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Imperfect Price Discrimination and Variety*

I analyze by geometric techniques a spatial model in which a monopolist uses location differences across consumers to price discriminate. The model is used to examine price discrimination in a model of variety in a setting in which consumers disagree on the definition of quality. The full burden of separation of types is not on binding self-selection conditions. Examples are presented in the paper to show the ambiguous output and welfare comparisons between imperfect discrimination and single-price monopoly. At various extremes discrimination can either (relative to single-price monopoly) make some consumers better off and none worse off, while raising firm profit, or, alternatively, reduce firm output. The effect of an increase in the cost of travel on output, variety, and consumer surplus is demonstrated. The implications of the model are also shown to be exhaustive under various assumptions about the set of observables.

I. Introduction

In many environments a monopolist can price discriminate even though consumers can select any option offered by the firm. Though consumers seem indistinguishable to the firm, the monopolist can discriminate by exploiting its knowledge of the joint distribution of agents' characteristics (in this paper, locations and reservation prices) to offer the most profitable menu of alternatives given the constraints imposed by the maximizing behavior of each type of consumer (see Chiang and Spatt [1982] for a partial bibliography of this research). In many of these models consumers agree typically on the definition of the best available good (exceptions include Adams and Yellin [1976] and Telser [1979]) but disagree on their incremental valuation of quality. The consumer's selection of a price-quality contract from the set offered by the monopolist can elicit information that the monopolist endogenously exploits. For example, the price-discriminating monopolist may charge a steeper price-quality gradient than predicted by cost considerations (see Mussa and Rosen 1978).

In a spatial market the monopolist can discriminate on the basis of differences in cost of travel or location of consumers. I analyze a model

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in which location differences are the basis for discrimination, assuming all consumers have the same cost of travel (in contrast, Chiang and Spatt [1982] assume a single location of consumers and use differences in the cost of travel, i.e., incremental valuation of quality, as the basis of discrimination). By using the abstraction of diverse consumer locations so that consumers prefer different goods (and thus, unlike most of the price discrimination literature, disagree on the definition of quality), I analyze issues involving *variety* (see, e.g., Spence 1976) rather than quality. For example, I show that some markets may be sufficiently far apart so that binding self-selection constraints are unnecessary to separate the markets (in sharp contrast to most of the models of quality and price discrimination), but whether markets are exogenously separated is endogenously determined. This model thus suggests the natural linkage between models of price discrimination with exogenously separated markets and the recent literature in which the binding self-selection constraints carry the full burden of separation.

The model is also linked to work on spatial pricing patterns in monopoly and other market structures (e.g., the pioneering study of Smithies [1941], the detailed treatment and excellent bibliography of Greenhut and Ohta [1975], and recent empirical work of Greenhut, Greenhut, and Li [1980]). The spatial pricing analysis typically assumes exogenous store locations (in fact, usually only one store) and seems less in the spirit of the modern work on imperfect information than our analysis. My model develops a natural link between the work on spatial pricing and imperfect information and emphasizes the application of geometric methods in analyzing price discrimination.

Section II of the paper develops the general model, including an analysis of the extent to which my results are exhaustive in several spaces of observables, in order to emphasize the manner in which theory limits observations. In Section III I present examples, including illustrations of the impact of changing the cost per variety, cost of travel, and cost of output; comparison of imperfect discrimination and single-price monopoly; and analysis of the resolution of uncertainty. I conclude in Section IV.

II. The General Model

I assume a finite number of types of consumers indexed by $i = 1, \ldots, N$. Consumers have a common cost of travel, c. (The travel cost is the cost all consumers pay to transport one unit of finished good one unit of length[1] and measures the consumer's relative intensity of preference toward his preferred product.) There are n_i consumers of type i characterized by a location on the compact interval $l_i \in [0, L]$ and reservation

1. No consumers can earn arbitrage profits from resale of any good because of the common travel cost.

price v_i (for expositional simplicity, I permit only one type of consumer per location).[2] Each consumer knows his type and demands at most one unit of the good. The monopolist knows the number and characteristics of each consumer type, though he is unable to identify ex ante the type of any particular consumer. The monopolist offers a set of contracts (location and price pairs) that maximizes profit subject to the maximizing behavior of consumers. In equilibrium, both the offered contracts and the purchasers (buyers) of each (it is possible for some consumers not to purchase any contract) are determined. The monopolist faces a zero fixed cost for each variety (equivalent to zero cost per store location) and a nonnegative constant unit cost of output identical at all locations.

