}$$

$$\overline{W}_{gt} DQ_{gt} = \gamma D_t + \lambda(1 - D_t) + \Theta_g DQ_t + \phi\Theta_g\left(\sum_{i=1}^{3} \theta_i' Dp_{it} - \sum_{j=1}^{n} \theta_j Dp_{jt}\right) + E_{gt}$$

Beverage	Constants: $\alpha_i \times 1{,}000$	Constants: $\beta_i \times 1{,}000$	Conditional Marginal Share (θ_i')	Conditional Slutsky Coefficients: $\pi_{i1}^{g} \times 100$	Conditional Slutsky Coefficients: $\pi_{i2}^{g} \times 100$	Conditional Slutsky Coefficients: $\pi_{i3}^{g} \times 100$	R^2	DW
Beer	−.065	−.196	.590	−.390	.150	.240	.67	1.72
	(.106)	(.115)	(.045)	(.145)	(.114)	(.097)		
Wine	.062	.517	.099	...	−.293	.144	.68	1.24
	(.078)	(.084)	(.035)		.124)	(.072)		
Spirits	.003	−.321	.311	...	...	−.384	.69	2.34
	(.104)	(.111)	(.042)			(.102)		
	$\gamma \times 1{,}000$	$\lambda \times 1{,}000$	Θ_g			ϕ		
Total alcohol	−.686	.209	.0576	...	...	−.595	.76	1.73
	(.332)	(.342)	(.0095)			(.130)		

NOTE.—R^2 refers to reduced form. See nn. to table 3.

TABLE 6 **Conditional and Composite Demand Elasticities for Alcoholic Beverages, Australia, 1955–56/1976–77**

Beverage	Conditional Income Elasticity ($\theta_i' \overline{W}_{gt}/\overline{w}_{it}$)	Conditional Price Elasticities			Composite Income Elasticity ($\Theta_g/\overline{W}_{gt}$)	Composite Own-Price Elasticity ($\phi\Theta_g/\overline{W}_{gt}$)
		$\pi_{i1}^g/\overline{w}_{it}$	$\pi_{i2}^g/\overline{w}_{it}$	$\pi_{i3}^g/\overline{w}_{it}$		
1956–57:						
Beer	.78	−.08	.03	.05	...	...
Wine	1.15	.29	−.56	.27	...	...
Spirits	1.98	.25	.15	−.40	...	...
Total alcohol	...	...	...	...	.94	−.56
1976–77:						
Beer	.90	−.10	.04	.06	...	...
Wine	.53	.13	−.26	.13	...	...
Spirits	1.99	.25	.15	−.40	...	...
Total alcohol	...	...	...	...	.95	−.56
Sample means:						
Beer	.83	−.09	.04	.06	...	...
Wine	.78	.20	−.39	.19	...	...
Spirits	1.97	.26	.15	−.41	...	...
Total alcohol	...	...	...	...	.97	−.58

SOURCE.—Based on table 5 estimates and table 2 data.

are unconditional Slutsky coefficients; and $\eta_{it} = \epsilon_{it} + \theta_i' E_{gt}$. In deriving (18) we have used the right side of (17) to substitute for $\Sigma_{j=1}^n \theta_j Dp_{jt}$ in (15). From (18) it can be seen that the unconditional income and price elasticities are $\theta_i/\overline{w}_{it}$, $\pi_{ij}/\overline{w}_{it}$, and $\pi_{i0}/\overline{w}_{it}$.

We use the table 5 estimates to evaluate (19) and (20) for $i, j = 1, 2, 3$. The results are given in table 7. Thus the direct and indirect effects of a \$1.00 rise in income is to increase expenditure on beer by 3.4 cents, wine by .6 cents and spirits by 1.8 cents. All the own-Slutsky coefficients are estimated quite precisely and each is more negative than the corresponding conditional coefficient of table 5. The reason is that an increase in the price of beverage i lowers total alcohol consumption, which also lowers the consumption of i as the estimate of θ_i' is positive for each i. This negative indirect effect is then added to the conditional Slutsky coefficient π_{ii}^g (<0) to give an unconditional coefficient π_{ii} larger in absolute value than π_{ii}^g. For a similar reason the estimates of π_{12} and π_{13} are negative, indicating that beer is an unconditional complement for wine and spirits, whereas these beverages are conditional substitutes (π_{12}^g, $\pi_{13}^g > 0$; see table 5). Finally, each of the three beverages is a substitute for all other goods.

Table 8 gives the unconditional elasticities. As the income elasticity for the group is about unity (see table 6), the unconditional income elasticities given in table 8 are quite close to the corresponding conditional elasticities. The high income elasticity for spirits agrees well with the notion that in Australia the more affluent tend to be spirits drinkers. For the reason given above, the own-price elasticities are substantially

TABLE 7 **Unconditional Demand Equations for Alcoholic Beverages, Australia, 1955–56/1976–77**

$$\bar{w}_{it}Dq_{it} = \delta_i D_t + \psi_i(1 - D_t) + \theta_i DQ_t + \sum_{j=1}^{3} \pi_{ij}Dp_{jt} + \pi_{i0}DP_{0t} + \eta_{it}$$

	Constants			Slutsky Coefficients			
Beverage	δ_i × 1,000	ψ_i × 1,000	Marginal Share (θ_i)	π_{i1} × 100	π_{i2} × 100	π_{i3} × 100	π_{i0} × 100
Beer	−.469	−.073	.0340	−1.514	−.039	−.353	1.906
	(.236)	(.235)	(.0062)	(.299)	(.125)	(.146)	(.377)
Wine	−.006	.537	.0057	...	−.325	.044	.319
	(.092)	(.089)	(.0023)		(.132)	(.080)	(.132)
Spirits	−.210	−.256	.0179	...	...	−.697	1.006
	(.139)	(.139)	(.0037)			(.130)	(.202)

NOTE.—The estimates from table 5 are used in eq. (19) and (20) to obtain the (unconditional) marginal shares and Slutsky coefficients. The constant term δ_i is defined as $\alpha_i + \theta_i'\gamma$, with estimates taken from table 5; and similarly for ψ_i. See nn. to table 3.

larger (in absolute value) than those of table 6. At sample means, the unconditional own-price elasticities are −.4, −.4, and −.7 for beer, wine, and spirits, respectively.

IV. Why Has Wine Grown So Rapidly?

As indicated in the previous section, wine consumption per capita grew by 4.5% per annum over our sample period; and the share of wine in total alcohol expenditure increased by 10 percentage points. In this section we analyze the reasons for this rapid growth by using the demand equations to (1) decompose the growth into a number of com-

TABLE 8 **Unconditional Demand Elasticities for Alcoholic Beverages, Australia, 1955–56/1976–77**

	Income Elasticity	Price Elasticities			
Beverage	($\theta_i/\bar{w}_{it}$)	$\pi_{i1}/\bar{w}_{it}$	$\pi_{i2}/\bar{w}_{it}$	$\pi_{i3}/\bar{w}_{it}$	$\pi_{i0}/\bar{w}_{it}$
1956–57:					
Beer	.73	−.33	−.01	−.08	.41
Wine	1.08	−.07	−.62	.08	.61
Spirits	1.87	−.37	.05	−.73	1.05
1976–77:					
Beer	.85	−.37	−.01	−.09	.48
Wine	.50	−.03	−.29	.04	.28
Spirits	1.89	−.37	.05	−.73	1.06
Sample means:					
Beer	.80	−.36	−.01	−.08	.45
Wine	.75	−.05	−.43	.06	.42
Spirits	1.91	−.38	.05	−.74	1.07

SOURCE.—Based on table 7 estimates and table 2 data.

ponents and (2) simulate alcohol consumption under several different scenarios.

