Gerald D. Gay
Georgia State University

Steven Manaster
University of Chicago

Hedging against Commodity Price Inflation: Stocks and Bills as Substitutes for Futures Contracts*

Several authors (Roll, Long, Merton, and Breeden) have recognized that changes in commodity prices represent a potential risk to consumer-investors. They have constructed models that allow for prices of financial securities to reflect the influence of these risks. While these models are theoretically elegant, the evidence presented in this paper suggests that using commodity prices as state variables is not empirically useful. In addition, the results of this study have implications for inflation hedging strategies, the role of future markets as a hedge against inflation, and the practical applicability of asset pricing models that rely on the supposed influence of specific commodity prices on asset returns.

I. Introduction

This study is concerned with the influence of changes in commodity prices on security prices. The study is motivated by the observation that stochastic changes in commodity prices represent a risk to each consumer-investor and that attempts to deal with these risks may be reflected in the demand for securities and ultimately in their prices and rates of return. If security rates of return do reflect information regarding relative price-change risk, it should be possible to extract that information from the price and returns history of the securities. Following this reasoning, we attempt to form portfolios of stocks and U.S. Treasury bills that hedge against price changes of various commodities. This approach differs from previous studies on hedging because we attempt to hedge against price changes in individual commodities, not macro aggregates. Our empirical results, however, indicate that the history of stock and bill returns contains very little information regarding relative price changes so that our attempts to hedge against commodity price

*Research support from Georgia State University, College of Business Administration, is gratefully acknowledged. We have benefited from discussions with Eugene Fama and John Long; however, we alone are responsible for any omissions or errors.

(*Journal of Business,* 1982, vol. 55, no. 3)

0021-9398/82/5503-0002$01.50

inflation had only limited success. Although these results might be interpreted as "negative" in that they indicate that financial securities are not useful hedging devices for commodity price changes, the paper provides a positive contribution in establishing this empircal result for investors and other researchers. Moreover, the results of this study have implications for inflation hedging strategies, the role of futures markets as a hedge against inflation, and the practical applicability of asset pricing models (see Merton 1973; Long 1974) that rely on the supposed influence of specific commodity prices on asset returns.

Outline and Summary

In Section II we review relevant theoretical work concerning the influence of commodity prices on security prices. We discuss briefly the traditional Sharpe-Lintner capital asset pricing model (CAPM) and extensions of it by Merton (1973), Roll (1973), Long (1974), and Breeden (1979). Particular attention is given to Long's model; his paper provides the most explicit role for commodity price inflation in determining the securities equilibrium expected rates of return and also suggests methods for creating the hedge portfolios that we employ to test for the influence of relative price changes.

In Section III the methodology used for the actual construction of hedge portfolios for six different commodities using asset data from the period January 1961 through December 1979 is presented. Selection of the various commodity, bill, and stock data to be used, as well as the formation of a market index, is discussed. The econometric difficulties encountered in creating the asset weights for the hedging portfolios are also discussed. Finally, the composition of the portfolios is examined with regard to the relative weights of the various stocks and bills contained in each period's hedge. These findings are compared with those of Fama and Schwert (1977*a*), who have investigated the success of various assets as hedging devices against the general inflation rate; and Schipper and Thompson (1981), who have investigated the stock market as a hedge against changes in GNP and general price level.

In Section IV the quality of the hedge portfolios as substitutes for their associated goods is examined. In general, the purchase of short-term Treasury bills provided as good a hedge against each commodity price change as did our hedge portfolios. We interpret these results to mean that there is little additional information in stock market data, regarding commodity price inflation, that is not already contained in Treasury bill data.

The final section presents the conclusions of the paper and draws the practical implications of the research for hedging. In addition, potential extensions of the paper are discussed.

II. Related Theoretical Work

Sharpe-Lintner Model

The capital asset pricing model as developed by Sharpe (1964) and Lintner (1965*a*, 1965*b*) abstracts from reality by assuming that consumers act to maximize the expected value of a utility function whose only arguments are consumption at time 0 and nominal wealth at time 1. To make this assumption consistent with consumer maximization of the expected utility of a lifetime consumption stream, it becomes necessary to assume that future relative prices of consumption goods are known with certainty and no unpredictable changes occur in investment opportunities over time. Uncertainty regarding relative price changes is thereby eliminated. In addition, the capital market cannot contain assets whose future rates of return over a time interval may depend on unanticipated events in the interim. Thus an economy with a bill market is not allowed. However, commodity price inflation and the term structure of interest rates may play an important role in the determination of equilibrium asset prices.

Roll's Model

Roll (1973) recognizes that inflation of commodity prices has been almost completely neglected in the literature of asset pricing. He develops a capital asset pricing model that includes the risk of currency inflation and attempts to show how assets and commodities acquire equilibrium prices in competitive markets. Roll points out that the simple Fisherian equation will not correctly specify the relation between nominal interest rates and expected rates of commodity price inflation. His analysis indcates that the nominal expected rate of return on assets will depend on the covariance between nominal asset returns and the rate of inflation. Roll's economy, however, has only one future date and thus the term structure issue is ignored.

Merton's Model

Merton (1973) presents a continuous-time analysis of the demand for assets in an economy where both consumption good prices and the investment opportunity set are allowed to vary randomly over time. He derives a formula that expresses the consumer's demand for risky assets in terms of parameters describing his consumption preferences and parameters of the stochastic process governing commodity and asset prices.

Merton's model is very general in that it can allow for the influence of relative price changes on equilibrium rates of return. In an example presented in his 1973 paper, Merton chose to illustrate the influence of changing investment opportunities on equilibrium asset prices rather than changing consumption opportunities; however, the potential to evaluate both can easily be incorporated into his model.

Breeden's Model

Breeden's model (1979) is a special case of the general Merton model. Breeden argues that changes in the consumption and investment opportunities can be summarized by changes in the aggregate consumption variable. This innovation makes the model very convenient since hedges against only one variable (aggregate consumption) need be determined in order to properly price securities. However, Cornell (1981) shows that the "consumption betas" of Breeden's model will be a function of the state variables of the Merton model. Therefore, the Breeden model consumption betas are likely to be nonstationary and empirical verification of the Breeden model will require identification and observation of Merton's state variables.

Long's Model

Long (1974) provides a realistic view of capital market equilibrium by presenting a multiperiod discrete-time analysis which deals directly with uncertainty in future commodity prices and future investment opportunities from which he develops empirically testable price formulas for common stock and long-term bills. These formulas are developed such that the relation between equilibrium prices and parameters describing consumer preferences can be analyzed directly. The price of an individual asset can be interpreted in terms of its marginal contributions to portfolio characteristics that concern investors such as mean, variance, and covariance.

The economy in Long's model contains three markets: a stock market where the shares of N firms are traded, a consumption goods market containing K nonstorable commodities available to consumers, and a market containing default-free bills for any maturity date up to and including time T. For each of these markets, the following assumptions are made: (1) markets are perfect in the sense that all items are infinitely divisible, there are no transaction costs, and all traders act as price takers; (2) on any date all traders have free and equal access to all information which is relevant to assessing the subjective joint probability distributions of prices which prevail on subsequent trading dates and, furthermore, all consumers form identical expectations regarding prices to be realized on subsequent dates; (3) markets are open only on trading dates which are equally spaced in time; (4) all production in the economy is accomplished by firms; and (5) the only inputs to production supplied directly by consumers are capital funds supplied by the purchase of the firm's shares and bills.