The variable t denotes the location of a variety that charges price p and Ω denotes the set of contracts selected by some consumer. The type i purchase the commodity if and only if

$$\min_{(t,p) \in \Omega} (p + |t - l_i| c_i) \leq v_i,$$

the individual rationality conditions, and if the type i consumer buys he purchases by a contract that

$$\min_{(t,p) \in \Omega} (p + |t - l_i| c_i),$$

the self-selection conditions (see fig. 1). In order that the specification is nontrivial, I assume the cost of production is low enough (i.e., below the maximum reservation price) so that some consumers purchase the good (i.e., Ω is nonempty). If more than one contract minimizes the consumer's total cost, then I assume the indifferent consumer purchases one at the highest price and any consumer indifferent between purchasing and not purchasing is assumed to purchase. The set of buyers and their equilibrium choice of contracts is endogenously determined by the solution to the firm's problem; that is, the monopolist determines who buys and the contracts $t(i)$, $p(i)$, selected by buyers of type i from the optimizing behavior of consumers.

The assumption of consumer maximization (without requiring firm profit maximization) implies some obvious restrictions on the set of contracts in the equilibrium set.

Property 2.1: The contract set Ω contains a finite number of elements.

This result exploits the hypothesized finite number of types and the convention that indifferent consumers purchase by the highest price contract that minimizes the consumer's expenditure.

Property 2.2: Any pair of contracts in the contract set must be at distinct locations.

2. Most of my results apply directly to the case in which consumers and products are located at points in k-dimensional Euclidean space and the standard Euclidean distance measure is used.

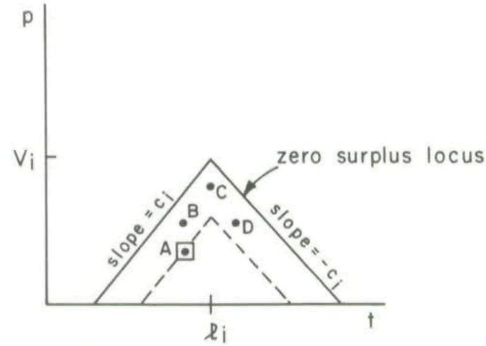

FIG. 1.—The consumer purchases the good if any contract lies in the triangle between the location axis and the zero surplus locus. Any purchasing consumer selects a contract on the closest possible iso-surplus curve to the axis (contract A in the figure).

Otherwise, two contracts at the same location would be offered at different prices and no maximizing consumer would purchase the more expensive one. Therefore, the relation mapping location to price is a function. Though properties 2.1 and 2.2 are trivial, they exhaust the restrictions in this model that depend only on the contract set (see the discussion at the end of this section).

The assumption of monopoly maximization is now introduced and properties of the monopoly solution examined. The monopolist lacks an incentive to offer a contract not selected by any consumer (and excessive offerings can be costly), so we interpret Ω as the profit-maximizing contract set. First, I show that firm maximization implies that each buyer purchases at his own location in the solution to the firm's profit maximization problem. This restriction is then imposed in our analysis of the firm's problem.

PROPERTY 2.3: Each buyer must purchase at his location.

Proof: If any buyer i buys at distance $d > 0$ from his location l_i, then profit can be increased by offering a contract at l_i and charging the old price he paid plus dc. These buyers switch to the new higher price contract under our indifference convention. No one switches to this new contract from an even higher priced one because that implies the consumer did not initially optimize. Q.E.D.