A Decomposition of the Change in the Budget Share

Recalling that the ith budget share is defined as $w_i = p_i q_i / M$, its change is $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log M)$. This can be expressed in terms of the relative price of i and real income by adding and subtracting from the right side the Divisia cost of living index $d(\log P) = \Sigma_{i=1}^n w_i d(\log p_i)$. This gives

$$dw_i = w_i d\left(\log \frac{p_i}{P}\right) + w_i d(\log q_i) - w_i d(\log Q), \tag{21}$$

where $d(\log p_i/P) = d(\log p_i) - d(\log P)$ is the change in the relative price of i and $d(\log Q) = d(\log M) - d(\log P) = \Sigma_{i=1}^n w_i d(\log q_i)$ is the change in real income.

Equation (21) states that dw_i is made up of relative price, quantity, and real income components. From the consumer's viewpoint prices and income are given, while the quantity demanded is to be determined. Thus we use the demand equation (1) to express the quantity component in terms of income and relative prices,

$$dw_i = (\theta_i - w_i) d(\log Q) + \sum_{j=1}^{n} v_{ij} d\left(\log \frac{p_j}{P'}\right) + w_i d\left(\log \frac{p_i}{P}\right). \tag{22}$$

Thus a rise in income causes the budget share of i to increase if the marginal share (θ_i) exceeds the average share (w_i); that is, if this good is a luxury. The second component on the right of (22) is the price substitution term, which gives the effect of changes in relative prices on w_i via q_i. The final term is the direct effect of the relative price of i on the budget share.[9]

To apply (22) to the finite-change data for alcoholic beverages, we use the right side of equation (18) to substitute for the quantity component in (21). This gives

$$\begin{aligned} \Delta w_{it} = {} & (\theta_i - \bar{w}_{it}) DQ_t + \sum_{j=1}^{3} \pi_{ij} Dp_{jt} + \pi_{i0} DP_{0t} \\ & + \bar{w}_{it} (Dp_{it} - DP_t) + \eta_{it} + 0_3, \end{aligned} \tag{23}$$

where $\Delta w_{it} = w_{it} - w_{i,t-1}$ and 0_3 is a remainder term of third degree (Theil 1975–76, pp. 37–40, 215). Note that (12) and (20) imply $\Sigma_{j=1}^3 \pi_{ij} + \pi_{i0} = 0$, which is a reflection of demand homogeneity. We use this to deflate the absolute prices in (23) Dp_{jt} and DP_{0t} by DP_t to give

$$\begin{aligned} \Delta w_{it} = {} & (\theta_i - \bar{w}_{it}) DQ_t + \sum_{j=1}^{3} \pi_{ij} (Dp_{jt} - DP_t) \\ & + \pi_{i0} (DP_{0t} - DP_t) + \bar{w}_{it} (Dp_{it} - DP_t) + \eta_{it} + 0_3. \end{aligned} \tag{24}$$

9. For a further analysis, see Theil (1975–76, pp. 31–32).

To evaluate (24) we use the estimates given in table 7 and the log change in the consumer price index for DP_t. The constant term in the demand equation $\delta_i D_t + \psi_i(1 - D_t)$ means that this is an additional component of Δw_i. The results are given in tables 9–11. Looking at table 9, on average the budget share of beer fell by .029 percentage points per annum. The shift in preferences away from beer accounts for .024 of this. The growth in real income, together with the fact that beer is a necessity, accounts for .019 of the fall. The rise in the relative price of beer accounts for .008, while the other relative prices have a negligible effect. Finally, offsetting the fall in the beer share is the direct relative price component of .022. The conclusion that emerges is that the shift in preferences away from beer and the growth in real income are the two most important reasons for the decline in the budget share of beer.

As can be seen from table 10, the most important component of growth in the wine share on average is the shift in preferences toward wine. This is then followed by the direct relative price component, caused by the price of wine rising more rapidly than the CPI. Offsetting these two terms are the effects due to growth in real income and own-price substitution. For spirits (table 11) on average the growth in income has the effect of almost offsetting the negative shift in preferences; and the other components are quite small.

The importance of the shift in preferences raises the question of what lies behind this shift. Possible explanations include (1) Southern Europeans migrating to Australia and bringing with them their alcohol consumption patterns; (2) changes in demographic structure, such as members of the postwar baby boom reaching drinking age over this period; and (3) advertising and innovation in packaging and marketing, such as the introduction of wine casks.

Simulation of Alcohol Consumption

In this subsection we attempt to isolate the key factors responsible for the rapid growth of wine consumption by using the demand model for counterfactual simulations. We simulate alcohol consumption with (1) alcohol tax rates held constant; (2) a wine tax rate equal to the beer tax rate; and (3) no constant terms in the demand equations. These simulations answer the question to what extent the growth in wine is due to (1) changes in all alcohol taxes; (2) the fact that beer is subject to a substantial tax, while wine is not; and (3) trend-like changes in tastes toward wine.

Table 12 gives the data on alcohol taxes, in terms of both revenue and tax rates.[10] As can be seen, although beer represents the most

10. These are customs and excise taxes. Full details and sources of these data are given in a separate appendix, available on request.

TABLE 9 **Decomposition of Change in Budget Share of Beer, Australia, 1955–56/1976–77**

Year	Change in Budget Share of Beer Δw_1 (1)	Constant $\delta_1 D + \psi_1 \times (1 - D)$ (2)	Income $(\theta_1 - \bar{w}_1)DQ$ (3)	Beer $\pi_{11} \times (Dp_1 - DP)$ (4)	Wine $\pi_{12} \times (Dp_2 - DP)$ (5)	Spirits $\pi_{13} \times (Dp_3 - DP)$ (6)	Total Alcohol $\sum_{j=1}^{3} \pi_{1j} \times (Dp_j - DP)$ (7)	All Other $\pi_{10} \times (DP_0 - DP)$ (8)	Direct Relative Price $\bar{w}_1(Dp_1 - DP)$ (9)	Demand Equation Residual η_1 (10)
		Components of Δw_1		Price Substitution						
1956–57	21.31	−4.69	1.77	−12.91	−.01	2.01	−10.91	−.69	39.50	−3.63
1957–58	−5.52	−4.69	−2.15	−.38	−.15	.34	−.19	−.05	1.19	.34
1958–59	−10.06	−4.69	−1.76	1.82	−.18	.55	2.19	.09	−5.57	−.23
1959–60	−20.50	−4.69	−5.07	2.47	−.20	.88	3.15	.14	−7.31	−6.78
1960–61	−9.39	−4.69	.31	3.34	−.13	−1.55	1.66	.07	−9.56	2.82
1961–62	−2.72	−4.69	−.43	−.40	−.04	−.31	−.74	−.05	1.12	2.05
1962–63	−13.91	−4.69	−3.88	−.55	−.05	−1.92	−2.52	−.15	1.53	−4.20
1963–64	−1.33	−4.69	−3.39	−1.64	−.01	−.00	−1.65	−.09	4.46	4.04
1964–65	−5.81	−4.69	−1.48	3.51	−.08	.60	4.02	.20	−9.44	5.58
1965–66	17.15	−.73	−.35	−6.96	−.15	−3.59	−10.71	−.63	19.01	10.62
1966–67	4.28	−.73	−2.60	−2.51	−.08	2.72	.13	−.03	7.02	.50
1967–68	1.97	−.73	−3.24	−1.31	−.14	1.15	−.31	−.07	3.70	2.56
1968–69	−.06	−.73	−3.06	−.67	−.38	−1.48	−2.53	−.28	1.90	4.70
1969–70	−6.47	−.73	−3.78	−.48	−.11	.11	−.49	−.08	1.36	−2.78
1970–71	3.06	−.73	−2.16	−2.47	−.18	.57	−2.08	−.20	6.90	1.34
1971–72	−12.64	−.73	−1.10	2.24	.03	2.23	4.50	.26	−6.19	−9.38
1972–73	−6.16	−.73	−2.68	.96	.26	1.76	2.98	.28	−2.59	−3.46
1973–74	−1.59	−.73	−1.86	6.89	.36	−2.83	4.41	.40	−18.42	14.65
1974–75	−13.95	−.73	−1.40	3.22	.02	−3.60	−.36	−.03	−8.45	−2.99
1975–76	17.17	−.73	−2.37	−15.86	−.25	−.26	−16.37	−1.00	41.77	−4.12
1976–77	−16.53	−.73	.48	6.05	.20	4.43	10.68	.68	−15.95	−11.70
Mean	−2.94	−2.43	−1.91	−.75	−.06	.09	−.72	−.06	2.19	−.00