Under these assumptions it is possible to derive the following reduced-form pricing model (see Long [1974], appendix B for details) which prices not only the systematic market risk of a share of stock but also the inflation risk from changing good prices and the interest rate risk from changes in the term structure:

$$\bar{\rho}_j = r_1 + \beta_{jM}(\bar{\rho}_M - r_1) + \sum_{k=1}^{K} \xi_{jk}[(1 + \bar{\imath}_k) - (1 + r_1)(F_{k0}/\pi_{k0})] + \sum_{m=2}^{T} \delta_{jm}(\bar{r}_m - r_1). \tag{1}$$

The notion employed in equation (1) is the same as in Long's paper. Table 1 provides a summary. In addition these conventions will be followed: (1) tildes (˜) indicate the value is a random variable at time zero; (2) bars (¯) indicate expected values as of time zero; (3) symbols without tildes or bars are nonstochastic; and (4) the symbols $\tilde{h}$, $\tilde{\rho}$, $\tilde{r}$,

TABLE 1 Notation

Variable	Current Price (Time = 0)	Next-Period's Price (Time = 1)	One-Period Rate of Return
Stock market portfolio*	P_{M0}	$\tilde{V}_{M1}$	$\tilde{\rho}_M$
One-period bill (risk-free asset)	B_{01}	1	r_1
Long-term bills ($m = 2, \ldots, T$)	B_{0m}	$\tilde{B}_{1m}$	$\tilde{r}_m$
Firms ($j = 1, \ldots, N$)	P_{j0}	$\tilde{V}_{j1}$	$\tilde{\rho}_j$
Commodities ($k = 1, \ldots, K$)	π_{k0}	$\tilde{\pi}_{k1}$	$\tilde{\imath}_k$
Quasi-futures contract ($k = 1, \ldots, K$)†	F_{k0}	$\tilde{F}_{k1}$	$\tilde{f}_k$
Hedge portfolios‡	H_{k0}	$\tilde{H}_{k1}$	$\tilde{h}_k$

*In Long's paper the stock market portfolio is defined to be a value-weighted index of all stocks. In this paper the market portfolio will be an equally weighted index.

†Long's quasi-futures contract for good k is a portfolio of stocks and bills such that the following hold:

$$\bar{\pi}_{k1} = \bar{F}_{k1},$$

$$\text{cov}\,(\tilde{F}_{k1}, \tilde{V}_{m1}) = \text{cov}\,(\tilde{\pi}_{k1}, \tilde{V}_{m1}),$$

$$\text{cov}\,(\tilde{F}_{k1}, \tilde{B}_{1m}) = \text{cov}\,(\tilde{\pi}_{k1}, \tilde{B}_{1m}), \quad m = 2, \ldots, T,$$

$$\text{cov}\,(\tilde{F}_{k1}, \tilde{\pi}_{j1}) = \text{cov}\,(\tilde{\pi}_{k1}, \tilde{\pi}_{j1}), \quad j = 1, \ldots, K.$$

‡Our hedge portfolio for good k is a combination of stocks and bills such that the following hold:

$$\text{cov}\,(\tilde{h}_k, \tilde{\rho}_M) = \text{cov}\,(\tilde{\imath}_k, \tilde{\rho}_M),$$

$$\text{cov}\,(\tilde{h}_k, \tilde{r}_m) = \text{cov}\,(\tilde{\imath}_k, \tilde{r}_m), \quad m = 2, \ldots, T,$$

$$\text{cov}\,(\tilde{h}_k, \tilde{\imath}_j) = \text{cov}\,(\tilde{\imath}_k, \tilde{\imath}_j), \quad j = 1, \ldots, K.$$

and $\tilde{\imath}$ without subscripts are used to indicate their associated column vectors; specifically

$$\tilde{h} = \begin{pmatrix} \tilde{h}_1 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{h}_k \end{pmatrix}, \quad \tilde{\rho} = \begin{pmatrix} \tilde{\rho}_1 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\rho}_N \end{pmatrix}, \quad \tilde{r} = \begin{pmatrix} \tilde{r}_2 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{r}_T \end{pmatrix}, \quad \text{and } \tilde{\imath} = \begin{pmatrix} \tilde{\imath}_1 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\imath}_k \end{pmatrix};$$

(5) the β_j, ξ_{jk} and δ_{jm} in equation (1) are constants; and (6) the symbol $\tilde{z}$ will be used to represent any element of the set $\{\tilde{\rho}_M, \tilde{\imath}, \tilde{r}\}$.

Equation (1) extends the traditional mean-variance CAPM which relates the return on a security to only its systematic market risk. The Long CAPM relates the return on a security to (1) the systematic risk of the security in the stock market; (2) the risk of the security due to expected price changes in each commodity k ($k = 1, \ldots, K$); and (3) the risk of the security with respect to a bill of maturity m ($m = 2, \ldots, T$), due to anticipated shifts in the yield curve. Since there are thus $K + T$ types of risk, each security can be represented by a point in $(K + T + 1)$ space. Thus a natural generalization of the security market line (SML) associated with the single-period CAPM is the security market hyperplane (SMH) on which each security in equilibrium must lie. The SMH will intersect the expected return axis at the riskless rate just as the SML does in the two-dimensional case.

Gouldey's work (1977, 1980) represents the only previous empirical investigation of the Long Model. The purpose of Gouldey's study was to test the implications of the Long model, which suggested that investors are concerned with three types of risk when making investment decisions: the traditional systematic market risk, the risk due to a stochastic consumption opportunities set, and the risk due to stochastically changing investment opportunities. Gouldey's results indicate that average rates of return on capital assets do contain risk premiums for systematic market risk and risk due to stochastic consumption and investment opportunities. This implies that investors can use stocks and bills to create a hedge against commodity price inflation. These hedges are referred to as "quasi-futures" contracts.

Even though quasi-futures contracts exist in principle, there needs to be further development and testing of these hedge portfolios.[1] This paper intensively examines the construction and composition of hedge portfolios for six consumption goods over the period 1965–79. The

1. Gouldey attempted to obtain estimates of the costs of forming quasi-futures contracts for food, housing, and transportation price indices; however, he failed to provide any information concerning the composition of the contracts, nor did he test the success of the portfolios as hedging instruments using subsequently observed data.

portfolios are also tested for their adequacy as hedging instruments against commodity price inflation.

III. Construction of Hedge Portfolios

Hedge Portfolios and Long's Quasi-Futures Contracts Compared

The differences between Long's quasi-futures contracts and our hedge portfolios are revealed in notes † and ‡ to table 1. The quasi-futures contracts and the hedge portfolios reveal the same economic information measured in different units. The reason for employing the hedge portfolios rather than the quasi-futures contracts is that the time series of rates of return is more likely to be stationary than the time series of prices. Therefore estimation and interpretation of the hedge portfolios is more convenient than for Long's quasi-futures contracts. Also, our hedge portfolios can be constructed without estimating any expected returns or expected prices.

Return Generating Process

The basic hypothesis underlying the concept of the hedge portfolios is that each security return is affected not only by the return to the market as a whole but also by changes in investment and consumption opportunities. Therefore we model the return generating process as shown in equation (2):

$$\tilde{\rho}_{jt} = \alpha_j + \beta_j\tilde{\rho}_{Mt} + \sum_{k=1}^{K} \xi_{jk}\tilde{\imath}_{kt} + \sum_{m=2}^{T} \delta_{jm}\tilde{r}_{mt} + \tilde{\epsilon}_{jt}, j = 1, \ldots, N. \quad (2)$$

Equation (2) can be interpreted in several different ways. In the framework of Merton's model (1972) the rates of price appreciation on the K goods ($\tilde{\imath}_k$) and the rates of return on the long-term bonds ($\tilde{r}_m$) may be regarded as state variables, the realizations of which affect investors' utility and therefore portfolio choices. In the framework of Ross's (1976, 1977) arbitrage pricing theory the $\tilde{\imath}_k$, $\tilde{\rho}_{Mt}$, and $\tilde{r}_{mt}$ can be thought of as the factors in a $K + T + 1$ factor model. The interpretations within the context of Long's model are similar to the Merton interpretations, although Long's model is in discrete rather than continuous time. The error term, $\tilde{\epsilon}_{jt}$ in equation (2) is serially uncorrelated and uncorrelated with $\tilde{\rho}_M$, and $\tilde{\imath}_k$'s and the $\tilde{r}_m$'s. The hedge portfolios for the K goods are derived from equation (2) using a procedure outlined in the next section.

Methodology for Creating the Hedge Portfolios

In order that a stock-bill portfolio can properly be called a "hedge portfolio," the covariance of its rate of return with the rates of return on market portfolio, on any good, and on long-term bills must equal the covariance of the associated good with these items:

$$\text{cov}\,(\tilde{h}_k, \tilde{Z}) = \text{cov}\,(\tilde{\imath}_k, \tilde{Z}) \text{ for all } k, \tag{3}$$

where the variables in equation (3) are defined in table 1.