The perturbation of the contract set used may add additional varieties and thus decrease profits if varieties were sufficiently costly. The importance of the costless variety assumption is illustrated by an example developed in Section III with two types of consumers and *costly* varieties in which one variety located between the traders is the optimal configuration. If the set of locations of buyers were known (the set of buyers is actually determined in the solution), then property 2.3 represents a restriction in terms of observables (and, as I later show, together with properties 2.1 and 2.2 represent the only restrictions for this environment when the contracts selected and the set of locations of buyers are the observables). In describing the solution to the model, we need only determine the set of buyers and the prices paid by each buyer, because each purchases at his own location.

In order further to ascertain the properties of the monopoly solution, I focus on an optimal set of buyers and order of prices charged to the buyers. The type of buyers are indexed $i = 1, \ldots, b$, so that $p(1) \geq p(2) \ldots \geq p(b)$.

PROPERTY 2.4: The monopoly solution has prices of the form

$$\begin{aligned}
p(b) &= v_b,\\
p(b-1) &= \min(v_{b-1}, p(b) + c|l_b - l_{b-1}|),\\
p(b-2) &= \min(v_{b-2}, p(b-1) + c|l_{b-1} - l_{b-2}|, p(b) + c|l_b - l_{b-2}|),\\
&\vdots\\
p(1) &= \min(v_1, p(2) + c|l_2 - l_1|, \ldots, p(b) + c|l_b - l_1|).
\end{aligned}$$

Proof: This solution is feasible because it satisfies $p(j) \leq v_j$ and $p(j) \leq p(k) + c|l_k - l_j|$ for $j < k$; and, also, we have by hypothesis $p(j) \leq p(k) \leq p(k) + c|l_k - l_j|$ for $j > k$. If we raised any price above those in the hypothesized solution we would no longer satisfy feasibility for that set of buyers. If any price is below these values no other price is raised (others can fall too), and so such a solution cannot be profit maximal. Given the optimal set of buyers and ordering of prices this solution is profit maximal. Q.E.D.

Inspection of this form of the solution shows that there are b binding surplus and self-selection constraints. The solution illustrates the endogenous determination of the binding self-selection constraints. Some markets might be sufficiently segmented so that the burden of separation is not upon the contracts offered. In contrast in models in which all consumers prefer the same variety the total burden of separation is typically upon the contract set. The present approach suggests a natural link between models of discrimination in which all markets analyzed are exogenously separated and those in which the firm's choice of contracts separates the markets.

By recursive substitution in the solution in property 2.4 and the application of the triangle inequality it follows that the prices in the monopoly solution can be rewritten as

$$\begin{aligned}
p(b) &= v_b,\\
p(b-1) &= \min(v_{b-1}, v_b + c|l_b - l_{b-1}|),\\
p(b-2) &= \min(v_{b-2}, v_{b-1} + c|l_{b-1} - l_{b-2}|, v_b + c|l_b - l_{b-2}|),\\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)\\
p(1) &= \min(v_1, v_2 + c|l_2 - l_1|, \ldots, v_b + c|l_b - l_1|).
\end{aligned}$$

It also follows from property 2.4 that those buyers with the minimum reservation price among buyers pay that price (and have zero surplus) and no buyer pays less.

Instead of ordering buyers by their prices, let us now order them by locations, so that $l_1 \leq l_2 \leq \ldots \leq l_b$.

PROPERTY 2.5: The self-selection constraints reduce to $p(i) \leq p(i + 1) + c(l_{i+1} - l_i)$ and $p(i + 1) \leq p(i) + c(l_{i+1} - l_i)$ for $i = 1, \ldots, b - 1$.

These conditions when combined imply the full set of self-selection conditions, meaning that feasibility is satisfied if each buyer prefers to purchase at his location over those contracts immediately to the left and right.

In much of my analysis I have focused on the characteristics of contracts given an optimal set of buyers; now I consider the characteristics of the set of buyers and nonbuyers. The results in this section and the examples in Section III are based on an explicit analysis of the optimal set of buyers. The previous results are interesting in their own right and are useful in determining the optimal set of buyers by enumerative methods.