NOTE.—Column (1) = (2) + (3) + (4) + (5) + (6) + (8) + (9) + (10) + a remainder term of third degree. The year 1956–57, e.g., refers to the transition from 1955–56 to 1956–57. All entries are to be divided by 10,000.

TABLE 10 Decomposition of Change in Budget Share of Wine, Australia, 1955–56/1976–77

		Components of Δw_2								
				Price Substitution						
Year	Change in Budget Share of Wine Δw_2 (1)	Constant $\delta_2 D + \psi_2 \times (1 - D)$ (2)	Income $(\theta_2 - \bar{w}_2)DQ$ (3)	Beer $\pi_{21} \times (Dp_1 - DP)$ (4)	Wine $\pi_{22} \times (Dp_2 - DP)$ (5)	Spirits $\pi_{23} \times (Dp_3 - DP)$ (6)	Total Alcohol $\sum_{j=1}^{3} \pi_{2j} \times (Dp_j - DP)$ (7)	All Other $\pi_{20} \times (DP_0 - DP)$ (8)	Direct Relative Price $\bar{w}_2(Dp_2 - DP)$ (9)	Demand Equation Residual η_2 (10)
1956–57	−.40	−.06	−.06	−.33	−.10	−.25	−.68	−.12	.16	.40
1957–58	.76	−.06	.07	−.01	−1.27	−.04	−1.32	−.01	2.06	.07
1958–59	2.06	−.06	.04	.05	−1.48	−.07	−1.50	.02	2.46	1.10
1959–60	.67	−.06	.07	.06	−1.64	−.11	−1.68	.02	2.79	−.45
1960–61	.48	−.06	−.00	.09	−1.09	.19	−.81	.01	1.88	−.61
1961–62	.55	−.06	.00	−.01	−.30	.04	−.27	−.01	.52	.33
1962–63	−.30	−.06	.01	−.01	−.44	.24	−.22	−.03	.77	−.78
1963–64	−.02	−.06	.02	−.04	−.06	.00	−.11	−.02	.11	.07
1964–65	.41	−.06	.01	.09	−.69	−.07	−.68	.03	1.21	−.14
1965–66	7.26	5.37	−.02	−.18	−1.25	.45	−.98	−.11	2.33	.74
1966–67	7.14	5.37	−.33	−.06	−.67	−.34	−1.08	−.00	1.41	1.75
1967–68	7.80	5.37	−.68	−.03	−1.20	−.14	−1.38	−.01	2.79	1.75
1968–69	12.71	5.37	−.99	−.02	−3.15	.18	−2.98	−.05	8.29	3.08
1969–70	6.13	5.37	−1.69	−.01	−.93	−.01	−.95	−.01	2.70	.74
1970–71	−.99	5.37	−1.05	−.06	−1.48	−.07	−1.61	−.03	4.44	−8.14
1971–72	−.26	5.37	−.56	.06	.24	−.28	.02	.04	−.71	−4.38
1972–73	−.54	5.37	−1.54	.02	2.19	−.22	2.00	.05	−6.50	.10
1973–74	−.63	5.37	−1.11	.18	3.01	.35	3.54	.07	−8.87	.37
1974–75	8.25	5.37	−1.04	.08	.15	.45	.68	−.00	−.46	3.74
1975–76	9.15	5.37	−2.07	−.41	−2.06	.03	−2.43	−.17	6.85	1.62
1976–77	.11	5.37	.46	.16	1.69	−.55	1.29	.11	−5.87	−1.25
Mean	2.87	3.04	−.50	−.02	−.50	−.01	−.53	−.01	.87	.00

NOTE.—Column (1) = (2) + (3) + (4) + (5) + (6) + (8) + (9) + (10) + a remainder term of third degree. The year 1956–57, e.g., refers to the transition from 1955–56 to 1956–57. All entries are to be divided by 10,000.

TABLE 11 Decomposition of Change in Budget Share of Spirits, Australia, 1955–56/1976–77

Year	Change in Budget Share of Spirits Δw_3 (1)	Components of Δw_3: Constant $\delta_3 D + \psi_3 \times (1 - D)$ (2)	Income $(\theta_3 - \bar{w}_3)DQ$ (3)	Price Substitution: Beer $\pi_{31} \times (Dp_1 - DP)$ (4)	Wine $\pi_{32} \times (Dp_2 - DP)$ (5)	Spirits $\pi_{33} \times (Dp_3 - DP)$ (6)	Total Alcohol $\sum_{j=1}^{3} \pi_{3j} \times (Dp_j - DP)$ (7)	All Other $\pi_{30} \times (DP_0 - DP)$ (8)	Direct Relative Price $\bar{w}_3(Dp_3 - DP)$ (9)	Demand Equation Residual η_3 (10)
1956–57	−13.98	−2.10	−1.20	−3.01	.01	3.96	.96	−.37	−5.45	−5.92
1957–58	−1.30	−2.10	1.49	−.09	.17	.68	.76	−.03	−.86	−.49
1958–59	1.77	−2.10	1.29	.42	.20	1.09	1.72	.05	−1.39	2.08
1959–60	1.12	−2.10	4.18	.58	.22	1.73	2.53	.07	−2.23	−1.32
1960–61	4.00	−2.10	−.29	.78	.15	−3.07	−2.14	.04	4.07	4.44
1961–62	.62	−2.10	.41	−.09	.04	−.60	−.66	−.03	.82	2.18
1962–63	−2.02	−2.10	4.18	−.13	.06	−3.79	−3.86	−.08	5.11	−5.21
1963–64	2.79	−2.10	4.03	−.38	.01	−.00	−.38	−.05	.01	1.22
1964–65	3.25	−2.10	1.79	.82	.09	1.19	2.10	.11	−1.66	3.06
1965–66	−2.18	−2.56	39	−1.62	.17	−7.10	−8.55	−.33	9.98	−1.16
1966–67	−10.59	−2.56	2.70	−.58	.09	5.37	4.88	−.01	−7.06	−8.49
1967–68	4.10	−2.56	3.37	−.31	.16	2.26	2.12	−.04	−2.87	4.05
1968–69	−.32	−2.56	3.08	−.16	.43	−2.92	−2.65	−.15	3.78	−1.83
1969–70	5.32	−2.56	3.85	−.11	.13	.21	.22	−.04	−.28	4.11
1970–71	−3.12	−2.56	2.22	−.58	.20	1.12	.74	−.11	−1.50	−1.86
1971–72	−1.85	−2.56	1.23	.52	−.03	4.41	4.90	.14	−5.78	.18
1972–73	3.07	−2.56	3.39	.22	−.30	3.48	3.41	.15	−4.60	3.31
1973–74	5.58	−2.56	2.37	1.61	−.41	−5.60	−4.40	.21	7.73	2.24
1974–75	3.54	−2.56	1.91	.75	−.02	−7.11	−6.38	−.01	10.30	.27
1975–76	−6.76	−2.56	3.22	−3.70	.28	−.52	−3.94	−.53	.74	−3.70
1976–77	−1.87	−2.56	−.69	1.41	−.23	8.75	9.93	.36	−11.92	3.01
Mean	−.42	−2.36	2.04	−.17	.07	.17	06	−.03	−.15	.01

Note.—Column (1) = (2) + (3) + (4) + (5) + (6) + (8) + (9) + (10) + a remainder term of third degree. The year 1956–57, e.g., refers to the transition from 1955–56 to 1956–57. All entries are to be divided by 10,000.