Define C as the $(N + T - 1) \times (K + T)$ covariance matrix of $(\tilde{\rho}, \tilde{r})$ with $(\tilde{\rho}_M, \tilde{\imath}, \tilde{r})$. Figure 1 displays the elements of the C matrix. Define Σ as the $(K + T) \times (K + T)$ variance-covariance matrix of $(\tilde{\rho}_m, \tilde{\imath}, \tilde{r})$ whose elements are presented in figure 2. Finally let Ω be a $[(N + T - 1) \times (K + T)]$ matrix whose elements are given in figure 3. Note the first N rows of Ω contain the coefficients taken from the multiple regression of equation (2) for each of the N stocks.

The matrices C, Ω, and Σ are related such that

$$C = \Omega\Sigma. \tag{4}$$

For example, this implies that the first element of C is

$$\beta_1 \,\text{cov}\,(\tilde{\rho}_M, \tilde{\rho}_M) + \sum_{k=1}^{K} \xi_{1k} \,\text{cov}\,(\tilde{\rho}_M, \tilde{\imath}_k) + \sum_{m=2}^{T} \delta_{1m} \,\text{cov}\,(\tilde{r}_m, \tilde{\rho}_M),$$

$$C \equiv \begin{bmatrix} \underset{(N\times 1)}{\sigma_M} & \underset{N\times K}{\phi} & \underset{N\times(T-1)}{\Gamma} \\ \\ \underset{(T-1)\times 1}{S_M} & \underset{(T-1)\times K}{\psi} & \underset{(T-1)\times(T-1)}{\textstyle\sum_B} \end{bmatrix}$$

$$\begin{bmatrix}
\begin{bmatrix} \text{Cov}(\hat{\rho}_1, \hat{\rho}_M) \\ \vdots \\ (\sigma_M\text{: } N\times 1) \\ \vdots \\ \text{Cov}(\hat{\rho}_N, \hat{\rho}_M) \end{bmatrix} &
\begin{bmatrix} \text{Cov}(\hat{\rho}_1, \hat{\imath}_1) & \cdots & \text{Cov}(\hat{\rho}_1, \hat{\imath}_k) \\ \vdots & & \vdots \\ \vdots & \phi\text{:Cov}(\hat{\rho}, \hat{\imath}) & \vdots \\ \vdots & N\times K & \vdots \\ \text{Cov}(\hat{\rho}_N, \hat{\imath}_1) & \cdots & \text{Cov}(\hat{\rho}_N, \hat{\imath}_k) \end{bmatrix} &
\begin{bmatrix} \text{Cov}(\hat{\rho}_1, \hat{r}_2) & \cdots & \text{Cov}(\hat{\rho}_1, \hat{r}_T) \\ \vdots & & \vdots \\ \vdots & \Gamma\text{:Cov}(\hat{\rho}, \hat{r}) & \vdots \\ \vdots & N\times(T-1) & \vdots \\ \text{Cov}(\hat{\rho}_n, \hat{r}_2) & \cdots & \text{Cov}(\hat{\rho}_N, \hat{r}_T) \end{bmatrix} \\
\begin{bmatrix} \text{Cov}(\hat{r}_2, \hat{\rho}_M) \\ \vdots \\ (S_M\text{:}(T-1)\times 1) \\ \vdots \\ \text{Cov}(\hat{r}_T, \hat{\rho}_M) \end{bmatrix} &
\begin{bmatrix} \text{Cov}(\hat{r}_2, \hat{\imath}_1) & \cdots & \text{Cov}(\hat{r}_2, \hat{\imath}_k) \\ \vdots & & \vdots \\ \vdots & \psi\text{: Cov}(\hat{r}, \hat{\imath}) & \vdots \\ \vdots & (T-1)\times K & \vdots \\ \text{Cov}(\hat{r}_T, \hat{\imath}_k) & \cdots & \text{Cov}(\hat{r}_T, \hat{\imath}_k) \end{bmatrix} &
\begin{bmatrix} \text{Cov}(\hat{r}_2, \hat{r}_2) & \cdots & \text{Cov}(\hat{r}_2, \hat{r}_T) \\ \vdots & & \vdots \\ \vdots & \textstyle\sum_B\text{: Cov}(\hat{r}, \hat{r}) & \vdots \\ \vdots & (T-1)\times(T-1) & \vdots \\ \text{Cov}(\hat{r}_T, \hat{r}_2) & \cdots & \text{Cov}(\hat{r}_T, \hat{r}_T) \end{bmatrix}
\end{bmatrix}$$

FIG. 1.—Elements of covariance matrix C

$$
\sum \equiv \begin{bmatrix} \underset{(1\times 1)}{\sigma_M{}'Q} & \underset{(1\times K)}{Q'\phi} & \underset{1\times(T-1)}{S'_M} \\[2ex] \underset{(K\times 1)}{\phi' Q'} & \underset{(K\times K)}{\sum_\pi} & \underset{K\times(T-1)}{\psi'} \\[2ex] \underset{(T-1)\times 1}{S_M} & \underset{(T-1)\times K}{\psi} & \underset{(T-1)\times(T-1)}{\sum_B} \end{bmatrix}
$$

$$
\underset{(1\times N)}{Q'} \equiv (1/N,\ 1/N,\ .\ .\ .,\ 1/N)
$$

$$
\begin{bmatrix}
\begin{bmatrix} \mathrm{Cov}(\hat\rho_M, \hat\rho_M) \\ \sigma'_M Q\text{: } 1\times 1 \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat\rho_M, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat\rho_M, \hat i_K) \\ & \phi'\phi\text{: } 1\times K & \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat r_2, \hat\rho_M) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat\rho_M) \\ & S'_M\text{: } 1\times(T-1) & \end{bmatrix} \\[2ex]
\begin{bmatrix} \mathrm{Cov}(\hat\rho_M, \hat i_1) \\ \vdots \\ \phi' Q \ \ . \\ (K\times 1). \\ \vdots \\ \mathrm{Cov}(\hat\rho_M, \hat i_k) \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat i_1, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat i_1, \hat i_k) \\ \vdots & & \vdots \\ . & \underset{(K\times K)}{\sum_\pi} & . \\ \vdots & & \vdots \\ \mathrm{Cov}(\hat i_k, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat i_k, \hat i_k) \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat r_2, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat i_1) \\ \vdots & & \vdots \\ . & \underset{K\times(T-1)}{\psi'\text{:}\mathrm{Cov}(\hat r_T, \hat i)} & . \\ \vdots & & \vdots \\ \mathrm{Cov}(\hat r_2, \hat i_k) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat r_k) \end{bmatrix} \\[2ex]
\begin{bmatrix} \mathrm{Cov}(\hat r_2, \hat\rho_M) \\ \vdots \\ S_M \ \ . \\ (T-1)\times 1 \\ \vdots \\ \mathrm{Cov}(\hat r_T, \hat\rho_M) \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat r_2, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat r_2, \hat i_k) \\ \vdots & & \vdots \\ . & \underset{(T-1)\times K}{\psi} & . \\ \vdots & & \vdots \\ \mathrm{Cov}(\hat r_T, \hat i_1) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat i_K) \end{bmatrix} &
\begin{bmatrix} \mathrm{Cov}(\hat r_2, \hat r_2) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat r_2) \\ \vdots & & \vdots \\ . & \underset{(T-1)\times(T-1)}{\sum_B} & . \\ \vdots & & \vdots \\ \mathrm{Cov}(\hat r_2, \hat r_T) & . \ . \ . & \mathrm{Cov}(\hat r_T, \hat r_T) \end{bmatrix}
\end{bmatrix}
$$

FIG. 2.—Elements of the covariance matrix Σ

which is equivalent to the covariance of equation (2) with $\tilde{\rho}_M$.

