Carl Alan Batlin
The Chase Manhattan Bank, N.A

Susan Hinko
Merrill Lynch Pierce Fenner & Smith, Inc.

A Game Theoretic Approach to Cash Management*

This paper utilizes the methodology of game theory to analyze the optimal use of cash management tools by interactive firms. The distribution of the float between debtors and creditors is examined, and it is found that, in the absence of strategies which alter the total size of the float, the equilibrium is more favorable to creditors. Conditions are given under which the equilibrium will be obtained through the use of pure strategies or mixed strategies, and an explanation is provided for the sometimes random observed behavior on the part of firms in this area.

The cash management function is a general designation for two distinct corporate activities: cash flow acceleration and liquidity account allocation. The former problem concerns procedures for speeding cash collections from customers and slowing cash disbursements to suppliers so as to optimize the firm's use of cash. The latter problem, on the other hand, takes the firm's cash flow as given and determines the minimum cash balance, the excess to be held in the form of marketable securities. Although a well-developed literature exists for the liquidity account allocation problem (e.g., Baumol 1952; Tobin 1956; Miller and Orr 1966; Eppen and Fama 1968), analysis of the cash flow acceleration problem has been less extensive. Some papers examine techniques for speeding cash collections, such as lock box systems (e.g., Kraus, Janssen, and McAdams 1970) while others explore methods for slowing cash disbursements, such as remote disbursement systems (e.g., Gitman, Forrester, and Forrester 1976), but no one has investigated the interaction of the two. This is a serious deficiency, because, as a practical matter, firms simultaneously utilize both types of procedures, so that the cash management efforts of other firms tend to counteract each firm's own efforts. An inherently

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static framework, therefore, which assumes that other firms will not react to a firm's attempts to accelerate its own cash flow, obscures the essential nature of the cash management problem. This paper proposes to use game theory to analyze the cash management behavior of interactive firms.

I. The Game Theoretic Structure of the Problem

Cash management activities directed at cash flow acceleration generally consist of the attempts by firms to influence the magnitude of their float with other firms. Float arises in customer-supplier relationships when debtor firms extinguish their obligations to creditors by mailing them checks drawn on demand deposit balances. A debtor can then retain the use of the remitted funds because of two types of systemic lags: mail float, or the delays in the postal system which preclude the instantaneous delivery of checks between two locations, and bank float, or the delays in the banking system associated with the clearing of checks through the Federal Reserve System. Control over the use of cash is valuable because it can be held in the form of interest-bearing assets. Consequently, debtors attempt to increase the length of time over which they can retain the use of remitted funds, while creditors try to reduce this float. Lock box systems, in which creditors open collection accounts in strategic locations, and remote disbursement systems, in which debtors write remittance checks on strategically located accounts, are the tools through which these firms try to achieve these goals.

To illustrate how firms might use these tools, consider two firms in two geographically distinct locations, one a customer of the other. Without considering float, both parties might pursue naive strategies: the creditor would request that the payment be sent directly to its home office, and the debtor would write the check on a local bank account. Because the check would be mailed and cleared between two separate locations, the debtor would obtain a total float of $M + B$ days, where M is the mail float between the two locations and B is the bank float. To reduce this float, the creditor would have an incentive to implement a lock box system and require that the payment be sent to some other location. If the debtor maintained its naive strategy, the most effective strategy for the creditor would be to open a collection account in the debtor's own "backyard" (i.e., in the debtor's geographical area). This policy would effectively reduce the total float to zero, since the check would be both mailed and cleared within the same geographical location.[1] Given that the creditor could adhere to this backyard strategy,

1. Strictly speaking, the float would still be positive since intercity mailing and clearing times are not instantaneous. Since this float is very small relative to the intracity float, however, we consider it to be negligible.

however, the debtor would have in incentive to implement a remote disbursement system by opening disbursement accounts in other locations. A policy which would greatly reduce the impact of the creditor's choice of strategies would be for the debtor to open an account in the creditor's own backyard. By mailing checks written on a bank located in the creditor's geographical area, the debtor would be certain of preserving one of the types of float regardless of the creditor's behavior. If the creditor maintained its naive strategy of not instituting a lock box, the debtor would retain the mail float, since the check must be mailed between the two locations, although the bank float would be lost, since the check would be cleared within the creditor's geographical area. On the other hand, if the creditor adopted its backyard strategy of requiring that the debtor remit to a local lock box, the debtor would retain the bank float, since, ironically, the check would have to be sent back to the debtor's bank in the creditor's geographical area for clearing, but the mail float would be eliminated. These ideas can be summarized by representing the magnitude of the float as a function of both parties' choice of strategies in matrix form as follows:

$$(\pi_{ij}) = \begin{pmatrix} \pi_{nn} & \pi_{nb} \\ \pi_{bn} & \pi_{bb} \end{pmatrix} = \begin{pmatrix} M+B & 0 \\ M & B \end{pmatrix}, \tag{1}$$

where π_{ij} denotes the size of the total float resulting from the debtor's ith strategy and the creditor's jth strategy, and the subscripts n and b denote naive and backyard strategies, respectively.