TABLE 12 Alcohol Taxes, Australia, 1955–56/1976–77

	Tax Revenues ($ per capita)				Tax Rates × 100 (% pretax prices)		
Year	Beer $\left(\frac{t_1}{1+t_1}p_1q_1\right)$	Wine $\left(\frac{t_2}{1+t_2}p_2q_2\right)$	Spirits $\left(\frac{t_3}{1+t_3}p_3q_3\right)$	Total Alcohol $\left(\sum_{i=1}^{3}\frac{t_i}{1+t_i}p_iq_i\right)$	Beer (t_1)	Wine (t_2)	Spirits (t_3)
1955–56	18.33	0	2.14	20.47	117.198	0	38.351
1956–57	21.72	0	2.27	23.99	141.132	0	48.401
1957–58	21.77	0	2.27	24.04	137.090	0	47.588
1958–59	21.17	0	2.39	23.56	126.086	0	47.704
1959–60	21.59	.01	2.63	24.23	124.296	.201	48.524
1960–61	21.51	.01	2.64	24.16	119.236	.192	43.421
1961–62	21.41	.01	2.67	24.09	117.250	.189	43.133
1962–63	21.72	.01	2.59	24.32	116.272	.180	39.602
1963–64	22.45	.01	2.78	25.24	111.358	.170	38.773
1964–65	23.17	.02	3.00	26.19	108.372	.320	37.927
1965–66	26.34	.02	3.31	29.67	119.674	.273	42.436
1966–67	27.43	.02	3.67	31.12	112.929	.232	53.891
1967–68	28.45	.03	4.09	32.57	104.365	.292	53.254
1968–69	29.26	.04	3.96	33.26	97.793	.316	46.589
1969–70	29.88	.05	4.41	34.34	90.491	.343	44.862
1970–71	30.08	.75	4.38	35.21	79.073	5.064	41.995
1971–72	30.53	.96	4.63	36.12	74.356	6.057	41.749
1972–73	31.65	.32	5.23	37.20	68.654	1.767	41.213
1973–74	34.31	.09	7.70	42.10	61.598	.425	53.584
1974–75	34.81	.11	10.73	45.65	50.353	.400	64.601
1975–76	50.17	.15	11.83	62.15	64.761	.426	64.965
1976–77	53.23	.18	13.40	66.81	62.615	.452	67.507
Mean	28.23	.13	4.67	33.02	100.225	.786	47.730

important source of revenue, its tax *rate* has almost halved over this period (from 117% to 63%). Aside from the early 1970s, the tax on wine is negligible, while the tax rate for spirits has increased.

We simulate consumption with the tax rates held constant at their 1955–56 values of 117%, 0, and 38% for beer, wine, and spirits, respectively. With p_i the posttax price of i, p_i^0 the pretax price and t_i the tax rate, we have $p_{it} = (1 + t_{it})p_{it}^0$, so that $Dp_{it} = D(1 + t_{it}) + Dp_{it}^0$, where D is the log-change operator (as before). Accordingly, we can simulate the constant tax rates by replacing in the demand equations the observed price log change Dp_{it} with $Dp_{it} - D(1 + t_{it})$, which is the price change purged of its tax change component. Using equation (18), the simulated quantity log change is

$$Dq_{it}^s = (\theta_i/\overline{w}_{it})DQ_t + \sum_{j=1}^{3} (\pi_{ij}/\overline{w}_{it})[Dp_{jt} - D(1 + t_{jt})] + (\pi_{i0}/\overline{w}_{it})DP_{0t} + \eta_{it}/\overline{w}_{it}.$$

Subtracting from this the observed quantity log change Dq_{it} and using (18) gives

$$Dq_{it}^s - Dq_{it} = - \sum_{j=1}^{3} (\pi_{ij}/\overline{w}_{it})D(1 + t_{jt}). \tag{25}$$

Converting from changes to levels, the simulated quantity is $q_{it}^s = \exp(Dq_{it}^s + \log q_{i,t-1}^s)$. Finally, to evaluate (25) we use the estimates given in table 7.

The results of this simulation are given in the first three columns of table 13 (for changes) and in columns 4–6 of table 14 (levels). As the observed tax rate for beer fell over this period and that for spirits rose, the constant tax rate policy causes beer consumption to be lower than otherwise and spirits to be higher. By 1976–77, simulated per capita consumption is 125 liters, 13.4 liters, and 3.23 liters for beer, wine, and spirits (see table 14). Accordingly, this tax package causes beer consumption to be (124.57 − 136.14)/136.14 = 8.5% lower than otherwise, wine to be (13.3616 − 13.6533)/13.6533 = 2.1% lower, and spirits to be (3.2300 − 3.1663)/3.1663 = 2.0% higher. The reasons for the lower consumption of wine are (1) that it is an unconditional complement for beer (see table 7) and in the simulation we increase the price of beer; and (2) wine and spirits are unconditional substitutes and the price of spirits increases less rapidly in the simulation.

The conclusion from this simulation is that although changes in all alcohol taxes have contributed to the rapid growth of wine, this contribution is a small component of the overall growth.

To simulate the effects of taxing wine at the same rate as beer, we impose a wine tax equal to the beer tax in 1956–57 and then adjust it in all subsequent years to keep it equal to the beer tax. To do this, we

TABLE 13 **Simulation of Consumption of Alcoholic Beverages, Australia, 1955–56/1976–77**