It can be shown that the average of the first N rows of C is equal to the first row of Σ by applying the following weighting scheme:

$$
\begin{aligned}
[Q' \ \vdots \ 0']C &= [1' \ \vdots \ 0']\Omega\Sigma \\
&= [1\ 0\ 0\ .\ .\ .\ 0]\Sigma \\
&= e_1{}'\Sigma,
\end{aligned} \tag{5}
$$

where Q' is a $(1\times N)$ vector of $(1/N)$ and $0'$ is a $[1\times(T-1)]$ vector of

$$\Omega = \begin{bmatrix} \underset{(N\times 1)}{\beta} & \underset{(N\times K)}{\Xi} & \underset{(N\times T-1)}{\Delta} \\ \underset{(T-1)\times 1}{\underline{0}} & \underset{(T-1)\times K}{\underline{\underline{0}}} & \underset{(T-1)\times(T-1)}{I} \end{bmatrix}$$

$$\begin{array}{ccc}
\begin{bmatrix} \beta_{1M} \\ \beta_{2M} \\ \cdot \\ \cdot \\ \underset{(N\times 1)}{\beta} \\ \cdot \\ \cdot \\ \beta_{NM} \end{bmatrix}
&
\begin{bmatrix} \xi_{11} & \xi_{12} & \cdot & \cdot & \cdot & \xi_{1K} \\ \xi_{21} & \xi_{22} & \cdot & \cdot & \cdot & \xi_{2K} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & \underset{(N\times K)}{\Xi} & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \xi_{N1} & \xi_{N2} & \cdot & \cdot & \cdot & \xi_{NK} \end{bmatrix}
&
\begin{bmatrix} \delta_{12} & \delta_{13} & \cdot & \cdot & \cdot & \delta_{1T} \\ \delta_{22} & \delta_{23} & \cdot & \cdot & \cdot & \delta_{2T} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & \underset{N\times(T-1)}{\Delta} & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \delta_{N2} & \delta_{N3} & \cdot & \cdot & \cdot & \delta_{NT} \end{bmatrix}
\\
\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \underset{(T-1)\times 1}{\underline{0}} \\ \cdot \\ \cdot \\ 0 \end{bmatrix}
&
\begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & \underset{(T-1)\times K}{\underline{\underline{0}}} & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}
&
\begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & \underset{(T-1)\times(T-1)}{I} & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{bmatrix}
\end{array}$$

FIG. 3.—Elements of the augmented coefficient matrix Ω

zeros. The relation above implies the following:

$$\frac{1}{N}\sum_{j=1}^{N} \beta_{jm} = 1, \tag{6}$$

$$\sum_{j=1}^{N} \xi_{jk} = 0,\ k = 1, \ldots, K, \tag{7}$$

and

$$\sum_{j=1}^{N} \delta_{jm} = 0,\ m = 2, \ldots, T. \tag{8}$$

The validity of these equations can be demonstrated by averaging equation (2) across all N securities and by recalling our definition of the return on the market portfolio,

$$\tilde{\rho}_M = 1/N \sum_{i=1}^{N} \tilde{\rho}_i.$$

Let e'_{1+k} be a $[1\times(K + T)]$ vector whose $(1 + k)$th element is 1, with all other elements equal to 0. The index k on e'_{1+k}, $k = 1, \ldots, K$, indicates that e'_{1+k} is associated with the kth consumption good. For example, when $k = 0$, vector e'_1 will refer to the "market" as was previously shown in equation (5). There exist K weighted combinations of the rows of Ω that will solve for the K row vectors, e'_{1+k}, $k = 1, \ldots, K$. These weights can be found by solving the following system of equations for b'_k where b'_k is a $[1\times(N + T - 1)]$ vector:[2]

$$b'_k\Omega = e'_{1+k},$$
$$b'_k = e'_{1+k}\Omega^{-1}, \qquad k = 1, \ldots, K. \tag{9}$$

Each of the K row vectors b'_k represents a combination of stocks and risky bills that has the same covariance with $(\tilde{\rho}_m, \tilde{\imath}, \tilde{r})$ as does $\tilde{\imath}_k$; for example,

$$\text{cov}\left[b'_k\begin{pmatrix}\tilde{\rho}\\ \tilde{r}\end{pmatrix}, \tilde{Z}\right] = \text{cov}\,(\tilde{\imath}_k, \tilde{Z}), \tag{10}$$

where $\tilde{Z}$ is any element from the set of $\{\tilde{\rho}_m, \tilde{\imath}, \tilde{r}\}$.

The vector b'_k is not a portfolio proportions vector, since the sum of the elements of b'_k are not required to equal 1. However, by augmenting the b'_k vector by $1 - b'_k\ell$ where ℓ is a column vector of ones and using this amount as the weight on the risk-free asset, r_1, we obtain a portfolio whose return is equal to

$$(b'_k \,\vdots\, 1 - b'_k\ell)\begin{pmatrix}\tilde{\rho}\\ \tilde{r}\\ r_1\end{pmatrix} = \tilde{h}_k, \tag{11}$$

with the desired property that[3]

$$\text{cov}\,(\tilde{h}_k, \tilde{Z}) = \text{cov}\,(\tilde{\imath}_k, \tilde{Z}). \tag{12}$$

In solving equation (9) for the b'_ks we assumed that the Ω matrix was square and of full rank. This assumption requires that the number of stocks (N) is one greater than the number of goods (K). Whenever $N = K + 1$, there will be a unique solution for each b'_k. However, if $N < K +$

2. Eqq. (9) require that Ω is a square matrix and that Ω^{-1} exists. These requirements are discussed below.

3. Schipper and Thompson (1981) construct portfolios to hedge against unexpected changes in GNP or the CPI. The properties of the Schipper-Thompson hedges are slightly different from our $\tilde{h}_k$'s. Their portfolios are designed so that the correlation coefficients between the returns on each asset and the state variables (GNP and CPI changes) are equal to the correlation coefficients between the rate of return on each asset and the hedge portfolio; they are concerned with only one state variable at a time, not K consumption goods simultaneously; and the composition of the hedging portfolios consists of stocks only—the potential of Treasury bills as a hedging instrument is not evaluated.

1, there are not enough stocks to solve for the b'_k and the concept of "hedge portfolio" will not have any empirical content. If $N > K + 1$ there will be $N - K$ linearly independent solutions for the b'_k. Each of the $N - K$ solutions can be used to generate a portfolio return $\tilde{h}_k$ that will satisfy equation (12). Moreover, any linear combination of the $N - K$ solutions will also satisfy (12). From this infinite number of solutions it is convenient to choose the hedge portfolio that satisfies (12) and at the same time minimizes var $(\tilde{h}_k - \tilde{i}_k)$. (This is discussed in detail in the section below on Parameter Estimation.)

When the hedging portfolios are being constructed, it is interesting that the number of long-term bills in the economy assumed does not impose any empirical restrictions on the model as does the number of shares and consumption goods assumed. However, one cannot conclude that bills are unimportant in calculating the share weights. Exclusion of bills in the multiple regression (2) would change the values of β and ξ and thus would have an effect on the elements of b'_k.

The Data

In order to select the proper quantities of stocks and long-term bills such that the covariance property of each hedge portfolio is attained, the multiple regression equations (2) are estimated. The following time-series data are required to perform the regression: (1) monthly returns for N common stocks, (2) monthly returns for stock market portfolio, (3) monthly rates of price appreciation for K consumption goods, and (4) monthly returns for $T - 1$ default-free long-term bills. The return data for the assets above were collected for the period January 1961 through December 1979.[4]

Previous studies concerning commodity price inflation (Dusak 1973; Gouldey 1977, 1980) were somewhat limited in the number of different commodities used. If several goods are used in the analysis, and all the goods were to experience the same relative price changes or inflation rates, one would expect the same hedging portfolio to be derived for each good. However, it appears that all goods do not experience the same relative price changes (see Fama and Schwert 1977*b*), and thus the composition of the hedge portfolios may differ for each good. Therefore, to test the success of hedges properly, the study should include a wide variety of consumption goods in the analysis.

The data concerning the consumption goods to be used presented two main problems. First, to be theoretically consistent with the Long model, all consumption goods should be included in the analysis. Including all these variables as independent variables in the regression

4. August 1971–December 1974 corresponds to a period of wage and price controls. All the empirical work reported in this paper has been repeated excluding data from the price control period. None of the conclusions we report are sensitive to the inclusion or exclusion of these data.

(2) obviously is not possible. However, failure to include enough of these variables may result in misspecification of the model and missing-variable problems. The second problem concerns finding accurate and available commodity price data. For this purpose prices of six various groupings containing closely related bundles of consumption goods used by the Bureau of Labor Statistics in compiling the CPI will be used to proxy for commodity prices. Price indices for these various groupings are readily available. The components of the CPI employed in this study are listed under the column labeled "Consumption Bundle" in table 2.