The monopolist may prefer to arrange the contracts so that some consumers are excluded from purchase. For example, the monopolist may wish to exclude consumers with moderate reservation values who live in a region populated by high reservation consumers, because servicing the moderate reservation consumers he would forgo substantial profits on most of the other agents. (Yet isolated consumers with very low reservation prices might be sold the commodity so that the good is not allocated to those who value it most.)

PROPERTY 2.6: If the type j agent does not purchase, then either $v_j < \max[p(k) - c|l_k - l_j|, p(i) - c|l_i - l_j|]$, where i and k denote the neighboring purchasers, or v_j is at most the unit cost of production.

Proof: If the unit cost of production is less than v_j, then the type j agent's not purchasing implies that the monopolist is unable to prevent other agents from purchasing any contract intended for j at l_j. In fact, from property 2.5 we know that the type j agent's not purchasing implies that the monopolist is unable to prevent some neighboring purchaser from buying the contract intended for j. Therefore, either $p(k) > v_j + c|l_k - l_j|$ or $p(i) > v_j + c|l_i - l_j|$, so that

$$v_j < \max\,[p(k) - c|l_k - l_j|, p(i) - c|l_i - l_j|].$$

Q.E.D.

Weaker forms of the condition include $v_j < \max\,(v_k - c|l_k - l_j|, v_i - c|l_i - l_j|)$, $v_j < \max\,[p(k), p(i)]$, and $v_j < \max\,(v_k, v_i)$. The last form also implies that the highest reservation price consumers *must* purchase (if anyone does). In at least some cases the relative numbers of the different types are irrelevant; if the cost of travel is high enough then the monopolist can sell to each consumer (whose reservation price exceeds unit cost) at his location and reservation value and extract all the surplus (independent of the relative proportions).

Thus far I have addressed the implications of the model without considering whether the results are exhaustive.[3] I now turn to that issue and show, under some alternate restrictions on what is observable, that my treatment is complete. In several spaces of observables I show that my conditions are sufficient as well as necessary. The technique is constructive; that is, given any set of observations satisfying the assumed restrictions, it generates a supporting economy.

The question whether the implications of a model are exhaustive has been addressed in other contexts. For example, integrability theory exhaustively identifies the restrictions utility maximization imposes on demand functions. In finance, the consequences of the linearity of the valuation operator exhaust the restrictions on rational option prices, without special assumptions on preferences or the distribution of returns (see Cox and Ross 1976). By establishing the complete set of implications (in an appropriate space) generated by a model, we understand the extent to which the theory limits observations.

First, assume the economist observes only price-location contracts (but nothing else about the structure of the economy). In this model we know that there are a finite number of such contracts and at most one at any location (properties 2.1 and 2.2). These conditions also prove sufficient.

PROPERTY 2.7: Any finite set of price-location contracts with at most one store per location is a solution to the model for some set of parameters.

Proof: Take one set of points in (t, p) space that satisfies the hypotheses. These points can be assigned to satisfy $t_1 < t_2 < \ldots < t_h$. Let $l_i = t_i$ and $v_i = p_i$ for all i. The unit cost of output is nonnegative and less than min p_i. If the cost of travel is high enough (determined by the maximum slope of the contract set), then each consumer can eliminate contracts at locations other than his own and each purchases at his location (i.e., consumers maximize given the contracts). The monopolist has extracted the entire surplus and so must be maximizing. Hence, any set of points satisfying our hypotheses solves some form of the model and no further restrictions in the contract space can be obtained. Q.E.D.