Year	Alcohol Tax Rates Held Constant			Wine Tax Rate equal to Beer Tax Rate			No Constant Terms in Demand Equations		
	Beer	Wine	Spirits	Beer	Wine	Spirits	Beer	Wine	Spirits
1956–57	3.95	.19	8.95	−.74	−54.43	4.04	1.01	.11	2.19
1957–58	−.58	−.08	−1.11	.01	1.04	−.08	1.00	.11	2.38
1958–59	−1.55	−.35	−1.83	.04	2.85	−.24	1.01	.11	2.37
1959–60	−.22	.02	.11	.01	.58	−.05	1.05	.11	2.34
1960–61	−1.08	.11	−3.51	.02	1.32	−.11	1.08	.11	2.27
1961–62	−.34	−.05	−.49	.01	−.52	−.04	1.10	.11	2.22
1962–63	−.37	.16	−2.02	.00	.25	−.02	1.12	.11	2.23
1963–64	−.90	−.12	−1.30	.02	1.31	−.11	1.14	.11	2.22
1964–65	−.58	.04	−.96	.02	.90	−.07	1.15	.11	2.15
1965–66	2.21	.08	4.19	−.05	−2.86	.24	.18	−8.86	2.61
1966–67	−.47	−.70	4.68	.03	1.48	−.15	.17	−7.92	2.79
1967–68	−1.49	−.16	−1.97	.04	1.80	−.21	.17	−7.14	2.90
1968–69	−1.52	.09	−4.71	.03	1.25	−.16	.17	−6.28	2.84
1969–70	−1.44	−.09	−2.32	.03	1.30	−.18	.17	−5.66	2.76
1970–71	−2.34	1.38	−4.02	.10	3.59	−.51	.17	−5.51	2.73
1971–72	−.97	.22	−1.21	.03	1.21	−.17	.17	−5.54	2.80
1972–73	−1.30	−1.51	−1.37	−.01	−.27	.04	.18	−5.57	2.78
1973–74	−.88	−1.01	4.57	.03	1.00	−.13	.18	−5.60	2.66
1974–75	−2.13	−.60	2.26	.07	2.34	−.31	.18	−5.39	2.54
1975–76	3.49	.33	3.41	−.09	−2.74	.40	.18	−4.95	2.58
1976–77	−.36	−.10	.63	.01	.38	−.06	.18	−4.75	2.70
Mean	−.42	−.10	.09	−.02	−1.77	.10	.56	−3.44	2.53

NOTE.—The entries are $Dq_{it}^s - Dq_{it}$, where Dq_{it}^s is the simulated log change in the quantity consumed per capita of i and Dq_{it} is the actual log change. The year 1956–57, e.g., refers to the transition from 1955–56 to 1956–57. All entries are to be divided by 100.

TABLE 14 **Actual and Simulated Consumption of Alcoholic Beverages, Australia, 1955–56/1976–77** (Liters per Capita)

Year	Actual Consumption			Simulated Consumption with Alcohol Tax Rates Constant			Simulated Consumption with Wine Tax Rate equal to Beer Tax Rate			Simulated Consumption with No Constant Terms in Demand Equations		
	Beer	Wine	Spirits	Beer	Wine	Spirits	Beer	Wine	Spirits	Beer	Wine	Spirits
1955–56	105.98	5.3374	1.9977	105.98	5.3374	1.9977	105.98	5.3374	1.9977	105.98	5.3374	1.9977
1956–57	100.45	5.2084	1.7999	104.50	5.2182	1.9684	99.71	3.0223	1.8741	101.47	5.2144	1.8398
1957–58	100.66	5.1705	1.8220	104.11	5.1762	1.9705	99.93	3.0317	1.8955	102.70	5.1823	1.9072
1958–59	101.14	5.2060	1.9126	103.00	5.1936	2.0309	100.45	3.1409	1.9851	104.24	5.2237	2.0501
1959–60	102.91	5.2531	2.0808	104.57	5.2415	2.2119	102.21	3.1878	2.1586	107.18	5.2766	2.2832
1960–61	102.61	5.0999	2.0727	103.14	5.0943	2.1274	101.94	3.1359	2.1479	108.03	5.1282	2.3265
1961–62	102.19	5.1244	2.0787	102.37	5.1162	2.1232	101.53	3.1674	2.1532	108.78	5.1583	2.3855
1962–63	103.46	5.2825	2.0256	103.25	5.2824	2.0276	102.79	3.2734	2.0977	111.37	5.3231	2.3771
1963–64	106.99	5.5316	2.1864	105.82	5.5250	2.1603	106.32	3.4731	2.2619	116.49	5.5800	2.6234
1964–65	110.34	5.5709	2.3513	108.51	5.5662	2.3011	109.67	3.5294	2.4307	121.53	5.6256	2.8827
1965–66	110.39	6.0795	2.0859	110.98	6.0793	2.1288	109.66	3.7431	2.1615	121.80	5.6185	2.6250
1966–67	113.13	6.8205	2.0711	113.20	6.7726	2.2150	112.42	4.2618	2.1430	125.04	5.8231	2.6802
1967–68	116.93	7.5703	2.3255	115.27	7.5049	2.4386	116.24	4.8161	2.4013	129.46	6.0181	3.0979
1968–69	120.53	8.2549	2.3008	117.03	8.1909	2.3016	119.85	5.3178	2.3720	133.67	6.1629	3.1531
1969–70	123.72	8.9508	2.5549	118.41	8.8734	2.4971	123.07	5.8414	2.6292	137.45	6.3148	3.5993
1970–71	125.84	8.6849	2.5790	117.66	8.7291	2.4213	125.30	5.8753	2.6406	140.04	5.7988	3.7337
1971–72	125.67	8.8518	2.7290	116.36	8.9161	2.5313	125.17	6.0612	2.7893	140.10	5.5915	4.0631
1972–73	129.51	9.7910	3.0846	118.36	9.7144	2.8224	128.99	6.6861	3.1540	144.64	5.8499	4.7222
1973–74	138.94	10.9766	3.1045	125.87	10.7814	2.9734	138.42	7.5710	3.1701	155.45	6.2011	4.8807
1974–75	140.42	12.2775	2.9748	124.53	11.9875	2.9145	139.99	8.6691	3.0281	157.39	6.5722	4.7969
1975–76	137.46	13.0573	2.8715	126.24	12.7908	2.9108	136.92	8.9709	2.9348	154.36	6.6518	4.7513
1976–77	136.14	13.6533	3.1663	124.57	13.3616	3.2300	135.62	9.4165	3.2341	153.15	6.6327	5.3823
Mean	116.16	7.6251	2.3716	112.75	7.6722	2.3955	116.01	5.0568	2.4601	127.35	5.7594	3.2458

write t_2^s for the simulated value of the tax rate for wine and define it as follows. In 1955–56 it takes the value zero (which is also the observed value of t_2 in that year: see table 12) and in all subsequent years it is equal to the observed beer tax rate t_1. Applying the same argument that led to equation (25), simulated minus actual consumption can then be expressed as

$$Dq_{it}^s - Dq_{it} = (\pi_{i2}/\overline{w}_{it})[D(1 + t_{2t}^s) - D(1 + t_{2t})].$$

The results of this simulation are given in columns 4–6 of table 13 and columns 7–9 of table 14. On average, the tax causes the annual growth in wine consumption to be about 1.8 percentage points lower than actual. Simulated consumption of wine in 1976–77 is 9.4 liters per capita, which is $(9.4165 - 13.6533)/13.6533 = 31.0\%$ lower than actual. The effect of the wine tax is to lower beer consumption by a small amount to 135.6 liters in 1976–77 and to increase spirits consumption to 3.23 liters. Hence, the fact that wine escaped a substantial tax does account for a large part of the observed growth in consumption over this period.

In the final simulation we take out the trendlike changes in tastes by setting to zero the constant terms in the demand equations of table 7. Simulated minus actual consumption can be expressed for this case as

$$Dq_{it}^s - Dq_{it} = -[(\delta_i/\overline{w}_{it})D_t + (\psi_i/\overline{w}_{it})(1 - D_t)],$$

and the results are given in the last three columns of tables 13 and 14. From table 13, taking out this trend has the effect on average of beer consumption growing by .6 percentage points per annum more than actual, wine growing by 3.4 percentage points less, and spirits 2.5 points more.

To summarize, the simulations indicate that there are two reasons for the rapid growth of wine consumption. First, wine has not attracted a substantial tax, whereas beer has done so. Second, there has been a significant trendlike shift in preferences toward wine, away from beer and spirits. We also found that the observed changes in all alcohol taxes contributed very little to the growth of wine.