To retain the uniqueness of each price series, monthly price data for the selected consumption bundles were used. Fama and Schwert (1979) show that inflation rates of different goods can be broken into a part common to all goods and a part peculiar to each good itself. An example of a part peculiar to a particular good is one that may be due to seasonal variation. If production for a good is seasonal whereas demand occurs smoothly throughout the year, seasonal variation in the price of such a good may be observed to offset the cost of storing the output. However, as one goes from using monthly prices to prices of longer intervals, the variability of the price series of a component becomes more and more like that of the CPI. Thus the variability of price indices of various goods will become more and more similar as they are measured over longer intervals.

Final selection of the consumption bundles used in this study depends in many ways on the construction of the CPI. An overall breakdown of the major groupings composing the CPI is given in table 3. Since the data to be used in the study are on a monthly basis, it will be desirable to have the selected CPI components sampled and priced monthly. In addition, it is desirable for each component chosen to be a fairly homogeneous grouping of goods. Approximately 50% of the items and locations used to construct the CPI are sampled each month. The rest of the component prices are collected on a quarterly but rotating basis so that there is some monthly revision of prices. However, components which are sampled in quarterly intervals will reflect changes in prices which actually occurred during the preceding 2 months. Therefore, such components at times will be incorrectly calculated as being unchanged. These lags in the updating of prices may introduce autocorrelation in the price series (for a thorough discussion of these issues see Fama and Schwert [1979]).

In summary, monthly price data for the period January 1961 through December 1979 were gathered for $K = 6$ consumption bundles taken from the CPI to be used as proxies for various commodity prices. The principal reason for selecting January 1961 as a starting point for the study was an important change in the way the CPI was constructed beginning in 1964. At this time the coverage of the index was extended

TABLE 2 Selected Consumption Bundles and Industry Portfolios

Consumption Bundle	Symbol	Associated Industry Portfolio	Symbol	Compustat Industry Code
Food	FD	Retail, food chains	F	5411
Gas and electricity	GE	Electric utilities, normalized	EU	4912
		Electric utilities, flow through		4911
		Natural gas companies	NG	4924
Apparel and upkeep	AU	Textile apparel MGF	TA	2300
Private transportation	PT	Motor vehicles	MV	3711
		Auto trucks		3713
Medical care	MC	Drugs, ethical	D	2835
		Drugs, proprietary		2836
		Drugs, medical supply		2837
Household furnishings	HFO	Home furnishings	HF	2510
		Others:		
		Forest products	FP	2400
		Chemicals, major	CH	2801
		Chemicals, intermediate		2802
		Steel, major	ST	3310
		Steel, minor		3311

TABLE 3 A Breakdown of the Consumer Price Index

Component	Percentage of All Items, December 1963	Component	Percentage of All Items, December 1963
1. Food:*	22.43	20. Transportation:	13.88
2. Food at home:	17.89	21. Private:*	12.64
3. Cereals and bakery products	2.45	22. Autos and related goods	3.28
4. Meats, poultry, and fish	5.63	23. Automobile services	3.62
5. Dairy products	2.80	24. Public	1.24
6. Fruits and vegetables	3.02	25. Health and recreation:	19.45
7. Other food	3.99	26. Medical care:*	5.70
8. Food away from home	4.54	27. Drugs	1.14
9. Housing	33.23	28. Professional services	2.59
10. Shelter	20.15	29. Hospital services	.36
11. Homeownership	14.27	30. Health insurance	1.61
12. Rent	5.50	31. Personal care	2.75
13. Hotels and motels	.38	32. Reading and recreation	5.94
14. Fuel and utilities	5.26	33. Other goods and services	5.06
15. Fuel oil and coal	.73	34. Tobacco products	1.89
16. Gas and electricity*	2.71	35. Alcoholic beverages	2.64
17. Telephone and water	1.82	36. Personal expenses	.53
18. Household furnishings and operation*	7.82	37. Miscellaneous	.38
19. Apparel and upkeep*	10.63		

SOURCE.—Condensed from appendix table IX of U.S. Department of Labor 1966.

NOTE.—Percentage of component 1 = percentage of components 2 + 8; 2 = 3 + 4 + 5 + 6 + 7; 9 = 10 + 14 + 18; 10 = 11 + 12 + 13; 14 = 15 + 16 + 17; 20 = 21 + 24; 25 = 26 + 31 + 32 + 33; 26 = 27 + 28 + 29 + 30; 33 = 34 + 35 + 36.

*Refers to those components included in the study.

and the weights assigned to the different items were updated in line with the 1960–61 Consumer Expenditure Survey. However, it was still possible to gather revised data for the selected price series back to January 1961 according to the 1964 updating scheme.

Based on the decision to use $K = 6$ commodities in the study, monthly return data on at least $N = 7\ (K + 1)$ common stocks are required. This empirical restriction that the number of stocks to be used equals at least one plus the number of goods was discussed in Section III under Return Generating Process. If $N > K + 1$, there will exist an infinite number of hedging portfolios which have the desired properties. However, only that portfolio which minimizes the variance of $(\tilde{h}_k - \tilde{\imath}_k)$ will be chosen as the hedge portfolio.

The risk parameters using portfolio data tend to be more stable than those of individual stocks, and much of the risk peculiar to individual stocks is diversified away in portfolios. Instead of using individual securities in the analysis, industry portfolios are formed. Various industry classifications as provided by the Compustat files will be used to create the industry portfolios. The 10 industry groupings to be used are listed in table 2. The industries selected vary widely, and an attempt was made to have each industry correspond to a particular consumption bundle. For example, the motor vehicles and auto trucks portfolio corresponds to the private-transportation component, the consumption bundle apparel and upkeep is associated with textile manufacturing, and so on. Our purpose is to test the intuition that the best hedging strategy against inflation in a particular good consists of buying the stock of a firm in a related industry.

The following procedure was used to create each industry return. First, each company within an industry grouping was checked to see if it had a monthly return (RET1) listed in the monthly returns file of the Center for Research in Security Prices (CRSP) at the University of Chicago. For all companies that had a monthly return listed in a given month, the returns were summed and an average monthly return was computed for that industry. This procedure was repeated for each month from January 1961 through December 1979. Firms were frequently observed to enter and leave the sample when computing each industry average monthly return. This method allowed the maximum number of companies to be used each month in the calculation of the industry average monthly return. To be consistent with the algebra presented in Section II, the return on the market portfolio is defined as an equally weighted return of all the industry portfolios, that is,

$$\tilde{\rho}_{Mt} = \frac{1}{N} \sum_{j=1}^{N} \tilde{\rho}_{jt}$$

for all months t.

Finally, the returns on 1-month and 6-month U.S. Treasury bills are employed in the study. The monthly return on a 1-month Treasury bill r_1 is risk free. It is used to adjust the portfolio proportions vector so that the sum of the weights equals one, following the procedure illustrated in equation (12). In this study, the only risky interest rate we employ is the 1-month return on a 6-month Treasury bill. Although returns on 2–5-month bills are available, the inclusion of more than one risky bill in the study did not appear to add much information beyond what is contained in the 6-month rate alone. Moreover, the inclusion of additional interest rates would needlessly exacerbate the multicollinearity problem already present in equation (2).

Parameter Estimation

For the readers' convenience, the version of equation (2) that we estimate is

$$\tilde{\rho}_{jt} = \alpha_j + \beta_j \tilde{\rho}_{Mt} + \sum_{k=1}^{6} \xi_{jk} \tilde{l}_{kt} + \delta_{j6} \tilde{r}_6 + \epsilon_{jt} \qquad \text{for } j = 1, \ldots, 10. \quad (2')$$

The estimated coefficients from equations (2′) become elements of the Ω matrix which is used to compute the hedge portfolios according to equation (9).

Five years of monthly observations are used in each time-series regression. The first set of N regressions is performed on 60 months of observations extending from January 1961 through December 1965. The estimated coefficients from this time interval are then used to compute the hedge portfolios for each of the goods according to the method discussed in Section II. Thus the first 60 months of observations are used to create portfolios which serve as hedging devices against price inflation in the various selected goods during January–June 1966. The portfolio weights are rebalanced at 6-month intervals. This is done by repeating the regression after dropping the first six observations from the sample and adding observations from the latest 6 months.