In this construction, the solution perfectly sorts all types of traders and is in fact equivalent to perfect discrimination with exogenously separated markets (i.e., for big enough c there are no binding self-selection constraints). We can slightly perturb the construction to show that a supporting economy can always be obtained without com-

3. Steve Ross suggested examining this question in the context of the related Chiang and Spatt (1982) model.

plete separation and, therefore, inspection of the contract space is never adequate to distinguish these from other environments. The necessity of the restrictions follows from consumer maximization even in a more general environment with diverse c_i, costly varieties, and multiple reservation prices at some locations. Therefore, these restrictions must be necessary and sufficient in that more general environment. In contrast, the necessary and sufficient conditions on the contract space in Chiang and Spatt (1982) (where all consumers live at one point) are much more restrictive. Imposing additional assumptions (as in the consumer theory and options pricing settings) yields stronger results. An alternative approach that I take here is to enrich the set of observables.

We suppose that the price-location contracts as well as the set of consumer (potential buyers) locations are observable.

PROPERTY 2.8: Any finite set of price-location contracts with at most one contract per location and where the set of locations of consumers includes all locations of contracts is a solution to the model for some set of parameters.

Proof: Take any set of points in (t, p) space that satisfies the hypotheses. These points can be arranged to satisfy $t_1 < t_2 < \ldots < t_b$. Let $l_i = t_i$ and $v_i = p_i$ for all $i = 1, \ldots, b$, where the cost of output is positive and less than min p_i. At any location of a consumer without a contract we merely assign a consumer whose reservation price is less than the cost of output. As in the proof of property 2.7, for a high enough cost of travel each buyer maximizes by purchasing at his location, and the monopolist extracts all surplus and is therefore maximizing. Hence, any set of points satisfying the hypotheses solves some form of the model and no further restrictions in the contract space can be obtained. Q.E.D.

In the case where the economist observes price-location contracts and the locations of all buying consumers we obtain a related result.

COROLLARY 2.1: Any finite set of price-location contracts with at most one store per location and where the set of locations of buying consumers is identical to all locations of contracts is a solution to the model for some set of parameters.

The proof of property 2.8 extends to this case after eliminating the discussion of the location of nonbuying consumers. The necessity of the restrictions is implied by properties 2.1, 2.2, and 2.3.

III. Examples and Comparative Statics

In this section of the paper I present a series of examples to illustrate the structure of the model and to obtain ambiguous comparative statics. My discussion of comparative statics properties includes analysis of the resolution of uncertainty and the comparison between imperfect price discrimination and single-price monopoly.

The cost of travel measures the intensity of the preference of consumers for their preferred good compared with others. Though one might conjecture (falsely) that increasing the cost of travel would increase the variety of goods supplied in equilibrium, the cost of travel is not an appropriate measure of the equilibrium variety. The cost of travel does, however, measure the extent to which markets are separated and therefore the degree of price discrimination. Interpret c as measuring the degree of price discrimination (ranging between single-price monopoly in which the firm is constrained to charge the same price in all markets and perfect discrimination). For sufficiently large values of c the monopolist is able to price discriminate perfectly (the monopolist extracts the entire consumer surplus), while if $c = 0$, the monopolist is constrained to charge the same (single) price in all markets (just the monopoly price if the diverse markets were integrated to one point).[4] Monopoly profits are nondecreasing in the degree of imperfect price discrimination with costless varieties, though the example developed later, with costly locations, shows the result is not robust.

PROPERTY 3.1: Monopoly profit is nondecreasing in the cost of travel.

Proof: For any fixed set of buyers and order of their prices (denoted by z) the prices described in (1) (following property 2.4) yield profit $\pi_z(c)$ that is nondecreasing in c. The monopoly profit (defined for the optimal set of buyers and order of their prices) is given by

$$\pi(c) = \max_z \pi_z(c).$$

The function $\pi(c)$ is the outer envelope of nondecreasing functions and is also nondecreasing. Q.E.D.