V. The Welfare Cost of Alcohol Taxes

In this section we measure the welfare cost of alcohol taxes by the reduction in consumer surplus not offset by government revenue from the taxes. This welfare cost can be expressed as (Harberger 1964)

$$W = -\tfrac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} T_i\, S_{ij}\, T_j, \tag{26}$$

where $T_i = t_i p_i^0$ is the tax per unit of i measured in terms of dollars and $S_{ij} = \partial q_i / \partial p_j$ with real income constant. Dividing (26) by income M, the cost can be formulated as a fraction of M as (see App.)

$$\frac{W}{M} = -\tfrac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\pi_{ij}\frac{t_j}{1+t_j}, \tag{27}$$

where π_{ij} is the (i,j)th unconditional Slutsky coefficient and $t_i = T_i/p_i^0$ is the tax rate on i. Using (20), (27) can be decomposed into the cost *within* the alcoholic beverages group and *between* alcoholic beverages and all other goods,

$$-\tfrac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\pi_{ij}\frac{t_j}{1+t_j} \qquad \text{(total cost)}$$

$$= -\tfrac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\pi_{ij}^{g}\frac{t_j}{1+t_j} \qquad \text{(cost within alcoholic beverages group)}$$

$$-\tfrac{1}{2}\phi\Theta_g(1-\Theta_g)\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\theta_i'\theta_j'\frac{t_j}{1+t_j} \qquad \text{(cost between alcoholic beverages and all other goods).} \tag{28}$$

We evaluate (28) with the tax data given in table 12 and the estimates given in tables 5 and 7. The results are given in table 15. In 1976–77 the welfare cost is .2% of income (total consumption expenditure) or $7.91 per capita (in current dollars). This represents 3.7% of expenditure on alcoholic beverages and 11.8% of government revenue from alcohol taxes. The within group component represents about 10% of the total cost.

In the previous section we simulated the effects of the imposition of a tax on wine at the same rate as that on beer. The simulated taxes are given in table 16. We now apply the above analysis to measure the welfare cost of this tax package; the results are given in table 17. As the imposition of the wine tax goes in the direction of having uniform tax rates, the within group cost falls. However, the between group and all other goods cost rises sufficiently to increase the total cost to $8.72 per capita in 1976–77.

VI. Concluding Comments

In this paper we have indicated how the systemwide approach to consumer demand can be extended so that it can be applied to quite narrowly defined commodity groups. Such applications have a number of

TABLE 15 Welfare Cost of Actual Alcohol Taxes, Australia, 1955–56/1976–77

Year	Welfare Cost as Fraction of Total Consumption Expenditure × 100: Within Alcoholic Beverages Group $-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\pi_{ij}^{g}\frac{t_j}{1+t_j}$ (1)	Between Alcoholic Beverages and All Other Goods $-\frac{1}{2}\phi\Theta_g(1-\Theta_g)\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\theta_i'\theta_j'\frac{t_j}{1+t_j}$ (2)	Total = Within + Between $-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i}{1+t_i}\pi_{ij}\frac{t_j}{1+t_j}$ (3)	Total Welfare Cost: Dollars per Capita $\frac{(3)}{100}\times M$ (4)	Percentage of Alcohol Expenditure $100\times(4)/M_g$ (5)	Percentage of Revenue from Alcohol Taxes $100\times(4)/R$ (6)
1955–56	.036	.264	.300	2.25	4.93	10.99
1956–57	.041	.322	.364	2.85	5.91	11.88
1957–58	.040	.315	.355	2.85	5.83	11.86
1958–59	.037	.298	.335	2.78	5.56	11.80
1959–60	.037	.297	.334	2.97	5.71	12.26
1960–61	.036	.278	.314	2.90	5.42	12.00
1961–62	.035	.275	.310	2.88	5.36	11.96
1962–63	.035	.266	.301	2.95	5.36	12.13
1963–64	.034	.256	.289	3.00	5.14	11.89
1964–65	.033	.249	.282	3.10	5.02	11.84
1965–66	.036	.277	.313	3.59	5.37	12.10
1966–67	.034	.288	.321	3.90	5.50	12.53
1967–68	.031	.271	.302	3.93	5.06	12.07
1968–69	.029	.247	.276	3.81	4.52	11.46
1969–70	.027	.229	.256	3.82	4.17	11.12
1970–71	.019	.206	.225	3.61	3.66	10.25
1971–72	.016	.197	.213	3.70	3.56	10.24
1972–73	.018	.179	.197	3.78	3.31	10.16
1973–74	.019	.180	.199	4.44	3.33	10.55
1974–75	.019	.165	.185	4.93	3.10	10.80
1975–76	.022	.203	.226	7.07	3.66	11.38
1976–77	.022	.201	.224	7.91	3.74	11.84
Mean	.030	.248	.278	3.77	4.69	11.51

SOURCE.—Based on data given in table 12 and parameter estimates given in tables 5, 7.
NOTE.—$M_g = \Sigma_{i=1}^{3} p_i q_i$ is expenditure on alcoholic beverages and $R = \Sigma_{i=1}^{3} [t_i/(1 + t_i)] p_i q_i$ is total revenue from alcohol taxes.

TABLE 16 Simulated Alcohol Taxes, Australia, 1955–56/1976–77

	Tax Revenues ($ per capita)				Tax Rates × 100 (% pretax prices)		
Year	Beer $\left(\frac{t_1^s}{1+t_1^s}p_1^s q_1^s\right)$	Wine $\left(\frac{t_2^s}{1+t_2^s}p_2^s q_2^s\right)$	Spirits $\left(\frac{t_3^s}{1+t_3^s}p_3^s q_3^s\right)$	Total Alcohol $\left(\sum_{i=1}^{3}\frac{t_i^s}{1+t_i^s}p_i^s q_i^s\right)$	Beer (t_1^s)	Wine (t_2^s)	Spirits (t_3^s)
1955–56	18.33	0	2.14	20.47	117.198	0	38.351
1956–57	21.56	3.36	2.36	27.28	141.132	141.132	48.401
1957–58	21.61	3.43	2.36	27.41	137.090	137.090	47.588
1958–59	21.02	3.48	2.48	26.98	126.086	126.086	47.704
1959–60	21.44	3.74	2.73	27.91	124.296	124.296	48.524
1960–61	21.37	3.81	2.74	27.91	119.236	119.236	43.421
1961–62	21.27	3.83	2.77	27.87	117.250	117.250	43.133
1962–63	21.58	3.99	2.68	28.25	116.272	116.272	39.602
1963–64	22.31	4.10	2.88	29.28	111.358	111.358	38.773
1964–65	23.03	4.29	3.10	30.42	108.372	108.372	37.927
1965–66	26.17	5.41	3.43	35.01	119.674	119.674	42.436
1966–67	27.26	6.10	3.80	37.15	112.929	112.929	53.891
1967–68	28.28	6.82	4.22	39.32	104.365	104.365	53.254
1968–69	29.10	7.98	4.08	41.15	97.793	97.793	46.589
1969–70	29.72	8.60	4.54	42.87	90.491	90.491	44.862
1970–71	29.95	7.92	4.48	42.36	79.073	79.073	41.995
1971–72	30.41	8.07	4.73	43.21	74.356	74.356	41.749
1972–73	31.52	8.49	5.35	45.36	68.654	68.654	41.213
1973–74	34.18	9.00	7.86	51.05	61.598	61.598	53.584
1974–75	34.70	9.79	10.92	55.42	50.353	50.353	64.601
1975–76	49.97	15.68	12.09	77.74	64.761	64.761	64.965
1976–77	53.03	17.19	13.69	83.91	62.615	62.615	67.507
Mean	28.08	6.59	4.79	39.47	100.225	94.898	47.730

NOTE.—t_i^s is the simulated tax rate on i; $p_i^s = p_i(1 + t_i^s)/(1 + t_i)$ is the simulated posttax price of i, where p_i is the actual (posttax) price and t_i the actual tax rate; and q_i^s is the simulated per capita consumption given in cols. 7–9 of table 14.