This representation of the use of these cash management tools differs markedly from other papers on lock box and remote disbursement systems, which rely upon a static formulation that implicitly assumes that the other firm will follow a naive strategy while the tool is being implemented. In actuality, however, the debtor and the creditor are free to react to each other's policies, so that the true effectiveness of each party's efforts in terms of their influence on the size of the float depends on the other party's policy as well.[2] Because of this interdependence between the debtor and creditor, the cash management problem can be analyzed as a noncooperative two-person game.[3]

2. This interdependence between debtor and creditor is a major reason why creditors do not automatically institute lock box accounts, even though they appear in a static context to have the capacity for very sizable gains in terms of float reduction. The other reason, of course, is that lock boxes are costly, so that any gains from the use of lock boxes must be weighed against their costs. For a static analysis of this lock-box decision within a value-maximizing context, see Batlin and Hinko (1981).

3. This characterization assumes the absence of collusion between debtors and creditors, which, given their adversary roles, is appropriate. The possibility of collusion and bargaining could arise, however, when considering the competitive equilibrium of a system of interacting debtors and creditors in an n-person game. The main outcome of this generalization would be to possibly alter the magnitudes of the float given in eq. (1). In Section III we illustrate how this same effect occurs within the context of a noncooperative two-person game by increasing the number of strategies available to each party.

If the feasible strategies for both parties are restricted to naive and backyard strategies, the game will be constant-sum. That is, the activities of the parties will be able to affect only the distribution of the total float, not its total size. The matrix in equation (1), by specifying the size of the float in each contingency, can be interpreted as the debtor's payoff matrix.[4] Because this is a constant-sum game, moreover, the creditor's payoff matrix is simply the difference between the total available float, $M + B$, and the float obtained by the debtor under each contingency. When the parties have access to other strategies, it will generally be possible to affect the size of the total float as well, changing the character of the game to a non-constant-sum game.[5] This possibility will be explored later.

II. The Game Theoretic Solution to the Problem

While equation (1) describes the structure of the cash management game, it gives no indication of the nature of the equilibrium solution (i.e., the set of optimal strategies of the debtor and creditor). To derive the solution, game theory makes two key assumptions, namely, that both participants are rational and risk averse and that neither participant knows a priori which strategy the other will adopt. These assumptions jointly imply that optimal behavior for each party consists of choosing a strategy so as to insure itself the maximum guaranteed payoff, or, in other words, the maximum of the set of minimum payoffs

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We provide here no additions to the methodology of game theory but rather concentrate on the application of standard analysis (see, e.g., Intrilligator 1971, chap. 6) to a new context—cash management. Accordingly, although the solution which we derive to the problem is straightforward, its interpretation and economic implications for optimal cash management policies differ significantly from the existing literature in this area.

4. We have formulated the payoffs in terms of days of float, because these magnitudes are common to both participants and thus provide the basis for a constant-sum game. The actual dollar payoff to each party resulting from each pair of strategies, however, is a linear function of the number of float days, D. For each debtor firm f choosing strategy s, its dollar payoff will be $r_f D - T_{fs}$, where r_f is the effective interest rate it can earn on uncollected cash balances and T_{fs} is its transaction cost of implementing strategy s (this amount will be positive for all strategies other than the naive one). In the case of creditor firms, the relevant payoff function would be $r_f(M + B - D) - T_{fs}$. Since different firms vary in the transaction costs and interest opportunity costs they face, the actual dollar value of the game will also vary among firms. In modeling the game as a struggle over the magnitude of D, we are implicitly assuming that r_f is sufficiently large relative to T_{fs} to make participation in the game profitable for both parties—i.e., the expected float gains outweigh the transactions costs of utilizing the appropriate strategies. If this is indeed the case, it will be true that competition over float days will be equivalent to competition over the corresponding dollar payoffs.