Consider an example with two types of traders in which there are n_i agents with reservation price v_i for $i = 1, 2$, the reservation prices satisfy $v_1 > v_2$, the traders are located one unit apart (let $l_1 = 1$ and $l_2 = 2$), each has cost of travel c, varieties are costless for the firm, and output is costless. Under these assumptions the monopolist always sells to the highest reservation price consumer, sells to each buyer at his location (property 2.3), and charges the lowest reservation price buyer his reservation value. The contract set of the monopolist is $\{(1, v_2 + c), (2, v_2)\}$ or $\{(1, v_1)\}$ if $c < v_1 - v_2$, and $\{(1, v_1), (2, v_2)\}$ otherwise.[5] I suppose $c < v_1 - v_2$. If $n_1/(n_1 + n_2) < v_2/(v_1 - c)$, then the contract set

4. My definition of single-price monopoly imposes the restriction that the same price is charged for all varieties. One alternative is to change the environment so that $c = 0$, which constrains the monopolist to charge one price to prevent arbitrage by consumers. The price, output allocation, and welfare distribution coincide under the definitions above if stores are costless. A third definition of single-price monopoly is the contract offered by a monopolist constrained to offer one variety. Many of the ambiguities discussed between the single-price solution and imperfect discrimination extend to these other two definitions.

5. Application of (1) yields the form of the solution.

is $\{(1, v_2 + c), (2, v_2)\}$ and both agents buy, and if $n_1/(n_1 + n_2) > v_2/(v_1 - c)$, the contract set is $\{(1, v_1)\}$ and only the high reservation agents buy. Therefore, increasing c can increase output and variety and make some consumers better off without making any worse off. Similarly, if the price discrimination solution is $\{(1, v_2 + c), (2, v_2)\}$ and the single-price monopoly solution is to charge price v_1, then discrimination, as compared with single-price monopoly, can increase output and variety and make some consumers better off without making any worse off. However, as the following example emphasizes, these comparative statics are not robust.

In my next example there are three types of traders with $v_1 > v_2 > v_3$; the traders are located one unit apart with $l_1 = 1$, $l_2 = 2$ and $l_3 = 3$; each consumer has cost of travel c; varieties are costless for the firm; and output is costless. Suppose $v_2 - v_3 \leq c \leq (1/2)(v_1 - v_3)$, so that the contract set is $\{(1, v_1)\}$, $\{(1, v_3 + 2c), (3, v_3)\}$ or $\{(1, v_2 + c), (2, v_2), (3, v_3)\}$, depending on the relative numbers of each type.[6] Hypothesize that most of the consumers are type 3, so that the equilibrium contract set is either $\{(1, v_3 + 2c), (3, v_3)\}$ (excluding type 2) or $\{(1, v_2 + c), (2, v_2), (3, v_3)\}$ (selling to all), depending on the relative numbers of types 1 and 2. Increasing c can cause the monopolist to substitute the contract set $\{(1, v_3 + 2c), (3, v_3)\}$ for $\{(1, v_2 + c), (2, v_2), (3, v_3)\}$, decreasing output and variety and making some consumers (i.e., type 1) worse off without making any consumer better off. At the switching point between contract sets we note that consumer surplus declines substantially, while firm profit is continuous in c, so that total surplus declines. Similarly, if the price discrimination solution is $\{(1, v_3 + 2c), (3, v_3)\}$ and the single-price monopoly solution is to charge v_3 at all locations, then discrimination can decrease output and variety and make some consumers worse off without making any consumer better off. The discrimination solution excludes some consumers and incurs deadweight loss, yielding less total surplus than single-price monopoly. Combining this with the earlier example shows that increasing the cost of travel has an ambiguous impact on variety, output, and consumer welfare.

Let us turn our attention to comparative statics of the firm's cost per location. Denote by F the firm's cost per location and denote by $\pi_i(F) = \hat{\pi}_i - iF$ the monopolist's maximum profit when the monopolist is constrained to offer i varieties. The monopolist's unconstrained profit is

$$\pi(F) = \max_i \pi_i(F),$$

which is the envelope of linear nonincreasing functions, so that firm profits are nonincreasing, convex, and piecewise linear in the cost per

6. Application of (1) again yields the form of the solution.

variety. By a standard substitution argument,[7] it is easy to establish that increasing the cost per variety cannot induce an increase in the number of varieties offered.