TABLE 17 Welfare Cost of Simulated Alcohol Taxes, Australia, 1955–56/1976–77

Year	Welfare Cost as Fraction of Total Consumption Expenditure × 100: Within Alcoholic Beverages Group $-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i^s}{1+t_i^s}\pi_{ij}^g\frac{t_j^s}{1+t_j^s}$ (1)	Between Alcoholic Beverages and All Other Goods $-\frac{1}{2}\phi\Theta_g(1-\Theta_g)\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i^s}{1+t_i^s}\theta_i'\theta_j'\frac{t_j^s}{1+t_j^s}$ (2)	Total = Within + Between $-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{t_i^s}{1+t_i^s}\pi_{ij}\frac{t_j^s}{1+t_j^s}$ (3)	Total Welfare Cost: Dollars per Capita $\frac{(3)}{100}\times M$ (4)	Percentage of Alcohol Expenditure $100\times(4)/M_g^s$ (5)	Percentage of Revenue from Alcohol Taxes $100\times(4)/R^s$ (6)
1955–56	.036	.264	.300	2.25	4.93	10.99
1956–57	.013	.411	.424	3.32	6.66	12.17
1957–58	.012	.402	.414	3.33	6.58	12.15
1958–59	.010	.379	.390	3.23	6.26	11.97
1959–60	.010	.377	.387	3.45	6.41	12.36
1960–61	.011	.355	.366	3.38	6.11	12.11
1961–62	.011	.350	.361	3.36	6.03	12.06
1962–63	.012	.340	.352	3.46	6.07	12.25
1963–64	.012	.327	.339	3.52	5.83	12.02
1964–65	.011	.318	.330	3.63	5.69	11.93
1965–66	.012	.354	.366	4.19	6.03	11.97
1966–67	.006	.363	.370	4.49	6.09	12.09
1967–68	.005	.342	.347	4.51	5.58	11.47
1968–69	.006	.312	.318	4.40	5.01	10.69
1969–70	.005	.290	.295	4.40	4.62	10.26
1970–71	.004	.254	.258	4.13	4.09	9.75
1971–72	.003	.240	.243	4.23	3.98	9.79
1972–73	.002	.223	.225	4.32	3.70	9.52
1973–74	.000	.222	.223	4.96	3.65	9.72
1974–75	.001	.201	.202	5.37	3.34	9.69
1975–76	.000	.250	.250	7.83	3.96	10.07
1976–77	.000	.246	.247	8.72	4.03	10.39
Mean	.008	.310	.319	4.30	5.21	11.16

SOURCE.—Based on data given in table 16 and parameter estimates given in tables 5 and 7.
NOTE.—$M_g^s = \Sigma_{i=1}^3 p_i^s q_i^s$ is simulated expenditure on alcoholic beverages and $R^s = \Sigma_{i=1}^3 [t_i^s/(1+t_i^s)] p_i^s q_i^s$ is simulated total revenue from alcohol taxes.

attractions from the viewpoint of policy analysis for business and government. We analyzed the consumption of beer, wine, and spirits to illustrate the general principles and used the demand model for (1) a number of simulations designed to analyze the rapid growth of wine consumption and (2) to measure the welfare cost of alcohol taxes.

The approach of this paper focused on income, prices, and a time trend as the systematic determinants of demand. It is clear, however, that the methodology could be extended to allow for other factors such as product quality, demographic characteristics, advertising, and so on; as indicated in Section IV, these factors may well lie behind the time trend in our demand equations. As the effects of advertising are likely to be distributed over time, the incorporation of this factor would require a dynamic formulation of the approach. As a possible area of future research, the methodology could be applied to individual product data, including the analysis of the demand for different brands of the same commodity.

Appendix

Unconditional Demand Equations

Letting $\mathbf{p} = [p_1, \ldots, p_n]'$ and $\mathbf{q} = [q_1, \ldots, q_n]'$, the consumer chooses $\mathbf{q}$ to maximize the utility function $u(\mathbf{q})$ subject to the budget constraint $\mathbf{p}'\mathbf{q} = M$. The first-order conditions are the budget constraint and $\partial u/\partial \mathbf{q} = \lambda \mathbf{p}$, where λ is the marginal utility of income. Differentiating these conditions with respect to $\mathbf{p}$ and M gives Barten's (1964) fundamental matrix equation,

$$\begin{bmatrix} \mathbf{U} & \mathbf{p} \\ \mathbf{p}' & 0 \end{bmatrix} \begin{bmatrix} \partial \mathbf{q}/\partial M & \partial \mathbf{q}/\partial \mathbf{p}' \\ -\partial\lambda/\partial M & -\partial\lambda/\partial \mathbf{p}' \end{bmatrix} = \begin{bmatrix} 0 & \lambda \mathbf{I} \\ 1 & -\mathbf{q}' \end{bmatrix}, \tag{A1}$$

where $\mathbf{U} = \partial^2 u/\partial \mathbf{q}\partial \mathbf{q}'$ and $\partial \mathbf{q}/\partial M$ and $\partial \mathbf{q}/\partial \mathbf{p}'$ are the income and price derivatives of the demand functions. Solving (A1) gives

$$\frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{U}^{-1} - \frac{\lambda}{\partial\lambda/\partial M}\frac{\partial \mathbf{q}}{\partial M}\frac{\partial \mathbf{q}'}{\partial M} - \frac{\partial \mathbf{q}}{\partial M}\mathbf{q}' \tag{A2}$$

for the price derivatives and

$$\frac{\partial \mathbf{q}}{\partial M} = \frac{\partial \lambda}{\partial M}\mathbf{U}^{-1}\mathbf{p} \tag{A3}$$

for the income derivatives.

Defining $\mathbf{P} = \operatorname{diag}[\mathbf{p}]$, we premultiply (A2) by $\mathbf{P}$ and postmultiply by $\mathbf{P}/M$ to give

$$\mathbf{P}\frac{\partial \mathbf{q}}{\partial \mathbf{p}'}\mathbf{P}/M = \boldsymbol{\nu} - \phi\boldsymbol{\theta}\boldsymbol{\theta}' - \boldsymbol{\theta}\mathbf{w}', \tag{A4}$$

where $\boldsymbol{\nu} = \lambda \mathbf{P}\mathbf{U}^{-1}\mathbf{P}/M = [\nu_{ij}]$, $\phi = \lambda/M/\partial\lambda/\partial M = (\partial \log \lambda/\partial \log M)^{-1}$, $\boldsymbol{\theta} = \mathbf{P}\partial \mathbf{q}/\partial M = [\theta_i]$ and $\mathbf{w} = \mathbf{P}\mathbf{q}/M = [w_i]$. Dividing (A3) by $\partial\lambda/\partial M$ and premultiplying by $\lambda \mathbf{P}/M$ gives

$$\frac{\lambda/M}{\partial\lambda/\partial M}\,\mathbf{P}\,\frac{\partial \mathbf{q}}{\partial M} = \lambda \mathbf{P}\mathbf{U}^{-1}\mathbf{p}/\mathbf{M} = (\lambda \mathbf{P}\mathbf{U}^{-1}\mathbf{P}/M)\boldsymbol{\iota},$$

where $\boldsymbol{\iota} = [1, \ldots, 1]'$, or

$$\phi\boldsymbol{\theta} = \boldsymbol{\nu}\boldsymbol{\iota}, \tag{A5}$$

which is equation (3) in vector form.