Following the estimation of each set of regression coefficients the Ω matrix is constructed. For each 6-month time interval the regression coefficients for each set of $N = 10$ regressions are placed in the first N rows of Ω. The last row consists of 10 zeros and a 1. Recall that equation (9) is to be solved for the vector b_k':

$$b_k' \Omega = e_{1+k}'. \quad (9)$$

The Ω matrix is rank 8, $(K + 2)$, with dimension (8×11), $[K + 2 \times N + 1]$; therefore there are four linearly independent solutions for b_k'. Each of the four solutions is obtained by deleting three columns from the Ω matrix and the e_{1+k}' vector and inverting the reduced Ω matrix. This is equivalent to eliminating thee industries from the market. This procedure produces four intermediate solutions for the b'_k vector $b'^{(i)}_k$ $i = 1, \ldots, 4$. Each $b^{(i)}_k$ satisfies equation (10) and in addition meets an artificial requirement that there be a zero investment in each of the three excluded industries. Each of the four $b'^{(i)}_k$ solutions is augmented by $1 - b'^{(i)}_k \ell$ of r_1 as shown in equation (11). Four repetitions of this procedure are performed, each time excluding a different set of three industries. This provides four intermediate hedge portfolios with return $\tilde{h}^{(i)}_k$, each satisfying equation (12).

To find the linear combination of the four $\tilde{h}^{(i)}_k$'s that will minimize the variance of $\tilde{\imath}_k - \tilde{h}_k$, the following restricted regression is estimated:

$$\tilde{\imath}_{kt} = \alpha_1 \tilde{h}^{(1)}_{kt} + \alpha_2 \tilde{h}^{(2)}_{kt} + \alpha_3 \tilde{h}^{(3)}_{kt} + \alpha_4 \tilde{h}^{(4)}_{kt} + \tilde{\epsilon}_{kt}, \tag{13}$$

with $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. The coefficients α_1, α_2, α_3, and α_4 represent the weights placed on each of the intermediate hedge portfolios to solve for the minimum variance portfolio that also has the desirable covariance properties (12).

The calculation of the portfolio weights makes extensive use of the 5-year history prior to the testing period. However, the final portfolio weights calculated are "clean" in the sense that in no case is information from the test period employed in the construction of the portfolios themselves.

IV. Results

Portfolio Composition: Industries and the Associated Goods

Table 4 shows the proportions of the associated industries in the hedge portfolios for each of the goods tested. The most revealing aspect of the table is that the weights on the industries are highly variable. This variability is doubly alarming when one considers that the column entries in table 4 are not independent. Adjacent entries in each column have 54 out of 60 months' data in common.

There are two potential explanations for the observed variability. First, the data may be nonstationary. Nonstationarity makes attempts to hedge against commodity price inflation difficult. The key feature of hedging in a nonstationary world is to anticipate the change in the parameters of an equation like (2′) rather than merely to estimate the coefficients from a time-series regression. In this paper we will maintain the assumption of stationarity. Despite the extreme variability of the portfolio weights in table 4, evidence will be presented below which suggests that the assumption of stationarity is reasonable.

The second explanation for the variability of the portfolio weights is the problem of multicollinearity. If movement in the variables on the right-hand side of equation (2′) is similar over time, the ability to detect the unique influence of each one is limited. Each variable may appro-

TABLE 4 **Proportions of Associated Industry Portfolio in Quasi-Futures Contracts for Various Consumption Bundles**

Time Period	FD	GE		AU	PT	MC	HFO
	F	EU	NG	TA	MV	D	HF
01/61–12/65	−.23	−.60	.78	.22	−.19	.02	−.21
07/61–06/66	−.44	−.25	.54	.17	−.01	.04	−.12
01/62–12/66	−.29	−.10	.17	.31	−.21	.06	−.23
07/62–06/67	−.14	.02	.08	.51	−.23	.07	−.07
01/63–12/67	−.69	.55	−.22	.60	−.19	.06	−.15
07/63–06/68	−.32	−.27	.11	.47	−.30	.11	−.02
01/64–12/68	−.34	−.05	−.16	.20	−.27	.15	−.00
07/64–06/69	−.11	−.15	−.03	−.07	−.02	.16	.04
01/65–12/69	−.20	−.31	−.02	−.30	.03	.11	.03
07/65–06/70	−.15	−.06	−.79	.09	−1.24	.05	.08
01/66–12/70	−.19	−.29	−.02	.02	−.18	−.05	.04
07/66–06/71	.45	−.10	−.04	.02	−.29	−.30	.13
01/67–12/71	.29	−.08	.28	−.21	−.03	−.01	.01
07/67–06/72	.19	−.09	.26	−.12	−.14	−.08	−.01
01/68–12/72	.09	−.14	.24	.20	−.13	−.08	.01
07/68–06/73	.10	−.03	.00	−.11	.07	−.09	.11
01/69–12/73	−.07	.09	−.07	−.11	−.04	−.01	.13
07/69–06/74	.03	.09	−.24	.24	.14	−.03	.06
01/70–12/74	−.21	.10	−.34	.19	−.12	−.06	.18
07/70–06/75	.23	−.06	−.25	.08	−.27	−.06	.10
01/71–12/75	.48	.07	−.34	.10	−.13	.03	.17
07/71–06/76	−.28	−.52	.72	.17	−.16	.04	−.18
01/72–12/76	−.40	−.33	.54	.34	−.00	.08	−.10
07/72–06/77	−.28	−.20	.19	.32	−.21	.09	−.17
01/73–12/77	−.12	−.05	.15	.48	−.28	.09	−.07
07/73–06/78	−.53	.32	−.28	.60	−.25	.09	−.23
01/74–12/78	−.20	−.40	.10	.47	−.27	.11	−.00
07/74–06/79	−.29	−.13	−.09	.00	−.22	.11	.07
01/75–12/79	.07	−.23	−.05	−.17	−.50	.15	.05

NOTE.—Abbreviations defined in table 2.

priately be included in the regression, but none of the coefficients are estimated very precisely. The standard errors are large, and the point estimates will change dramatically with small changes in the data. Since the portfolio weights are functions of the estimated coefficients of equation (2′), multicollinearity in equation (2′) could be responsible for the variability observed in table 4. The problem of collinear data is neither a shortcoming of the model nor an error in the estimation technique. It merely reflects the fact that our ability to detect separate influences of similar explanatory variables is limited.

Some caution should be used in making any inference from table 4. For example, from inspection of table 4 it appears that to hedge against the private-transportation component of the CPI one should sell short stocks in the motor vehicle industry. Only two out of 29 entries in the column labeled PT/MV in table 4 are positive. However, since the entries in the table are not independent, no reliable inference can be made.

It can be concluded from table 4 that to employ a particular industry's stocks to hedge against the price inflation of an associated commodity is probably not useful. An investor is likely to do as well by using the stocks of nonassociated industries. This conclusion is based on the observation that the portfolio weights are highly variable, that the pattern of the weights for nonassociated industries (not shown) cannot be distinguished from the pattern of the associated industry weights, and that no consistent pattern is observed in any column in table 4.

The lack of any obvious usefulness of the associated industries as hedging devices for the commodity, although counterintuitive, is easily explained. First, the correspondence between the industries and the consumption goods is not always direct. For example, consider private transportation and the automobile industry. On the one hand, the private-transportation component of the CPI is intended to measure the price of transportation services, not just the price of automobile ownership and use. On the other hand, returns to the motor vehicle industry are related to the demand not only for automobiles but also for trucks and heavy equipment—items that are not directly related to private transportation. Since both the industry portfolios and the CPI components relate to more than one good, it is unreasonable to expect specific conclusions regarding any one good to come from analysis of data related to general indices. This may explain in part why no reliable pattern of portfolio weights is observed in table 4.

Even if there is a close correspondence between the consumption good and the industry, the impact of price changes in the good on returns to the industry portfolio is uncertain. The effect of a price change on an industry's profitability will depend, in general, on the causes of the price change and on the elasticities of the industry supply and demand curves. For example, in an industry with an elastic demand, an upward shift in the supply curve will result in a higher commodity price but lower total revenue and presumably a lower rate of return on the industry stock. However, if the demand curve is inelastic, an upward shift in the supply curve will increase industry revenues. Similar arguments can be made relating to shifts in demand curves. Also, changes in price may reflect only transitory changes in the structure of the industry. The value of an industry is related to all future incomes that will be earned. The current rate of return earned by any stock will depend more on the changes in the long-run prospects of the company than on short-run price fluctuations. Therefore, without additional information, it is impossible to predict what influence commodity price changes will have on the rates of return of their associated industry portfolios.