5. In this interpretation, in which the parties are viewed as competing over the distribution of the float, the creditor's payoff is the amount of float reduction. An alternative interpretation of the game, which is equally valid, is that this conflict is a zero-sum game, with all positive float (i.e., the debtor's payoffs) being viewed as coming at the creditor's expense. In this interpretation, the distinction in strategies as to their ability to affect the size of the total float is moot.

that would result from each possible strategy of the other party (i.e., the maximin choice). Moreover, because of the constant sum nature of the game, the creditor's problem is dual to the debtor's problem, so that the creditor's maximin payoff is equivalent to its minimax payoff (i.e., the minimum of the set of possible maxima for each of the debtor's strategies) on the debtor's payoff matrix (i.e., eq. [1]). This behavior will result in a stable equilibrium solution to the game if neither party has an incentive to change its strategy. This state of affairs can occur in one of two ways: if a saddlepoint exists, such that the debtor's strategy choice and the creditor's strategy choice lead to the same payoff, or

$$\max_i \min_j (\pi_{ij}) = \min_j \max_i (\pi_{ij}), \tag{2}$$

both parties will choose those pure (i.e., unique) strategies which result in the realization of the saddlepoint. If the payoff matrix is such that a saddlepoint does not exist, on the other hand, both parties will use mixed strategies (i.e., they will randomize their choice of strategies on repeated trials and choose the optimal probabilities, or long-run relative frequencies, to assign to each strategy in order to maximin their expected payoff). In equilibrium, both parties will choose mixed strategies that result in the same expected payoff.

To apply these concepts to the cash management problem, it is necessary to determine whether the matrix in equation (1) contains a saddlepoint. As it turns out, this characteristic depends on the relative magnitudes of the two types of float. If the mail float is greater than the bank float, a saddlepoint will exist, and a stable equilibrium in pure strategies will obtain. If the bank float is the larger of the two, however, mixed strategies will be required in equilibrium. The determinants of these relative sizes are primarily geographical: due to technological factors, mail float is approximately proportional to the distance between the two locations, while bank float is a decreasing function of the size of the municipalities in which the firms are located. Consequently, a saddlepoint equilibrium would be most likely in cases in which the parties are located in large metropolitan areas which are geographically far apart; mixed strategies would probably be used, on the other hand, in cases where the debtor and creditor are located in small towns that are relatively close together.

To investigate the saddlepoint case, it must be assumed that $M > B$. Then, using equation (1), the debtor's problem is

$$\max_i \min_j (\pi_{ij}) = \max_i \begin{pmatrix} 0 \\ B \end{pmatrix}, \tag{3}$$

while the creditor's problem is

$$\min_j \max_i (\pi_{ij}) = \min_j (M+B \quad B). \tag{4}$$

The solution to both problems is the saddlepoint solution B. The debtor's optimal strategy is always to follow a backyard strategy of disbursing from an account in the creditor's geographical area, since by doing so, it is guaranteed a float of at least B, which is more favorable than the zero float which could materialize if it followed a naive strategy. Furthermore, the creditor's optimal strategy is always to follow a backyard strategy of instituting a lock box in the debtor's geographical area in order to guarantee a float of no more than B, which is more favorable than the float of $M + B$ which could occur from its adherence to a naive strategy. This solution is stable, since deviation from the optimal strategy by either party would result in a less favorable float and would therefore be self-correcting. It is interesting to note that the equilibrium payoff, B, known as the value of the game, represents a compromise situation for both parties. In the absence of lock box and remote disbursement opportunities, the debtor would obtain the total float $M + B$; by instituting a local lock box, the creditor would expropriate the entire float, but by writing checks on an account in the creditor's geographical area, the debtor is able to recapture a float of B. Since it is assumed that $M > B$ in this case, the saddlepoint solution divides the float between the parties but gives the debtor the smaller share.

Although the saddlepoint equilibrium predicts that firms should always use pure strategies, this behavior is often inconsistent with observed cash management activities. It is common for firms qua debtors to rotate disbursement accounts and qua creditors to use lock box accounts sporadically. One explanation of this behavior is that firms have imperfect information about the effectiveness of cash management technology (i.e., they are ignorant of the magnitudes of the elements of the matrix in eq. [1]), and they experiment with these tools until they learn which strategies are most effective. This explanation, however, fails to account for this type of behavior on the part of firms with established cash management departments which have been functioning over a long period of time. In such cases, it must be concluded that such behavior is a conscious management policy. Although static treatments of the use of these cash management tools are unable to explain this phenomenon, game theory would predict this type of activity as a rational response by both parties to a situation in which the size of the bank float exceeded the size of the mail float.