We write the demand equations in differential form as $d\mathbf{q} = (\partial\mathbf{q}/\partial M)\, dM + (\partial\mathbf{q}/\partial\mathbf{p}')\, d\mathbf{p}$. Premultiplying by $\mathbf{P}/M$ gives

$$\mathbf{W}d(\log \mathbf{q}) = \boldsymbol{\theta}d(\log M) + \left(\mathbf{P}\,\frac{\partial\mathbf{q}}{\partial\mathbf{p}'}\,\mathbf{P}/M\right)d(\log \mathbf{p}),$$

where $\mathbf{W} = \text{diag}[\mathbf{w}]$. Substituting the right side of (A4) for $(\mathbf{P}\,\partial\mathbf{q}/\partial\mathbf{p}')\,\mathrm{P}/M$, we obtain

$$\begin{aligned}\mathbf{W}d(\log \mathbf{q}) &= \boldsymbol{\theta}d(\log M) + (\boldsymbol{\nu} - \phi\boldsymbol{\theta}\boldsymbol{\theta}' - \boldsymbol{\theta}\mathbf{w}')d(\log \mathbf{p}) \\ &= \boldsymbol{\theta}[d(\log M) - d(\log P)] + \boldsymbol{\nu}(\mathbf{I} - \boldsymbol{\iota}\boldsymbol{\theta}')d(\log \mathbf{p}), \end{aligned} \tag{A6}$$

where $d(\log P) = \mathbf{w}'d(\log \mathbf{p})$ and where we have used (A5) for substitute $\boldsymbol{\nu}\boldsymbol{\iota}$ for $\phi\boldsymbol{\theta}$. The term $d(\log M) - d(\log P)$ is the change in real income, which we write as $d(\log Q)$. Taking the differential of the budget constraint, we obtain $\mathbf{w}'d(\log \mathbf{p}) + \mathbf{w}'d(\log \mathbf{q}) = d(\log M)$, so that $d(\log Q) = \mathbf{w}'d(\log \mathbf{q})$ (see below eq. [1]). Writing $d(\log P') = \boldsymbol{\theta}'d(\log \mathbf{p})$ for the Frisch price index, (A6) can be expressed as

$$\mathbf{W}d(\log \mathbf{q}) = \boldsymbol{\theta}d(\log Q) + \boldsymbol{\nu}[d(\log \mathbf{p}) - \boldsymbol{\iota}d(\log P')],$$

which is equation (1) in vector form.

Derivation of Equations (7) and (8)

Equation (7) is obtained by summing both sides of (6) over $i \in S_g$. The derivation of the variable on the left and the first term on the right of equation (7) is straightforward. We obtain the substitution term as follows. The sum over $i \in S_g$ of the substitution term of equation (6) is

$$\sum_{i\in S_g}\sum_{j\in S_g} \nu_{ij}d\left(\log \frac{p_j}{P'}\right) = \phi \sum_{j\in S_g} \theta_j[d(\log p_j) - d(\log P')], \tag{A7}$$

where we have used $\Sigma_{i\in S_g}\, \nu_{ij} = \phi\theta_j$, which follows from the symmetry of $[\nu_{ij}]$ and (5). Since $\Theta_g = \Sigma_{j\in S_g}\, \theta_j$, the second member of (A7) can be expressed as

$$\phi\Theta_g\left[\sum_{j\in S_g} \frac{\theta_j}{\Theta_g} d(\log p_j) - d(\log P')\right] = \phi\Theta_g[d(\log P_g') - d(\log P')], \tag{A8}$$

where $d(\log P_g') = \Sigma_{j\in S_g}(\theta_j/\Theta_g)d(\log p_j)$ is the Frisch price index of the group. The second member of (A8) is the substitution term of equation (7).

To derive equation (8), we rearrange (6) and (7) to give

$$d(\log Q) = \frac{w_i}{\theta_i} d(\log q_i) - \sum_{j\in S_g} \frac{\nu_{ij}}{\theta_i} d\left(\log \frac{p_j}{P'}\right) \tag{A9}$$

and

$$d(\log Q) = \frac{W_g}{\Theta_g} d(\log Q_g) - \phi d\left(\log \frac{P_g'}{P'}\right). \tag{A10}$$

Equating the right side of (A9) with that of (A10) and rearranging gives

$$\begin{aligned} w_i d(\log q_i) &= \frac{\theta_i}{\Theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} \nu_{ij} d(\log p_j) \\ &\quad - \sum_{j \in S_g} \nu_{ij} d(\log P') - \phi\theta_i d(\log P'_g) + \phi\theta_i d(\log P') \\ &= \frac{\theta_i}{\Theta_g} W_g d(\log Q_g) + \sum_{j \in S_g} \nu_{ij} [d(\log p_j) - d(\log P'_g)], \end{aligned}$$

which follows from (5). This is equation (8).

The Construction of DP_0

We write the log change in the consumer price index (DP^*) as a weighted average of the price log changes of the n goods, with the averages of the budget shares as weights. With the alcoholic beverages the first three goods, we have

$$DP_t^* = \sum_{i=1}^{3} \overline{w}_{it}\, Dp_{it} + \sum_{i=4}^{n} \overline{w}_{it}\, Dp_{it} = \overline{W}_{gt}\, DP_{gt} + (1 - \overline{W}_{gt}) DP_{0t}, \qquad \text{(A11)}$$

where $\overline{W}_{gt} = \Sigma_{i=1}^{3} \overline{w}_{it}$, $DP_{gt} = \Sigma_{i=1}^{3} (\overline{w}_{it}/\overline{W}_{gt}) Dp_{it}$ is the Divisia price index of the group and $DP_{0t} = \Sigma_{i=4}^{n} [\overline{w}_{it}/(1 - \overline{W}_{gt})] Dp_{it}$ is the Divisia price index of all other goods. We express DP_{0t} in terms of observables by rearranging (A11) to give

$$DP_{0t} = (DP_t^* - \overline{W}_{gt}\, DP_{gt})/(1 - \overline{W}_{gt}),$$

which is the equation used to define DP_{0t}.

Derivation of Equation (27)

We divide both sides of (26) by M and then multiply and divide the right by $p_i p_j$ to give

$$\begin{aligned} \frac{W}{M} &= -\tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{T_i}{p_i} \frac{p_i p_j}{M} S_{ij} \frac{T_j}{p_j} \\ &= -\tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{t_i}{1 + t_i} \frac{p_i p_j}{M} S_{ij} \frac{t_j}{1 + t_j}, \end{aligned}$$

as $T_i = t_i p_i^0$ and $p_i = (1 + t_i) p_i^0$. To establish that this is equivalent to (27) we need to show that $\pi_{ij} = (p_i p_j / M) S_{ij}$. To do this, consider the change in demand for i when real income is constant and when only p_j changes. From (A6), this is

$$w_i d(\log q_i) = (\nu_{ij} - \phi\theta_i\theta_j) d(\log p_j) = \pi_{ij} d(\log p_j),$$

where the second step is based on (10) and (20). This can be written as

$$\frac{p_i}{M} dq_i = \pi_{ij} \frac{dp_j}{p_j},$$

so that

$$dq_i = (M/p_i p_j) \pi_{ij} dp_j.$$

As real income is constant, $(M/p_i p_j)\pi_{ij} = S_{ij}$, so that $\pi_{ij} = (p_i p_j / M) S_{ij}$.

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