Portfolio Composition: Stocks versus Bills

Fama and Schwert (1977*a*) investigate the question of which assets provide effective hedges against the general inflation rate during the 1953–71 period. By regressing the returns on various assets against both the anticipated and unanticipated components of the inflation rate, they conclude that U.S. government bonds and bills were a "complete" hedge against expected inflation and that common stocks were negatively related to the expected component of the inflation rate and probably also to the unexpected component. These results imply that bills served as a better hedge against inflation than did common stocks during the 1953–71 period. To be consistent with the Fama and Schwert observations, our hedge portfolios should consist mostly of bills. In fact, if returns on common stock were negatively related to specific commodity inflation rates, one would expect to observe a short position in common stocks in our hedge portfolios. Table 5 shows the proportions of stocks and bills in each of the hedge portfolios. For all the commodities the bill portion of the portfolio has by far a larger proportion of the asset weight. Often the total weight of the stock proportion is close to zero. Frequently it is negative. As in table 4, the entries in each column of table 5 are not independent since they are estimated from overlapping time intervals. The information in table 5 extends the results of Fama and Schwert. They tested various assets as hedges against the overall (CPI) inflation, whereas we are testing the ability of stocks and bills to provide hedges against specific commodity price inflation. Also, Fama and Schwert were only concerned with the responsiveness of the assets they tested with the overall inflation rate. Our hedge portfolios are constructed to mimic, to the extent possible, not only the rate of commodity price inflation but also the covariances of the commodity's return with other goods stocks and bills.

A comparison of table 4 and table 5 indicates that the numbers in table 5 appear to be much more stable than those in table 4. Recall that the portfolio weights of associated industries in table 4 are highly variable. Yet the total net investment in stock as a whole appears to be small and quite stable in table 5. This is consistent with the observation that the variables in equation (2′) are collinear. The collinearity makes the individual stock weights difficult to estimate as shown in table 4. However, the sums of the stock and bill weights are estimated more precisely. This is reflected in the stability exhibited in table 5. It also supports the claim that the instability observed in table 4 is more apt to be due to the multicollinearity than a problem of nonstationarity.

Portfolio Composition: Conclusion

From evidence in tables 4 and 5 two principal facts emerge. (1) The use of stock of a specific industry to hedge against commodity price

TABLE 5 Proportions of Stocks and Bills in the Quasi-Futures Contracts for the Various Consumption Bundles

Time Period*	FD		GE		AU		PT		MC		HFO	
	Stocks	Bills	Stocks	Bills	Stocks	Bills	Stocks	Bills	Stocks	Bills	Stocks	Bills
01/61–12/65	.09	.91	.07	.93	−.06	1.06	−.01	1.01	.03	.97	.05	.95
07/61–06/66	.10	.90	.07	.93	−.01	1.01	.00	1.00	.02	.98	.05	.95
01/62–12/66	.01	.99	.00	1.00	−.16	1.16	.09	.91	.04	.96	−.01	1.01
07/62–06/67	−.22	1.22	.03	.97	−.05	1.05	.23	.77	.05	.95	−.04	1.04
01/63–12/67	−.64	1.64	.34	.66	.22	.78	.17	.83	.09	.91	.13	.87
07/63–06/68	−.05	1.05	−.04	1.04	−.21	1.21	.22	.78	.01	.99	−.03	1.03
01/64–12/68	−.06	1.06	.02	.98	−.12	1.12	.16	.84	.03	.97	−.01	1.01
07/64–06/69	−.10	1.10	−.05	1.05	−.21	1.21	.25	.75	.02	.98	−.04	1.04
01/65–12/69	−.47	1.47	−.25	1.25	.21	.79	.08	.92	.25	.75	.02	.98
07/65–06/70	.02	.98	−.42	1.42	.56	.44	.76	.24	.34	.66	.08	.92
01/66–12/70	−.27	1.27	−.04	1.04	.35	.65	−.03	1.03	.14	.86	.10	.90
07/66–06/71	−.26	1.26	.04	.96	−.01	1.01	−.47	1.47	−.26	1.26	.02	.98
01/67–12/71	−.0	1.01	.23	.77	−.04	1.04	−.24	1.24	−.07	1.07	.06	.94
07/67–06/72	−.10	1.10	.24	.76	.03	.97	−.02	1.02	−.09	1.09	.05	.95
01/68–12/72	−.18	1.18	.16	.84	.06	.94	−.07	1.07	−.15	1.15	.00	1.00
07/68–06/73	−.13	1.13	.11	.89	−.30	1.30	−.01	1.01	−.14	1.14	−.07	1.07
01/69–12/73	−.45	1.45	.00	1.00	−.68	1.68	.28	.72	−.11	1.11	−.13	1.13
07/69–06/74	−.32	1.32	−.16	1.16	−.43	1.43	−.12	1.12	−.05	1.05	−.18	1.18
01/70–12/74	−.30	1.30	−.16	1.16	−.32	1.32	−.05	1.05	−.10	1.10	−.11	1.11
07/70–06/75	−.46	1.46	−.21	1.21	−.39	1.39	.19	.81	−.13	1.13	−.04	1.04
01/71–12/75	−.35	1.35	−.17	1.17	−.42	1.42	.31	.69	−.23	1.23	−.02	1.02
07/71–06/76	−.02	1.02	.07	.93	−.03	1.03	−.01	1.01	.02	.98	.12	.88
01/72–12/76	−.08	1.08	.03	.97	−.03	1.03	.03	.97	−.01	1.01	.01	.99
07/72–06/77	−.04	1.04	−.02	1.02	−.14	1.14	.11	.89	.02	.98	.08	.92
01/73–12/77	−.19	1.19	.02	.98	−.07	1.07	.20	.80	.02	.98	−.04	1.04
07/73–06/78	−.53	1.53	.24	.76	.11	.89	.18	.82	.05	.95	.09	.91
01/74–12/78	−.06	.94	−.15	1.15	−.35	1.35	.20	.80	.01	.99	−.10	1.10
07/74–06/79	−.12	1.12	.02	.98	−.16	1.16	.21	.79	.07	.93	−.03	1.03
01/75–12/79	−.17	1.17	−.12	1.12	−.09	1.09	.14	.86	.05	.95	.00	1.00

NOTE.—Abbreviations defined in table 2.

*Each time period represents the months used to compute the hedge portfolio investments to be in effect the subsequent 6 months.

inflation of an associated commodity did not appear to be of any special value. The stocks from all industries appeared to have equal nonimportance. (2) For any particular commodity's inflation rate, Treasury bills are by far the largest component of the hedging portfolio. Individual stocks may represent large relative portions of the portfolio, but for all stocks the proportion held long is offset by a nearly equal proportion held short. The net stock proportion is nearly zero. In the next section we will show that a bills only strategy does not reduce the quality of the hedges.

Evaluation of Hedge Performance

In table 6 the quality of the hedge portfolios is compared with a portfolio consisting of only a 1-month Treasury bill. Under equation (1) of the table are the results of regressions of each commodity on the constructed hedge portfolio; under equation (2), the results for the regression of each commodity against the rate of return on a 1-month Treasury bill; and under equation (3), the results of the regression of the commodity returns against both the hedge portfolio and the Treasury bill.

An inspection of the R^2 values under equations (1) and (2) of the table reveals that Treasury bills alone explain slightly more of the variation in the commodity returns than does the hedge portfolio. This result may seem paradoxical, since Treasury bills may be included in the hedge portfolio. However, the quantity of short-term Treasury bills in the hedge portfolio was not estimated directly. It was an amount added to the hedge vector as a by-product to assure that the hedge portfolio proportions summed to unity. Also, the hedge portfolio was constructed by attempting to mimic not only the return of the associated commodity but also the covariances of the commodity with other commodities and the stock market. It is possible that by constructing the hedge portfolio to satisfy the covariance conditions, its direct usefulness as a hedging device is reduced.

A comparison of the slope coefficients for the three equations of table 6 is revealing. Fama and Schwert had evaluated hedging success as how close the coefficients in regressions like those reported in table 6 are to unity. By this criterion the Treasury bills dominate the constructed hedge portfolios. For the Treasury bill regressions (eq. [2]) all the coefficients are positive and reasonably close to unity. In equation (1), however, the coefficients are small and, except for AU (apparel and upkeep), not statistically significantly different from zero at the 5% level. Finally, when both the Treasury bills and the hedge portfolio are included, the addition of the hedge portfolio did not improve the quality of the regressions.