To understand why the use of randomized strategies is necessary for a stable equilibrium when no saddlepoint exists, consider the case where $M \leq B$ and firms attempt to use pure strategies. The debtor's problem becomes

$$\max_i \min_j (\pi_{ij}) = \max_i \begin{pmatrix} 0 \\ M \end{pmatrix}, \tag{3'}$$

while the creditor's problem remains

$$\min_j \max_i (\pi_{ij}) = \min_j (M+B \quad B). \tag{4}$$

The debtor will select the backyard strategy, expecting a float of M, while the creditor will use its backyard strategy, expecting a float of B. As the result of this joint choice of strategies, the actual float that materializes will be B. Because there is no saddlepoint, the expectations of at least one party will not be realized; in this case, the debtor will be pleasantly surprised to obtain a float greater than expected. However, this situation will not represent a stable equilibrium, because it will encourage cycling behavior. That is, on successive trials, the parties will have incentives to change their strategies in a cyclical pattern, making the value of the game indeterminate. In this case, the creditor will observe that it can reduce the float to M by using a naive strategy on the next bill-paying occasion, assuming that the debtor maintains its backyard strategy. However, on the following trial, the debtor will switch to a naive strategy, so that it can increase the float to $M + B$, provided the creditor maintains its naive strategy. On their next interaction, the creditor will retaliate by switching back to a backyard strategy to reduce the float to zero. However, after that the debtor will also change back to a backyard strategy, raising the float back to B and completing the cycle which will then continue to repeat itself. Because there is no tendency for this pattern to converge, the use of pure strategies under the circumstances of $M \leqslant B$ will not lead to a stable equilibrium. Rather, what is required is the use of mixed strategies.

To circumvent the problem of cycling when no saddlepoint exists, both parties can implement their strategies in such a way that the particular policy used on a particular occasion is random. Optimal behavior for each party then consists of choosing the probabilities of each strategy so as to maximin its expected payoff in the long run. Formally, the debtor's problem becomes

$$\max_{p_i^d} \min_j \sum_i p_i^d \pi_{ij} \tag{5a}$$

$$\text{s.t.} \min_j \sum_i p_i^d \pi_{ij} \leqslant \sum_i p_i^d \pi_{ij}, \; j = n,b \tag{5b}$$

$$p_i^d \geqslant 0, \, i = n,b \tag{5c}$$

$$\sum_i p_i^d = 1, \tag{5d}$$

where p_i^d is the probability that the debtor will choose the ith strategy (either naive or backyard). Constraint (5b) defines the minimum set of expected payoffs over the creditor's set of possible strategies, while constraints (5c) and (5d) imply that $p_b^d = 1 - p_n^d$.

To find the minimum expected payoff set (and its maximum) in constraint (5b), it is helpful to derive the functions $E(\pi_{in})$ and $E(\pi_{ib})$, the debtor's expected payoff from the creditor's naive and backyard strategies, respectively. These are given by

$$[E(\pi_{in})\ E(\pi_{ib})] = (p_n^d\ \ 1-p_n^d)\begin{pmatrix} M+B & 0 \\ M & B \end{pmatrix} = (M+Bp_n^d\ \ B-Bp_n^d). \qquad (6)$$

In figure 1, these expected payoffs are graphed as a function of p_n^d, the probability that the debtor will choose the naive strategy. The two functions specify the debtor's expected payoff from each of the creditor's pure strategies. Consequently, the area between the two lines represents combinations of the creditor's pure strategies (i.e., the debtor's expected payoff when the creditor uses mixed strategies) and the lower boundary of that set is the minimum expected payoff set. Since the maximum of this latter set occurs at the intersection of the two lines, p_n^{d*}, the probability that maximins the debtor's expected payoff is found by solving the equation $E(\pi_{in}) = E(\pi_{ib})$. It can then be verified that