The introduction of costly variety ($F > 0$) can affect in a fundamental fashion the model's solution. With costly variety it is possible that no store is located at the location of any buyer (compare property 2.3). Suppose $v_1 > v_2$, $l_1 = 0$, $l_2 = 1$, and both types have travel cost c; permit varieties to be costly (but production is otherwise costless); and assume $v_1 - v_2 < c < v_2$. In order for this structure to be nontrivial and different from the $F = 0$ case it must entail a single location. If the one location services only one type of consumer, then it is optimally situated at the location of that consumer. Because I want the location to be interior (i.e., $0 < t < 1$), I focus on an interior location that services both types. In such a configuration both consumers must have zero surplus; otherwise the firm can raise profit by a locational movement away from an agent with positive surplus accompanied by a price hike. The corresponding contract is then $.5 + (v_1 - v_2)/2c$, $(v_1 + v_2 - c)/2$.[8] This is the most profitable single contract that services both consumers.[9] It can be shown for some parameter values (e.g., $n_1/n_2 = v_2/v_1$) that this contract is also more profitable than any contract servicing just one type.[10] For an appropriate fixed cost per location this contract set is most profitable.[11] It is useful to observe that

7. The monopolist's profit with fixed cost F_i per location is given by $\hat{\pi}_i - iF_i$, where i is the optimal number of locations for fixed cost F_i. Thus, $\hat{\pi}_i - iF_i \geq \hat{\pi}_j - jF_i$, and similarly, $\hat{\pi}_j - jF_j \geq \hat{\pi}_i - iF_j$. These imply $(j - i)F_i \geq \hat{\pi}_j - \hat{\pi}_i \geq (j - i)F_j$. Then it follows that $F_i < F_j$ implies $i \geq j$. Increasing the cost per location cannot increase the number offered.

8. In order to determine the contract, just solve the equations $p + ct = v_1$ and $p + c(1 - t) = v_2$, to obtain $p = (v_1 + v_2 - c)/2$ and $t = [(v_1 - v_2)/2c] + 1/2$. Under our assumptions, $p > 0$ and $1/2 < t < 1$.

9. Contract (t, p) with $t < 0$ is dominated in profit by $(0, p + \epsilon)$ and (t, p) with $t > 1$ is dominated in profit by $(1, p + \epsilon)$. Hence, the profit-maximizing contract that services both consumers has $0 \leq t \leq 1$. However, if $t = 0$ we obtain $p = v_2 - c$, and if $t = 1$ we obtain $p = \min(v_2, v_1 - c) = v_1 - c < v_2$, so that a higher price and profit are obtained if $0 < t < 1$.

10. The profit of the interior store is

$$\pi = (n_1 + n_2)\left(\frac{v_1 + v_2 - c}{2}\right) = \frac{n_1 v_1 + n_2 v_2}{2} + \frac{n_2(v_1 - c)}{2} + \frac{n_1(v_2 - c)}{2} > \frac{n_1 v_1 + n_2 v_2}{2}.$$

The profit of the best contract that services one type is either $n_1 v_1$ or $n_2 v_2$. If $(n_1/n_2) = (v_2/v_1)$, then $n_1 v_1 = n_2 v_2$ and the right-hand side of the inequality is just the profit of the best location that services only one type. Therefore, for some parameter values, the interior location is more profitable.

11. I take the fixed cost to be less than the profit of the interior location but to exceed the excess of the profits in the best contract set with costless variety over the interior location. Such a fixed cost level is well defined, because by the analysis in n. 10 we know that the profits in the interior location exceed half those of the most profitable contract set with costless variety.

the interior contract is optimal even though the travel cost is proportional to distance, rather than for example, quadratic cost. This occurs because the interior solution is one in which all surplus is extracted, while any corner solution in which both types are serviced gives some consumers positive surplus.