Based on the evidence in table 6, it appears that the use of separate industry returns from the stock market, the use of 6-month Treasury

TABLE 6 Hedge Portfolios and 1-month Bills as Substitutes for Consumption Bundles: Cochrane-Orcutt Estimation (January 1966–December 1979, $T = 168$)

Consumption Bundle ($\tilde{i}_k$)	(1)				(2)				(3)				
	γ_{k0}	γ_{k1}	R^2	D-W	γ'_{k0}	γ'_{k1}	R^2	D-W	γ''_{k0}	γ''_{k1}	γ''_{k2}	R^2	D-W
FD	.006	−.001	.08	2.04	−.000	1.157	.11	2.02	−.000	−.006	1.177	.11	2.02
	(6.75)	(−.08)			(−.01)	(2.26)			(−.04)	(−.34)	(2.29)		
GE	.006	.017	.20	2.10	.002	.909	.21	2.07	.002	.018	.904	.21	2.08
	(6.58)	(.82)			(.61)	(1.64)			(.60)	(.82)	(1.64)		
AU	.004	.028	.07	1.79	.000	.671	.07	1.81	.000	.028	.672	.08	1.80
	(1.65)	(4.89)			(.18)	(1.39)			(.14)	(1.65)	(1.39)		
PT	.005	−.21	.27	2.07	−.003	1.721	.31	1.99	−.003	−.026	1.806	.31	1.97
	(5.75)	(−1.36)			(−1.23)	(3.53)			(1.33)	(−1.65)	(3.67)		
MC	.006	.008	.24	2.11	.004	.489	.26	2.07	.004	.004	.483	.26	2.07
	(13.70)	(.43)			(3.09)	(2.03)			(3.08)	(.22)	(1.98)		
HFO	.005	−.13	.50	2.44	.000	.898	.51	2.36	.000	−.015	.924	.52	2.37
	(6.80)	(−1.13)			(.21)	(2.86)			(.17)	(−1.31)	(2.90)		

NOTE.—(1) $\tilde{i}_k = \gamma_{k0} + \gamma_{k1}\tilde{h}_k + \tilde{\epsilon}_{kt}$, (2) $\tilde{i}_k = \gamma'_{k0} + \gamma'_{k1} r_1 + \tilde{\epsilon}'_{kt}$, (3) $\tilde{i}_k = \gamma''_{k0} + \gamma''_{k1}\tilde{h}_k + \gamma''_{k2} r_1 + \tilde{\epsilon}''_{kt}$; t-statistics in parentheses.

bills, and the sample covariances among commodities and financial markets did not improve the ability to hedge against particular commodity price inflation. The hedges that were constructed using this additional information are inferior to a simple hedge of a 1-month Treasury bill.

V. Conclusions and Implications

Several authors (Roll, Long, Merton, and Breeden) have recognized that changes in commodity prices represent a potential risk to consumer-investors. They have constructed models that allow for prices of financial securities to reflect the influence of these risks. While these models are theoretically elegant, the evidence presented in this paper suggests that using commodity prices as state variables is not empirically useful. Financial securities of industries closely related to specific commodities did not appear to have any advantage as hedging devices for price changes of the related commodity. Moreover, portfolios of stocks and 6-month Treasury bills did not appear to contain any additional information regarding commodity price inflation beyond that contained in the return on a 1-month bill.

These findings are consistent with and extend previous empirical research. The work of Bodie (1976), Jaffee and Mandelker (1976), and Nelson (1976) has documented negative correlations between the rate of return on the stock market and the rate of inflation. The conclusions from these studies are that stocks have not been a useful hedge against inflation. Fama (1975) and Fama and Schwert (1977*a*) have investigated the use of Treasury bills and various commodities as hedges against the general level of inflation. They find that Treasury bills provide a hedge against anticipated inflation and that real estate seems to provide a hedge against both anticipated and unanticipated inflation.

Schipper and Thompson (1981) have attempted to use stock industry portfolios to hedge against changes in GNP and the general price level. Their tests differ from ours in that they employ quarterly rather than monthly data, they do not include the use of Treasury bills as hedging instruments, and they do not attempt to hedge against specific commodity price inflation. They found that out of sample their hedge portfolios did not provide successful hedges against GNP and price level changes.

Fama (1975) has documented that the return on a 1-month bill is a good predictor of the general inflation rate. From our evidence it appears that, compared with stocks and long-term bills, the 1-month Treasury bill is also the best hedge against specific commodity price inflation. Evidently investors cannot successfully employ the stock market to hedge against relative price changes. For the six consumption bundles we have tested, an investor's best hope of hedging commodity price inflation is to purchase Treasury bills, which will hedge against the general inflation level, and ignore the relative price changes. The stock and bill markets by themselves do not provide any additional information regarding commodity prices beyond what is contained in the short-term Treasury bills.

This conclusion, that the stock market cannot be used to hedge against specific commodity price inflation, does not mean that hedging against such price changes is impossible. Assets other than those considered here may be suitable as hedges. Fama and Schwert (1977*a*) found that real estate and human capital provided some hedging capabilities against the general price level. Whether these assets can also provide hedging power for specific commodities has not been examined. Investors may also hedge against commodity price changes by purchases of the good for storage, or with futures contracts.

References

Bodie, Z. 1976. Common stocks as a hedge against inflation. *Journal of Finance* 31 (May): 459–70.

Breeden, D. 1979. An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics* 7 (September): 265–96.

Cornell, B. 1981. The consumption based asset pricing model: a note on potential tests and applications. *Journal of Financial Economics* 9 (March): 103–8.

Dusak, K. 1973. Futures trading and investor returns: an investigation of commodity market risk premiums. *Journal of Political Economy* 81 (December): 1387–1406.

Fama, E. F. 1975. Short-term interest rates as predictors of inflation. *American Economic Review* 65 (June): 269–82.

Fama, E. F., and Schwert, G. F. 1977*a*. Asset returns and inflation. *Journal of Financial Economics* 5 (November): 115–46.

Fama, E. F., and Schwert, G. F. 1977*b*. The behavior of relative and money prices of consumption goods. Working Paper, University of Chicago, February.

Fama, E. F., and Schwert, G. F. 1979. Inflation, interest, and relative prices. *Journal of Business* 52 (April): 183–209.

Gouldey, B. K. 1977. Empirical test of a multiperiod capital asset pricing model. Doctoral dissertation, University of Pittsburgh.

Gouldey, B. K. 1980. Evidence of nonmarket risk premiums in common stock returns. *Journal of Financial Research* 3 (Fall): 243–60.

Jaffe, J. F., and Mandelker, G. 1976. The 'Fisher effect' for risky assets: an empirical investigation. *Journal of Finance* 31 (May): 447–58.

Lintner, J. 1965*a*. Security prices, risk and maximal gains from diversification. *Journal of Finance* 20 (December): 587–615.

Lintner, J. 1965*b*. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics* 47 (February): 13–37.

Long, J. B. 1974. Stock prices, inflation, and the term structure of interest rates. *Journal of Financial Economics* 1 (July): 131–70.

Merton, R. C. 1973. An intertemporal capital asset pricing model. *Econometrica* 41 (September): 867–87.

Nelson, C. R. 1976. Inflation and rates of return on common stocks. *Journal of Finance* 31 (May): 471–83.

Roll, R. 1973. Assets, money, and commodity price inflation under uncertainty. *Journal of Money, Credit, and Banking* 5 (November): 903–23.

Ross, S. A. 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13 (December): 341–60.

Ross, S. A. 1977. Return, risk, and arbitrage. In Irwin Friend and James L. Bicksler (eds.), *Risk and Return in Finance*. Vol. 1. Cambridge, Mass.: Ballinger.

Schipper, K., and Thompson, R. 1981. Common stocks as hedges against shifts in the consumption or investment opportunity set. *Journal of Business* 54 (April): 305–28.

Sharpe, W. F. 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance* 19 (September): 425–42.

U.S. Department of Labor. 1966. The consumer price index: history and techniques. Bureau of Labor Statistics Bulletin no. 1517. Washington, D.C.: Government Printing Office.