$$p_n^{d*} = \frac{1}{2}(1 - M/B) \qquad (7a)$$

and

$$p_b^{d*} \equiv 1 - p_n^{d*} = \frac{1}{2}(1 + M/B). \qquad (7b)$$

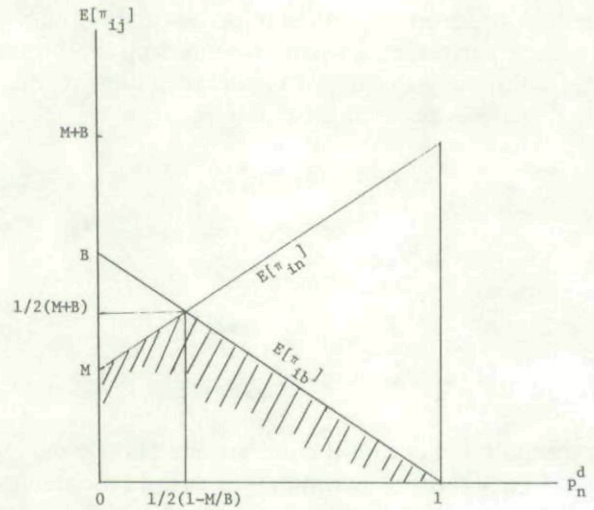

FIG. 1

Before investigating the solution to the debtor's problem further, it is useful to solve the creditor's problem. This can be stated formally as

$$\min_{p_j^c} \max_i \sum_j \pi_{ij} p_j^c \tag{8a}$$

$$\text{s.t. } \max_i \sum_j \pi_{ij} p_j^c \geq \sum_j \pi_{ij} p_j^c,\ i = n,b \tag{8b}$$

$$p_j^c \geq 0,\ j = n,b \tag{8c}$$

$$\sum_j p_j^c = 1, \tag{8d}$$

where p_j^c is the probability that the creditor will use its jth strategy, and, because of (8c) and (8d), $p_b^c = 1 - p_n^c$. Analogous to the debtor's problem, constraint (8b), defining the creditor's maximum expected loss set, can best be specified through the functions $E(\pi_{nj})$ and $E(\pi_{bj})$, the creditor's expected loss from each of the debtor's pure strategies. These are given by

$$\begin{bmatrix} E(\pi_{nj}) \\ E(\pi_{bj}) \end{bmatrix} = \begin{pmatrix} M+B & 0 \\ M & B \end{pmatrix} \begin{pmatrix} p_n^c \\ 1-p_n^c \end{pmatrix} = \begin{bmatrix} (M+B)p_n^c \\ B-(B-M)p_n^c \end{bmatrix}, \tag{9}$$

which are graphed in figure 2 as a function of p_n^c, the probability that the creditor will use its naive strategy. Through arguments analogous to those presented in the case of the debtor and as depicted on the graph, the minimum of the maximum expected loss set occurs at the intersec-

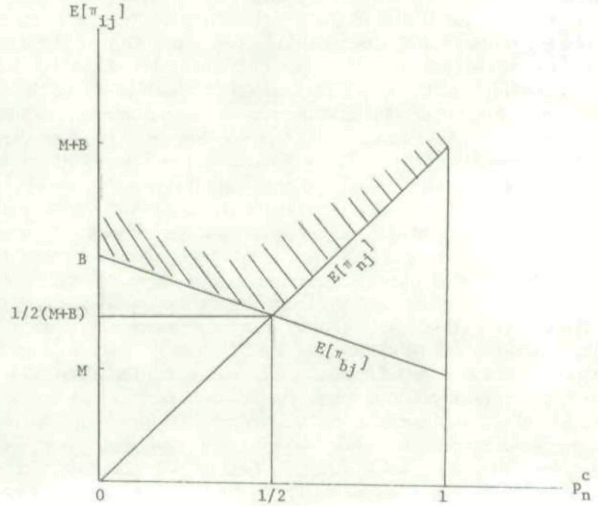

FIG. 2

tion of the two functions, so that p_n^{c*}, the probability that minimaxes the expected loss, is given by the equation $E(\pi_{nj}) = E(\pi_{bj})$ and found to be

$$p_n^{c*} = p_b^{c*} \equiv 1 - p_n^{c*} = \frac{1}{2}. \tag{10}$$

From the debtor's and creditor's choices of optimal probabilities to assign to each pure strategy, the equilibrium expected value of the game can be expressed as

$$V = (p_n^{d*} \; 1-p_n^{d*}) \begin{pmatrix} M+B & 0 \\ M & B \end{pmatrix} \begin{pmatrix} p_n^{c*} \\ 1-p_n^{c} \end{pmatrix} = \frac{1}{2}(M+B), \tag{11}$$

and it can be verified by substituting the debtor's optimal probabilities (eqq. [7a] and [7b]) into either of the functions $E(\pi_{in})$ or $E(\pi_{ib})$ (eq. [6]) and by substituting the creditor's optimal probabilities (eq. [10]) into either of the functions $E(\pi_{nj})$ or $E(\pi_{bj})$ (eq. [9]) that this value—the expected float—is both the debtor's expected payoff and the creditor's expected loss. Since the two are equal, the solution is stable, and neither party will have an incentive to alter its strategy from this optimum.[6]