The analysis of the comparative statics of the firm's unit cost of output, denoted by g, is similar. I describe by $\pi_w(g) = \hat{\pi}_w - wg$ the monopolist's maximum profit when the monopolist is constrained to sell exactly w units of output. From my earlier analysis for the cost per location it follows that firm profit is nonincreasing, convex, and piecewise linear in the cost of output and that output is nonincreasing in its cost. It is easy to construct examples to illustrate that increasing the cost of output (variety) can decrease variety (output), make some consumers worse off and none better off, and reduce total surplus.

The ambiguous impact of resolution of uncertainty can be illustrated by an example that contrasts the case in which the monopolist must decide on his contracts before the cost of travel (the state) is revealed with the solution when the monopolist determines contracts after the cost of travel (the state) is revealed. I examine a structure with two types of consumers with $v_1 > v_2$, $l_1 = 0$, $l_2 = 1$, variety and output costless for the firm, and the cost of travel depending on the state of nature (two states) with $c_{\text{high}} > c_{\text{low}}$ and $v_2 + c_{\text{high}} < v_1$. Suppose that the ex post contract set with c_{high} travel cost is $\{(0, v_2 + c_{\text{high}}), (1, v_2)\}$ and with c_{low} travel cost is $\{(0, v_1)\}$. If most of the state probability is on c_{high}, then the ex ante contract set is $\{(0, v_2 + c_{\text{high}}), (1, v_2)\}$. The resolution of uncertainty then reduces expected output, makes some consumers worse off and none better off, and reduces combined firm and consumer surplus. If most of the state probability is on c_{low}, then the ex ante contract set is $\{(0, v_1)\}$ and the resolution of uncertainty increases expected output and expected variety and makes some consumers better off and none worse off. The resolution of uncertainty generally increases firm profit.

It is also easy to establish that the impact on output, variety, and welfare of the resolution of uncertainty on the cost per location (or output) is ambiguous. I assume that in one state the cost per store location is zero and positive output, variety, and consumer surplus (to some consumers) result and that in the other state the cost of variety is sufficiently big so that no sales occur. If most of the prior probability is on the high cost state, then the ex ante solution does not entail any sales and the resolution of uncertainty increases expected output and variety and makes some consumers better off without making any worse off. If most of the prior probability is on the low cost state, then the resolution of uncertainty decreases expected output and variety and makes some consumers worse off without making any better off. Similar arguments establish the ambiguity of the impact of the resolution of uncertainty on the cost of output.

IV. Conclusion

In this paper I analyze imperfect price discrimination in an environment in which consumers prefer different varieties; I contrast the qualitative features of the solution with environments in which all consumers agree on the definition of the best but disagree on the valuation of quality. Geometric methods are employed and some sets of results are shown to be exhaustive within appropriate spaces of observables. Both these approaches should be useful in many related environments with asymmetric information.

The spatial approach used here is employed also in analyzing problems involving monopolistic competition and oligopoly (e.g., Hotelling 1929; Prescott and Visscher 1977; and Salop 1979). My spatial approach to monopoly price discrimination suggests a linkage in analyzing discrimination with monopolistic competition and oligopoly. The interrelation between market structure and price discrimination is certainly important. Though many of the examples of discrimination observed are not set in monopoly contexts, most of the analysis of discrimination in the literature exploits a monopoly framework (exceptions include Jaynes [1978], Spence [1978], Wilson, Oren, and Smith [1980], Gal-or [1980, 1981], and Spulber [1980*a*, 1980*b*, 1981]).

The determination and characteristics of the variety of goods offered in markets is also important (as illustrated by the work of Spence [1976], Dixit and Stiglitz [1977], and Salop [1979]). This paper attempts to apply the modern price discrimination perspective to this problem in a monopoly environment, but insight on variety also could be gained from models of discrimination in other market structures. Spence (1976) argues that the introduction of perfect price discrimination at the margin in his monopolistic competition setting causes the equilibrium and optimal variety levels to coincide. This exploits the perfect discrimination structure, and I suspect that the introduction only of imperfect discrimination (in which consumers must be given incentives to reveal their private parameters) may make things worse (for some parameter values).

Our understanding of the effects of discriminatory pricing is at an early stage and many issues remain unexplored.

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