In this mixed strategy equilibrium, the creditor will use its naive strategy as often as its backyard strategy. So long as the mail float is positive, however, the debtor will rely less heavily on its naive strategy than on its backyard strategy. Thus, unless the two firms are in such close proximity that the mail float is zero, the debtor will use its cash

6. The solutions in eqq. (3), (4), and (11) represent the pure and mixed strategy equilibria for the two-person game, where each party takes the payoff matrix in eq. (1) as given. The determination of the actual magnitudes of the elements in the matrix is the result of a competitive equilibrium of many interacting debtors and creditors, which, although beyond the scope of this paper, could be regarded as an n-person game. In such a game, each debtor would seek out creditors which allowed the largest float for each pair of strategies, while each creditor would favor debtors which provided the smallest float for each contingency. Moreover, the strict noncooperative nature of the game would be relaxed, as each group of participants could form coalitions to enhance their bargaining positions, and there arose the potential for collusion and side payments between groups. The equilibrium float values which would emerge from this process would also reflect the facts that each debtor behaves as if it represents an insignificant portion of each creditor's portfolio, and all firms which are creditors in one context are also debtors in another. For instance, there might be institutional mechanisms for reducing bank float to zero (e.g., electronic funds transfer systems). It can be seen from eq. (1) that these have the effect of insulating the creditor from the debtor's choice of strategies—i.e., regardless of the debtor's choice, the creditor's naive strategy will result in a float of M, and its backyard strategy will produce zero float. Naturally, such a game implies complete creditor dominance and would remove cash management as a subject of concern for firms. However, since those creditor firms which would be in a position to expropriate the entire float would also find themselves in the role of debtors with other firms, it is not clear whether such developments would benefit them. Although we do not attempt to resolve these issues here, we do illustrate in Section III how this struggle over the magnitude of the float provides an incentive for both parties to create new strategies within the two-person game framework.

management tool—the remote disbursement account—more frequently than the creditor will use its tool—the lock box account.[7] Moreover, from equations (7a) and (7b), it can be seen that as the size of the mail float increases, the debtor will rely less on its naive strategy, and in the limit, as the size of the mail float approaches that of the bank float, the debtor will adopt the pure strategy of relying exclusively upon its backyard strategy of remote disbursement. In contrast, so long as $M \leq B$ so that mixed strategies are appropriate, the creditor's choice of probabilities to assign to its strategies is independent of the relative sizes of the components of the total float. This analysis of the optimal behavior of debtors and creditors provides a reasonable explanation of the apparently random activities that are sometimes observed in the use of cash management tools. In circumstances in which the size of the bank float exceeds the size of the mail float, a mixed strategies equilibrium will obtain, in which it is optimal for debtors to rotate disbursement accounts and for creditors to use their lock box accounts sporadically.

As in the saddlepoint equilibrium case, the (expected) value of the game represents a compromise solution between complete creditor dominance (i.e., zero float) and complete debtor dominance (i.e., maximum float of $M + B$). In the saddlepoint case, however, the solution favored the creditor, since the amount of float reduction M was larger than the size of the float B. In the mixed strategies case, on the other hand, the solution divides the total float evenly between the two parties. To the extent that these considerations impact upon the decision of choosing a geographical location for the firm, there will be a tendency to locate far from one's debtors and close to one's creditors.[8] By following this policy, the firm will increase the likelihood that in its relationships as a creditor the float will be divided in its favor, while in its relationships as a debtor the float will not be divided adversely.

7. Another example in which the mail float does not exist is the case in which the creditor requires that payment must be received, not merely postmarked, by a given date. This phenomenon, which has recently been occurring with more frequency, forces the debtor to absorb the mail float, thereby making the payoff matrix in eq. (1) symmetric, with B on the main diagonal and zero elsewhere. The solution is a special case of eqq. (7), (10), and (11): Mixed strategies are still appropriate, both parties rely equally on each of their strategies, and the total float B is evenly divided between them. Naturally, this represents an improvement of $\frac{1}{2}M$ for the creditor over the general case solution.

8. The counterintuitive nature of this result arises from the notion that mail float increases with geographical distance, so that, for parties located far apart, the mail float is likely to exceed the bank float, leading to a pure strategies equilibrium favoring the creditor. However, as mentioned above, the actual dollar payoff to a firm from a particular equilibrium division of the float will depend on relative transactions and interest opportunity costs. It is these costs, as well as other non-cash-management-related considerations outside the model, which prevent corner solutions (e.g., always locate the firm as far away as possible from one's debtors). Thus, if the transactions costs of instituting a lock box were high relative to the interest income obtainable by recovering the float, it would not necessarily benefit a creditor to behave in this way since the

III. Some Extensions

As long as the debtor and creditor are restricted to naive and backyard strategies, their actions will jointly determine the distribution of the total float but cannot affect its size, which is fixed at $M + B$. In these circumstances, the debtor can never obtain more than half the float and can easily receive less than half (i.e., when a saddlepoint exists). There is therefore an incentive for the debtor to devise new strategies that increase the total size of the float. Although the use of other remote disbursement accounts cannot affect the size of the mail float, it can directly increase the magnitude of the bank float, due to inefficiencies in the check-clearing process outside of large urban areas. This strategy's most profund effect is felt when the debtor writes its checks on a country bank located in a remote rural area. The major problem with implementing this rural disbursement strategy, however, is that the Federal Reserve Board discourages this use of country bank accounts. A strategy which is almost as effective, though, known as controlled disbursement, is to write checks on a regional check-processing center bank (RCPC) located outside of a large metropolitan area, often in a suburb of the debtor's city. The object of using rural or controlled disbursement strategies is to improve the debtor's payoff by increasing the bank float regardless of the creditor's location or choice of strategies.

Although there will always be a float advantage to a rural or RCPC disbursement location for the debtor, creditors can retaliate by developing aggressive direct-send programs. This strategy involves establishing lock box accounts in locations where the banks bypass the Federal Reserve System and use couriers to clear checks directly with the banks on which they are written.[9] Because of these rapid collection procedures, the use of lock box accounts in such locations might easily result in a lower total float than the implementation of a collection account in a location closer to the debtor. This phenomenon occurs whenever the direct send account strategy results in a bank float advantage that more than compensates for the increase in mail float.

The addition of these other strategies can be represented by the

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cost of reducing the mail float (i.e., opening a lock box) might exceed the benefit (i.e., the float reduction multiplied by the interest rate). See Batlin and Hinko (1981) for a discussion of this point.

9. Another alternative available to the creditor is to demand payment by wire transfer to eliminate the bank float entirely (as well as the mail float). As with the use of any of these strategies, the advisability of this policy would depend upon relative transactions and interest opportunity costs. Since the cost of the wire transfer might be quite high, it would probably only be worthwhile if interest rates were quite high and if the amount owed were quite large. In this event, however, it provides opportunities for cooperative behavior between debtors and creditors. For instance, if the float savings were substantial, it might benefit the creditor to offer a side payment in return for payment by wire transfer.

debtor's augmented payoff matrix

$$(\pi_{ij}') = \begin{pmatrix} \pi_{nn} & \pi_{nb} & \pi_{no} \\ \pi_{bn} & \pi_{bb} & \pi_{bo} \\ \pi_{on} & \pi_{ob} & \pi_{oo} \end{pmatrix}, \tag{1'}$$

where the subscript o refers to the party's "other" strategy (i.e., the debtor's rural or controlled disbursement strategy and the creditor's direct send strategy). The solution to this extended game will generally depend upon the relative magnitudes of the elements of the matrix in equation (1′). However, the impact of these additional strategies on the behavior of the two parties can be understood by examining two examples of actual float times, using data on mail and clearing times collected by the Domestic Cash Management Consulting Department of The Chase Manhattan Bank.

Consider first the case of a debtor located in Los Angeles and a creditor in New York City. Beyond their naive and backyard strategies, the debtor can disburse from a rural account in Odessa, Texas, and the creditor can open a lock box account in Chicago, which is known for the effectiveness of its direct send programs. Summing the mail times and clearing times for each pair of strategies, we can represent the number of total float days for this first example in matrix form as follows:

$$(\pi_{ij}')_1 = \begin{pmatrix} 6.42 & 2.82 & 4.44 \\ 5.52 & 3.48 & 3.98 \\ 8.12 & 5.97 & 6.22 \end{pmatrix} \tag{1''}$$

If we restrict our attention just to naive and backyard strategies, it is clear that a saddlepoint exists at 3.48, which is the debtor's maximin choice and the creditor's minimax choice, so that both parties would always use their backyard strategies. When the entire matrix is considered, however, a new saddlepoint emerges at 5.97, and in equilibrium, when both parties have access to all three of their strategies, the debtor's maximin strategy would be always to disburse from its Odessa account, while the creditor would minimax its losses by always collecting remittances from its Los Angeles lock box (i.e., the backyard strategy).

There are three aspects of this example that are noteworthy. First, the addition of a rural disbursement strategy is beneficial to the debtor, since it increases the value of the game (i.e., the equilibrium float level) by 2.5 days. Second, in no circumstance does the creditor's direct send strategy enter into the equilibrium. In this situation, the use of a Chicago lock box is a dominated strategy, since the creditor can always bring about a lower float level, regardless of the debtor's choice of strategies, by relying on its backyard strategy. The superior check-processing capacity of the Chicago banks is always more than offset by the lower mail times assured by the use of a Los Angeles lock box. Finally, the solution to this example is a saddlepoint equilibrium, such that both parties will implement pure strategies. This results because the bank float between Odessa and anywhere else significantly exceeds the original New York–Los Angeles bank float. In fact, the differences between the Odessa total float levels that materialize from different creditor strategies originate largely from differences in mail float between Los Angeles and the other collection points. Thus, regardless of the relative magnitudes of the original bank float and mail float (the factor which determines whether a saddlepoint will exist in the two strategy case), there will always be a saddlepoint when the debtor has access to its rural disbursement strategy. By altering the relative magnitudes of π_{bn}, π_{bb}, and π_{bo}, so long as none of them is larger than π_{ob} (which is reasonable), it can be verified that the solution is unaltered.

There do exist situations, however, in which the debtor's "other" strategy does not so totally dominate the game. Consider, for instance, a second example of a debtor in Cleveland and a creditor in Detroit, with the debtor having access to a controlled disbursement account in a Cleveland RCPC bank in Ashtabula, a suburb of Cleveland, and the creditor having access to a Chicago lock box account in a bank that employs direct send procedures. The summed mail times and clearing times in this second example can be represented in matrix form as follows:

$$(\pi_{ij}')_2 = \begin{pmatrix} 3.57 & 1.88 & 2.15 \\ 3.19 & 2.59 & 2.07 \\ 3.10 & 2.33 & 2.51 \end{pmatrix}. \qquad (1''')$$

It can be seen immediately that this game does not possess a saddlepoint: the debtor's maximin strategy is to disburse from the Ashtabula account, whereby it expects a float of 2.33 days, while the creditor's minimax choice, which it expects will bring about a float of 2.51 days, is to direct the remittance to its Chicago lock box. Since these two payoffs are not equal, a stable equilibrium will not be possible using pure strategies. Rather, the parties must use mixed strategies, solving the problems in equation systems (5) and (8). Using linear programming methods, it can be verified that at the optimum,

$$\begin{aligned} &p_n^{d^*} = 0 \quad p_n^{c^*} = 0 \\ &p_b^{d^*} = .26 \quad p_b^{c^*} = .63 \quad V = 2.4. \\ &p_o^{d^*} = .74 \quad p_o^{c^*} = .37 \end{aligned} \qquad (12)$$

For both parties, the naive strategy is dominated. The debtor will write checks on both Detroit and Ashtabula accounts, using the latter almost three times as often, due to the additional bank float engendered by controlled disbursement. The creditor, on the other hand, will use lock box accounts in Cleveland and Chicago, relying on the former almost twice as often. Although the direct send strategy is generally very effective in reducing bank float, it does not counteract the debtor's controlled disbursement strategy as well as a Detroit lock box does, because of the great reduction in mail time associated with this backyard strategy. Since the debtor will rely more heavily on its "other" strategy, the creditor must depend more frequently on the strategy that works best against it. The expected float value of 2.4 days, finally, is a compromise between the debtor's maximin choice and the creditor's minimax strategy.

IV. Summary

This paper utilized the methodology of game theory to analyze the optimal use of cash management tools by interactive firms. Strategies for both debtor and creditor firms were examined whereby both parties attempt to bring about a level of float most favorable to themselves, assuming that the other party also behaves optimally. The equilibrium float level and its distribution were characterized, and conditions under which the equilibrium would be achieved with the use of pure strategies or mixed strategies were specified. Finally, an explanation was offered for the seemingly random behavior in the use of these tools that is often observed.

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